CMB Spectral Distortions: a Robust Probe of Primordial Non-Gaussianity

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Review of CMB (μ-type) Spectral Distortions

Photon thermodynamics: early times

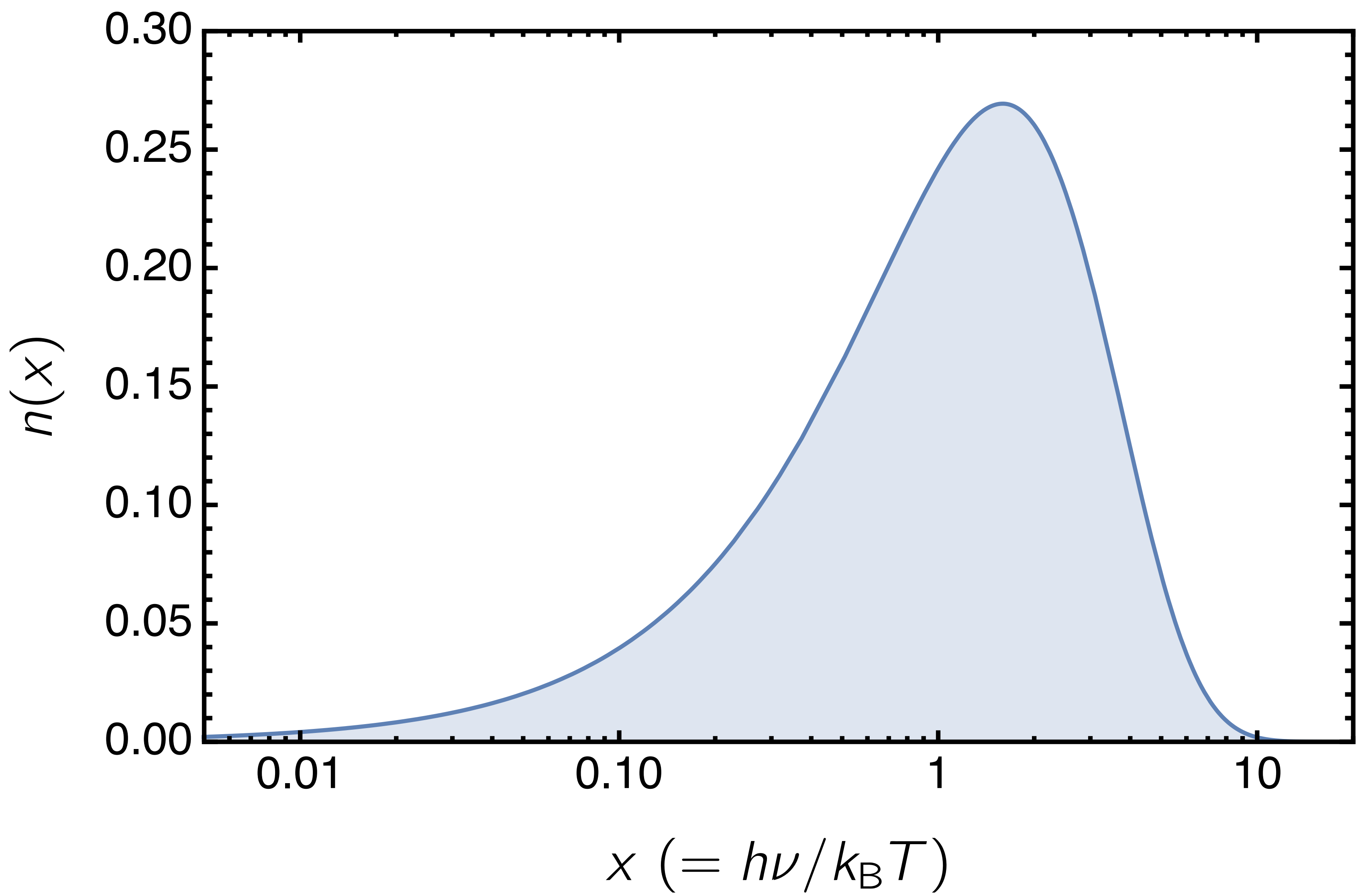
- early universe: hot photon-baryon-electron plasma;
- before $z_{\mu,i} \simeq 2 \times 10^6$, double Compton scattering ($e^- + \gamma \to e^- + 2\gamma$) and Bremsstrahlung are very efficient;
- then: perfect thermodynamical equilibrium. Any perturbation to the system is thermalized.

photons can be created at low ν and rescattered to high ν

black-body spectrum

$$n(x) \propto \frac{x^2}{e^x - 1}$$
$$(x = h\nu/k_{\mathrm{B}}T)$$

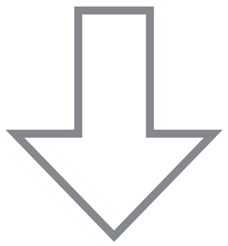
is achieved



Photon thermodynamics: μ-era

- between $z_{\mu,i} \simeq 2 \times 10^6$ and $z_{\mu,f} \simeq 5 \times 10^4$ double Compton scattering and Bremsstrahlung are not efficient enough to create photons;
- elastic Compton scattering $(e^- + \gamma \to e^- + \gamma)$ maintains kinetic equilibrium;
- photon number effectively frozen (except at low ν).

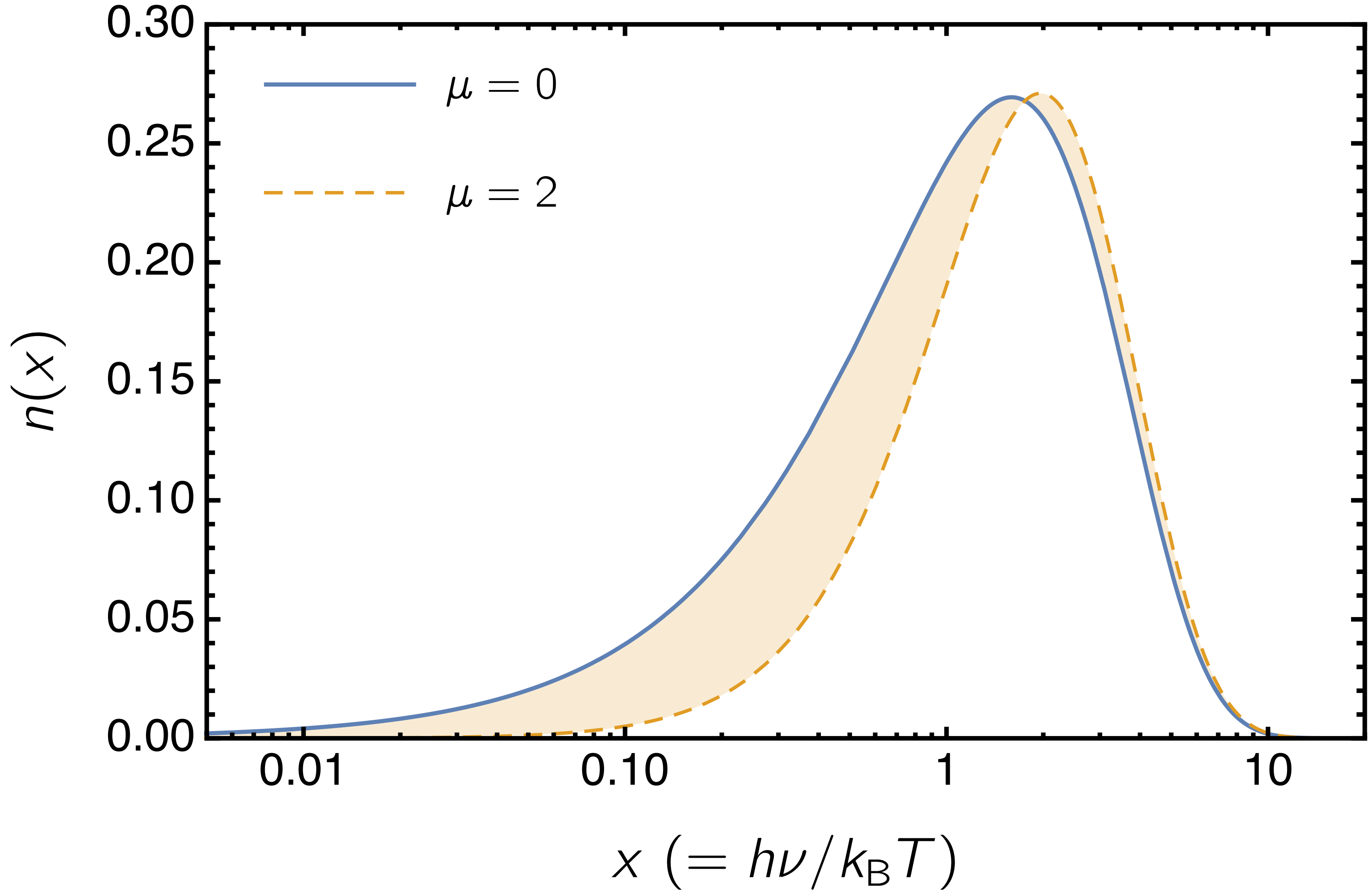
energy injections
in primordial plasma
will distort the BB spectrum



$$n(x) \propto \frac{x^2}{e^{x+\mu} - 1}$$

Bose-Einstein spectrum with
chemical potential μ

Sunyaev, Zel'dovich, 1970; Danese, de Zotti, 1982; Burigana, Danese, de Zotti, 1991
Hu, Silk, 1994; Chluba, Sunyaev, 2011; Kathri, Sunyaev, 2012; Chluba, 2016



Generation of μ-distortions

$$N_{\text{B-E}}\big(T(E+\delta E,\mu),\mu\big) = N_{\text{P}}(T(E)) \quad \Rightarrow \quad \mu = 1.40066 \times \frac{\delta E}{E}$$

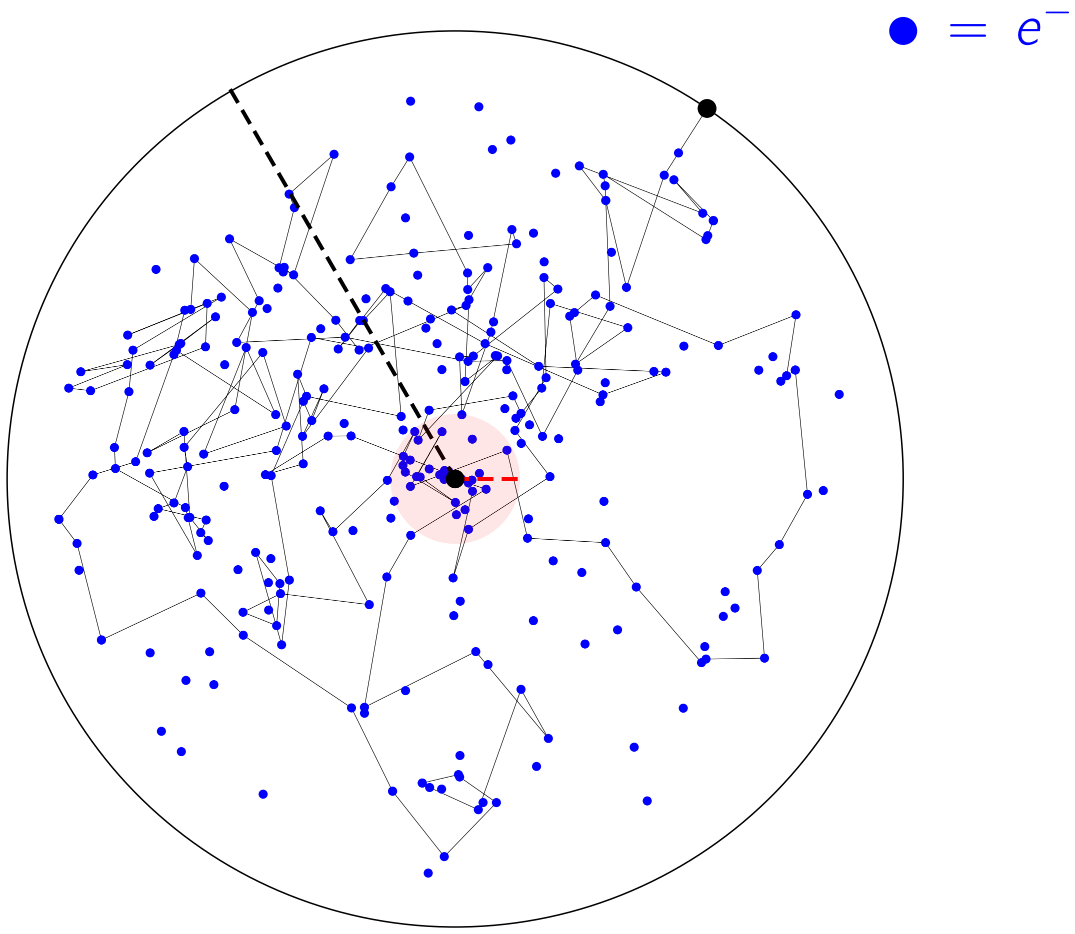
What $\delta E/E$?

Silk damping

$$\delta_\gamma(\boldsymbol{k}) \propto \zeta_{\boldsymbol{k}} \cos(k r_s) e^{-\frac{k^2}{k_D^2}}$$

$$\frac{\delta E}{E} \approx \frac{1}{4} \langle \delta_\gamma^2(t,\boldsymbol{x})\rangle_{p}\Big|_{z_{\mu,f}}^{z_{\mu,i}}$$

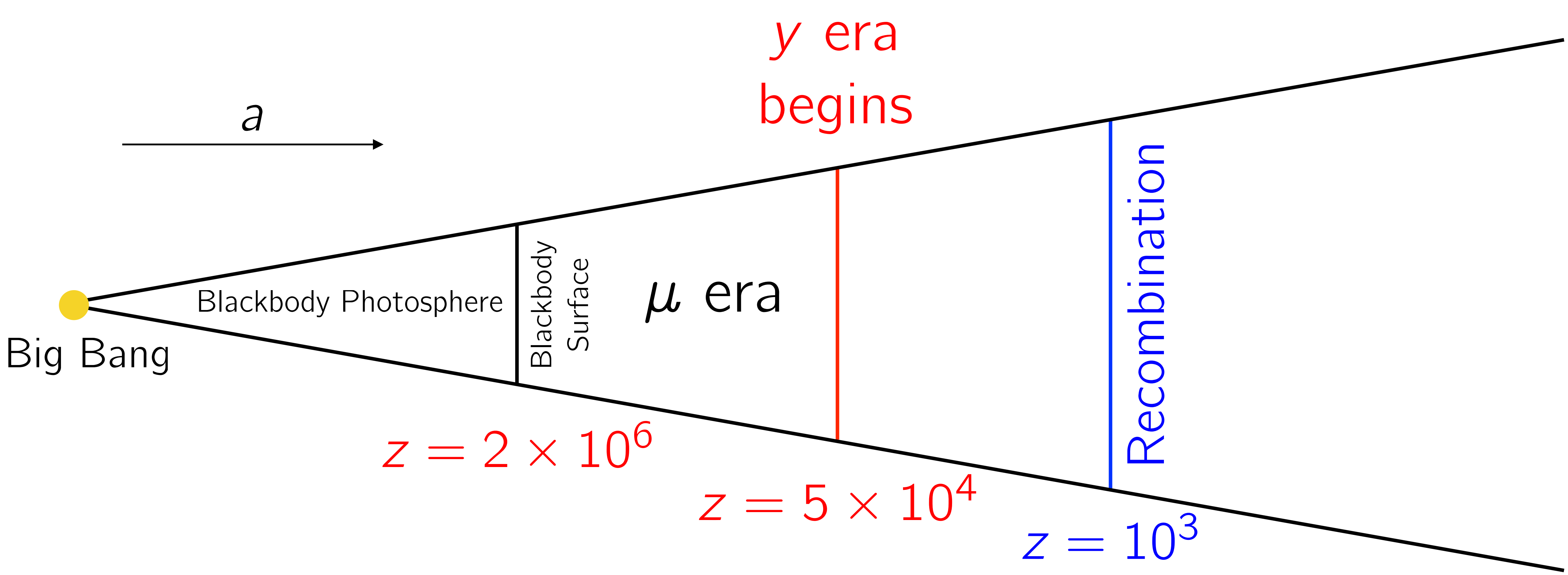
average over a period of oscillation



----- λ_{D}: Silk damping scale ----- λ_{mfp}: photon mean free path

$$k_D(z_{\mu,f}) \approx 50\,\text{Mpc}^{-1}\,,\ k_D(z_{\mu,i}) \approx 10^4\,\text{Mpc}^{-1}$$

After end of the μ era



- after $z_{\mu,f}$: inefficient energy exchange between electrons and photons
- photon number is still effectively frozen $\Rightarrow$ y-distortion of the spectrum
- (several astrophysical foregrounds create y-dist. (*e.g.* SZ effect): not as clean as μ-dist.)

but… See Ravenni et al. (2017)

→ μ is conserved from $z = z_{\mu,f}$ to the last-scattering surface!

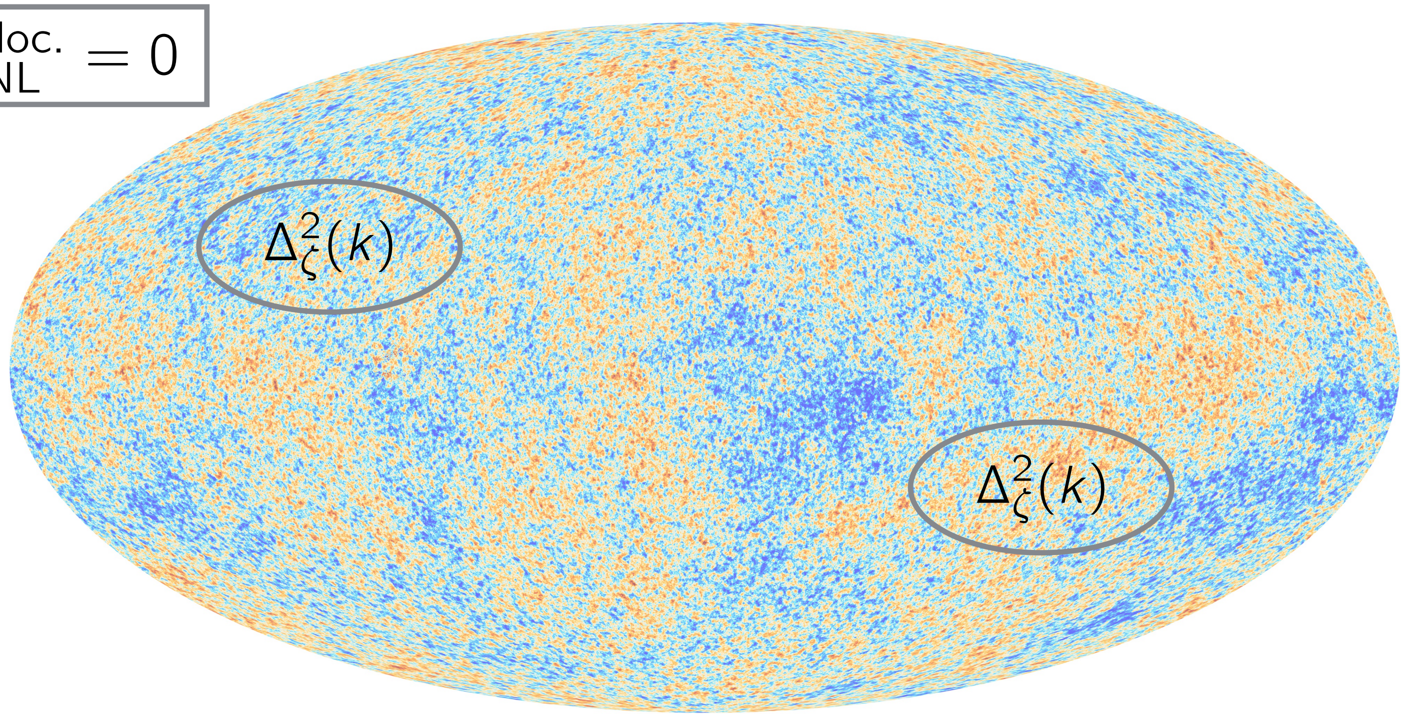
we will go back to this later…

μ anisotropies and non-Gaussianity

With local non-Gaussianity

In presence of local NG, the heating rate becomes spatially dependent!

$f_{\rm NL}^{\rm loc.} = 0$

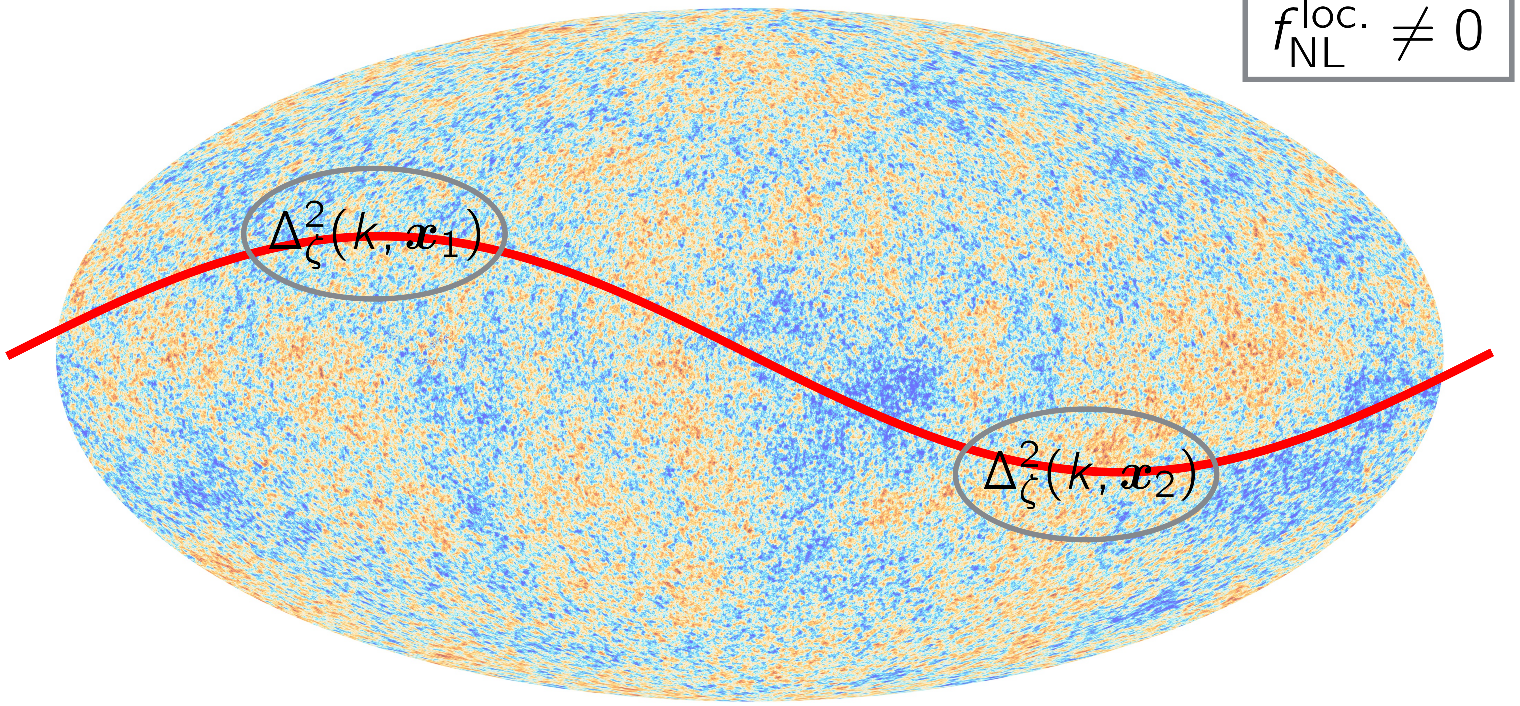


amplitude $\Delta^2_\zeta(k_{\rm small})$
will be the same
everywhere

$f_{\rm NL}^{\rm loc.} \neq 0$

amplitude $\Delta^2_\zeta(k_{\rm small})$
will differ from
one patch to another

$\to$ correlated with ζ_ℓ



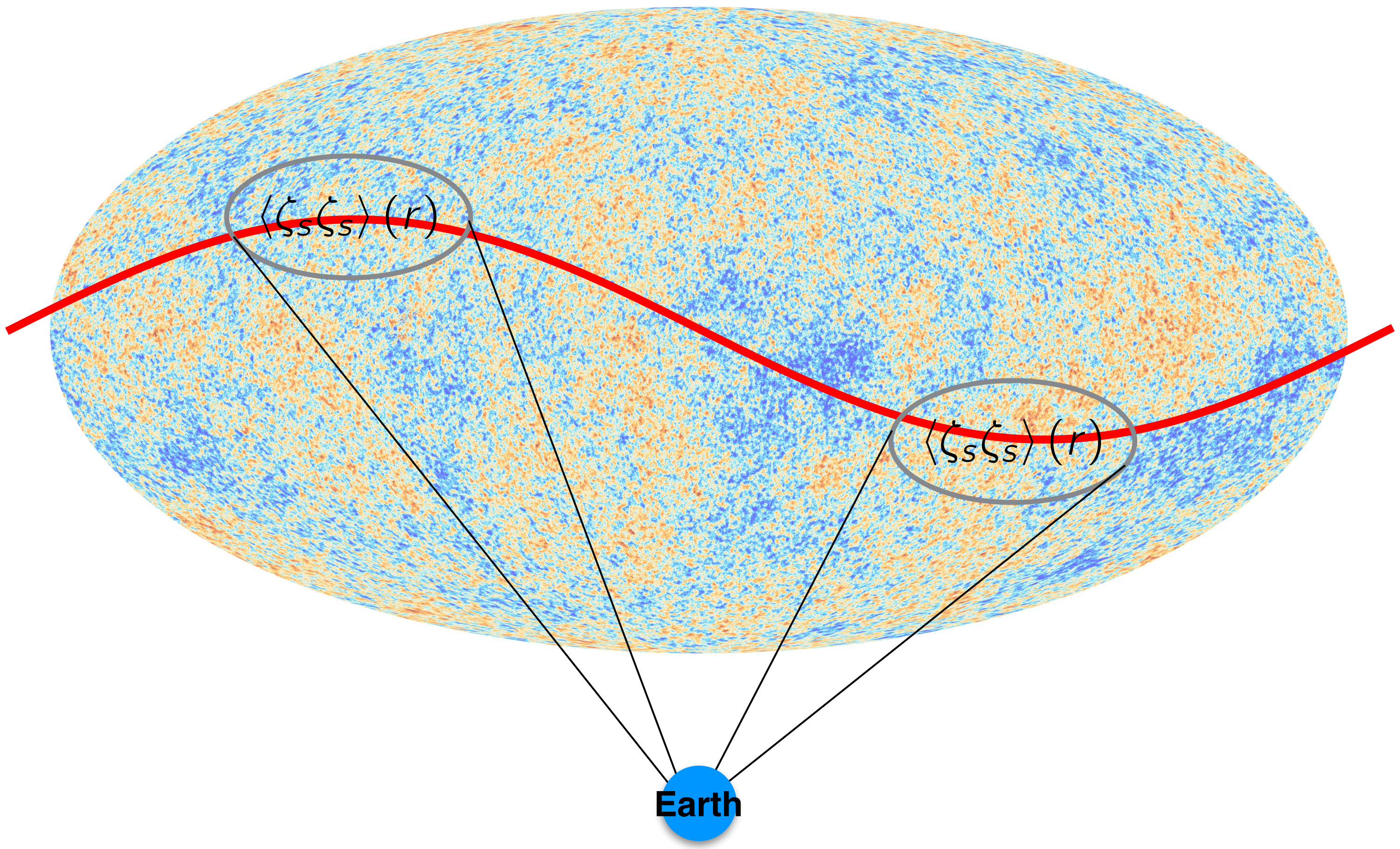
μ-T correlations

Pajer and Zaldarriaga, 2012; Emami, Dimastrogiovanni, Chluba, Kamionkowski, 2015

Short-scale power spectrum is correlated with the long mode → is correlated with the long-wavelength T perturbation! *Angular power spectrum* $C^{\mu T}_{\ell\lesssim 100} \approx 12 f^{\rm loc.}_{\rm NL} C^{TT}_\ell$

However! In every patch a local observer cannot see the coupling with ζ_ℓ in single-field inflation! No μ-T correlations in this case?

But → be careful about projection effects! E.g.: CMB squeezed bispectrum



$$B^{TTT}_{\ell_L,\ell_S} = C^{TT}_{\ell_L} C^{TT}_{\ell_S}\left(2 - \frac{{\rm d}\log(\ell_S^2 C^{TT}_{\ell_S})}{{\rm d}\log\ell_S}\right)$$

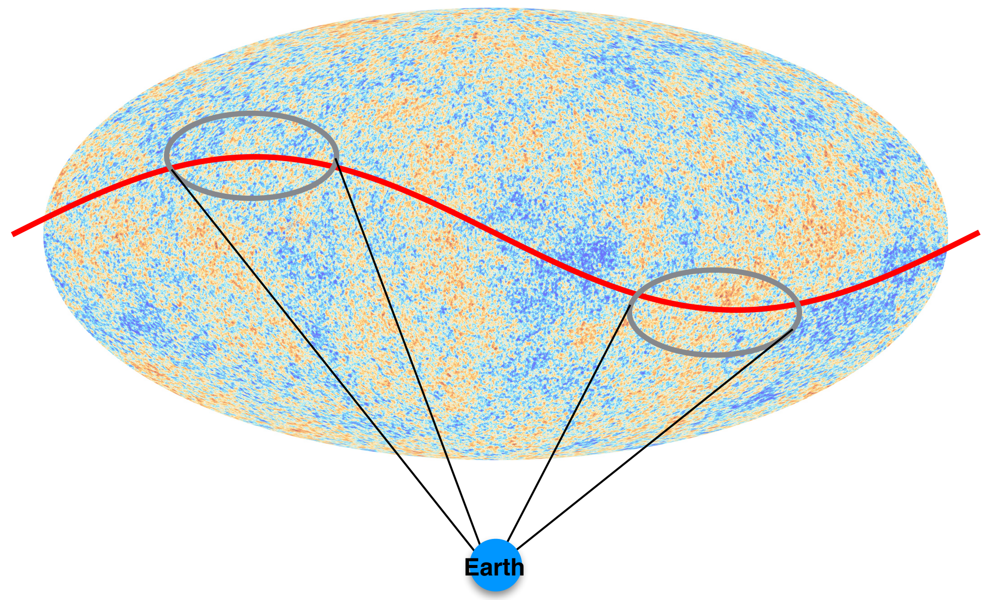
$\ell_L \lesssim 100$

Creminelli, Pitrou, Vernizzi (2011)
Bartolo, Matarrese, Riotto (2011)

any deviation is
$\propto f_{\rm NL} - (1-n_{\rm s})$

e.g.:
Pajer, Schmidt, Zaldarriaga (2013)
Mirbabayi and Zaldarriaga (2014)

Projection effects



1. same physical length in the local patch at recombination appears at different angular sizes to the observer
2. photons experience a different redshift in different directions due to the presence of the long mode

⇓ for $C_\ell^{\mu T}$

1. we are looking at the average μ in a patch $\to$ no "ruler" whose length the long mode can perturb
2. μ does not redshift

$$\frac{\mathrm{D}f_\gamma}{\mathrm{d}\lambda} = 0 \Rightarrow \begin{cases} T_{\rm obs} = \frac{E_{\rm obs}}{E_{\rm rec}} T_{\rm rec} \\ \mu_{\rm obs} = \mu_{\rm rec} \end{cases}$$

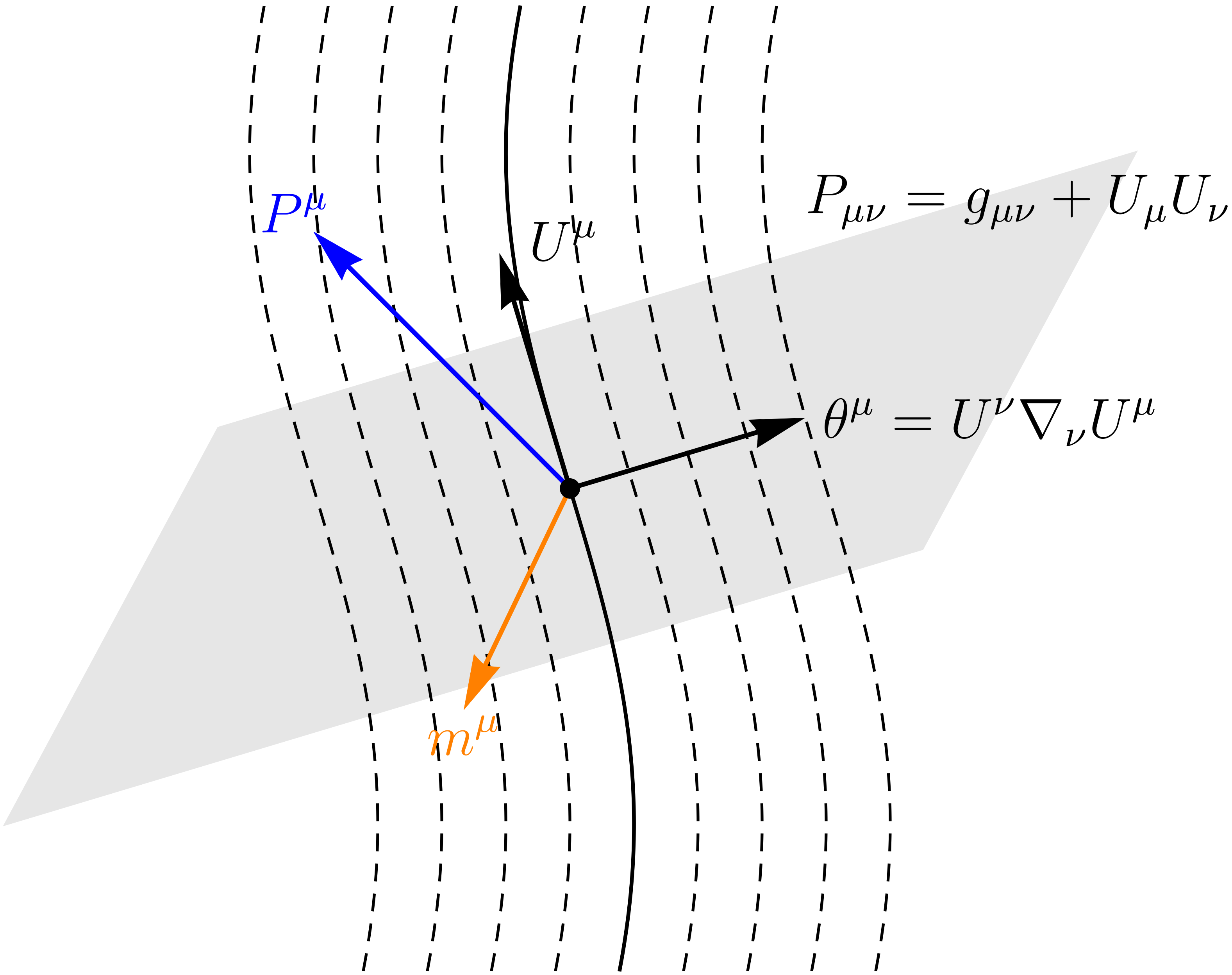
No correlation from evolution from the LSS to the observer. . . What is the leading effect?

Evolution after the end of the μ-era

damping of μ **inhomogeneities**! Pajer and Zaldarriaga, 2012

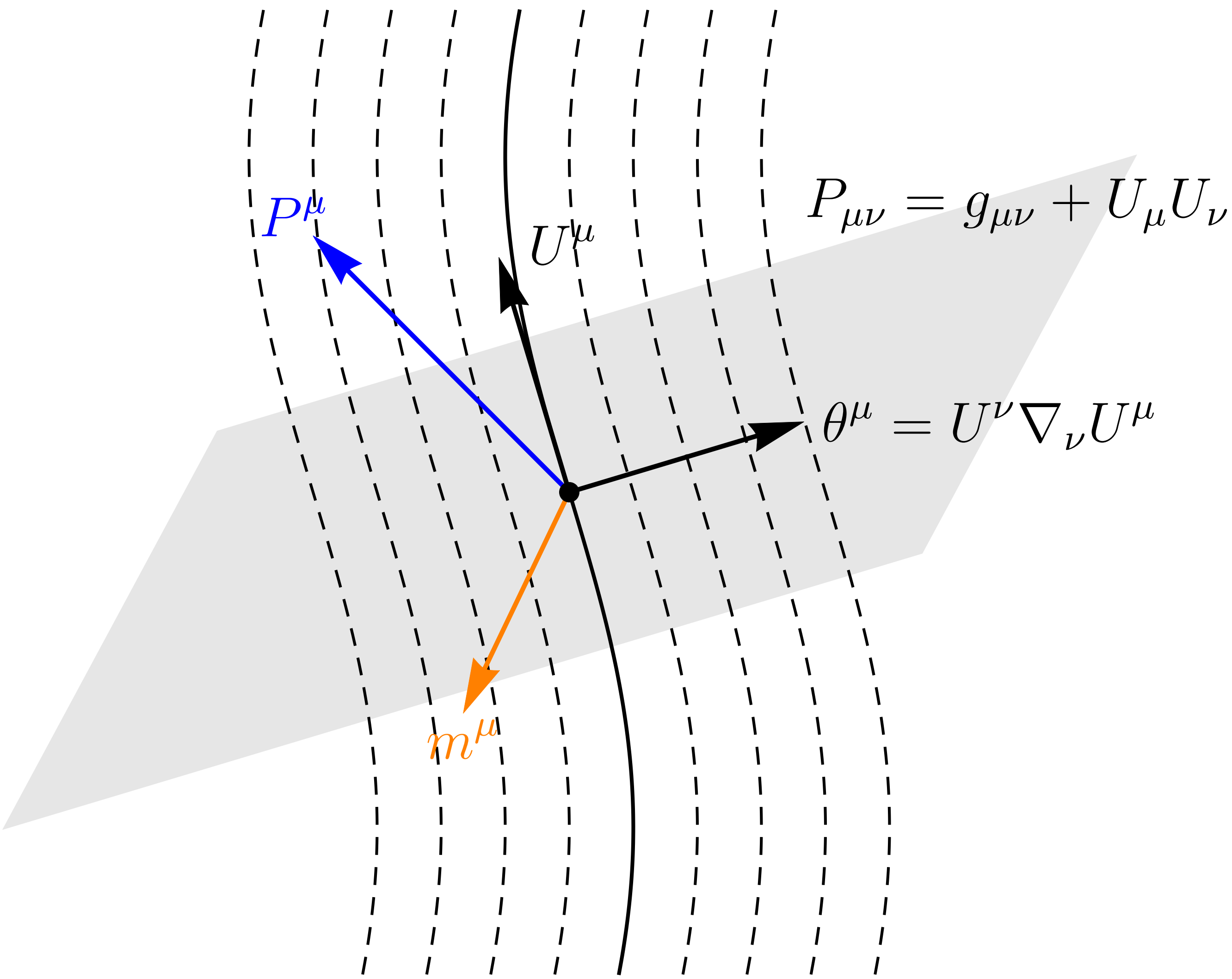
no effect on $\langle\mu\rangle\ldots$

Let's check the fluid limit of the Boltzmann equation



$$P^\mu = E(U^\mu + m^\mu)\ , \qquad \mu = \mu(x^\nu, m^\nu)\ , \qquad \mu_0 = \int \frac{\mathrm{d}\hat{\boldsymbol{m}}}{4\pi}\mu$$

Fluid limit of Boltzmann equation – 1



for $t_\gamma \to 0$ the distribution is brought to equilibrium

$$\frac{1}{E}\frac{\mathrm{D}\mu}{\mathrm{d}\lambda} = \frac{\mu - \mu_0}{t_\gamma}$$

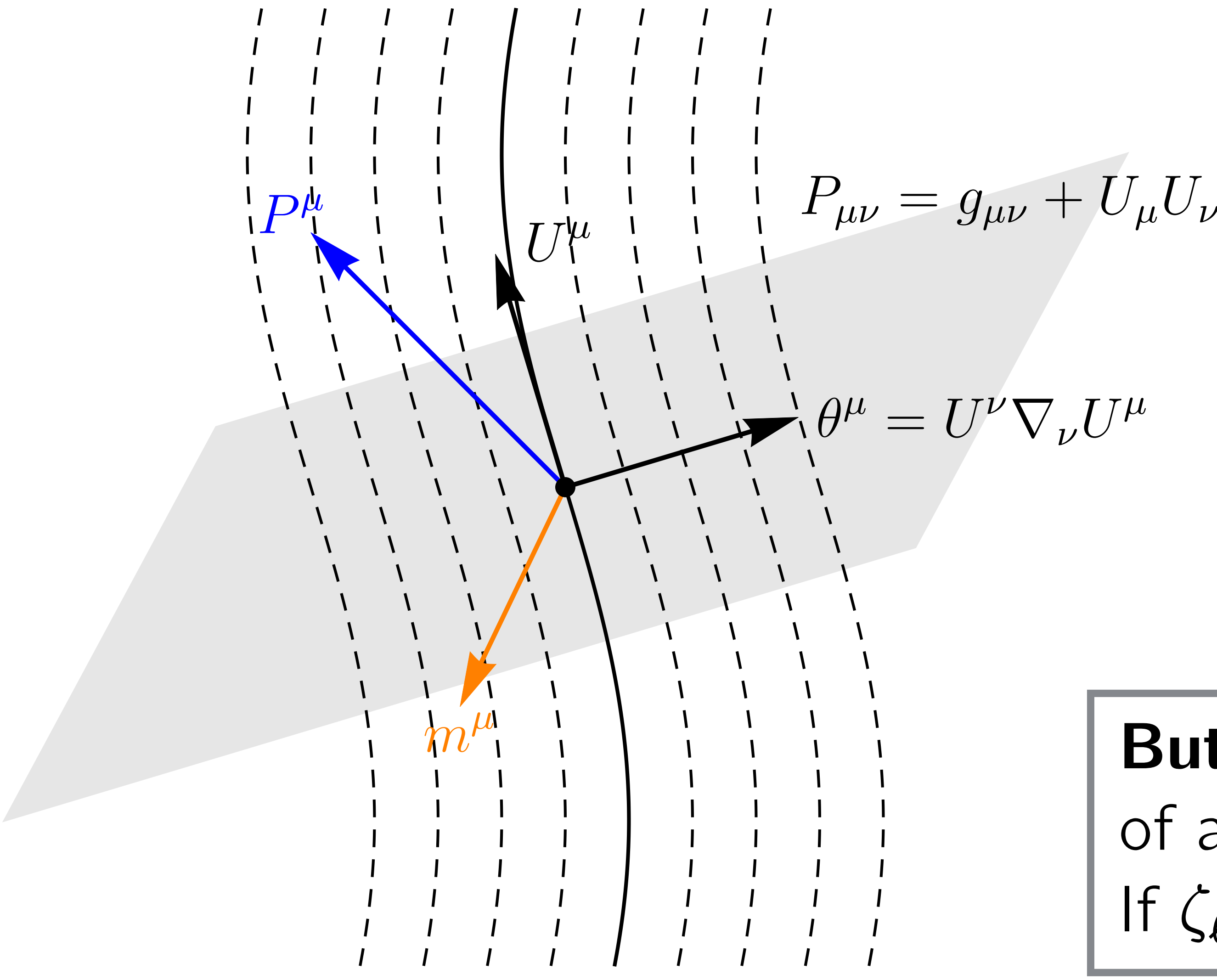
t_γ: photon m.f.p.

$$U^\nu \nabla_\nu \mu + m^\nu \nabla_\nu \mu + (U^\rho + m^\rho)(\nabla_\rho m^\nu) P^\lambda{}_\nu \frac{\partial \mu}{\partial m^\lambda} = \frac{\mu - \mu_0}{t_\gamma}$$

In the tight-coupling limit, only the monopole will be relevant:

- expand $\mu = \mu_0 - 3 m^\nu P_\nu{}^\rho \nabla_\rho \mu_1$ (monopole + dipole: higher multipoles will be suppressed)
- take moments $\int \frac{\mathrm{d}\hat{\boldsymbol{m}}}{4\pi} m^\mu m^\nu \ldots$: only first two moments are needed

Fluid limit of Boltzmann equation – 2



mixed equations
for μ_0 and μ_1
$\downarrow$
acceleration θ^μ gives
complicated mixing
at 2nd order

But! We are interested in the effect of a long mode on short modes!
If ζ_ℓ is practically uniform $\rightarrow$ $(\theta^\mu)_\ell = 0$

We can obtain an equation for μ_0 only!

$$\underbrace{\left[U^\nu \nabla_\nu (U^\rho \nabla_\rho) - \frac{D_\nu D^\nu}{3} - \left(\frac{\nabla_\mu U^\mu}{3} - \frac{1}{t_\gamma}\right) U^\nu \nabla_\nu\right]}_{= \mathcal{O}(1) + \mathcal{O}(\zeta_\ell)} \mu_0 = 0$$

$(D_\mu = P_\mu{}^\nu \nabla_\nu)$

⇨ an homogeneous μ_0 is conserved along the fluid lines: $U^\nu \nabla_\nu \mu_0 = 0$!

What happens during the μ-era?

Effect of the long mode on μ production! In principle, complicated non-linear dynamics. . .

Exploit another nice difference with CMB bispectrum: horizon at end of μ era is $\approx 10^{-1}\,\mathrm{Mpc}^{-1}$

Up to very high $\ell \gg 100$ we can treat the effect with the separate universe approach!

similarly to halo bias… E.g.: Dai, Pajer, Schmidt, 2015

Long mode modifies the expansion history and adds curvature $K_F \propto \partial^2\mathcal{R}$ to the FLRW:

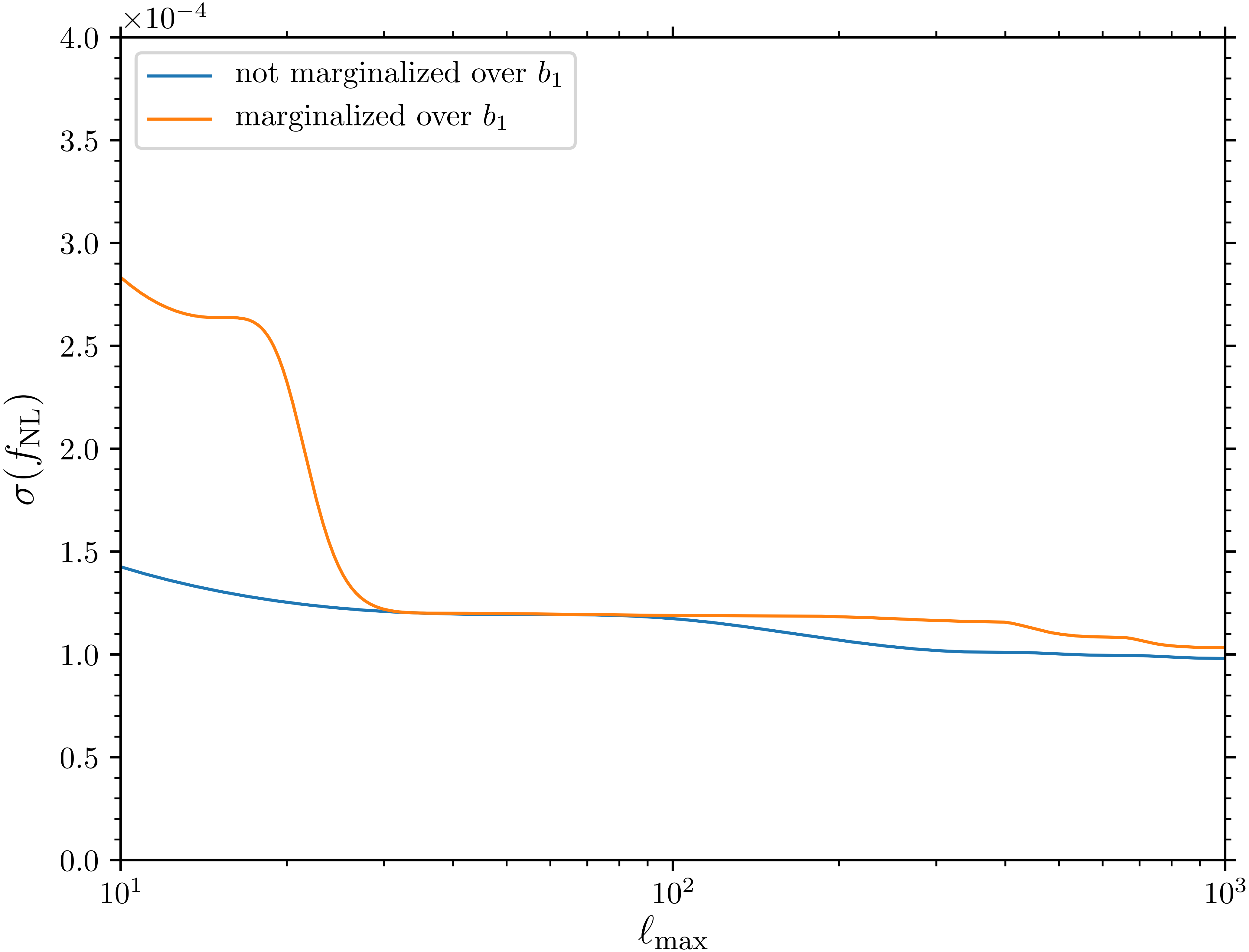
1. modification the duration of the μ era
2. direct effect on the evolution of short modes
3. additional coupling to $K_F \propto \partial^2\mathcal{R}$ from inflation. It is of order $1 - n_\mathrm{s}$: can be neglected unless there are (unlikely) cancellations of 1. and 2. GC, Pajer, Schmidt, 2016

Model this by a bias parameter: $\langle\mu_S\rangle_{\zeta_L}(\eta,\boldsymbol{x}) = \langle\mu_S\rangle_{\zeta_L=0} - \frac{b_1}{\Lambda^2}\partial^2\zeta_L(\eta,\boldsymbol{x})\,\langle\mu_S\rangle_{\zeta_L=0}$

with: $\Lambda = \mathcal{H}(z_{\mu,f})$, $b_1 = \mathcal{O}(1)$

Will this effect bias constraints on $f_{\rm NL}^{\rm loc}$?

check with a forecast for full-sky cosmic variance-limited experiment!



Conclusions

- CMB μ-type distortions probe small scales, $50\,\mathrm{Mpc}^{-1} \lesssim k \lesssim 10^4\,\mathrm{Mpc}^{-1}$
- very sensitive to primordial local non-Gaussianity through μ anisotropies
- in single-field inflation (practically) no μ-T correlations:
  - no projection effects from last-scattering surface to observer
  - no effects from tight-coupling evolution
  - bias effect on μ production: absolutely negligible impact on $f_{\mathrm{NL}}^{\mathrm{loc}}$ forecasts!

Thanks!

Backup slides

Hydrodynamical description of μ generation

Viscous corrections $\to$ entropy production: $U^\mu\nabla_\mu\left(\frac{s}{n}\right) = -\frac{\Delta T^{\mu\nu}\nabla_\nu U_\mu}{nT}$

s, n: entropy and number density for observer U^μ. For a black-body $\to$

$$\frac{s}{n} \propto 1 + (A_s - A_n)\mu \implies U^\nu\nabla_\nu\mu = \frac{8t_\gamma}{15(A_n - A_s)}\sigma_{\mu\nu}\sigma^{\mu\nu}$$

$$\sigma_{\mu\nu}\sigma^{\mu\nu} \sim a^{-2}(\partial_i v_j)^2\ , \qquad t_\gamma \approx a^2 H/k_{\rm D}^2$$

➡ $\frac{\dot\mu}{H} \sim \frac{(\partial_i v_j)^2}{k_{\rm D}^2} \implies$ μ production suppressed by $\partial^2/k_{\rm D}^2$

Some details of tight-coupling evolution

$$U^\nu\nabla_\nu\mu + m^\nu\nabla_\nu\mu + \big[(m^\rho\theta_\rho + m^\rho m^\sigma\theta_{\rho\sigma})m^\lambda - \theta^\lambda - m^\rho\theta_\rho{}^\lambda\big]\frac{\partial\mu}{\partial m^\lambda} = \frac{\mu-\mu_0}{t_\gamma}$$

$\theta^\mu = U^\nu\nabla_\nu U^\mu$ $\qquad$ $\theta_{\mu\nu} = P_\mu{}^\rho\nabla_\rho U_\nu$ $\qquad$ $P_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu$

Assume $\mu = \mu_0 - 3m^\nu P_\nu{}^\rho\nabla_\rho\mu_1$ and take moments: $\int\frac{\mathrm{d}\hat{\boldsymbol{m}}}{4\pi}$ and $\int\frac{\mathrm{d}\hat{\boldsymbol{m}}}{4\pi}m^\mu$

$$U^\nu\nabla_\nu\mu_0 + 4\theta^\nu D_\nu\mu_1 - D_\nu D^\nu\mu_1 = 0$$

$$\frac{D^\nu\mu_0}{3} - \frac{4\theta^{(\nu\rho)}D_\rho\mu_1}{5} - \frac{2\theta D^\nu\mu_1}{5} + 3\theta^{\nu\rho}D_\rho\mu_1 - \theta^\nu U^\rho\nabla_\rho\mu_1 - D^\nu(U^\rho\nabla_\rho\mu_1) = \frac{D^\nu\mu_1}{t_\gamma}$$

- assume uniform $\zeta_\ell \Rightarrow \theta_{\mu\nu} = \frac{\nabla_\rho U^\rho}{3}P_{\mu\nu},\ \theta^\mu = 0$
- take D_ν of second equation

$$\boxed{\begin{aligned} &U^\nu\nabla_\nu\mu_0 - D_\nu D^\nu\mu_1 = 0 \\ &U^\nu\nabla_\nu(U^\rho\nabla_\rho\mu_0) - \frac{D_\nu D^\nu\mu_0}{3} - \left(\frac{\theta}{3} - \frac{1}{t_\gamma}\right)D_\nu D^\nu\mu_1 = 0 \end{aligned}}$$