NO-GO THEOREMS FOR EKPYROSIS FROM TEN-DIMENSIONAL SUPERGRAVITY

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[1] Introduction

- The strong no-go theorems which exclude tree-level de Sitter compactifications have been much explored.

  (S. Kachru et al., hep-th/0301240)
  (O. DeWolfe et al., hep-th/0505160)
  (T. Wrase & M. Zagermann, arXiv:1003.0029 [hep-th])

- However, the no-go theorems for ekpyrotic scenario which is alternative to inflation model in string theory is much less extensive.

- One motivation for the present work is to improve this situation.

- The Ekpyrosis inspired by string theory and brane world model suggests alternative solutions to the early universe puzzles such as inflation and dark energy.

  (P. Horava, E. Witten, hep-th/9510209)
  (A. Lukas et al., hep-th/9710208)
  (J. Khoury et at., hep-th/0103239)

- Since the big bang is described as a collision of branes, there is also a new ekpyrotic phase or a cyclic universe due to another brane collision with the creation of new matter.

- The potential during ekpyrosis is negative and steeply falling; it can be modeled by the exponential form $V(\phi)=-V_0 \exp(-c\phi)$ ($c \gg 1$).
  (J. Khoury et at., hep-th/0103239)

$$a(t) = (-t)^p , \quad p = \frac{2}{c^2}$$

- There was plenty of time before the big bang for the universe to be in causal contact over large regions.

- The scalar potential obeys fast-roll condition.
  (S. Gratton, et al., astro-ph/0301395)

$$\varepsilon_\mathrm{f} = \frac{V^2}{\sum_i \left(\partial_{\phi_i} V\right)^2} , \quad \eta_\mathrm{f} = 1 - \frac{V \sum_i \partial^2_{\phi_i} V}{\sum_i \left(\partial_{\phi_i} V\right)^2}$$

If the potential form for the ekpyrotic scenario gives the negative and steep, the "fast-roll" parameters for the ekpyrosis are given by

$$\varepsilon_{\mathrm{f}} \equiv \kappa^2 \frac{V^2}{\left(\partial_{\bar{\tau}} V\right)^2 + \left(\partial_{\bar{\rho}} V\right)^2} \ll 1 \,,$$

$$|\eta_{\mathrm{f}}| \equiv \left| 1 - \frac{V \left(\partial_{\bar{\tau}}^2 V + \partial_{\bar{\rho}}^2 V\right)}{\left(\partial_{\bar{\tau}} V\right)^2 + \left(\partial_{\bar{\rho}} V\right)^2} \right| \ll 1$$

- In ekpyrotic models with single scalar field, the spectrum of the curvature perturbation is blue, in disagreement with observations.
(D. H. Lyth, hep-ph/0110007)

- It is necessary to consider two scalar fields at least in the 4-dimensional theory.
$\Rightarrow$ new Ekpyrotic scenario
(E. I. Buchbinder, et al., hep-th/0702154)
(K. Koyama, et al., arXiv:0704.1152 [hep-th])
(K. Koyama, et al., , arXiv:0708.4321 [hep-th])

- Ekpyrotic scenario

  … The low energy effective field theory of the Ekpyrotic scenario is given by Einstein gravity minimally coupled to a scalar field.

The embedding in string theory

(1) Heterotic theory – OK
(with non perturbative correction)

(2) Type II theory – ?

- Our work:

- We investigate whether the Ekpyrosis can be embedded into 10D string theory (no go theorem).

- We use that the scalar potential obtained from compactifications of type II string with sources has a universal scaling with respect to the dilaton and the volume mode.

[2] Compactifications of the type II theory

Compactifications of the type II theory to 4-diemnsional spacetime on compact manifold

☆10-dimensional action

$$S = \frac{1}{2\bar{\kappa}^2} \int d^{10}x \sqrt{-g} \left[\mathrm{e}^{-2\phi} \left(R + 4g^{MN} \partial_M \phi \partial_N \phi - \frac{1}{2} |H|^2 \right) - \frac{1}{2} \sum_p |F_p|^2 \right]$$

$$- \sum_p (T_{\mathrm{D}p} + T_{\mathrm{O}p}) \int d^{p+1}x \sqrt{-g_{p+1}}\, \mathrm{e}^{-\phi} \,,$$

F_p : R-R p-form field strengths

T_{Dp} , (T_{Op}) : Dp-brane (Op-plane) tension

★To compactify the theory to 4 dimensions, we consider the metric ansatz of the form:

6-dimensional internal space

$$ds^2 = g_{MN}dx^M dx^N = q_{\mu\nu}dx^\mu dx^\nu + \rho\, u_{ij}(\mathrm{Y})dy^i dy^j$$

4-dimensional universe

$q_{\mu\nu}$: 4-dimensional metric

ρ : volume modulus of the compact space

• 10-dimensional action

$$S = \frac{1}{2\bar{\kappa}^2} \int d^{10}x \sqrt{-g} \left[\mathrm{e}^{-2\phi} \left(R + 4g^{MN} \partial_M \phi \partial_N \phi - \frac{1}{2} |H|^2 \right) - \frac{1}{2} \sum_p |F_p|^2 \right]$$

$$- \sum_p (T_{\mathrm{D}p} + T_{\mathrm{O}p}) \int d^{p+1}x \sqrt{-g_{p+1}} \, \mathrm{e}^{-\phi} \, ,$$

- The 3-form H and p-form field strengths F_p can have a non-vanishing integral over any closed 3-, p-dimensional internal manifold of the compact space Y.

- **Field strengths have to obey generalized Dirac charge quantization conditions.**

$$\int_{\Sigma} H = h_{\Sigma}\,, \qquad \int_{\mathcal{C}_p} F_p = f^{(p)}_{\mathcal{C}_p}$$

- **h_{Σ} and $f^{(p)}{}_{Cp}$ are integers associated with number of quanta of H and F_p through each 3-, p-dimensional homology cycles, Σ, Cp in the internal manifold.**

- Orientifold planes occupy $(p-3)$-dimensional internal space due to extending our 4-dimensional universe.

- The contribution of Op-plane ($p \geq 3$) to moduli potential will survive.

- The moduli potential arises from the compactification of the terms in 10-dim action associated with the various field strengths, Dp-branes and Op-planes as well as the gravity, the dilaton.

4-dimensional effective action (Einstein frame)

(M. P. Hertzberg, arXiv:0711.2512)

$$S_{\rm E} = \int d^4x\sqrt{-\bar{q}}\left[\frac{1}{2\kappa^2}\bar{R} - \frac{1}{2}\bar{q}^{\mu\nu}\partial_\mu\bar{\rho}\partial_\nu\bar{\rho} - \frac{1}{2}\bar{q}^{\mu\nu}\partial_\mu\bar{\tau}\partial_\nu\bar{\tau} - V(\bar{\tau}\,,\bar{\rho})\right]$$

$\bar{R}$: Ricci scalar constructed from $\bar{q}_{\mu\nu}$

$$q_{\mu\nu} = \left(\frac{\bar{\kappa}}{\tau\kappa}\right)^2 \bar{q}_{\mu\nu}$$

κ^2 : 4-dimensional gravitational constant

τ : dilaton modulus

$$\bar{\rho} = \sqrt{\frac{3}{2}}\kappa^{-1}\ln\rho\,, \quad \bar{\tau} = \sqrt{2}\kappa^{-1}\ln\tau\,, \quad \tau = {\rm e}^{-\phi}\rho^{3/2}$$

☆ moduli potential

$$V(\bar{\tau}\,,\bar{\rho}) = V_{\mathrm{Y}} + V_{\mathrm{H}} + V_p + V_{\mathrm{DO}}$$

$$V_{\mathrm{Y}}(\bar{\tau}\,,\bar{\rho}) = -A_{\mathrm{Y}}\,(\phi_i)\exp\left[-\kappa\left(\sqrt{2}\bar{\tau} + \frac{\sqrt{6}}{3}\bar{\rho}\right)\right]R(\mathrm{Y})\,,$$

$$V_{\mathrm{H}}(\bar{\tau}\,,\bar{\rho}) = A_{\mathrm{H}}\,(\phi_i)\exp\left[-\kappa\left(\sqrt{2}\bar{\tau} + \sqrt{6}\bar{\rho}\right)\right],$$

$$V_p(\bar{\tau}\,,\bar{\rho}) = \sum_p A_p\,(\phi_i)\exp\left[-\kappa\left\{2\sqrt{2}\bar{\tau} + \frac{\sqrt{6}}{3}(p-3)\bar{\rho}\right\}\right],$$

$$V_{\mathrm{DO}}(\bar{\tau}\,,\bar{\rho}) = \sum_p \left[A_{\mathrm{D}p}\,(\phi_i) - A_{\mathrm{O}p}\,(\phi_i)\right]\exp\left[-\kappa\left\{\frac{3\sqrt{2}}{2}\bar{\tau} + \frac{\sqrt{6}}{6}(6-p)\bar{\rho}\right\}\right]$$

$$\times\int d^{p-3}x\sqrt{g_{p-3}}$$

positive

A_Y, A_H, A_p, A_{Dp}, A_{0p} : coefficients

[3] The scenario with vanishing flux

☆ Statement :

Ekpyrosis are prohibited in string theory with D-branes, O-planes source and zero fluxes.

☆ moduli potential with vanishing flux

$$
\begin{aligned}
V(\bar{\tau}\,,\bar{\rho}) &= V_{\mathrm{Y}} + V_{\mathrm{D}p} + V_{\mathrm{O}p} \\
&= -A_{\mathrm{Y}}\left(\phi_i\right)\exp\left[-\kappa\left(\sqrt{2}\bar{\tau} + \frac{\sqrt{6}}{3}\bar{\rho}\right)\right] R(\mathrm{Y}) \\
&\quad + \sum_p \left[A_{\mathrm{D}p}\left(\phi_i\right) - A_{\mathrm{O}p}\left(\phi_i\right)\right]\exp\left[-\kappa\left\{\frac{3\sqrt{2}}{2}\bar{\tau} + \frac{\sqrt{6}}{6}\left(6-p\right)\bar{\rho}\right\}\right]\int d^{p-3}x\sqrt{g_{p-3}}
\end{aligned}
$$

★Ekpyrosis:

(E. Meeus & T. Riet, (2016), K. Uzawa, JHEP06 (2018) 041)

For the case of R(Y)=0, A_Y=1, A_H=A_p=A_{Dp}=0, and $A_{0p} \int d^{p-3}x (g_{p-3})^{1/2}$=1, (p=4, 6, 8 for IIA and p=3, 5, 7, 9 for IIB), in the moduli potential, the fast roll parameters ε_f satisfy

$$\varepsilon_{\rm f} = \kappa^2 \frac{V^2}{(\partial_{\bar{\tau}} V)^2 + (\partial_{\bar{\rho}} V)^2} > \frac{6}{31}\,, \qquad \text{For IIA}$$

$$\varepsilon_{\rm f} = \kappa^2 \frac{V^2}{(\partial_{\bar{\tau}} V)^2 + (\partial_{\bar{\rho}} V)^2} > \frac{1}{6}\,, \qquad \text{For IIB}$$

The result gives the contradiction with the fast-roll condition for ekpyrosis.

- ❑ Our results:

❀ We find strong constraints ruling out ekpyrosis from analyzing the fast-roll conditions.

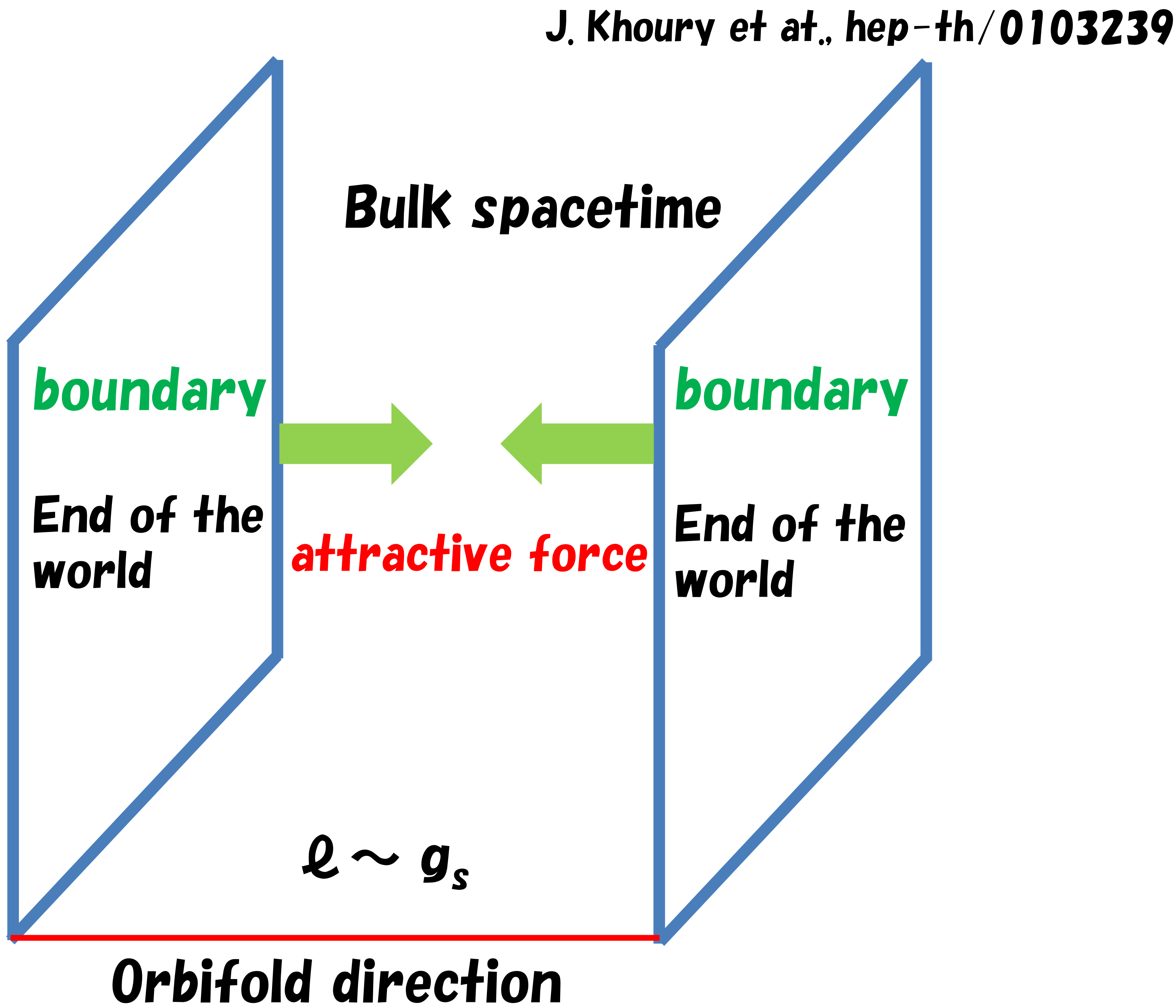
☞ We conclude that a compactification in type II string theory tend to provide potentials that are not too steep and negative.

[4] Summary and comments

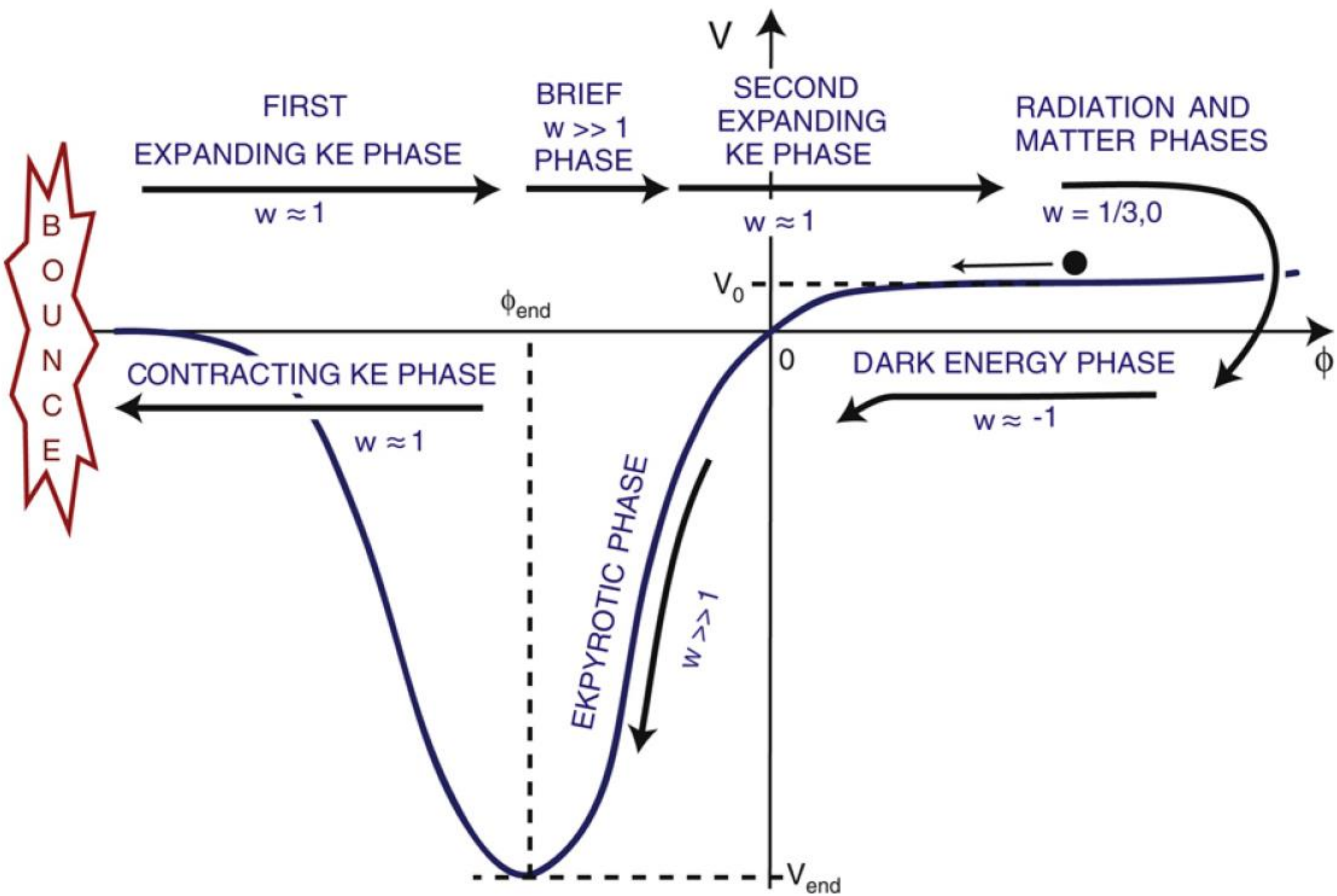
(1) We studied the No-Go theorem of the ekpyrosis for string theory with vanishing flux.

(2) The 4-dimensional effective potential of two scalar fields can be constructed by postulating suitable emergent gravity, orientifold planes in terms of the compactification with smooth manifold.

(3) Since the fast-roll parameter is not small during the ekpyrotic phase, the explicit nature of the dynamics has made it impossible to realize the ekpyrotic scenario.


J. Khoury et at., hep-th/0103239
Bulk spacetime
boundary
End of the world
attractive force
boundary
End of the world
$\ell \sim g_s$
Orbifold direction

(Joel K. Erickson, et al., hep-th/0607164)



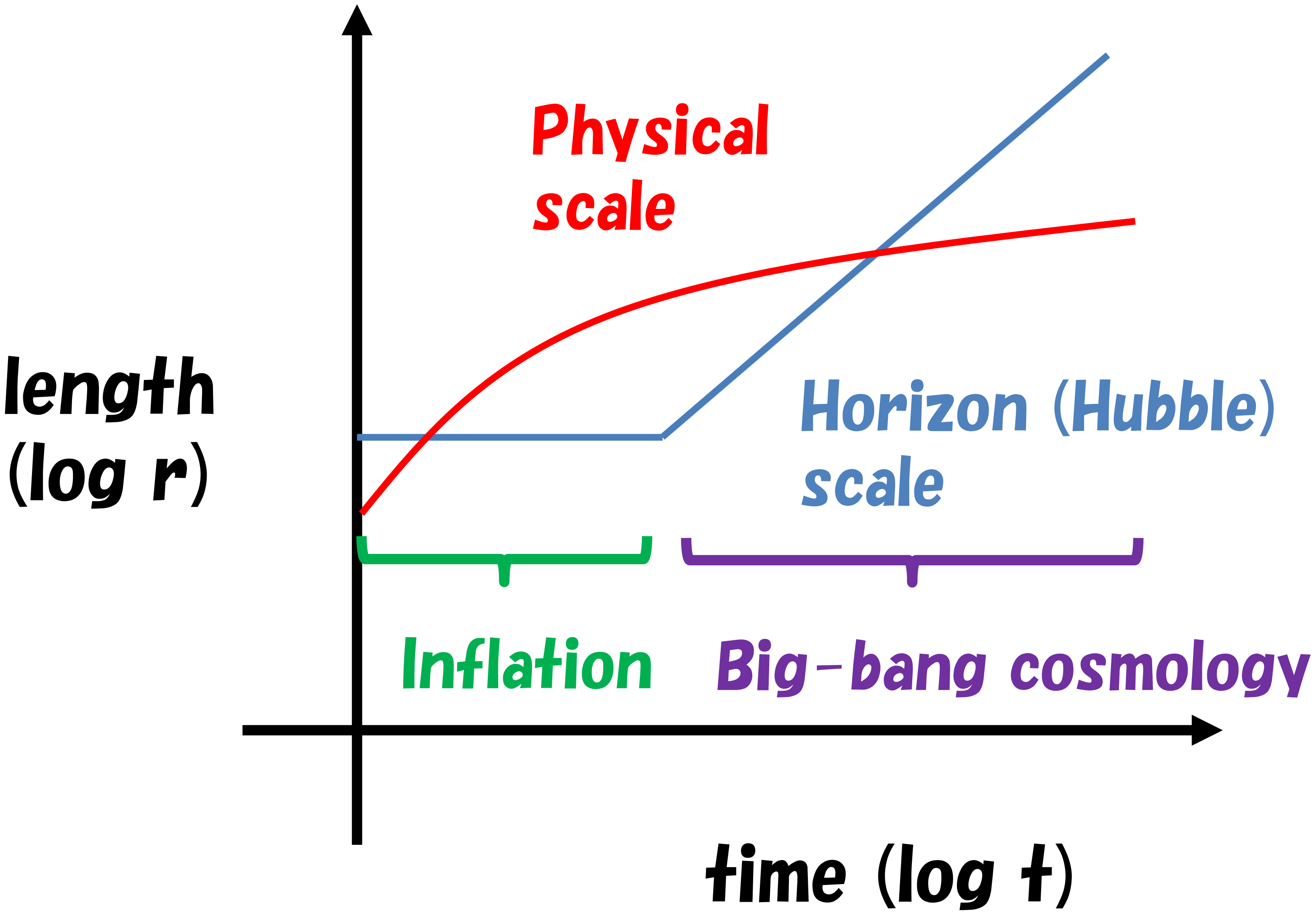
🕶 Property of eKypirosis
(J. Khoury et at., hep-th/0103239)

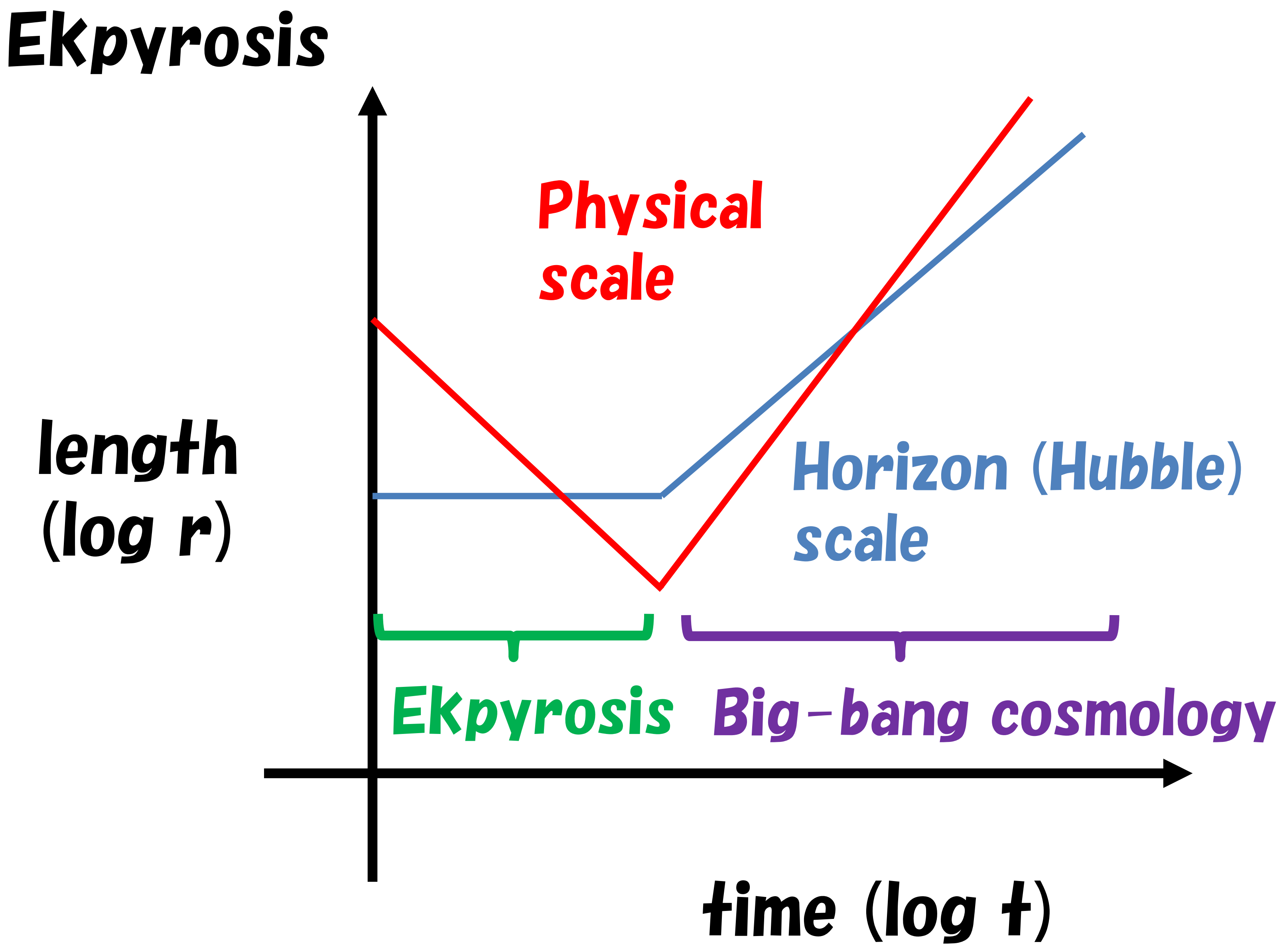
eKpyrosis – the world would continuously be consumed by a great inferno only to arise again like phenix.

- According to this scenario, the universe is in a slowly contracting phase before big bang, and universe undergoes a slow expansion.

- To take place eKpyrosis the scalar field rolls down its potential and Kinetic energy of the scalar increases.

Inflation


Physical scale
length (log r)
Horizon (Hubble) scale
Inflation
Big-bang cosmology
time (log t)


Ekpyrosis
Physical scale
length (log r)
Horizon (Hubble) scale
Ekpyrosis
Big-bang cosmology
time (log t)