Grand Unification and Weak Gravity Conjecture

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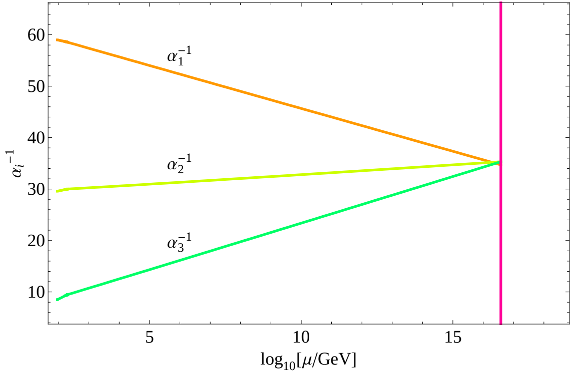
Grand Unified Theories (GUTs)

- Unification of SM/MSSM gauge couplings
- Unification of matter/quark-lepton multiplets
- Electric charge quantization, Magnetic monopoles predicted (as Dirac wanted)
- Proton Decay
- $b - \tau$ Yukawa unification in realistic models.
- Seesaw physics, neutrino oscillations
- Baryogenesis/leptogenesis
- Inflation/gravity waves, $\delta\rho/\rho$ and cosmic strings

Gauge Coupling Unification in Non-SUSY $SU(5)$



Gauge Coupling Unification in the $SU(5)$ model with additional fermions $Q + \bar{Q} + D + \bar{D}$ at mass scale $\sim (1 - 10)$ TeV.

Non-SUSY SO(10)

Usually broken via one or more intermediate steps to the SM

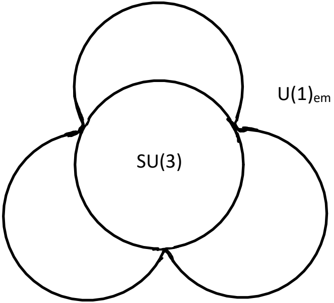
- $G = SO(10)/\text{Spin}(10)$
- $H = SU(3)_c \times U(1)_{e.m.}$
- $\Pi_2(G/H) \cong \Pi_1(H) \Rightarrow$ Monopoles
- $\Pi_1(G/H) \cong \Pi_0(H) = \mathbb{Z}_2 \Rightarrow$ Cosmic Strings (provided $G \to H$ breaking uses only tensor representations)
- $\mathbb{Z}_2 \subset \mathbb{Z}_4$ (center of $SO(10)$)
  [T. Kibble, G. Lazarides, Q.S., PLB, 1982]
- Intermediate scale monopoles and cosmic strings may survive inflation.
- Recent work suggests that this Z_2 symmetry can yield plausible cold dark matter candidates.
  [Mario Kadastik, Kristjan Kannike, and Martti Raidal Phys. Rev. D 81 (2010), 015002; Yann Mambrini, Natsumi Nagata, Keith A. Olive, Jeremi Quevillon, and Jiaming Zheng Phys.Rev. D91 (2015) no.9, 095010 ; Sofiane M. Boucenna, Martin B. Krauss, Enrico Nardi Phys.Lett. B755 (2016) 168-17]

Magnetic Monopoles in Unified Theories

Any unified theory with electric charge quantization predicts the existence of topologically stable ('tHooft-Polyakov) magnetic monopoles. Their mass is about an order of magnitude larger than the associated symmetry breaking scale.

Examples:

1. SU(5) $\rightarrow$ SM (3-2-1)
   Lightest monopole carries one unit of Dirac magnetic charge even though there exist fractionally charged quarks;



monopole mass $\sim \frac{M_G}{\alpha_G}$

2. $SU(4)_c \times SU(2)_L \times SU(2)_R$ (Pati-Salam)

   Electric charge is quantized with the smallest permissible charge being $\pm(e/6)$; Lightest monopole carries two units of Dirac magnetic charge;

3. SO(10) $\rightarrow$ 4-2-2 $\rightarrow$ 3-2-1

   Two sets of monopoles: First breaking produces monopoles with a single unit of Dirac charge.
   Second breaking yields monopoles with two Dirac units.

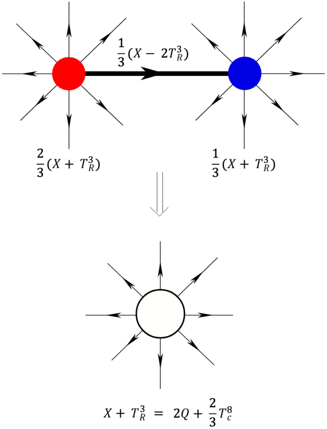
4. E_6 breaking to the SM can yield intermediate mass monopoles carrying three units of Dirac charge.

5. $E_6 \rightarrow SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \rightarrow$
   $SU(3)_c \otimes SU(2)_L \otimes U(1)_{em}$

   The discovery of primordial magnetic monopoles would have far-reaching implications for high energy physics $\&$ cosmology.

‘Schwinger’ Monopole

$$SU(4)_c \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R$$
$$\rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \tag{1}$$



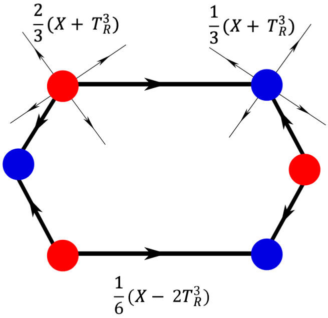


FIG. 2: Necklace with $SU(4)_c$ and $SU(2)_R$ monopoles from the symmetry breaking $SU(4)_c \times SU(2)_L \times SU(2)_R \to SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2$, where the last step is achieved by a **126**-plet of $SO(10)$. Notation as in Fig. 1. We display explicitly only the Coulomb magnetic flux of two of the monopoles and the magnetic flux along one of the tubes. This necklace may survive inflation.

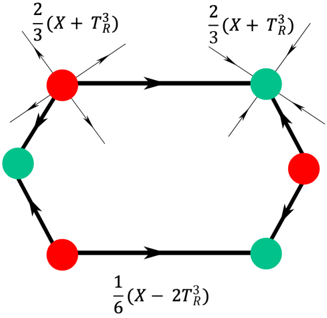


FIG. 3: Necklace with $SU(4)_c$ monopoles (red) and antimonopoles (green) from the symmetry breaking $SO(10) \to SU(4)_c \times SU(2)_L \times U(1)_R \to SU(3)_c \times U(1)_{B-L} \times SU(2)_L \times U(1)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y \times Z_2$, where the last step is achieved by a **126**-plet of $SO(10)$. We assume that the monopoles from the first step of symmetry breaking are inflated away. We display explicitly only the Coulomb magnetic flux of one monopole and one antimonopole and the magnetic flux along one of the tubes. This necklace may survive inflation.

Cosmic Strings from SO(10)

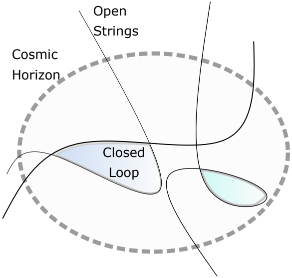
Cosmic Strings arise during symmetry breaking of $G \to H$ if $\pi_1(G/H)$ is non-trivial.

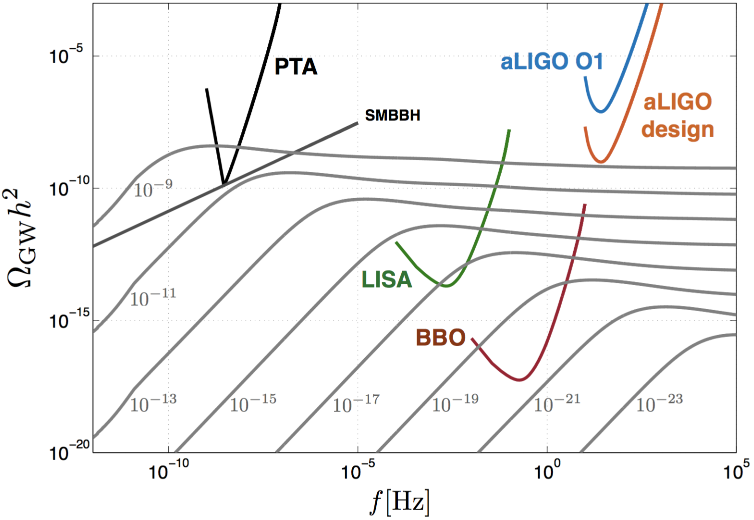
Consider $SO(10) \xrightarrow{M_{GUT}} SU(4) \times SU(2)_L \times SU(2)_R \xrightarrow{M_I} SM \times Z_2$

Mass per unit length of string is $\mu \sim M_I^2$, with $M_I \ll M_P$.

The strength of string gravity is determined by the dimensionless parameter $G\mu \ll 1$.

For this talk $G\mu \sim 10^{-12}$ or so, such that strings, analogous to monopoles, can survive inflation.





Stochastic gravitational wave backgrounds compared with present and future experiments. The grey lines show the background from cosmic strings with the indicated energy scales Gμ. The straight black line is the largest allowable background from SMBBH.[Blanco-Pillado, Olum and Siemens, arxiv: 1709.02434]

Inflationary Cosmology [Starobinsky, Mukhanov, Chibisov, Guth, Linde, Hawking, $\cdots$]

Successful Primordial Inflation should:

- Explain flatness, isotropy;
- Provide origin of $\frac{\delta T}{T}$;
- Offer testable predictions for n_s, r (gravity waves), $dn_s/d \ln k$;
- Recover Hot Big Bang Cosmology;
- Explain the observed baryon asymmetry;
- Offer plausible CDM candidate;

Slow-roll Inflation

- Inflation is driven by some potential $V(\phi)$:
- Slow-roll parameters:

$$\epsilon = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2,\ \eta = m_p^2\left(\frac{V''}{V}\right).$$

- The spectral index n_s and the tensor to scalar ratio r are given by

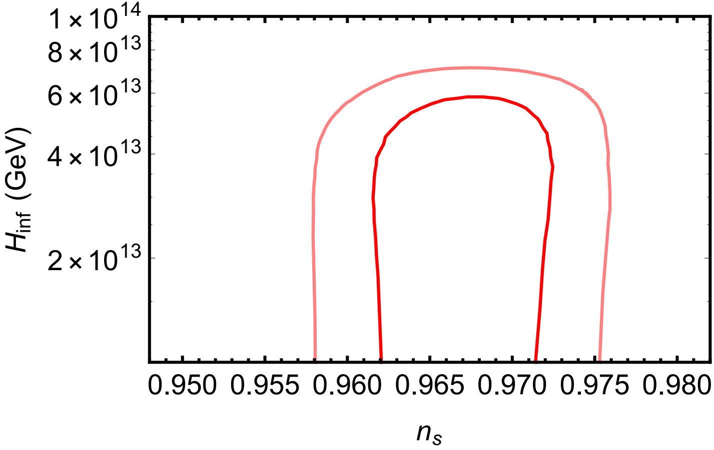
$$n_s - 1 \equiv \frac{d\ln\Delta_{\mathcal{R}}^2}{d\ln k},\ r \equiv \frac{\Delta_h^2}{\Delta_{\mathcal{R}}^2},$$

  where Δ_h^2 and $\Delta_{\mathcal{R}}^2$ are the spectra of primordial gravity waves and curvature perturbation respectively.

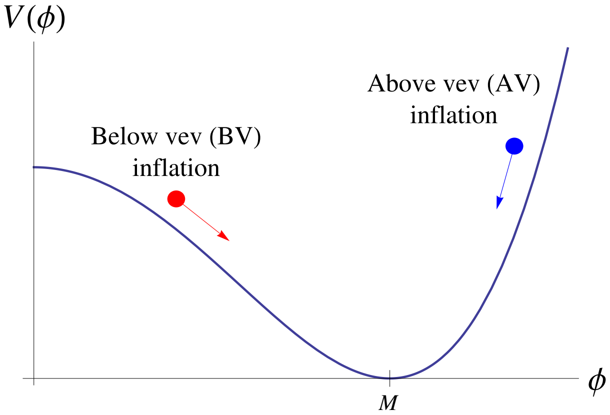
- Assuming slow-roll approximation (i.e. $(\epsilon, |\eta|) \ll 1$), the spectral index n_s and the tensor to scalar ratio r are given by

$$n_s \simeq 1 - 6\epsilon + 2\eta,\ r \simeq 16\epsilon.$$

Constraint on Inflation Planck (2018), BK (2015)

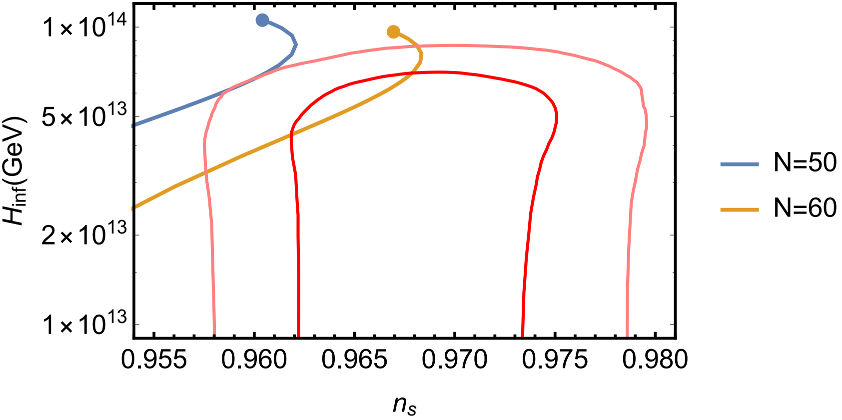


Inflation with a CW Higgs Potential



Note: This is for minimal coupling to gravity

Inflation with a CW Higgs Potential



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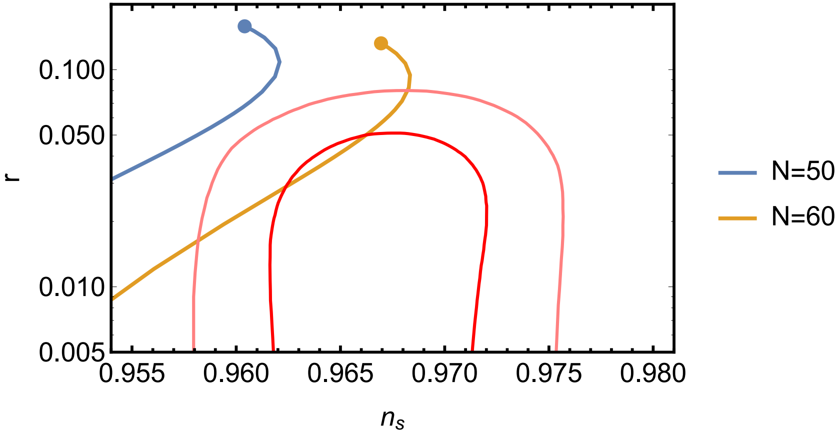
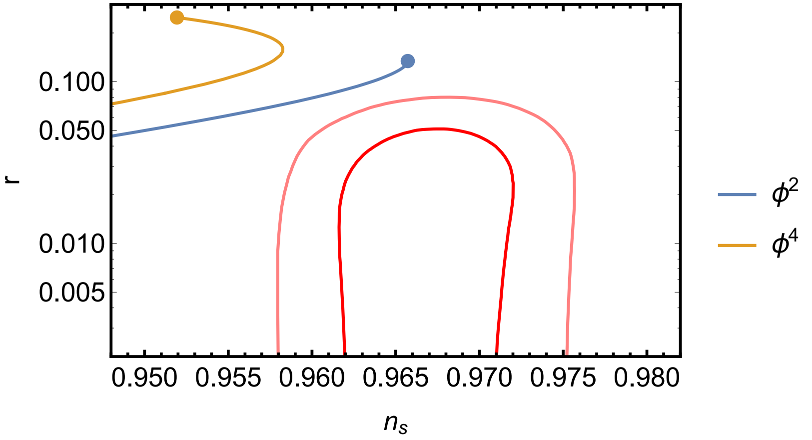
Inflation with a CW Higgs Potential



Note: This is for minimal coupling to gravity

$V_0^{1/4}$(GeV)	$V(\phi_0)^{1/4}$(GeV)	$A(10^{-14})$	v	ϕ_0	ϕ_e	n_s	r	$-\alpha(10^{-4})$
				solutions below the VEV ($\phi < v$)				
$1. \times 10^{16}$	9.92×10^{15}	4.37	12.7	3.38	11.4	0.954	0.008	5.97
1.5×10^{16}	1.43×10^{16}	2.41	22.1	10.2	20.8	0.964	0.036	4.87
1.75×10^{16}	1.58×10^{16}	1.43	29.4	16.5	28.0	0.967	0.055	4.95
$2. \times 10^{16}$	1.7×10^{16}	0.812	38.7	25.1	37.3	0.968	0.072	5.09

Inflation with ϕ^2 and ϕ^4 Potential



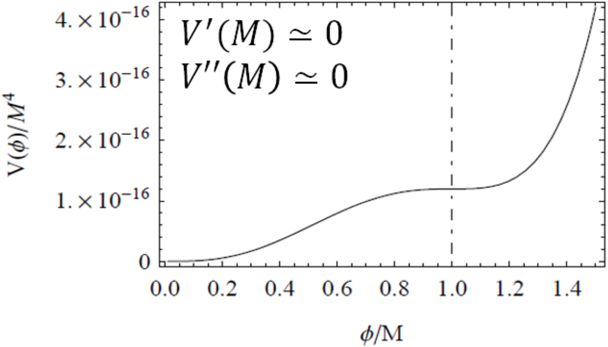
Note: This is for minimal coupling to gravity

Primordial Monopoles in SO(10) $\to 4-2-2 \to 3-2-1$

- Let's consider how much dilution of the monopoles is necessary. $M_I \sim 10^{13}$ GeV corresponds to monopole masses of order $M_M \sim 10^{14}$ GeV. For these intermediate mass monopoles the MACRO experiment has put an upper bound on the flux of $2.8 \times 10^{-16}\ \mathrm{cm}^{-2}$ s^{-1} sr^{-1}. For monopole mass $\sim 10^{14}$ GeV, this bound corresponds to a monopole number per comoving volume of $Y_M \equiv n_M/s \lesssim 10^{-27}$. There is also a stronger but indirect bound on the flux of $(M_M/10^{17}$ GeV$)10^{-16}\mathrm{cm}^{-2}$ s^{-1} sr^{-1} obtained by considering the evolution of the seed Galactic magnetic field.
- At production, the monopole number density n_M is of order H_x^3, which gets diluted to $H_x^3 e^{-3N_x}$, where N_x is the number of e-folds after $\phi = \phi_x$. Using
$$Y_M \sim \frac{H_x^3 e^{-3N_x}}{s},$$
where $s = (2\pi^2 g_S/45)T_r^3$, we find that sufficient dilution requires $N_x \gtrsim \ln(H_x/T_r) + 20$. Thus, for $T_r \sim 10^9$ GeV, $N_x \gtrsim 30$ yields a monopole flux close to the observable level.

Unique Low Scale Single Field Inflation Scenario: Inflection-point Inflation (IPI) [N. Okada and D. Raut, PRD, 2017]

- A unique realization of low-scale slow-roll inflation driven by a single scalar field



M is the value of the inflaton field ϕ at the start of inflation.

Inflection-point Inflation [N. Okada and D. Raut, PRD, 2017]

- How to realize an IPI?
  1. Consider a gauged-Higgs Model:
     Identify inflaton (ϕ) to be a Higgs field
     Inflaton has both gauge and Yukawa interaction
  2. $V'(M) \simeq 0$ & $V''(M) \simeq 0$:
     - $\lambda(M) \simeq 4.77 \times 10^{-16} \left(\frac{M}{M_P}\right)^2 \left(\frac{60}{N}\right)^4$
     - $\beta_\lambda(M) = \# g(M)^4 - \# Y(M)^4 \simeq 0$

       $g(M), Y(M) \ll 1$: Model Dependent

- IPI constraint and predictions:
  - Theoretical consistency: $M \leq 5.67 M_P$
  - Inflationary measurement fixes n_S
  - Unique prediction: $\alpha_S \simeq -2.742 \times 10^{-3} \left(\frac{60}{N}\right)^2$
  - (N=60) $H_{inf} < 1.5 \times 10^{10}\mathrm{GeV} \times \left(\frac{M}{M_P}\right)^3$

$$\Rightarrow r < 3.7 \times 10^{-9} \left(\frac{M}{M_P}\right)^6 \ll 1$$

$SO(10) \times U(1)_\psi$ w/ IPI [N. Okada, D. Raut, and Q. Shafi 2019 ()]

		$SO(10)$	$U(1)_\psi$	$\mathbf{Z}_2$	
$E_6 \supset$ Fermions ✓ Anomaly free	$16^{(i)}_{SM}$	**16**	$+1$	$-$	✓ SM fermions
	$10^{(i)}_{E}$	**10**	-2	$+$	✓ DM candidate
	$1^{(i)}_{E}$	**1**	$+4$	$+$	
Scalars	10_H	**10**	-2	$-$	✓ Symmetry breaking and Fermion Masses
	45_H	**45**	$+4$	$-$	
	126_H	**126**	$+2$	$-$	
	210_H	**210**	0	$-$	
	Φ_A	**1**	$+4$	$-$	✓ IPI inflation (Solves Monopole Problem)
	Φ_B	**1**	-8	$-$	

$$SO(10)\times U(1)_\psi \xrightarrow{\langle 210_H\rangle = M_{GUT}} SU(4)_c \times SU(2)_L \times SU(2)_R \times U(1)_\psi$$

$$\xrightarrow{\langle \overline{126}_H\rangle, \langle 45_H\rangle, \langle \Phi_{A,B}\rangle = M_I} SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$\xrightarrow{\langle 10_H\rangle} SU(3)_c \times U(1)_{EM}$$

SUSY Higgs (Hybrid) Inflation

[Dvali, Shafi, Schaefer; Copeland, Liddle, Lyth, Stewart, Wands '94]
[Lazarides, Schaefer, Shafi '97][Senoguz, Shafi '04; Linde, Riotto '97]
[Buchmüller, Domcke and Schmitz]

- Attractive scenario in which inflation can be associated with symmetry breaking $G \longrightarrow H$
- Simplest inflation model is based on

$$W = \kappa\, S\, (\Phi\, \overline{\Phi} - M^2)$$

  $S =$ gauge singlet superfield, $(\Phi\, , \overline{\Phi})$ belong to suitable representation of G

- Need $\Phi\, , \overline{\Phi}$ pair in order to preserve SUSY while breaking $G \longrightarrow H$ at scale $M \gg$ TeV, SUSY breaking scale.
- R-symmetry

$$\Phi\, \overline{\Phi} \rightarrow \Phi\, \overline{\Phi}, \;\; S \rightarrow e^{i\alpha}\, S, \;\; W \rightarrow e^{i\alpha}\, W$$

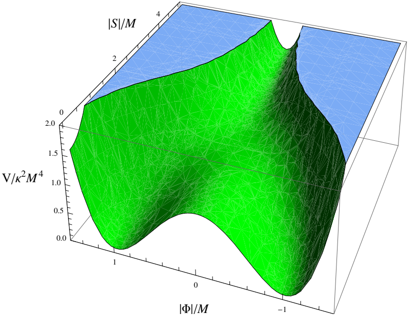
SUSY Higgs (Hybrid) Inflation

- Tree Level Potential

$$V_F = \kappa^2 \left(M^2 - |\Phi^2|\right)^2 + 2\kappa^2 |S|^2 |\Phi|^2$$

- SUSY vacua

$$|\langle\overline{\Phi}\rangle| = |\langle\Phi\rangle| = M,\ \langle S\rangle = 0$$



SUSY Higgs (Hybrid) Inflation

Take into account radiative corrections (because during inflation $V \neq 0$ and SUSY is broken by $F_S = -\kappa\, M^2$)

- Mass splitting in $\Phi - \overline{\Phi}$

$$m_\pm^2 = \kappa^2\, S^2 \pm \kappa^2\, M^2, \quad m_F^2 = \kappa^2\, S^2$$

- One-loop radiative corrections

$$\Delta V_{\mathsf{1loop}} = \frac{1}{64\pi^2}\mathsf{Str}[\mathcal{M}^4(S)(\ln\frac{\mathcal{M}^2(S)}{Q^2} - \frac{3}{2})]$$

- In the inflationary valley ($\Phi = 0$)

$$V \simeq \kappa^2\, M^4 \left(1 + \frac{\kappa^2 \mathcal{N}}{8\pi^2} F(x)\right)$$

where $x = |S|/M$ and

$$F(x) = \frac{1}{4}\left(\left(x^4+1\right)\ln\frac{\left(x^4-1\right)}{x^4} + 2x^2\ln\frac{x^2+1}{x^2-1} + 2\ln\frac{\kappa^2 M^2 x^2}{Q^2} - 3\right)$$

SUSY Higgs (Hybrid) Inflation

Tree level + radiative corrections + minimal Kähler potential yield:

$$n_s = 1 - \frac{1}{N} \approx 0.98.$$

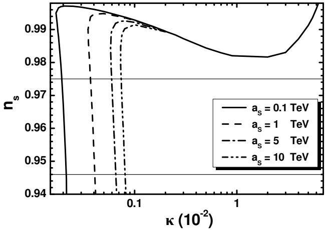
$\delta T/T$ proportional to M^2/M_p^2, where M denotes the gauge symmetry breaking scale. Thus we expect $M \sim M_{GUT}$ for this simple model. In practice, $M \approx (1-5) \times 10^{15}$ GeV

Since observations suggest that n_s lie close to 0.97, there are at least two ways to realize this slightly lower value:

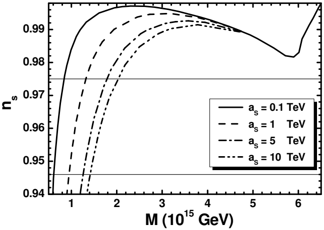
- include soft SUSY breaking terms, especially a linear term in S;
- employ non-minimal Kähler potential.

SUSY Higgs (Hybrid) Inflation

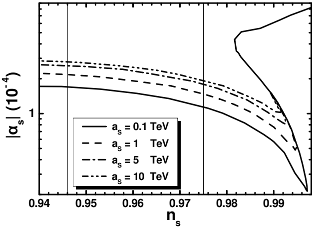
[Pallis, Shafi, 2013; Rehman, Shafi, Wickman, 2010]



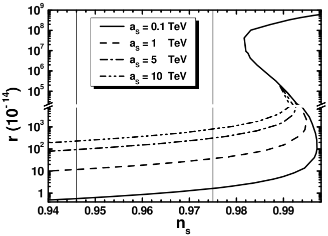
(a)



(b)



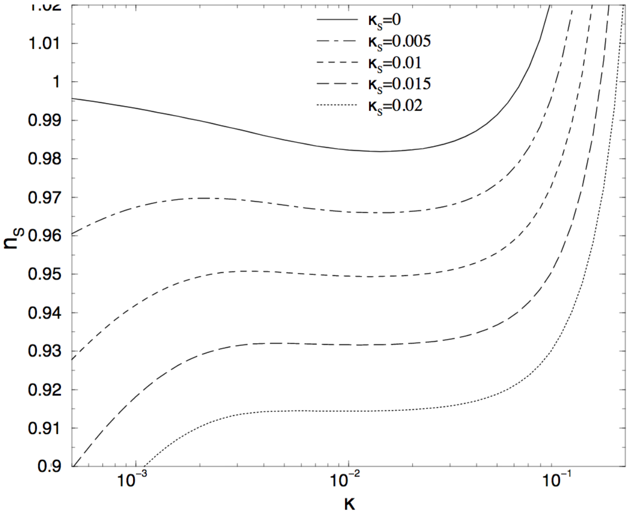
(a)



(b)

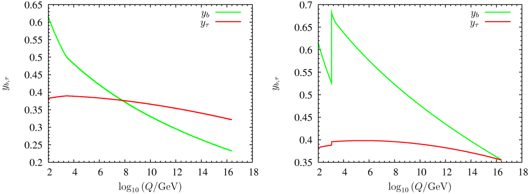
- $K \supset \kappa_s (S^\dagger S)^2$

[M. Bastero-Gil, S. F. King and Q. Shafi, 2006]

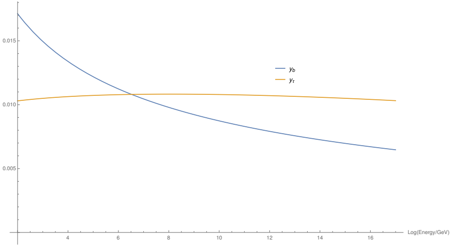


Yukawa Unification in GUTs

b$-\tau$ Yukawa coupling unification



Without Supersymmetry

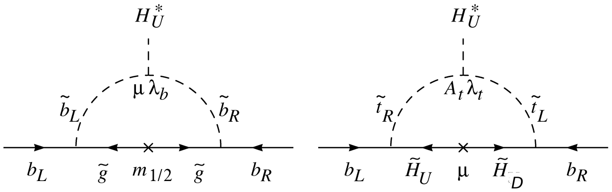


b-τ YU and finite threshold corrections [1]

Dominant contributions to the bottom quark mass from the gluino and chargino loop

$$\delta y_b \approx \frac{g_3^2}{12\pi^2}\frac{\mu m_{\tilde{g}} \tan\beta}{m_1^2} + \frac{y_t^2}{32\pi^2}\frac{\mu A_t \tan\beta}{m_2^2} + \ldots$$

where $m_1 \approx (m_{\tilde{b}_1} + m_{\tilde{b}_2})/2$ and $m_2 \approx (m_{\tilde{t}_2} + \mu)/2$

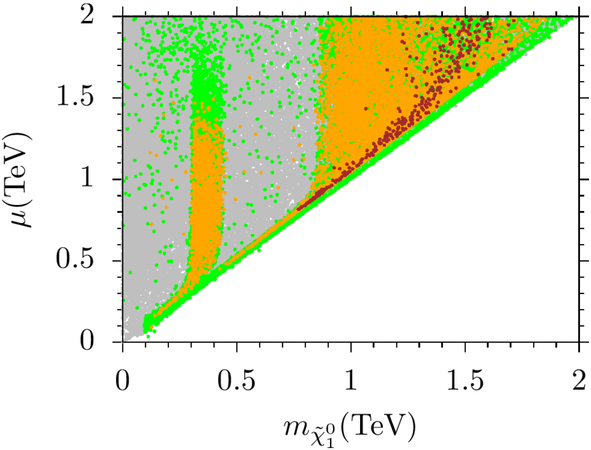


where $\lambda_b = y_b$ and $\lambda_t = y_t$

[1] L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev.D 50, 7048 (1994)

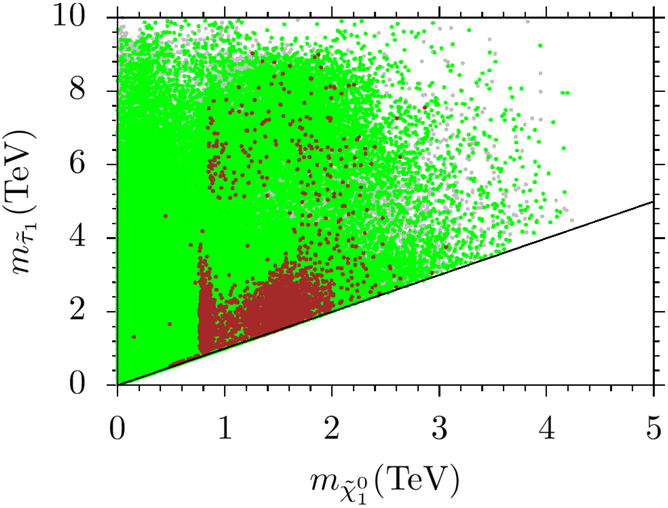
SUSY $SU(4)_C \times SU(2)_L \times SU(2)_R$ without Discrete $L \leftrightarrow R$ Symmetry M. Gomez, C. S Un, and Q. Shafi

- $\mu^2 = -\frac{{m_Z}^2}{2} + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1}$



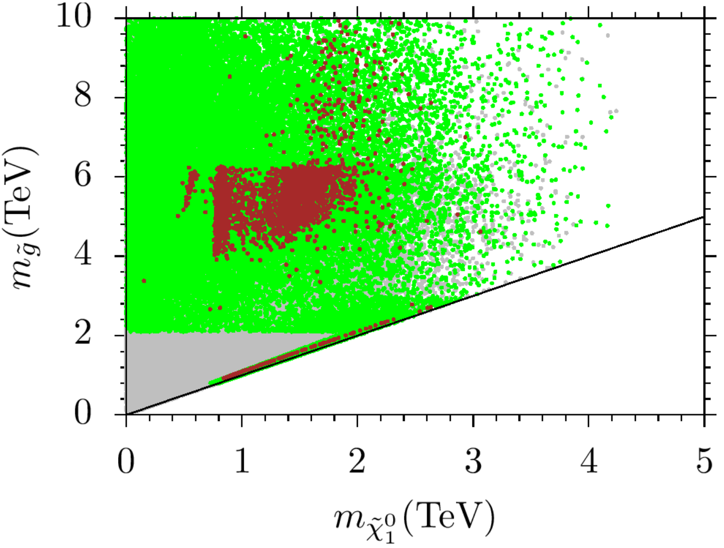
- Orange: $(t - b - \tau)$ Yukawa Unification
- Red ($\supset$ Orange $\supset$ Green $\supset$ Grey): Dark Matter

SUSY $SU(4)_C \times SU(2)_L \times SU(2)_R$ without Discrete $L \leftrightarrow R$ Symmetry M. Gomez, C. S Un, and Q. Shafi

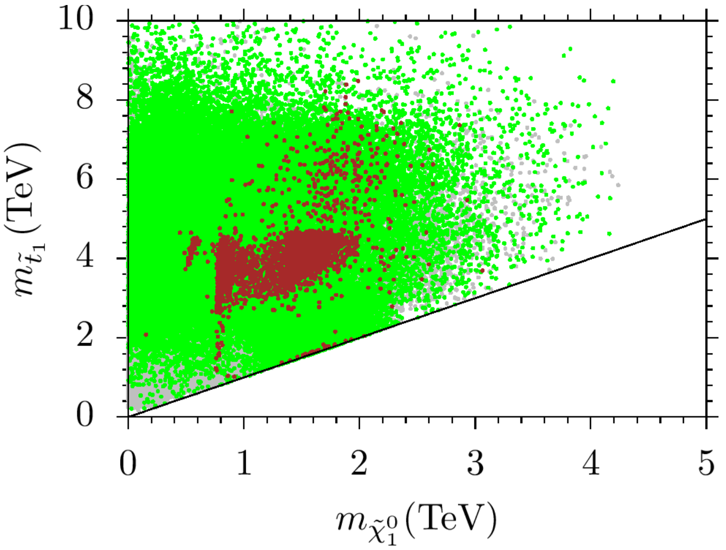


- Red: $(t - b - \tau)$ YU and Neutralino Dark Matter

SUSY $SU(4)_C \times SU(2)_L \times SU(2)_R$ without Discrete $L \leftrightarrow R$ Symmetry M. Gomez, C. S Un, and Q. Shafi



SUSY $SU(4)_C \times SU(2)_L \times SU(2)_R$ without Discrete $L \leftrightarrow R$ Symmetry M. Gomez, C. S Un, and Q. Shafi



SUSY $SU(4)_C \times SU(2)_L \times SU(2)_R$ without Discrete $L \leftrightarrow R$ Symmetry

M. Gomez, C. S Un, and Q. Shafi

	Point 1	Point 2	Point 3	Point 4	Point 5
$m_{\tilde{L}}$	9461	8031	781	1202	3714
M_1	-4810	1786	-1288	-3653	-4502
M_{2L}	3915	2859	758.5	1559	2348
M_3	-926	-316.7	-2904	-2802	-2537
$A_0/m_{\tilde{L}}$	-0.13	-1.06	1.57	1.16	1.30
$\tan\beta$	47.8	48.0	43.8	42.6	52.4
x_{LR}	0.78	1.43	0.83	1.45	0.84
y_{LR}	-1.89	1.11	-0.28	-2.71	-2.47
$m_{\tilde{R}}$	7398	11460	646.9	1740	3111
M_{2R}	-7399	3188	-210.1	-4221	-5811
μ	6401	8416	3431	3398	894.4
m_h	123.2	124.3	123.2	123.1	124.1
m_H	5568	6205	1088	1549	2432
m_A	5531	6164	1081	1539	2417
$m_{H^\pm}$	5568	6206	1093	1552	2434
$m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}$	2259, 3438	849.6, 2530	558.1, 711.6	1387, 1638	850.6, 853.3
$m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}$	5962, 5962	7757, 7757	3190, 3190	3165, 3165	2044, 2058
$m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}$	3439, 5962	2535, 7710	713.8, 3162	1389, 3137	871.5, 2018
$M_{\tilde{g}}$	2357	933.9	5954	5781	5360
$m_{\tilde{u}_L}, m_{\tilde{u}_R}$	9906, 7462	8183, 11379	5172, 5145	5163, 5294	6019, 5540
$m_{\tilde{t}_1}, m_{\tilde{t}_2}$	4763, 7707	4075, 9113	4371, 4506	4331, 4564	3523, 4338
$m_{\tilde{d}_L}, m_{\tilde{d}_R}$	9906, 7623	8183, 11558	5173, 5151	5164, 5258	6019, 5516
$m_{\tilde{b}_1}, m_{\tilde{b}_2}$	3491, 7690	4131, 8784	4340, 4490	4327, 4466	3556, 4327
$m_{\tilde{\nu}_{e,\mu}}, m_{\tilde{\nu}_\tau}$	9720, 8794	8104, 6498	909.3, 905.6	1668, 1449	4042, 3245
$m_{\tilde{e}_L}, m_{\tilde{e}_R}$	9714, 7811	8110, 11641	914.7, 843.7	1667, 2236	4043, 3600
$m_{\tilde{\tau}_1}, m_{\tilde{\tau}_2}$	5167, 8783	6509, 9396	605.6, 988.1	**1436**, 1896	1178, 3245
$\sigma_{\rm SI}$ (pb)	0.71×10^{-12}	0.40×10^{-14}	0.13×10^{-13}	0.47×10^{-10}	0.19×10^{-9}
$\sigma_{\rm SD}$ (pb)	0.48×10^{-10}	0.16×10^{-10}	0.68×10^{-9}	0.17×10^{-7}	0.29×10^{-6}
Ωh^2	0.124	0.116	0.122	0.120	0.125
$R_{tb\tau}$	1.08	1.04	1.09	1.08	1.09

Weak Gravity Conjecture [Arkani-Hamed, Motl, Nicolis, Vafa]

Weak gravity conjecture suggests the presence of an ultraviolet cutoff Λ that lies between M_{GUT} and M_{Planck}.

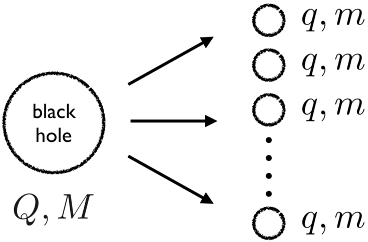
[Slogan: Gravity is the weakest force.]

Effective field theory description holds for scales below Λ.

For U(1) gauge theory there exists a particle with charge $q > m$, where m is measured in Planck units; this means U(1) breaking vev $< M_P$.

Motivation

- Requirement that extremal black holes can decay;



- Global symmetry not compatible with quantum gravity, so we cannot take $q \to 0$ limit;
- String theory only possesses gauge symmetries.

For GUTs, $\Lambda \sim \alpha_\Lambda^{1/2} \times M_P \sim 5 \times 10^{17}$ GeV for SUSY GUTs.

$SU(5) \times U(1)_\chi (\chi SU(5))$

Group	Representations					
	Matter					
$SU(5)$	$F_i(\bar{5})$	$T_i(10)$	$\nu_i^c(1)$			
$2\sqrt{10}U(1)_\chi$	3	-1	-5			
	Scalars					
$SU(5)$	$\Phi(24)$	$H(5)$	$\bar{H}(\bar{5})$	$\chi(1)$	$\bar{\chi}(1)$	$S(1)$
$2\sqrt{10}U(1)_\chi$	0	2	-2	10	-10	0

Table: Matter and Higgs content in minimal $SU(5) \times U(1)_\chi$. $\chi, \bar{\chi}$ fields implement $U(1)_\chi$ breaking and $\bar{\chi}$ provides masses to the right handed neutrinos, ν_i^c. The singlet S plays an important role during inflation.

$SU(5) \times U(1)_\chi(\chi SU(5))$: Salient Features

- $U(1)_\chi$ prevents rapid proton decay
- $U(1)_\chi \to Z_2$('matter parity'); Stable LSP.
- Observable Proton Decay
- Stable Cosmic Strings
- Yukawa Unification

$SU(5) \times U(1)_\chi (\chi SU(5))$: Gauge Coupling Unification

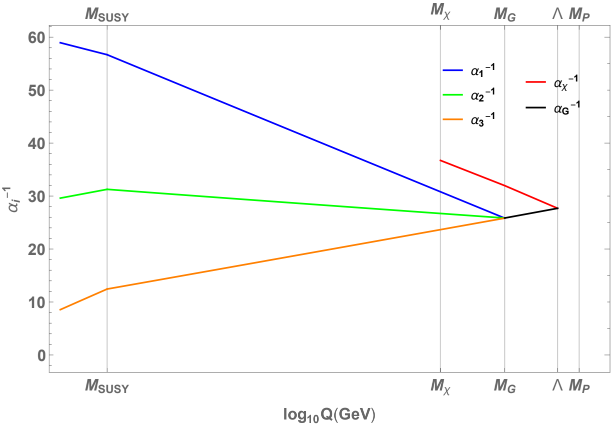


Figure: Running of gauge couplings in MSSM and $\chi SU(5)$. Unification of the $\chi SU(5)$ gauge couplings occurs at $\Lambda \approx 5 \times 10^{17}$GeV. $\mu_\chi = 10^{14}$GeV denotes the $U(1)_\chi$ symmetry breaking scale and $M_P = 2.4 \times 10^{18}$GeV is the reduced Planck scale.

$SU(5) \times U(1)_\chi (\chi SU(5))$: Dim-5 Operator and Proton Lifetime

- Dimension five operator $\frac{\eta}{\Lambda} Tr(F \cdot F\Phi)$ modifies conditions for GCU.

$$(1+\epsilon)^{1/2} g_1(M_X) = (1+6\epsilon)^{1/2} g_2(M_X) = (1-4\epsilon)^{1/2} g_3(M_X).$$

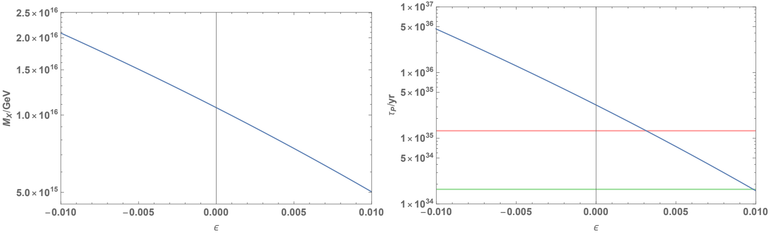


Figure: Left: Unification scale M_X as a function of ϵ. Right: Proton Lifetime vs. ϵ(blue line). The green line denotes the 2σ experimental bound on proton lifetime set by Super-K, and the red line is the expected 2σ sensitivity at Hyper-K.

$SU(5) \times U(1)_\chi (\chi SU(5))$: Dim-5 Operator and Yukawa Unification

Dimension Five terms can also affect the fermion masses [Panagiotakopoulos and Shafi, 1984; Wiesenfeldt, 2005; Calmet and Yang, 2011;].

Consider

$$\frac{\varepsilon_{\alpha\beta\mu\nu\delta}}{\Lambda}\left(f_{ij}F_i^{\alpha\beta}T_j^{\mu\nu}\Phi_\rho^\delta\bar{H}^\rho + f'_{ij}F_i^{\alpha\beta}T_j^{\mu\rho}\bar{H}^\nu\Phi_\rho^\delta\right) + h.c.$$

The condition for YU in the third family is modified to

$$y_b - y_\tau \approx 5 f'_{33}\frac{M_{GUT}}{\Lambda}.$$

$SU(5) \times U(1)_\chi (\chi SU(5))$: Dim-5 Operator and Yukawa Unification

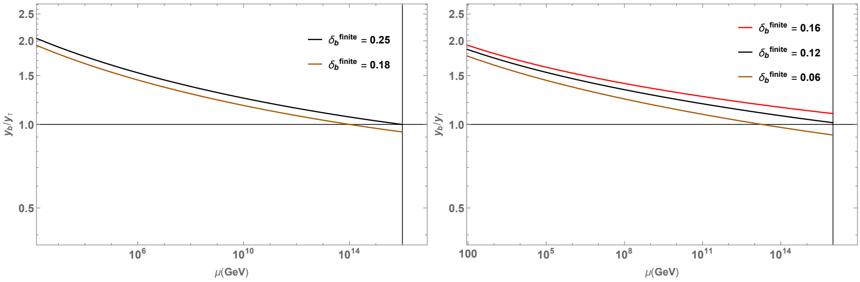


Figure: Left: y_b/y_τ versus μ, the energy scale, for tan$\beta = 20$. $y_b - y_\tau$ at M_{GUT} are 0 (top curve) and -0.01 (bottom curve). δ_b^{finite} denote the size of the finite one loop corrections to y_b.
Right: Corresponding figure for tan$\beta = 50$. $y_b - y_\tau$ at M_{GUT} are 0.06 (top), 0 (middle) and -0.04 (bottom).

Summary

- Unification of all forces remains a compelling idea.
- Grand unification explains charge quantization, predicts monopoles and proton decay.
- Also explains tiny neutrino masses via seesaw mechanism.
- Intermediate scale monopoles and cosmic strings may survive inflation.
- In non-SUSY inflation with Higgs potential, r $\gtrsim$ 0.01 (minimal coupling to gravity).
- SUSY and Non-SUSY models offer plausible dark matter candidates such as TeV mass higgsino, axions....
- Search for primordial gravity waves, monopoles, cosmic strings and dark matter.

Thank You!