(In)Stability of the EW Vacuum

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Stability analysis of the EW vacuum (few references)

I. V. Krive, A. D. Linde, Nucl.Phys. **B432** (1976) 265;

N. Cabibbo, L. Maiani, G. Parisi, R. Petronzio, Nucl.Phys. **B158** (1979) 295;

M. S. Turner, F. Wilczek, **Nature 298** (1982) 633;

R.A. Flores, M. Sher, Phys. Rev. **D27** (1983) 1679.

M. Lindner, Z. Phys. **31** (1986) 295.

D.L. Bennett, H.B. Nielsen and I. Picek, Phys. Lett. **B208** (1988) 275.

M. Sher, Phys. Rep. **179** (1989) 273.

M. Lindner, M. Sher, H. W. Zaglauer, Phys. Lett. **B228** (1989) 139.

G. Anderson, Phys. Lett. **B243** (1990) 265

P. Arnold and S. Vokos, Phys. Rev. **D44** (1991) 3620

C. Ford, D.R.T. Jones, P.W. Stephenson, M.B. Einhorn, Nucl.Phys. **B395** (1993) 17.

M. Sher, Phys. Lett. **B317** (1993) 159.

G. Altarelli, G. Isidori, Phys. Lett. **B337** (1994) 141.

J.A. Casas, J.R. Espinosa, M. Quirós, Phys. Lett. **B342** (1995) 171.

J.A. Casas, J.R. Espinosa, M. Quirós, Phys. Lett. **B382** (1996) 374.

J.R. Espinosa, M. Quiros, Phys.Lett. **B353** (1995) 257-266

C. D. Froggatt and H. B. Nielsen, Phys. Lett. **B 368** (1996) 96.

C.D. Froggatt, H. B. Nielsen, Y. Takanishi, Phys.Rev. **D64** (2001) 113014

G. Isidori, G. Ridolfi, A. Strumia, Nucl. Phys. **B609** (2001) 387.

J. R. Espinosa, G. F. Giudice, A. Riotto, JCAP 0805, 002 (2008).

After the discovery of the Higgs boson ... renewed interest ...

J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, A. Riotto, A. Strumia, Phys.Lett. **B709** (2012) 222.

G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G.Isidori, A. Strumia, JHEP **1208** (2012) 098.

S. Alekhin, A. Djouadi, S. Moch, Phys.Lett. B716 (2012) 214.

F. Bezrukov, M. Yu. Kalmykov, B. A. Kniehl, M. Shaposhnikov, JHEP 1210 (2012) 140.

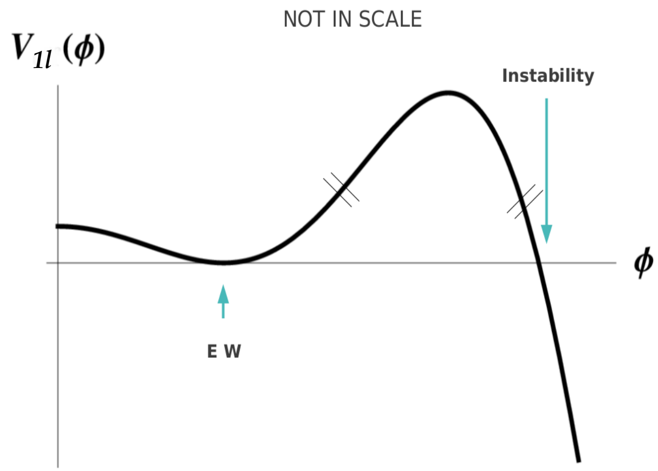
D. Buttazzo, G. Degrassi, P.P. Giardino, G.F. Giudice, F. Sala, A. Salvio, A. Strumia, JHEP 1312 (2013) 089.

... many other references ...

Stability analysis of the EW vacuum

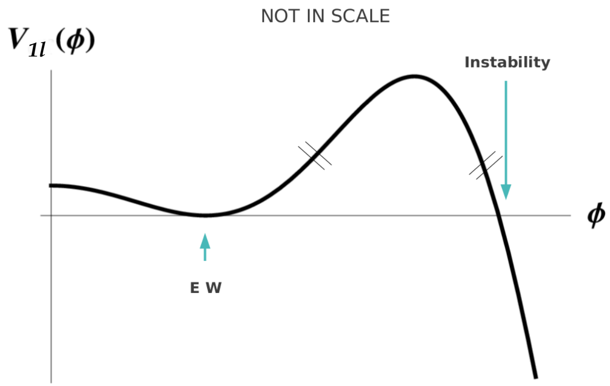
Key tool: Higgs Effective Potential

Top loop-corrections destabilize the EW Vacuum...



EW $= v \sim 246$ GeV ; For $M_H \sim 125$ GeV , $M_t \sim 173$ GeV : Instability $\sim 10^{11}$ GeV

One-Loop Higgs Effective Potential $V_{1l}(\phi)$

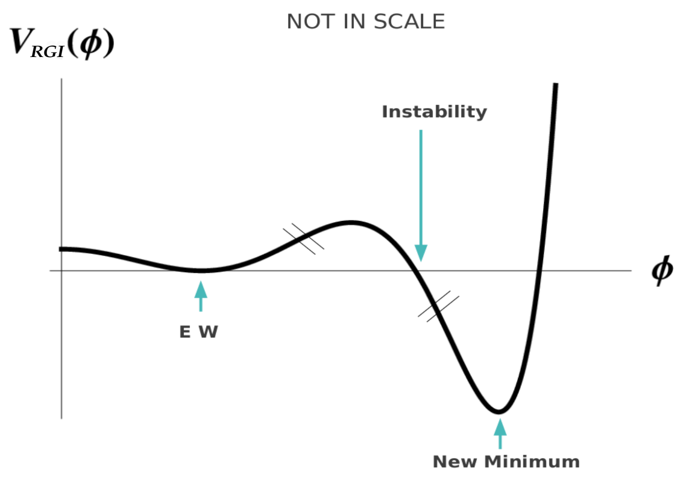


$$V_{1l}(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{24}\phi^4 + \frac{1}{64\pi^2}\Bigg[\left(m^2 + \frac{\lambda}{2}\phi^2\right)^2\left(\ln\left(\frac{m^2+\frac{\lambda}{2}\phi^2}{\mu^2}\right) - \frac{3}{2}\right)$$

$$+3\left(m^2 + \frac{\lambda}{6}\phi^2\right)^2\left(\ln\left(\frac{m^2+\frac{\lambda}{6}\phi^2}{\mu^2}\right) - \frac{3}{2}\right) + 6\,\frac{{g_1}^4}{16}\phi^4\left(\ln\left(\frac{\frac{1}{4}{g_1}^2\phi^2}{\mu^2}\right) - \frac{5}{6}\right)$$

$$+3\,\frac{\left({g_1}^2+{g_2}^2\right)^2}{16}\phi^4\left(\ln\left(\frac{\frac{1}{4}\left({g_1}^2+{g_2}^2\right)\phi^2}{\mu^2}\right) - \frac{5}{6}\right) - 12\,h_t^4\phi^4\left(\ln\frac{g^2\phi^2}{\mu^2} - \frac{3}{2}\right)\Bigg]$$

Running the RG eqs. for the SM couplings $\Rightarrow$ RGI Potential:



Depending on M_H and M_t, the second minimum can be :

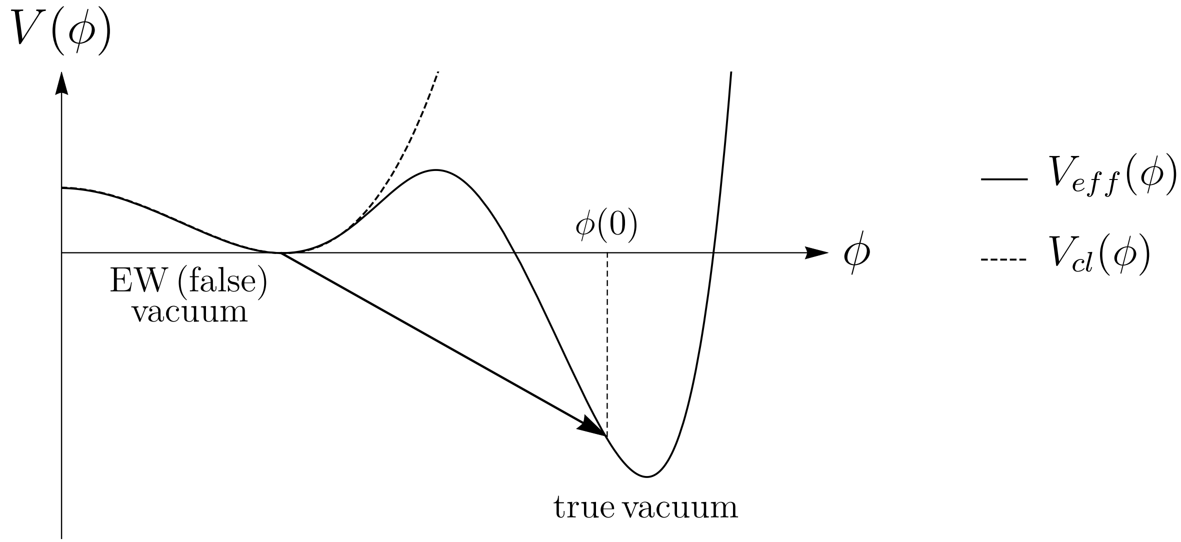
1. lower than the EW minimum (as in the figure) : This is the case for $M_H \sim 125$ GeV , $M_t \sim 173$ GeV (central values);

2. at the same height ... ;

3. higher ...

Case 1 (figure) : EW vacuum Metastable

Tunneling : EW vacuum (false) → True Vacuum



... If the EW vacuum lifetime larger than the age of the Universe ...

.... we may well live in such a Metastable Vacuum ....

Tunneling Rate

$$\Gamma = \frac{1}{\tau} = De^{-(S[\phi_b]-S[\phi_{\rm fv}])} \equiv D\, e^{-B}$$

$\phi_b(r)$ Bounce: Solution to the Euclidean EOM with appropriate b.c.

Euclidean equations of motion ($O(4)$ Symmetry)

$$-\,\partial_\mu\partial_\mu\phi + \frac{d\,V(\phi)}{d\,\phi} = -\frac{d^2\phi}{dr^2} - \frac{3}{r}\frac{d\phi}{dr} + \frac{d\,V(\phi)}{d\,\phi} = 0$$

Boundary conditions : $\phi'(0) = 0\ ,\qquad \phi(\infty) = v \to 0\ .$

A well known example: $V(\phi) = \frac{\lambda}{4}\phi^4$ **with constant and negative** λ

Bounce (Fubini instanton) : $\phi_b(r) = \sqrt{\frac{2}{|\lambda|}}\frac{2R}{r^2+R^2}$ $\quad$ ($R =$ **size**)

Degeneracy : $S[\phi_b] = \frac{8\pi^2}{3|\lambda|}$ **Bounce Action does not depend on R**

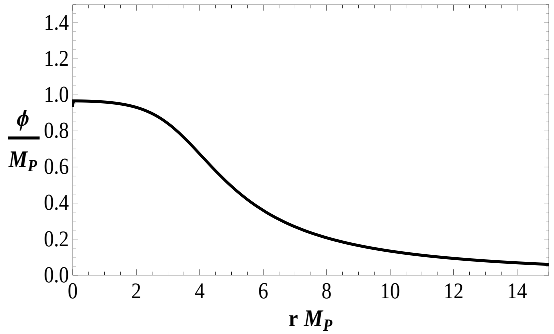
Classical Scale Invariance

Degeneracy removed at the Quantum Level

$$\Gamma = \frac{1}{\tau} = D e^{-(S[\phi_b] - S[\phi_{\rm fv}])} \equiv D\, e^{-S[\phi_b]}$$

A good estimate for Γ is obtained by approximating the prefactor D in terms of the bounce size R, defined as the value of r such that:

$$\phi_b(R) = \frac{1}{2}\phi_b(0)$$



and the age of the universe T_U.

For the EW vacuum lifetime $\tau = \Gamma^{-1}$ we get:

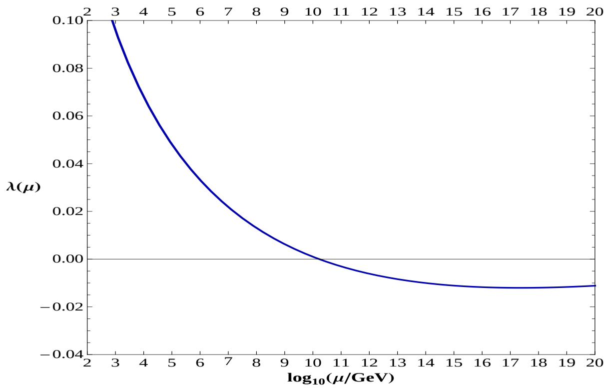
$$\tau \simeq \left(\frac{R^4}{T_U^3}\right) e^{S[\phi_b]}$$

For SM the instability occurs at large values of ϕ

$\Rightarrow$ $V_{eff}(\phi)$ **well approximated by keeping only the quartic term**

$$V_{SM}(\phi) \sim \lambda(\phi)\phi^4$$

$\lambda(\phi)$ depends on ϕ essentially as $\lambda(\mu)$ depends on μ

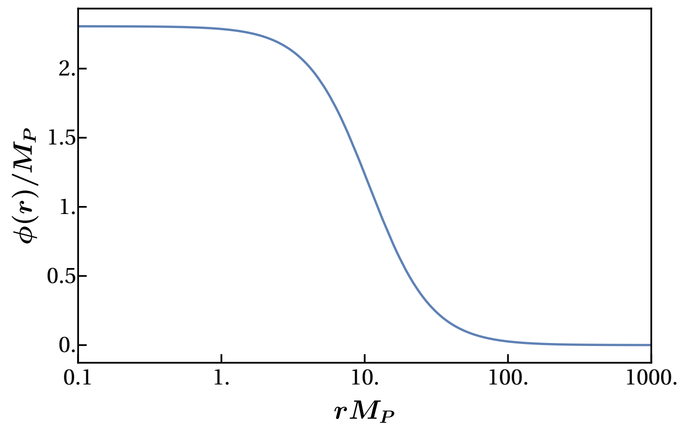


For large values of ϕ, the coupling λ becomes negative and almost constant in the region of interest ... close to the Fubini instanton case ... In fact people used analytical approximations, but we can do better ... we can calculate the bounce numerically ...

1. Stability Analysis - Flat Spacetime

Euclidean action $S[\phi] = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi)^2 + V_{SM}(\phi)\right]$

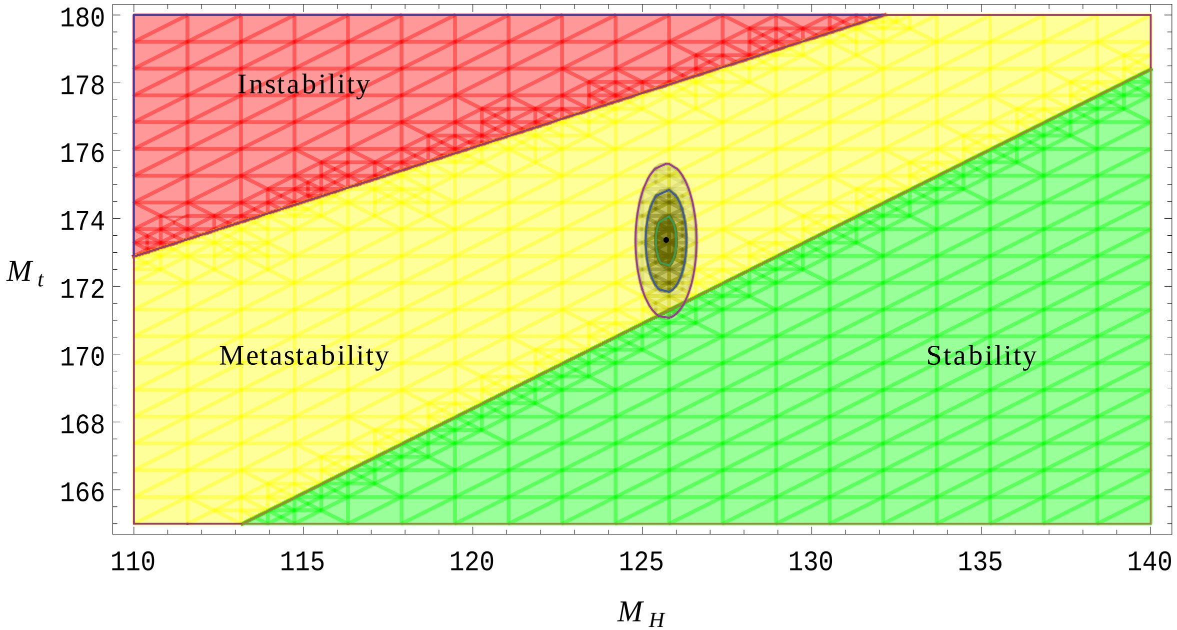
Bounce Solution



EW vacuum lifetime $\tau_{flat} \simeq \left(\frac{R^4}{T_U^3}\right) e^B \simeq 10^{640} T_U$

Obtained for $M_H \sim 125$ GeV and $M_t \sim 173$ GeV

More generally: Stability Diagram in the $M_H - M_t$ plane



Stability region : $V_{eff}(v) < V_{eff}(\phi_{min}^{(2)})$.

Meta-stability region : $V_{eff}(\phi_{min}^{(2)}) < V_{eff}(v)$ and $\tau > T_U$.

Instability region : $V_{eff}(\phi_{min}^{(2)}) < V_{eff}(v)$ and $\tau < T_U$.

Stability line : $V_{eff}(v) = V_{eff}(\phi_{min}^{(2)})$.

Instability line : M_H and M_t such that $\tau = T_U$.

2. Stability Analysis - Curved Spacetime

$$S[\phi, g_{\mu\nu}] = \int d^4x\sqrt{g}\left[-\frac{R}{16\pi G} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + V_{SM}(\phi)\right]$$

Requiring again $O(4)$ symmetry, the (Euclidean) metric:

$$ds^2 = dr^2 + \rho^2(r)d\Omega_3^2$$

Bounce $(\phi_b(r),\, \rho_b(r))$, solutions of the coupled equations: $(\kappa \equiv 8\pi G)$:

$$\ddot{\phi} + 3\,\frac{\dot{\rho}}{\rho}\,\dot{\phi} = \frac{d\,V_{SM}(\phi)}{d\phi} \qquad \dot{\rho}^2 = 1 + \frac{\kappa\rho^2}{3}\left(\frac{1}{2}\dot{\phi}^2 - V_{SM}(\phi)\right)$$

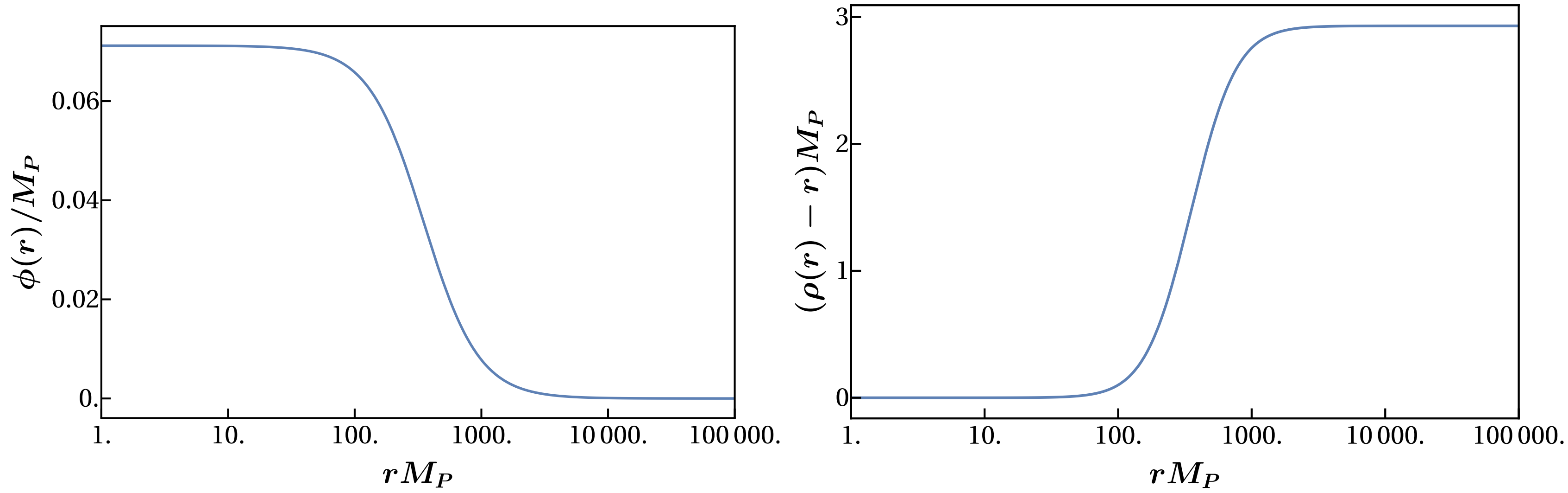
First equation: replaces the equivalent equation in flat spacetime;

Second equation: the only Einstein equation left by the symmetry.

For the decay of a Minkowski false vacuum to a true AdS vacuum (the case of interest to us) the boundary conditions are:

$$\phi_b(\infty) = 0 \qquad \dot{\phi}_b(0) = 0 \qquad \rho_b(0) = 0\,.$$

Bounce in Curved Spacetime Background



Profile of $\phi_b(r)$ and of the difference between $\rho(r)$ and its asymptotic value, $\rho(r) - r$ (asymptotically $\rho_b(r)$ reaches the Minkowskian $\rho_M(r) \sim r + Const$).

EW vacuum lifetime $\tau_{\rm grav} \simeq 10^{660} T_U$

Obtained for $M_H \sim 125$ GeV and $M_t \sim 173$ GeV

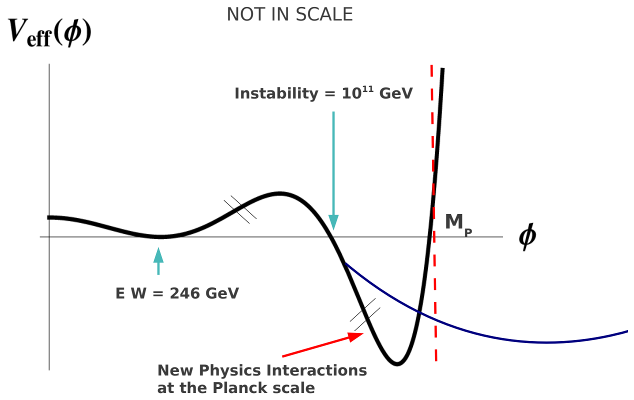
Crucial point - Calculation of τ under the assumptions

1. **No New Physics between Fermi scale and Planck scale**

2. **New Physics at Planck scale has no impact on the EW vacuum lifetime, so it can be neglected when computing τ.**

Argument: Instability scale, $\Lambda_{inst} \sim 10^{11}$ GeV, much lower than $M_P \Rightarrow$

$$\Rightarrow \quad \text{suppression} \quad \left(\frac{\Lambda_{inst}}{M_P}\right)^n \quad \text{expected}$$

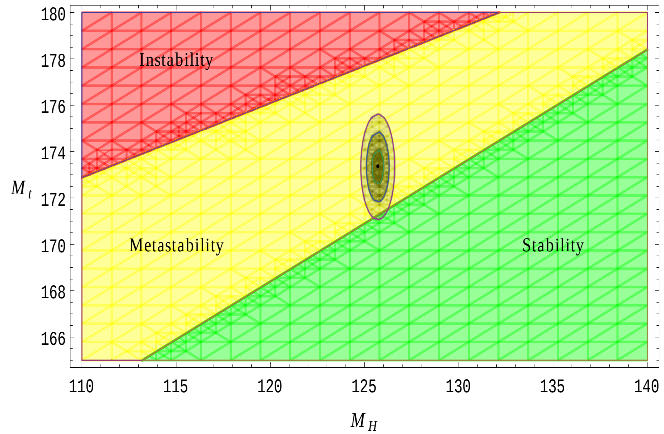


J.R. Espinosa, G.F. Giudice, A. Riotto, JCAP 0805 (2008) 002

Isidori, Ridolfi, Strumia, Nucl.Phys. B609 (2001) 387

... However, things are more subtle ...

The Stability Diagram is not universal



New Physics at Planck scale can strongly modify this Stability Diagram

VB, E. Messina, Phys.Rev.Lett.111, 241801 (2013)

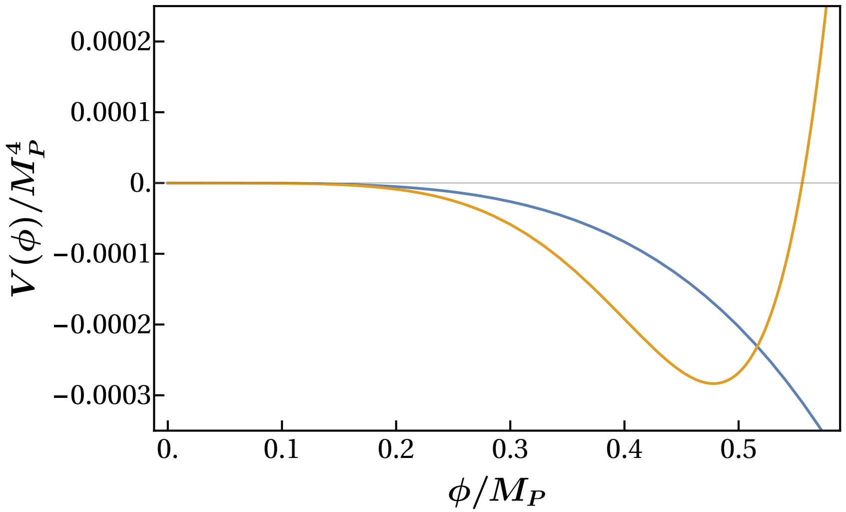
VB, E. Messina, M. Sher, Phys.Rev.D91 (2015) 1, 013003

E. Bentivegna, VB, F. Contino, D. Zappalà, JHEP 1712 (2017) 100

... Let's add New Physics around M_P ...

New Physics around M_P

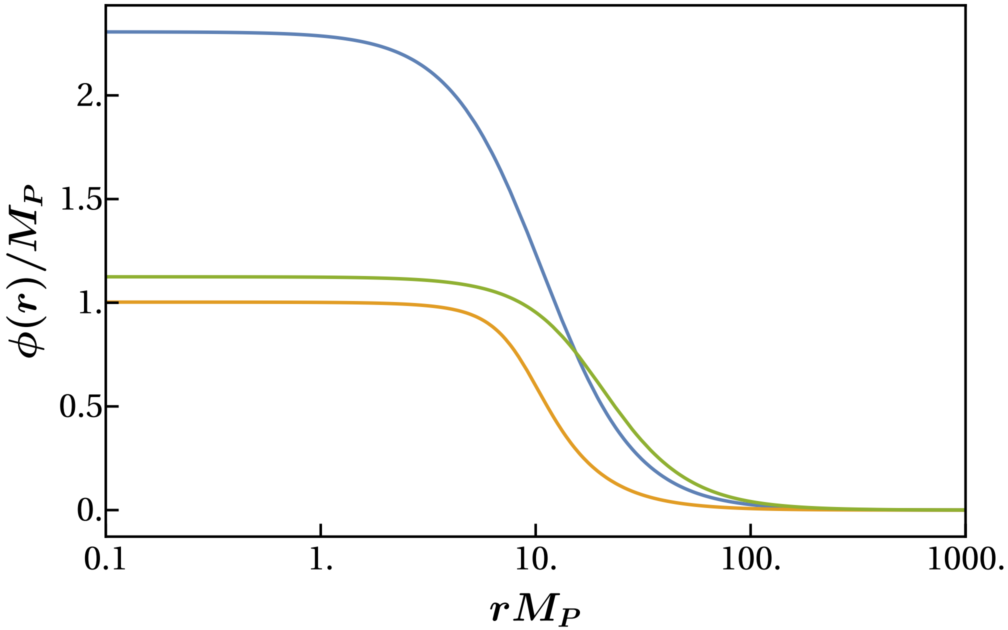
$$V(\phi) \;=\; \frac{\lambda(\phi)}{4}\phi^4 + \frac{\lambda_6}{6}\frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8}\frac{\phi^8}{M_P^4}$$



Yellow line: Potential with $\lambda_6 = -0.4$ and $\lambda_8 = 2$.

Blue line: SM alone.

Bounce profiles - Flat Spacetime



Blue curve: bounce obtained for the potential with $\lambda_6 = 0$ and $\lambda_8 = 0$ (**SM alone**).

Yellow curve: bounce for $\lambda_6 = -0.3$ and $\lambda_8 = 0.3$.

Green curve: bounce for $\lambda_6 = -0.01$ and $\lambda_8 = 0.01$.

Tunneling times for different values of λ_6 and λ_8

λ_6	λ_8	τ_{flat}/T_U
0	0	10^{639}
-0.05	0.1	10^{446}
-0.1	0.2	10^{317}
-0.3	0.3	10^{-52}
-0.45	0.5	10^{-93}
-0.7	0.6	10^{-162}
-1.2	1.0	10^{-195}
-2.0	2.1	10^{-206}

Remember : $\tau \sim e^{S[\phi_b]}$

New bounce $\phi_b^{(new)}(r)$, **New action** $S[\phi_b^{(new)}]$, **New** τ

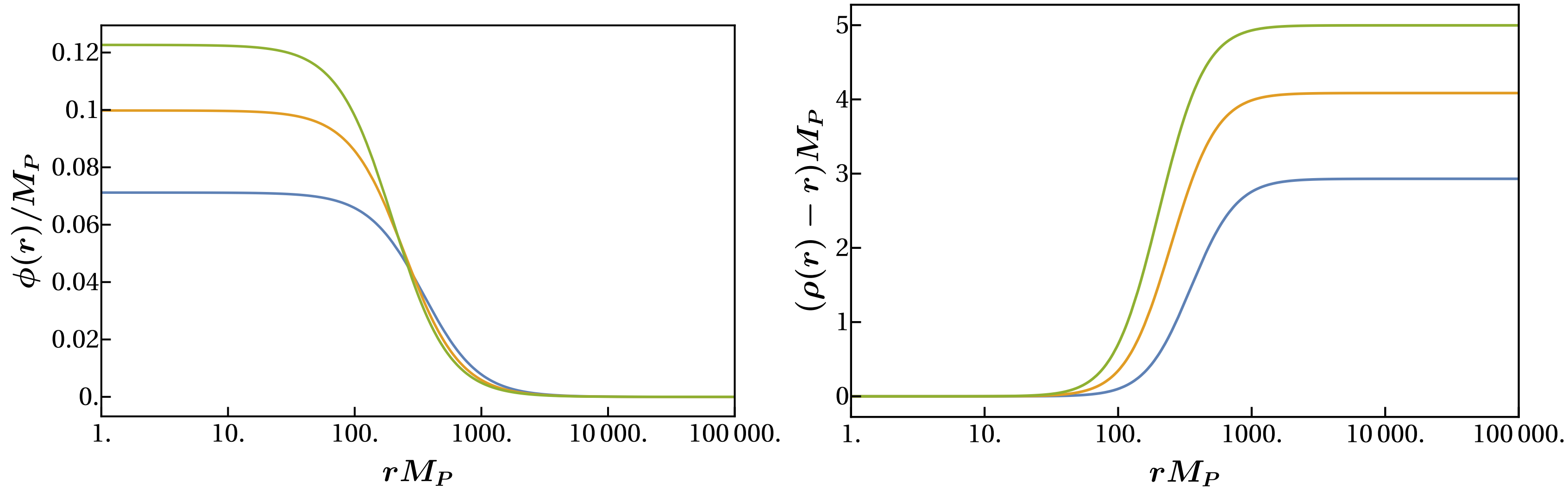
These results were however challenged

It was claimed that moving from **Flat Spacetime Background** $\rightarrow$ **Curved Spacetime Background**, i.e. taking into account the presence of gravity, the decay rate induced by the new bounce solutions presented above are suppressed ...

More precisely ... "the decay rate through the new instanton solution discussed in [..] is strongly suppressed when the all-important CDL gravitational effects are included ..."

J.R. Espinosa, J-F. Fortin, M. Trépanier, (arXiv:1508.05343) Phys.Rev.D 93, 124067 (2016).

Bounce profiles - Curved Spacetime



Left Panel - Blue curve: profile of the bounce solution with $\lambda_6 = 0$ and $\lambda_8 = 0$, i.e. in the absence of new physics. Yellow curve: profile of the bounce solution for $\lambda_6 = -0.03$ and $\lambda_8 = 0.03$. Green curve: profile of the bounce solution for $\lambda_6 = -0.04$ and $\lambda_8 = 0.04$.

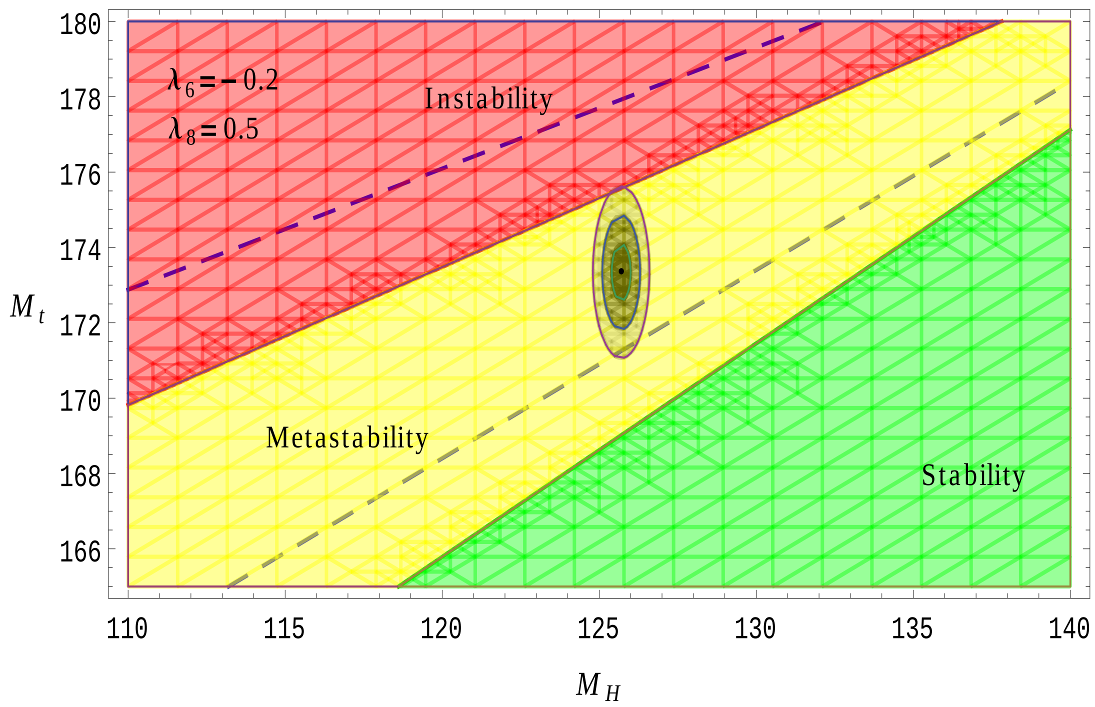
Right Panel - Profile of the difference between $\rho(r)$ and its asymptotic value: $\rho(r) - r$.

Tunneling times for different values of λ_6 and λ_8

λ_6	λ_8	$\tau_{\rm flat}/T_U$	$\tau_{\rm grav}/T_U$
0	0	10^{639}	10^{661}
-0.05	0.1	10^{446}	10^{653}
-0.1	0.2	10^{317}	10^{598}
-0.3	0.3	10^{-52}	10^{287}
-0.45	0.5	10^{-93}	10^{173}
-0.7	0.6	10^{-162}	10^{47}
-1.2	1.0	10^{-195}	10^{-58}
-2.0	2.1	10^{-206}	10^{-121}

Gravity tends to stabilize the EW vacuum ($\tau_{\rm grav}$ always higher than $\tau_{\rm flat}$). However, New Physics has always a strong (that can be even devastating) impact.

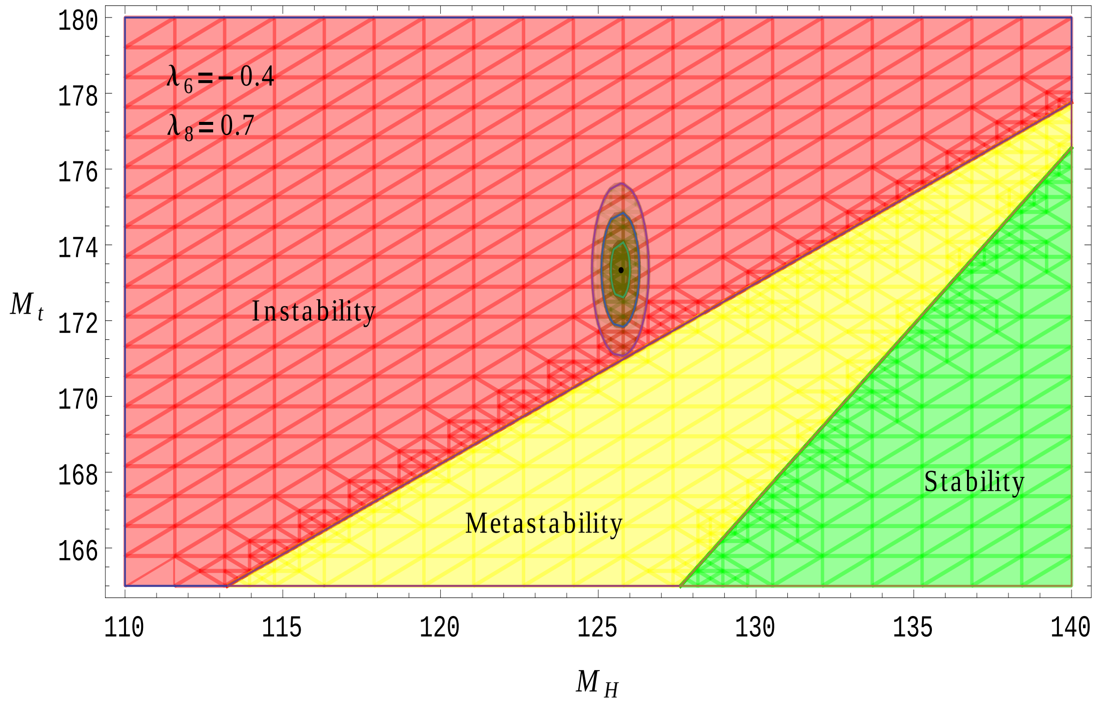
Stability Diagram in the (M_H, M_t) - plane for $\lambda_6 = -0.2$ and $\lambda_8 = 0.5$



The strips move downwards ... Central values no longer at $3\,\sigma$ from the stability line ...

... The Stability Diagram depends on new physics ...

Stability Diagram in the (M_H, M_t) - plane for $\lambda_6 = -0.4$ and $\lambda_8 = 0.7$



... The Stability Diagram depends on new physics ...

... These results came as a Surprise ...

It was thought that New Physics at the Planck scale should have no impact on the EW vacuum lifetime ... on the Stability Diagram

How comes that New Physics at M_P has such an impact on τ ?

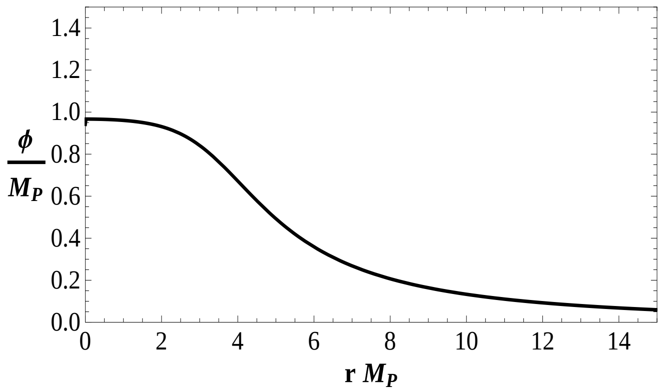
How comes that decoupling arguments do not apply ?

- As $\Lambda_{inst} \sim 10^{11}$ GeV, a decoupling effect was expected: the contribution of higher dimension operators $\frac{\phi^n}{M_P^n}$ was expected to be suppressed as $(\frac{\Lambda_{inst}}{M_P})^n$.
- However: Tunnelling is a non-perturbative phenomenon. We first select the saddle point, i.e. compute the bounce (tree level). Then, on the top of that, we compute the quantum fluctuations (loop corrections).
- Suppression in terms of inverse powers of M_P (power counting theorem) concerns the loop corrections, not the selection of the saddle point (tree level).

Once again : $$\tau \sim e^{S[\phi_b]} \quad \Rightarrow$$

$$\text{New bounce } \phi_b^{(new)}(r) \quad \Rightarrow \quad \text{New bounce action } S[\phi_b^{(new)}] \quad \Rightarrow$$

New τ



... It seems that the problem is there ...

Can we find

Physical Stabilization Mechanisms ?

... Let's see ...

... Following the same line of reasoning as before ...

- Consider a set of " ϕ^{2n}-models " that could describe unknown Planckian NP effects, by extending the SM effective potential as follows:

$$V_{2n}(\phi) \;=\; V_{\rm SM}(\phi) \,+\, V_{\rm NP}^{(2n)}(\phi)$$

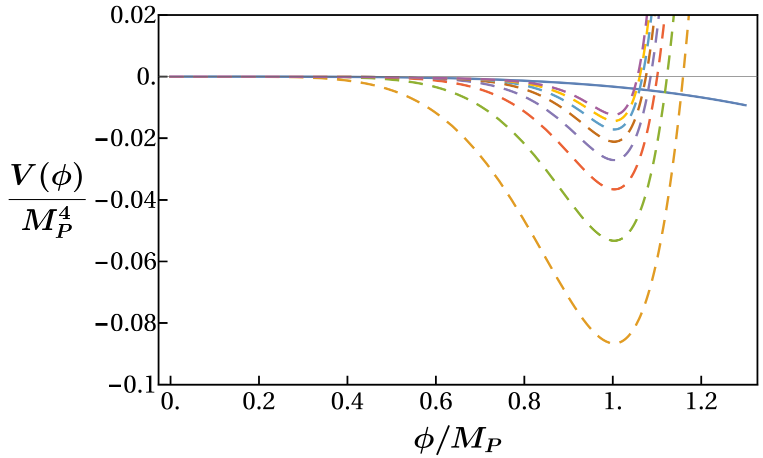
with $\;n \geq 3\;$ and

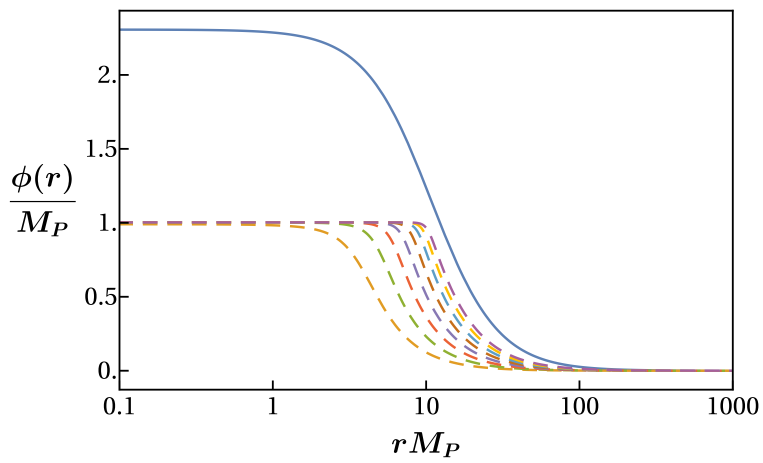
$$V_{\rm NP}^{(2n)}(\phi) \;=\; \frac{c_1}{2n}\,\frac{\phi^{2n}}{M^{2n-4}} + \frac{c_2}{2(n+1)}\,\frac{\phi^{2(n+1)}}{M^{2n-2}}$$

- All these potentials $V_{2n}(\phi)$ reduce to $V_{\rm SM}(\phi)$ for $\phi \ll M$

- Take c_1 negative and c_2 is positive (New Minimum around M and Potential Bounded Below)

- Repeating the same analysis as before for different values of n we get ...





$\frac{\phi^{2n}}{M^{2n-4}}$ n	τ/T_U (flat)	τ/T_U (curved)	$\frac{\phi^{2n}}{M^{2n-4}}$ n	τ/T_U (flat)	τ/T_U (curved)
3	10^{-208}	10^{-122}	7	10^{7}	8.8×10^{661}
4	10^{-166}	3.4×10^{661}	8	10^{71}	8.8×10^{661}
5	10^{-114}	8.8×10^{661}	9	10^{133}	8.8×10^{661}
6	10^{-55}	8.8×10^{661}	10	10^{193}	8.8×10^{661}

Can we construct "bona fide" Models where we can implement the suppression of lower ϕ^{2n} powers ... postpone the appearance of higher order operators ?

SUGRA Models

VB, F.Contino, A. Pilaftsis, Phys.Rev. D98 (2018) 075001

... A Minimal Supersymmetric extension of the SM ... $\widehat{\mathcal{W}}$ Effective Superpotential containing Planck-scale suppressed operators involving Higgs chiral superfields $\widehat{H}_{1,2}$

$$\widehat{\mathcal{W}} \;=\; \widehat{\mathcal{W}}_0 \;+\; \mu\, \widehat{H}_1 \widehat{H}_2 \;+\; \sum_{n=2}^{\infty} \frac{\rho_{2n}}{2n}\, \frac{(\widehat{H}_1 \widehat{H}_2)^n}{M_{\rm P}^{2n-3}}$$

$$\widehat{\mathcal{W}}_0 \;=\; h_l\, \widehat{H}_1 \widehat{L} \widehat{E} \;+\; h_d\, \widehat{H}_1 \widehat{Q} \widehat{D} \;+\; h_u\, \widehat{H}_2 \widehat{Q} \widehat{U}$$

SUGRA embedding based on a minimal Kaehler potential

$$\widehat{\mathcal{K}} \;\equiv\; \mathcal{K}(\widehat{\varphi}^*_i\,,\, \widehat{\varphi}_i) \;=\; \widehat{H}_1^\dagger \widehat{H}_1 \;+\; \widehat{H}_2^\dagger \widehat{H}_2 \;+\; \ldots$$

Scalar SUGRA potential : $V = V_F + V_D + V_{\rm br}$

F-terms + D-terms + SUSY-breaking terms ($V_{\rm br}$) induced by spontaneous breakdown of SUGRA, that may occur in the hidden sector of the theory (Nilles, 1984)

$$V_F = e^{\mathcal{K}/M_{\rm P}^2} \left[\left(\mathcal{W}_{,\,i} \, + \, \frac{\mathcal{K}_{,\,i}}{M_{\rm P}^2} \, \mathcal{W} \right) G^{-1,\,i\bar{j}} \left(\mathcal{W}_{,\,\bar{j}} \, + \, \frac{\mathcal{K}_{,\,\bar{j}}}{M_{\rm P}^2} \, \mathcal{W}^* \right) \, - \, 3 \, \frac{|\mathcal{W}|^2}{M_{\rm P}^2} \right]$$

$$V_D = \frac{g^2}{2} f_{ab}^{-1} D^a D^b$$

SUSY-breaking Higgs potential $V_{\rm br}^H$ generated from $\widehat{\mathcal{W}}$

$$V_{\rm br}^H = m_1^2 \, |H_1|^2 + m_2^2 \, |H_2|^2 + \left(B\mu \, H_1 H_2 + \sum_{n=2}^{\infty} \, A_{2n} \, \frac{(H_1 H_2)^n}{M_{\rm P}^{2n-3}} + {\rm H.c.} \right)$$

- We assume that:

1. μ-term, soft mass parameters $m_{1,2}^2$, $B\mu$ are $\sim M_{\mathcal{S}}$

2. The SUSY-breaking A_{2n}-terms could be as large as $M_{\rm P}$

(The mechanism that causes this large hierarchy depends on the details of the hidden sector, where SUSY is spontaneously broken).

$\mathcal{W} \equiv \mathcal{W}(\varphi_i)$, $\mathcal{K} \equiv \mathcal{K}(\varphi_i^*, \varphi_i)$, $\mathcal{W}_{,\,i} \equiv \partial \mathcal{W}/\partial \varphi_i$, $\mathcal{K}_{,\,i} \equiv \partial \mathcal{K}/\partial \varphi_i$, $\mathcal{K}_{,\,\bar{i}} \equiv \mathcal{K}^*_{,\,i}$ etc, for a generic scalar field φ_i, and $G^{-1,i\bar{j}}$ is the inverse of the Kaehler-manifold metric: $G_{i\bar{j}} = \mathcal{K}_{,\,i\bar{j}} = \partial^2 \mathcal{K}/(\partial \varphi_i \partial \varphi_j^*)$. In addition, g is a generic gauge coupling, e.g. of $SU(2)_L$, f_{ab} is the gauge kinetic function taken to be minimal, i.e. $f_{ab} = \delta_{ab}$, and $D^a = \mathcal{K}_{,\,\varphi} T^a \varphi$ are the so-called D-terms, where T^a are the generators of the gauge group

- If we now consider the SUSY limit of the MSSM (ignore SUSY-breaking terms $V_{\rm br}^{H}$)

& assume that μ-term $\sim M_{\mathcal{S}}$ (negligible w.r.t. to $M_{\rm P}$) $\Rightarrow$ The renormalizable part of the MSSM potential , V_0 , has an F- and D-flat direction associated with $\widehat{H}_1\widehat{H}_2$.

- In the absence of μ-term, the configuration $H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi \\ 0\end{pmatrix}$, $H_2 = \frac{e^{i\xi}}{\sqrt{2}}\begin{pmatrix}0 \\ \phi\end{pmatrix}$

with $\xi \in [0, 2\pi)$ (all other scalar fields taken at the origin) gives rise to an exact flat direction for V_0, i.e. $\partial V_0/\partial\phi = 0$ (ϕ is a *positive* scalar field background with canonical kinetic term that parameterizes the D-flat direction).

- The CP-odd angle ξ indicates that the flat directions for H_1 and H_2 may differ by an arbitrary relative phase ξ : (ϕ, ξ) describe fully the D-flat direction of interest.

- In the flat-space limit $M_{\rm P} \to \infty$, V_{0F} is positive, implying that $V_0 = V_{0F} + V_{0D} \geq 0$, where the equality sign holds along a flat direction, such as the ϕ-direction.

- The observable sector $(V_F + V_D)$ of the potential $V(\phi)$ generically remains positive upon the inclusion of gauge-invariant non- renormalizable operators ... but ...

This changes drastically when the SUSY-breaking A-terms (V_{br}) are added

- Consider for instance the minimally extended MSSM superpotential

$$\widehat{\mathcal{W}} \;=\; \widehat{\mathcal{W}}_0 \;+\; \mu\, \widehat{H}_1 \widehat{H}_2 \;+\; \frac{\rho_4}{4}\,\frac{(\widehat{H}_1 \widehat{H}_2)^2}{M}$$

which induces the SUSY-breaking potential (for the Higgs sector)

$$V^H_{4,\rm br} = m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 + \left(B\mu\, H_1 H_2 + A_4\,\frac{(H_1 H_2)^2}{M} \;+\; \text{H.c.}\right)$$

- Take $m^2_{1,2} \ll B\mu$. Moving along the D-flat direction, and upon ignoring radiative corrections for $\phi > M_{\mathcal{S}}$ $\Rightarrow$ the leading part of the potential takes on the form:

$$V_4(\phi) = e^{\phi^2/M^2_{\rm Pl}} \left[-\,\frac{m^2}{2}\,\phi^2 \;+\; \frac{\text{Re}(e^{2i\xi} A_4)}{2M}\,\phi^4 + \frac{|\rho_4|^2}{8}\,\frac{\phi^6}{M^2}\left(1 + \frac{5}{32}\,\frac{\phi^2}{M^2_{\rm Pl}} + \frac{1}{32}\,\frac{\phi^4}{M^4_{\rm Pl}}\right)\right]$$

where higher-order terms proportional to $|\mu|/M \lesssim M_{\mathcal{S}}/M \ll 1$ were neglected and $m^2 = |e^{i\xi}\, B\mu - |\mu|^2|$ is arranged to be of the required EW order.

Let's have a closer look at the Potential that we obtained

$$V_4(\phi) = e^{\phi^2/M_{\rm Pl}^2}\left[-\frac{m^2}{2}\phi^2 + \frac{{\rm Re}(e^{2i\xi}A_4)}{2M}\phi^4 + \frac{|\rho_4|^2}{8}\frac{\phi^6}{M^2}\left(1+\frac{5}{32}\frac{\phi^2}{M_{\rm Pl}^2}+\frac{1}{32}\frac{\phi^4}{M_{\rm Pl}^4}\right)\right]$$

- Even if $A_4 > 0$, the flat field direction with $\xi = \pi/2$ makes the coefficient ${\rm Re}(e^{2i\xi}A_4)$ entering the potential $V_4(\phi)$ negative.

- If A_4 is comparable to M, the quartic term ϕ^4 can become both sizeable and negative, giving rise to a potential $V_4(\phi)$ that develops a new minimum of order $M/|\rho_4|$, far away from its SM value.

- The higher powers ϕ^6, ϕ^8 and ϕ^{10} are all proportional to the positive coefficient $|\rho_4|^2$, thereby ensuring that $V_4(\phi)$ is bounded below.

- **Typically SUSY is effective in protecting the stability of the EW vacuum from *unknown* Planck-scale gravitational effects ...**
- **... unless the induced SUSY-breaking coupling A_4 happens to be $\sim M_{\rm Pl}$**

... So potentially in these Models we have the same problem discussed before ...

... Protection Mechanism ...

- Actually SUSY may still protect the stability of the EW vacuum, even for $A_{2n} \sim M_{\rm Pl}$ (along the lines of split-SUSY)

- Consider the **Discrete Symmetry** transformations on the chiral superfields:

$$\left(\widehat{H}_1\,,\,\widehat{H}_2\,,\,\widehat{Q}\,,\,\widehat{L}\right)\;\rightarrow\;\omega\left(\widehat{H}_1\,,\,\widehat{H}_2\,,\,\widehat{Q}\,,\,\widehat{L}\right)$$

(the remaining iso-singlet chiral superfields, $\widehat{U}$, $\widehat{D}$ and $\widehat{E}$ do not transform)

$$\widehat{\mathcal{W}}\;\rightarrow\;\omega^2\,\widehat{\mathcal{W}}$$

- If $\omega^2 = 1$, these discrete transformations give rise to a global $\mathbf{Z}_2$ symmetry, automatically satisfied by $\mathcal{W}$ and by the Kaehler potential $\mathcal{K}$.

- If $\omega^2 \neq 1$, they represent a **non-trivial Discrete R Symmetry**, maintained by a rotation of the Grassmann-valued coordinates of the SUSY space.

- **Idea : exploit this discrete R symmetry** to **suppress lower powers** of the non- renormalizable operators in $\widehat{\mathcal{W}}$, and then the corresponding A_{2n} terms in $V_{\rm br}$ ...

- **Hope : postponing the appearence** of higher order terms ... their **destabilizing impact** becomes **less severe** ... **washed out** ... **but we already know that ...**

Require that under the R-symmetry transformation $\widehat{H}_1\widehat{H}_2 \to \omega^2 \widehat{H}_1\widehat{H}_2$

$$\omega^{2n} = \omega^2$$

for $n > 2$ (for $n = 1, 2$ no non-trivial restrictions on the form of $\widehat{\mathcal{W}}$ arises)

Case $n = 3$: $\mathbf{Z}_4^R$ R symmetry , with $\omega^4 = 1$ and $\omega^2 = -1 \neq 1$

$$\widehat{\mathcal{W}} = \widehat{\mathcal{W}}_0 + \mu\, \widehat{H}_1\widehat{H}_2 + \frac{\rho_6}{6}\frac{(\widehat{H}_1\widehat{H}_2)^3}{M^3} + \frac{\rho_{10}}{10}\frac{(\widehat{H}_1\widehat{H}_2)^5}{M^7} + \ldots$$

The induced SUSY-breaking potential for the Higgs sector is

$$V^H_{6,\mathrm{br}} = \left(B\mu\, H_1H_2 + A_6\frac{(H_1H_2)^3}{M^3} + A_{10}\frac{(H_1H_2)^5}{M^7} + \ldots \right) + \text{H.c.}$$

Assume for simplicity that the soft SUSY-breaking mass parameters $m^2_{1,2}$ are small, $m^2_{1,2} \ll B\mu$ (ignore) and that only the ρ_6 and A_6 terms are sizeable.

... Along the D-flat direction the **Scalar Potential** for $\phi > M_{\mathcal{S}}$ takes the form

$$V_6(\phi) = e^{\phi^2/M^2_{\mathrm{Pl}}}\left[-\frac{m^2}{2}\phi^2 + \frac{\mathrm{Re}(e^{3i\xi}A_6)}{4M}\frac{\phi^6}{M^2} + \frac{|\rho_6|^2}{32}\frac{\phi^{10}}{M^6}\left(1 + \frac{9}{72}\frac{\phi^2}{M^2_{\mathrm{Pl}}} + \frac{1}{72}\frac{\phi^4}{M^4_{\mathrm{Pl}}}\right)\right]$$

This can be generalized:

Discrete $\mathbf{Z}^R_{2n-2}$ R symmetry, with $\omega^{2(n-1)} = 1$ and $n \geq 3$

The leading form of the scalar potential V_{2n} for $\phi > M_{\mathcal{S}}$ becomes

$$V_{2n}(\phi) = e^{\phi^2/M_{\rm Pl}^2}\Bigg[-\frac{m^2}{2}\,\phi^2 + \frac{{\rm Re}(e^{n\,i\xi}A_{2n})}{2^{n-1}\,M}\,\frac{\phi^{2n}}{M^{2(n-2)}} + \frac{|\rho_{2n}|^2}{2^{2n-1}}\,\frac{\phi^{2(2n-1)}}{M^{2(2n-3)}}\left(1+\frac{4n-3}{2(2n)^2}\,\frac{\phi^2}{M_{\rm Pl}^2}+\frac{1}{2(2n)^2}\,\frac{\phi^4}{M_{\rm Pl}^4}\right)\Bigg]$$

(all small terms proportional to $|\mu|/M$ neglected)

Note that if $A_{2n} > 0$, the harmful D-flat direction is obtained for $\xi = \pi/n$

... We got what we were looking for ...

... Postponing the appearance of higher order operators ...

n	A_{2n}	V_{min}	ϕ_{min}	ϕ_0^{flat}	ϕ_0^{curved}	τ^{flat}	τ^{curved}
2	1	-4.1791	1.4310	1.4281	1.4253	10^{-238}	10^{-238}
3	1	-5.1768	1.4308	1.4308	1.4308	10^{-238}	10^{-237}
4	1	-5.6986	1.4264	1.4264	1.4264	10^{-238}	10^{-236}
2	1/10	-0.0014	0.5123	1.49×10^{-7}	1.47×10^{-7}	10^{-154}	10^{-154}
3	1/10	-0.0057	0.8268	0.8262	0.8261	10^{76}	10^{100}
4	1/10	-0.0108	0.9809	0.9809	0.9809	10^{218}	10^{260}
2	1/50	-9.8×10^{-6}	0.2307	1.10×10^{-7}	1.10×10^{-7}	10^{76}	10^{76}
3	1/50	-0.00008	0.5554	0.5543	0.5543	10^{4196}	10^{4354}
4	1/50	-0.00018	0.7519	0.7519	0.7519	10^{8006}	10^{9056}

... An alternative Protection Mechanism ...

SM Potential ... Non-minimal coupling ...

VB, E. Bentivegna, F. Contino, D. Zappalà, Phys.Rev. D99 (2019)

$$S[\phi] = \int d^4x\sqrt{g}\left[-\frac{R}{2\kappa} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\ \partial_\nu\phi + V_{SM}(\phi) + \frac{1}{2}\xi\phi^2 R\right]$$

Again $O(4)$ symmetry:

$$\ddot{\phi} + 3\ \frac{\dot{\rho}}{\rho}\ \dot{\phi} = \frac{dV}{d\phi} + \xi\phi R \qquad \dot{\rho}^2 = 1 - \frac{\kappa}{3}\rho^2\ \frac{-\frac{1}{2}\dot{\phi}^2 + V(\phi) - 6\xi\frac{\dot{\rho}}{\rho}\phi\dot{\phi}}{1 - \kappa\xi\phi^2}\,,$$

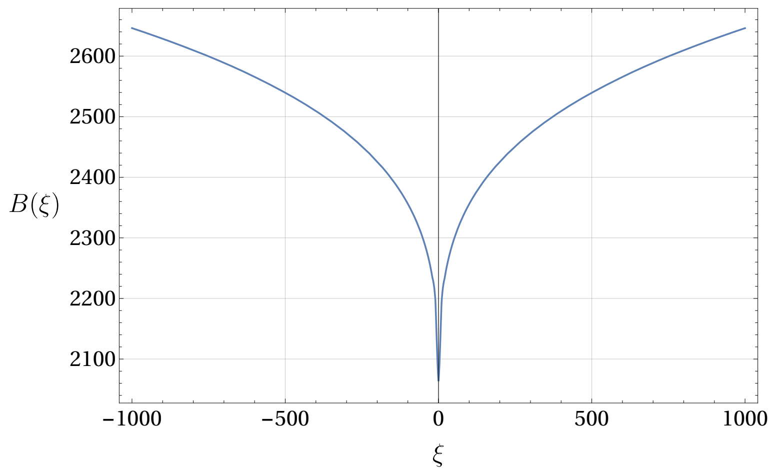
with R given by:

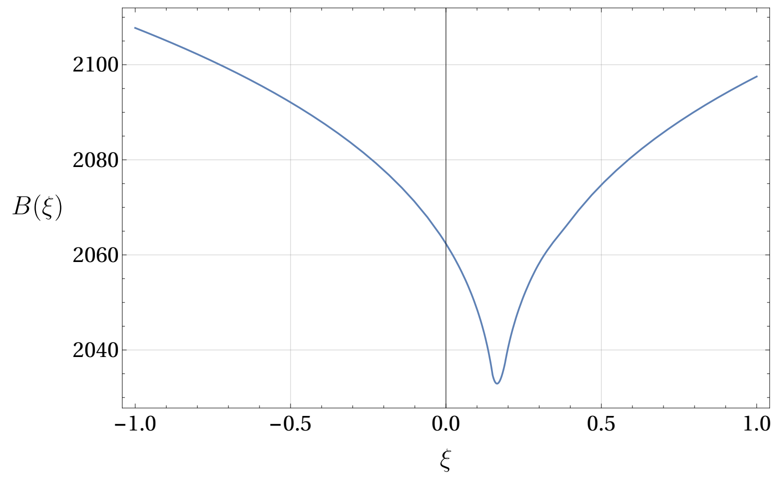
$$R = \kappa\ \frac{\dot{\phi}^2(1-6\xi) + 4V(\phi) - 6\xi\phi\, dV/d\phi}{1 - \kappa\xi(1-6\xi)\phi^2}\,.$$

For $\xi = 0$ these Equations become the minimal coupling ones.

Asymptotics: For $r \to \infty$, $\dot{\rho}_b^2 = 1$, so $\rho(r)$ approaches the flat spacetime metric. In the same limit, $R \to 0$.

SM Potential. Non-minimal coupling. $S_{bounce} \equiv B(\xi)$





B very sensitive to ξ. Outside the range $[\xi = 0, \xi = 1/3]$, $B(\xi)$ is greater than $B(\xi = 0)$, and non-minimal coupled gravity stabilizes the EW vacuum more than minimally coupled gravity.

Minimum at $\xi_{min} \simeq 0.17$, close to the conformal value $\xi = 1/6$. Actually for the scale invariant potential $V(\phi) = \frac{\lambda}{4}\phi^4$ (constant λ) the minimum is reached at $\xi = 1/6$.

What happens if we Add New Physics at M_P ?

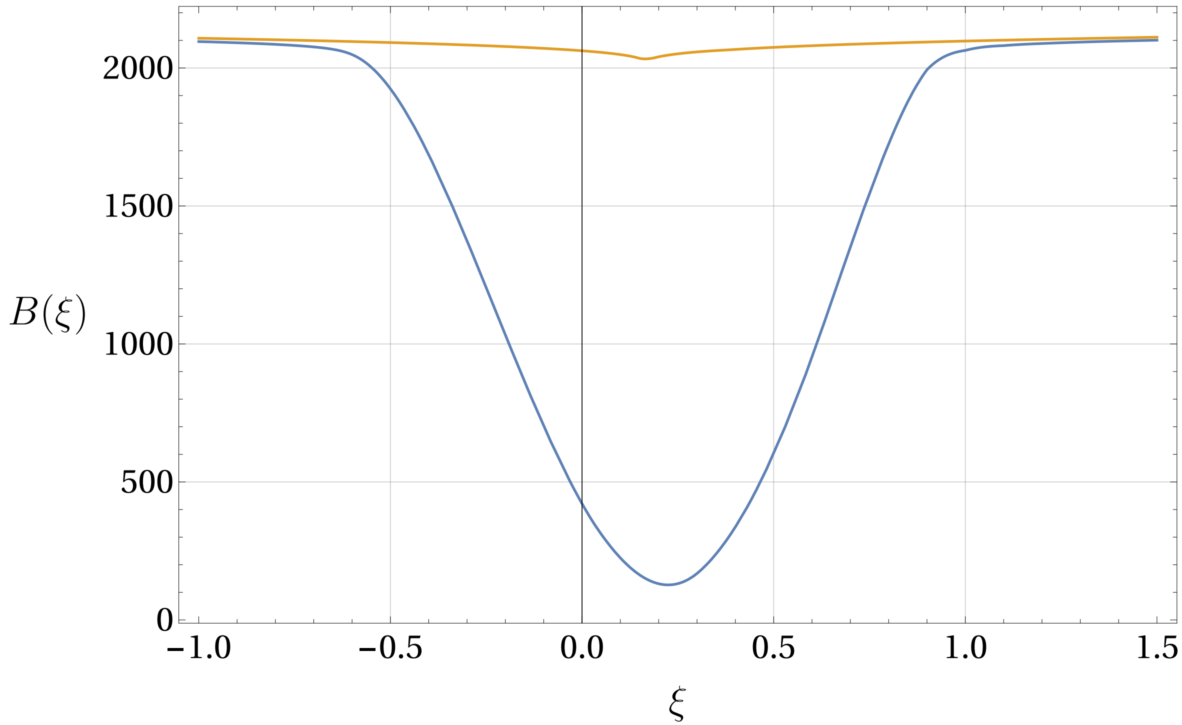
Add New Physics : $\lambda_6\,\phi^6$ and $\lambda_8\,\phi^8$

ξ	$(\tau/T_U)_{SM}$	$(\tau/T_U)_{NP}$
-15	10^{736}	10^{736}
-10	10^{726}	10^{726}
-5	10^{710}	10^{710}
-1	10^{684}	10^{680}
-0.5	10^{677}	10^{600}
-0.3	10^{672}	10^{358}
-0.1	10^{666}	10^{65}
0	10^{661}	10^{-58}

ξ	$(\tau/T_U)_{SM}$	$(\tau/T_U)_{NP}$
0.3	10^{660}	10^{-167}
0.5	10^{668}	10^{23}
0.7	10^{674}	10^{346}
0.8	10^{676}	10^{512}
1	10^{679}	10^{666}
5	10^{709}	10^{709}
10	10^{725}	10^{725}
15	10^{735}	10^{735}

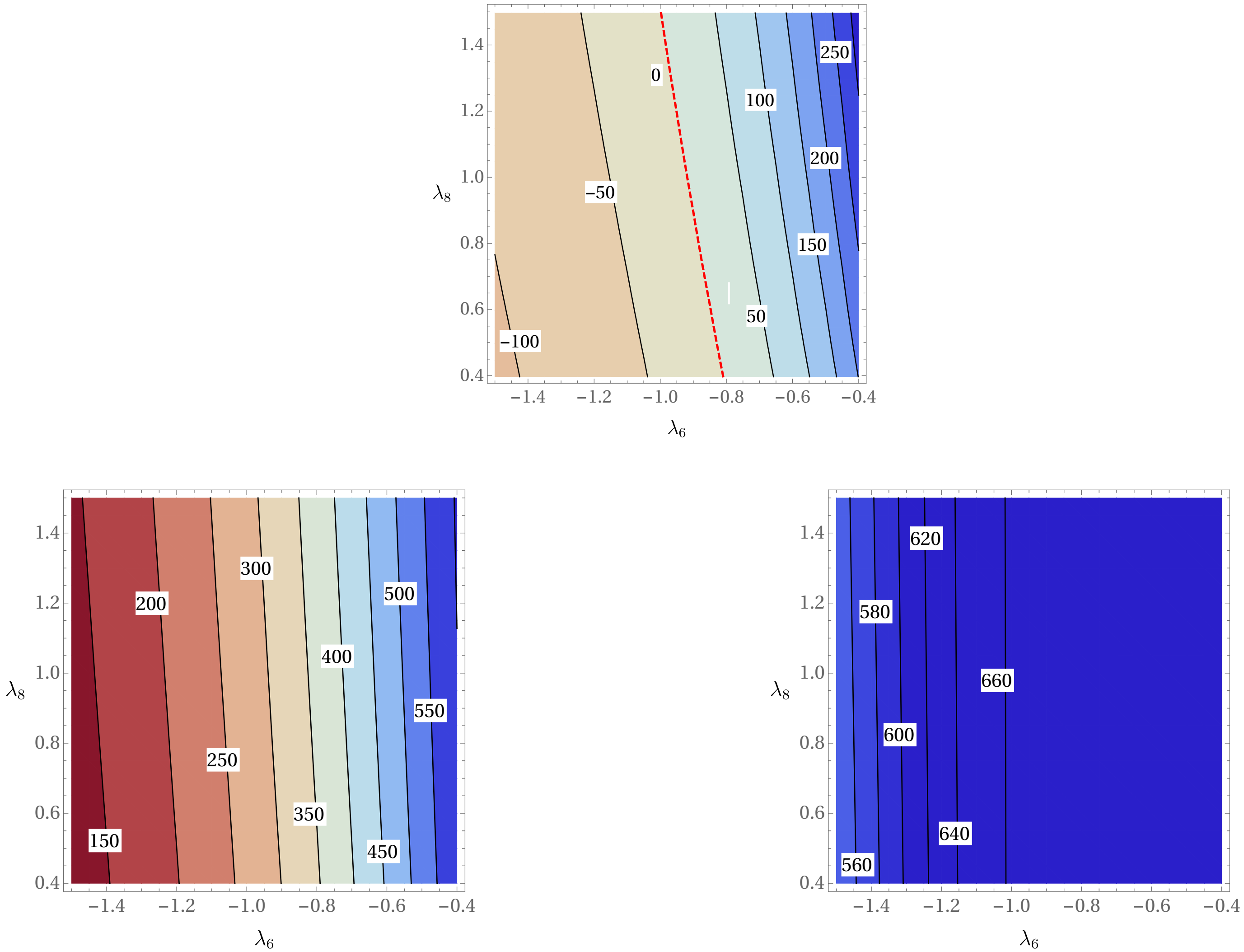
Values of τ with and without New Physics for different values of ξ, where $\lambda_6 = -1.2$ and $\lambda_8 = 1$.

Tunneling exponent $B(\xi)$ as a function of ξ



Yellow: $B(\xi)$ when the SM potential alone is considered. Blue: $B(\xi)$ when the New Physics potential with $\lambda_6 = -1.2$ and $\lambda_8 = 1$ is considered

Stability Diagrams for $\xi = 0\,,\,-0.2\,,\,0.9$



... An important Remark ...

The dimension four operator $\xi \phi^2 R$ naturally arises when quantization is carried out in a curved space-time background ... in the SM the term $\xi \, R \, H \, H^*$ is required in order to make the theory multiplicatively renormalizable in curved spacetime.

... Take home messages ...

- New Physics at Planckian scales, generically parametrized with the help of higher order operators in the Higgs potential (ϕ^6 and ϕ^8), can destabilize the EW vacuum.

- This result was first established in a flat spacetime background, and later confirmed by performing the analysis in a curved spacetime background (minimal coupling).

- Gravity shows a tendency toward stabilization, but still in a large portion of the parameter space destabilization wins against stabilization.

- Within the framework of a SUGRA embedding, and invoking a Discrete R symmetry, we can "postpone" the appearance of higher order terms, and this provides an effective protection mechanism for the stability of the EW vacuum.

- An alternative protection mechanism arises from the non-minimal coupling of the Higgs to gravity. Very minimalistic and efficient mechanism.

BACK UP SLIDES

Non-Renormalizable New Phyisics → Renormalizable New Physics

... It was also argued that the fact that New Physics was parametrized in terms of Non-Renormalizable operators actually could invalidate these results ...

New Physics around M_P in terms of renormalizable operators

Add to the SM potential a "New Boson S" and a "New Fermion ψ" :

$$\Delta V(\phi, S, \psi) = \frac{M_S^2}{2} S^2 + \frac{\lambda_S}{4} S^4 + \frac{g_S}{4} \phi^2 S^2 + M_f \bar{\psi}\psi + \frac{g_f}{\sqrt{2}} \phi \bar{\psi}\psi$$

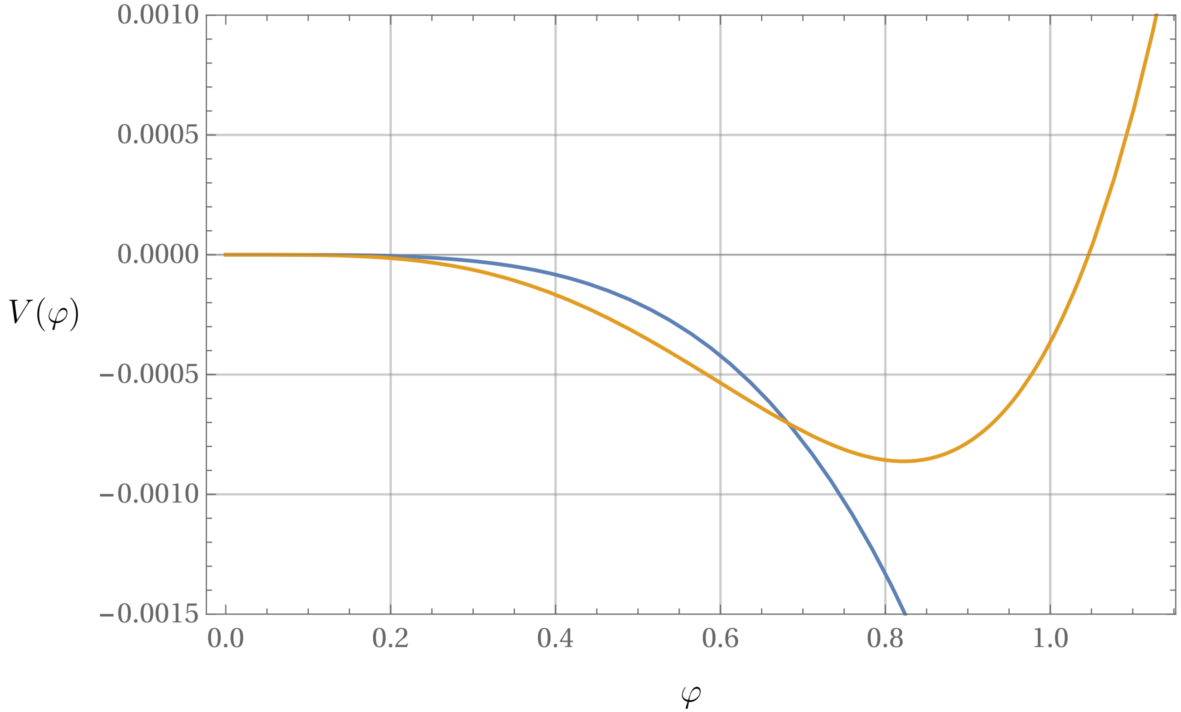
with $M_f \sim 10^{17}$ GeV and $M_S \sim 10^{18}$ GeV.

Integrating out this new scalar and fermion fields we get the

Modified Higgs Potential

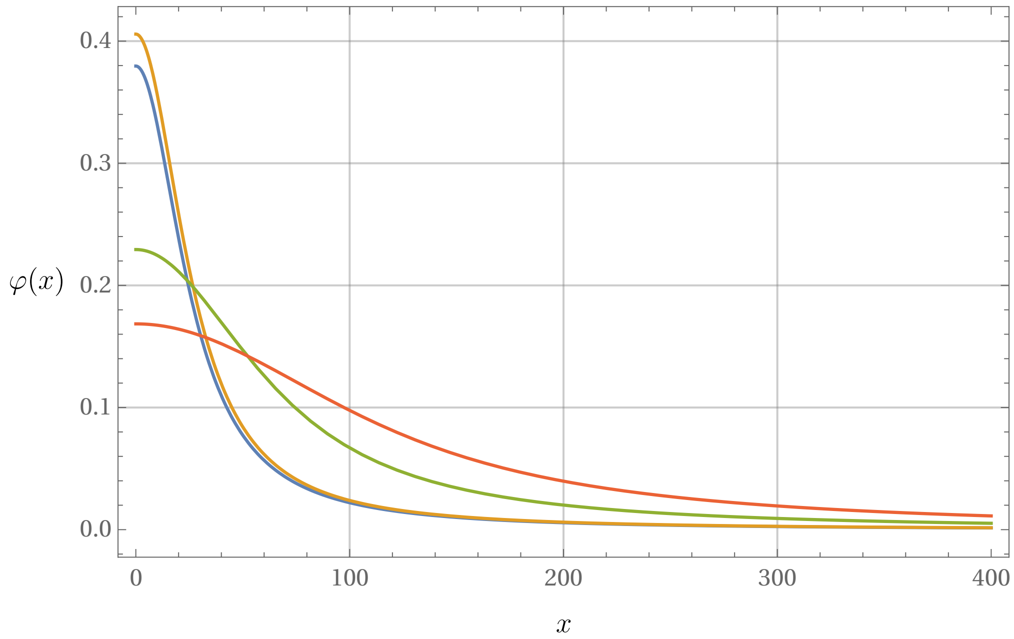
$$\begin{aligned} V(\phi) \;\; &= \;\; \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4} \phi^4 + \frac{1}{64\pi^2} \left(M_S^2 + \frac{g_S}{2} \phi^2 \right)^2 \left[\ln \left(\frac{M_S^2 + \frac{g_S}{2} \phi^2}{\mu^2} \right) - \frac{3}{2} \right] \\ &- \;\; \frac{1}{16\pi^2} \left(M_f^2 + \frac{g_f^2}{2} \phi^2 \right)^2 \left[\ln \left(\frac{M_f^2 + \frac{g_f^2}{2} \phi^2}{\mu^2} \right) - \frac{3}{2} \right] \end{aligned}$$

Modifided potential (yellow) against SM potential (blue)



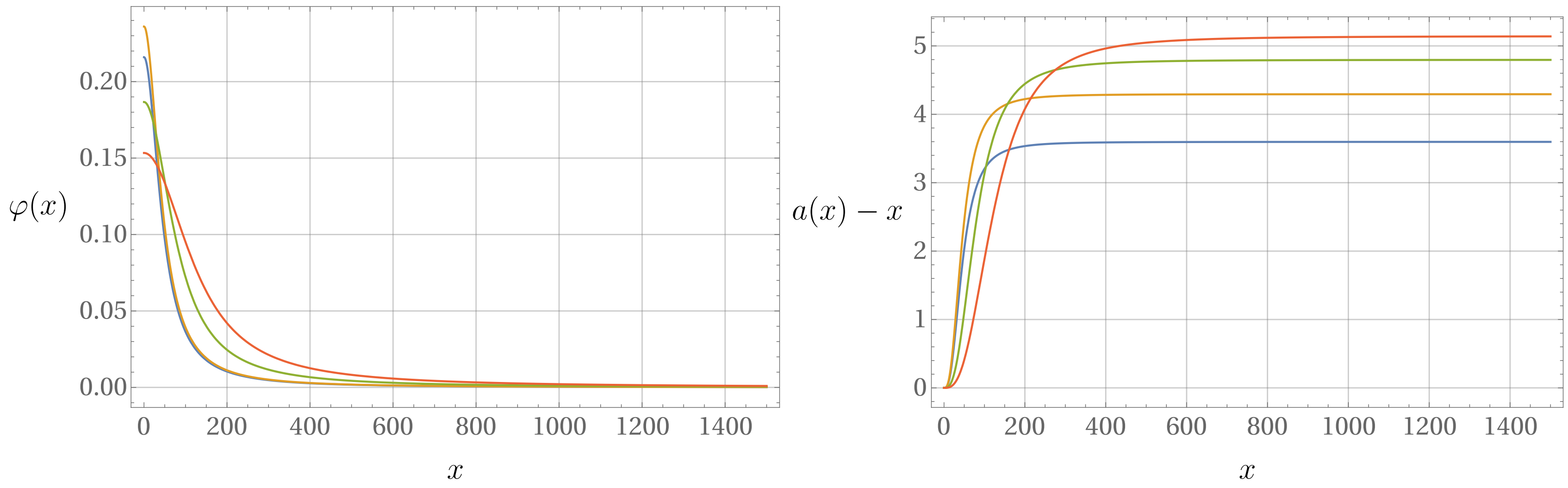
The values of the parameter are: $M_S = 2.0 \times 10^{-1} M_P$, $M_f = 10^{-3} M_P$, $g_S = 0.95$, $g_f^2 = 0.4$.

Bounce profiles for the Flat Spacetime Case



Profile of the bounce solutions $\varphi(x)$ relative to the four cases: $M_S = 2.5 \times 10^{-1}$, $M_f = 3 \times 10^{-4}$, $g_S = 0.96$, $g_f^2 = 0.5$ (yellow) ; $M_S = 2.0 \times 10^{-1}$, $M_f = 10^{-4}$, $g_S = 0.9$, $g_f^2 = 0.5$ (blue); $M_S = 2.0 \times 10^{-1}$, $M_f = 10^{-3}$, $g_S = 0.95$, $g_f^2 = 0.4$ (green); $M_S = 1.5 \times 10^{-1}$, $M_f = 5 \times 10^{-3}$, $g_S = 0.92$, $g_f^2 = 0.4$ (red).

Bounce profiles for the Curved Spacetime Case



Left panel: Profile of the bounce solutions $\varphi(x)$ relative to the four cases: $M_S = 2.5 \times 10^{-1}$, $M_f = 3 \times 10^{-4}$, $g_S = 0.96$, $g_f^2 = 0.5$ (yellow) ; $M_S = 2.0 \times 10^{-1}$, $M_f = 10^{-4}$, $g_S = 0.9$, $g_f^2 = 0.5$ (blue); $M_S = 2.0 \times 10^{-1}$, $M_f = 10^{-3}$, $g_S = 0.95$, $g_f^2 = 0.4$ (green); $M_S = 1.5 \times 10^{-1}$, $M_f = 5 \times 10^{-3}$, $g_S = 0.92$, $g_f^2 = 0.4$ (red).

Right panel: difference between the curvature radius and its asymptotic value, $a(x) - x$, for the same parameters as in the left panel.

Tunneling times for different values of the parameters

M_S	M_f	g_S	g_f^2	τ_{flat}/T_U	τ_{grav}/T_U
0	0	0	0	10^{639}	10^{661}
$1.5 \times 10^{-1} M_P$	$5 \times 10^{-3} M_P$	0.92	0.4	10^{293}	10^{307}
$2.0 \times 10^{-1} M_P$	$10^{-3} M_P$	0.95	0.4	10^{80}	10^{94}
$2.5 \times 10^{-1} M_P$	$3 \times 10^{-4} M_P$	0.96	0.5	10^{-80}	10^{-65}
$2.0 \times 10^{-1} M_P$	$10^{-4} M_P$	0.9	0.5	10^{-103}	10^{-93}

As for the case of the parametrization of New Phyiscs with

$$V_{NP}(\phi) = \frac{\lambda_6}{6} \frac{\phi^6}{M_P^2} + \frac{\lambda_8}{8} \frac{\phi^8}{M_P^4}$$

we again observe that Gravity tends to stabilize the EW vacuum (τ_{grav} always higher than τ_{flat}). However, New Physics has always a strong (that can be even devastating) impact.

… "Old Ideas" …

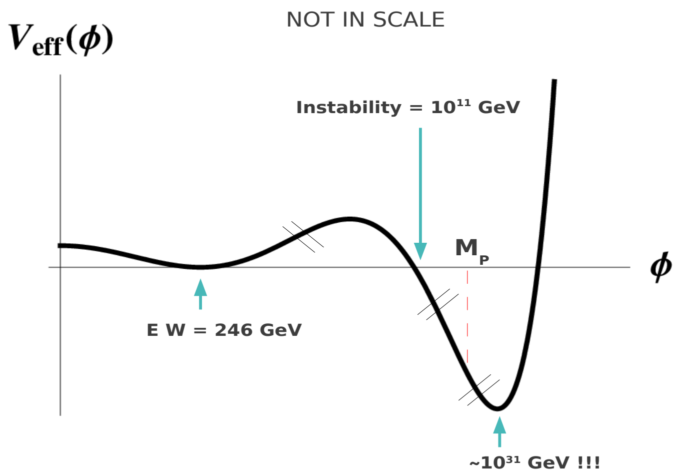
From: **J.R. Espinosa, G.F. Giudice, A. Riotto, JCAP 0805 (2008) 002**

"For most of the relevant values of the top and Higgs masses, the instability scale Λ_{inst} is sufficiently smaller than the Planck mass, **justifying the hypothesis of neglecting effects from unknown Planckian physics**."

From: **Isidori, Ridolfi, Strumia, Nucl.Phys. B609 (2001) 387**

"The SM potential is eventually stabilized by unknown new physics around M_P : because of this uncertainty, we cannot really predict what will happen after tunnelling has taken place. **Nevertheless, a computation of the tunnelling rate can still be performed, this result does not depend on the unknown new physics at the Planck scale.**"

Turning points...



This is QFT with "very many" dof, not 1 dof QM $\Rightarrow$ the potential is not $V(\phi)$ in figure with 1 dof, but...

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\vec{\nabla}\phi)^2 - V(\phi) = \frac{1}{2}\dot{\phi}(\vec{x},t)^2 - U(\phi(\vec{x},t))$$

where $U(\phi(\vec{x},t))$ is : $\quad U(\phi(\vec{x},t)) = V(\phi(\vec{x},t)) + \frac{1}{2}(\vec{\nabla}\phi(\vec{x},t))^2$

Very many dof, not 1 dof... The Potential is : $\sum_{\vec{x}} U(\phi(\vec{x},t))$

The bounce is not a constant configuration ... Gradients do matter a lot.