Perturbatively renormalizable quantum gravity

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TRM JHEP 1808 (2018) 024 [1802.04281], Int J Mod Phys D [1804.03834],
SciPost Phys. 5 (2018) no.4, 040 [1806.02206];
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<Y Igarashi, K Itoh & TRM, 1904.08231>

Quantum gravity does not have a perturbative continuum limit

$$S_{EH} = \int d^4x\, \mathcal{L}_{EH}\,, \qquad \mathcal{L}_{EH} = -2\sqrt{g}R/\kappa^2$$

$$\kappa = 2/M_{\text{Planck}}\,, \quad \kappa^2 = 32\pi G$$

$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa H_{\mu\nu}$$

$$\mathcal{L}_{EH} = \partial H \partial H + \sum_{n=1}^{\infty} \kappa^n H^n \partial H \partial H$$

$\mathcal{L}_{\text{free}}$

only continuum limit

irrelevant operators dim^n n+4

But it also has another problem ...

$$S_{EH} = \int d^4x \, \mathcal{L}_{EH} \,, \qquad \mathcal{L}_{EH} = -2\sqrt{g}R/\kappa^2$$

$$\mathcal{Z} = \int \mathcal{D}g_{\mu\nu} \; \mathrm{e}^{-S_{EH}}$$ does **not** converge

Gibbons, Hawking, Perry '78

Problem is in the conformal factor $g_{\mu\nu} = \varphi^2 \hat{g}_{\mu\nu}$

... key to solving the first problem

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$$g_{\mu\nu} = \delta_{\mu\nu} + \kappa H_{\mu\nu}$$

$$h_{\mu\nu} + \tfrac{1}{2}\varphi\, \delta_{\mu\nu}$$

$$\mathcal{L}_{\text{free}} = \tfrac{1}{2}\left(\partial_\lambda h_{\mu\nu}\right)^2 - \tfrac{1}{2}\left(\partial_\lambda \varphi\right)^2$$

(Feynman - De Donder)

traceless

... key to solving the first problem

Wilsonian RG with right sign kinetic term necessarily has polynomial interactions

$$\mathcal{L}_\Lambda = \tfrac{1}{2}(\partial_\mu \varphi)^2 + \epsilon\, V_\Lambda(\varphi) \qquad \Omega_\Lambda = \langle \varphi(x)\varphi(x)\rangle = \frac{\hbar \Lambda^2}{2a^2}$$

x

$$\Lambda \partial_\Lambda V_\Lambda(\varphi) = -\Omega_\Lambda \partial_\varphi^2 V_\Lambda(\varphi)$$

$$V_\Lambda(\varphi) = \Lambda^4 \tilde{V}_\Lambda(\tilde{\varphi} = \varphi/\Lambda)$$

$$\Lambda \partial_\Lambda \tilde{V}_\Lambda - \tilde{\varphi}\partial_{\tilde{\varphi}} \tilde{V}_\Lambda + 4\tilde{V}_\Lambda = -\tfrac{1}{2a^2}\partial_{\tilde{\varphi}}^2 \tilde{V}_\Lambda(\tilde{\varphi})$$

$$\tilde{V}_\Lambda(\tilde{\varphi}) = (\tfrac{\mu}{\Lambda})^\lambda \tilde{V}(\tilde{\varphi})$$

$$-\lambda \tilde{V}(\tilde{\varphi}) - \tilde{\varphi}\partial_{\tilde{\varphi}} \tilde{V} + 4\tilde{V} = -\tfrac{1}{2a^2}\partial_{\tilde{\varphi}}^2 \tilde{V}(\tilde{\varphi})$$

Sturm-Liouville $\tilde{V} = \tilde{\mathcal{O}}_n(\tilde{\varphi}) = \frac{H_n(a\tilde{\varphi})}{(2a)^n} = \tilde{\varphi}^n - \frac{n(n-1)}{4a^2}\tilde{\varphi}^{n-2} + \cdots$

$$[\tilde{\mathcal{O}}_n] = n = [\varphi^n]$$

Wilsonian RG with right sign kinetic term necessarily has polynomial interactions

$$\Omega_\Lambda = \langle \varphi(x)\varphi(x)\rangle = \frac{\hbar\Lambda^2}{2a^2}$$

$$\Lambda\partial_\Lambda V_\Lambda(\varphi) = -\Omega_\Lambda \partial^2_\varphi V_\Lambda(\varphi)$$

$$\int_{-\infty}^{\infty} d\tilde{\varphi}\ \mathrm{e}^{-a^2\tilde{\varphi}^2}\tilde{\mathcal{O}}_n(\tilde{\varphi})\tilde{\mathcal{O}}_m(\tilde{\varphi}) \propto \delta_{nm}$$

$$\int_{-\infty}^{\infty} d\tilde{\varphi}\ \mathrm{e}^{-a^2\tilde{\varphi}^2}\left(\tilde{V}(\tilde{\varphi}) - \sum_{n=0}^{N}\tilde{g}_n\tilde{\mathcal{O}}_n(\tilde{\varphi})\right)^2 \to 0 \quad \text{as} \quad N\to\infty\,.$$

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Wilsonian RG with wrong sign kinetic term

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$$-\lambda\tilde{V}(\tilde\varphi) - \tilde\varphi\partial_{\tilde\varphi}\tilde{V} + 4\tilde{V} = +\tfrac{1}{2a^2}\partial^2_{\tilde\varphi}\tilde{V}(\tilde\varphi)$$

Still Sturm-Liouville but $\displaystyle\int_{-\infty}^{\infty} d\tilde\varphi\ \mathrm{e}^{a^2\tilde\varphi^2}\,\delta_n(\tilde\varphi)\,\delta_m(\tilde\varphi) \propto \delta_{nm}$

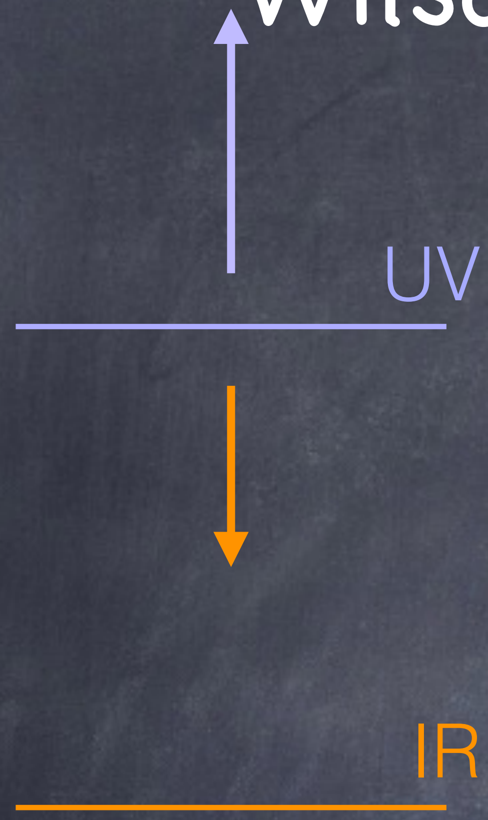
$$\int_{-\infty}^{\infty} d\tilde\varphi\ \mathrm{e}^{a^2\tilde\varphi^2}\left(\tilde{V}(\tilde\varphi) - \sum_{n=0}^{N}\tilde{g}_n\,\delta_n(\tilde\varphi)\right)^2 \to 0 \quad \text{as} \quad N\to\infty\,.$$

$$\delta^{(n)}_\Lambda(\varphi) = \frac{\partial^n}{\partial\varphi^n}\,\delta^{(0)}_\Lambda(\varphi)\,, \qquad \delta^{(0)}_\Lambda(\varphi) = \frac{1}{\sqrt{2\pi\Omega_\Lambda}}\,\exp\left(-\frac{\varphi^2}{2\Omega_\Lambda}\right)$$

$$[\delta^{(n)}_\Lambda(\varphi)] = -1-n$$

∞ tower super-relevant

Wilsonian RG with wrong sign kinetic term



$$\int_{-\infty}^{\infty} d\varphi \; \mathrm{e}^{\varphi^2/2\Omega_\Lambda} V_\Lambda^2(\varphi) < \infty \quad \forall \Lambda > \Lambda_0$$

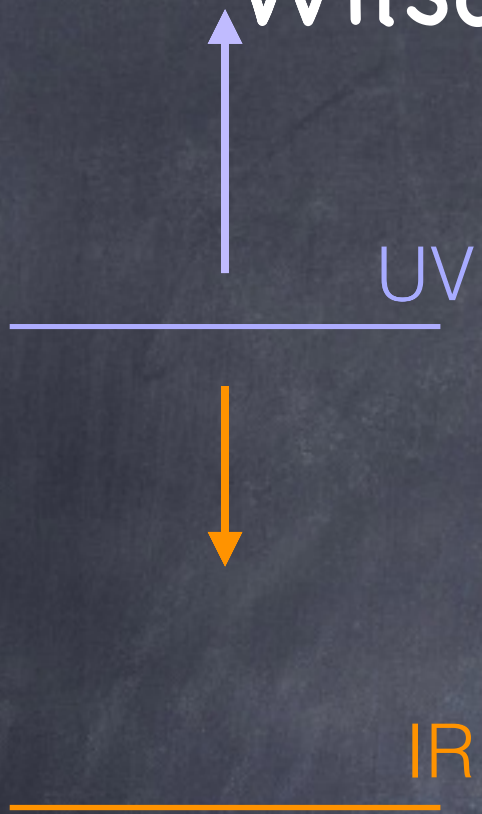
$$V_\Lambda(\varphi) = \sum_{n=0}^{\infty} g_n \, \delta_\Lambda^{(n)}(\varphi)$$

$$V(\varphi, \Lambda) = \int_{-\infty}^{\infty} \frac{d\pi}{2\pi} \mathcal{V}_{\mathrm{p}}(\pi) \, \mathrm{e}^{-\frac{\pi^2}{2}\Omega_\Lambda + i\pi\varphi}$$

$$\mathcal{V}_{\mathrm{p}}(\pi) = \sum_{n=0} g_n (i\pi)^n$$

is an entire function

Wilsonian RG with wrong sign kinetic term



$$\int_{-\infty}^{\infty} d\varphi \; \mathrm{e}^{\varphi^2/2\Omega_\Lambda} V_\Lambda^2(\varphi) < \infty \quad \forall \Lambda > \Lambda_0$$

amplitude suppression scale Λ_σ

$$V(\varphi, \Lambda) = \int_{-\infty}^{\infty} \frac{d\pi}{2\pi} \mathcal{V}_{\mathrm{p}}(\pi) \, \mathrm{e}^{-\frac{\pi^2}{2}\Omega_\Lambda + i\pi\varphi}$$

$$V_{\mathrm{p}}(\varphi) = \lim_{\Lambda \to 0} V_\Lambda(\varphi)$$

$$V_{\mathrm{p}}(\varphi) \sim \mathrm{e}^{-\varphi^2/\Lambda_\sigma^2}$$

Wilsonian RG of perturbative quantum gravity

Non-differentiated fields must be integrable under

$$\exp \frac{1}{2\Omega_\Lambda} \left(\varphi^2 - h^2_{\mu\nu} - 2\,\bar{c}_\mu c_\mu\right)$$

Interactions are $\delta^{(n)}_\Lambda(\varphi)$ polynomials

Operator: $f^\sigma_\Lambda(\varphi)\,\sigma(\partial_\alpha, \partial_\beta\varphi, h_{\gamma\delta}, \bar{c}_\varepsilon, c_\zeta, \Phi^*_A) + \cdots$

Renormalizability: $[\sigma] - 1 - n \leq 4$

tadpole corrections

Coefficient f^n: $f^\sigma_\Lambda(\varphi) = \sum_{n=n_\sigma}^{\infty} g^\sigma_n \delta^{(n)}_\Lambda(\varphi)$

What is the quantum version of diffeomorphism invariance?

Wilsonian RG & QME (Slavnov-Taylor identities)

$$\mathcal{A}[S] = 0 \qquad \mathcal{A}[S] = \tfrac{1}{2}(S,S) - \Delta S$$

$$(X,Y) = \frac{\partial_r X}{\partial \Phi^A} C^\Lambda \frac{\partial_l Y}{\partial \Phi^*_A} - \frac{\partial_r X}{\partial \Phi^*_A} C^\Lambda \frac{\partial_l Y}{\partial \Phi^A} \qquad \Delta X = (-)^A \frac{\partial_l}{\partial \Phi^A} C^\Lambda \frac{\partial_l}{\partial \Phi^*_A} X$$

$$S_0 = \tfrac{1}{2}\, \Phi^A (\triangle^\Lambda)^{-1}_{AB} \Phi^B - (Q_0 \Phi^A) \left(C^\Lambda\right)^{-1} \Phi^*_A \,.$$

$$\dot{\mathcal{A}} = \frac{\partial_r \mathcal{A}}{\partial \Phi^A} (\dot{\triangle}^\Lambda)^{AB} \frac{\partial_l (S - S_0)}{\partial \Phi^B} - \frac{1}{2} (\dot{\triangle}^\Lambda)^{AB} \frac{\partial_l}{\partial \Phi^B} \frac{\partial_l}{\partial \Phi^A} \mathcal{A}$$

What is the quantum version of diffeomorphism invariance?

$$\mathcal{A}[S] = \tfrac{1}{2}(S,S) - \Delta S = 0$$

$$S = S_0 + \kappa S_1 + \tfrac{1}{2}\kappa^2 S_2 + \cdots$$

$$s_0 S_1 = 0 \qquad \text{s.t.} \qquad S_1 \neq s_0 K$$

$$Q_0 + Q_0^- - \Delta^- - \Delta^=$$

$$Q_0 \Phi^A = (S_0, \Phi^A) \quad \Longrightarrow \quad Q_0 H_{\mu\nu} = \partial_\mu c_\nu + \partial_\nu c_\mu$$

$$Q_0^- \Phi^*_A = (S_0, \Phi^*_A) \quad \Longrightarrow \quad Q_0^- H^*_{\mu\nu} = -2G^{(1)}_{\mu\nu}\,, \quad Q_0^- c^*_\nu = -2\partial_\mu H^*_{\mu\nu}$$

Koszul-Tate

$$Q_0 f^\sigma_\Lambda(\varphi) = \partial\cdot c\, f^{\sigma\,\prime}_\Lambda(\varphi)$$

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Prove solution if & only if $f^\sigma_\Lambda(\varphi)$ independent of φ

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Koszul-Tate

$$Q_0 f^\sigma_\Lambda(\varphi) = \partial\cdot c\, f^{\sigma\,\prime}_\Lambda(\varphi)$$

But that can be done by sending $\Lambda_\sigma \to \infty$!

For $\sigma \sim H\partial H\partial H$ so $[\sigma] = 5$:

$$g^{\sigma}_{2m} = \frac{\sqrt{\pi}}{m!4^m}\,\kappa\,\Lambda_\sigma^{2m+1} \qquad (m = 0, 1, 2, \cdots)$$

$$f^{\sigma}_{\Lambda}(\varphi) = \frac{\kappa a \Lambda_\sigma}{\sqrt{\Lambda^2 + a^2\Lambda_\sigma^2}} \exp\left(-\frac{a^2\varphi^2}{\Lambda^2 + a^2\Lambda_\sigma^2}\right)$$

$$f^{\sigma}(\varphi) = \lim_{\Lambda\to 0} f^{\sigma}_{\Lambda}(\varphi) = \kappa\, \mathrm{e}^{-\varphi^2/\Lambda_\sigma^2}$$

$$f^{\sigma}_{\Lambda}(\varphi) \to \kappa \quad \text{as} \quad \Lambda_\sigma \to \infty$$

N.B. Newton's constant is a `collective' effect

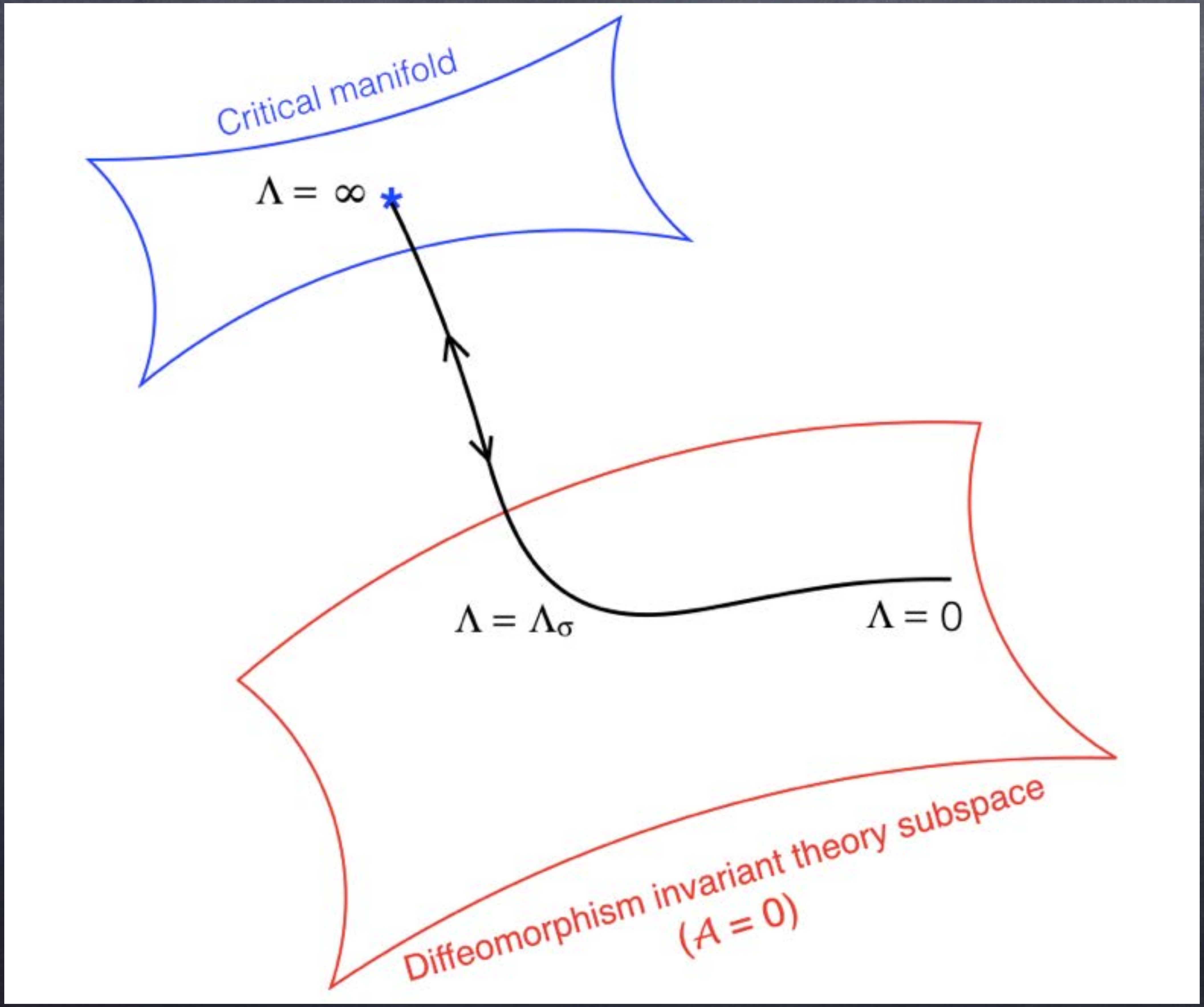
Construction establishes quantum gravity as a genuine continuum quantum field theory, at $O(\kappa)$, with all the correct properties.

Works at higher order in κ, with only one more free parameter: the cosmological constant...

(work in progress)

Inevitable logical consequence of insisting on Wilsonian RG applied to (unmodified) Einstein-Hilbert action

Construction crucially different from other QFTs, & other conceptions for QG.



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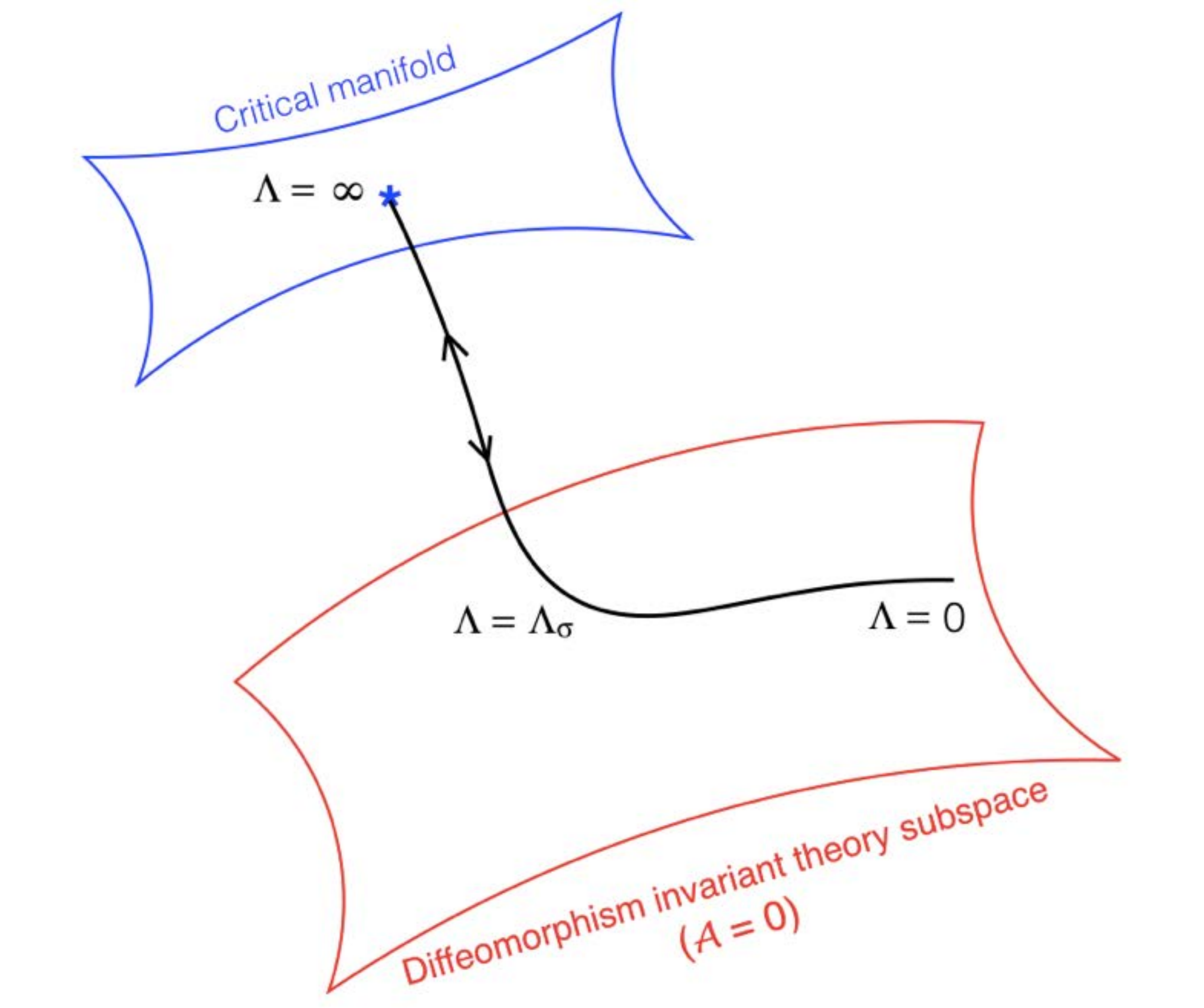
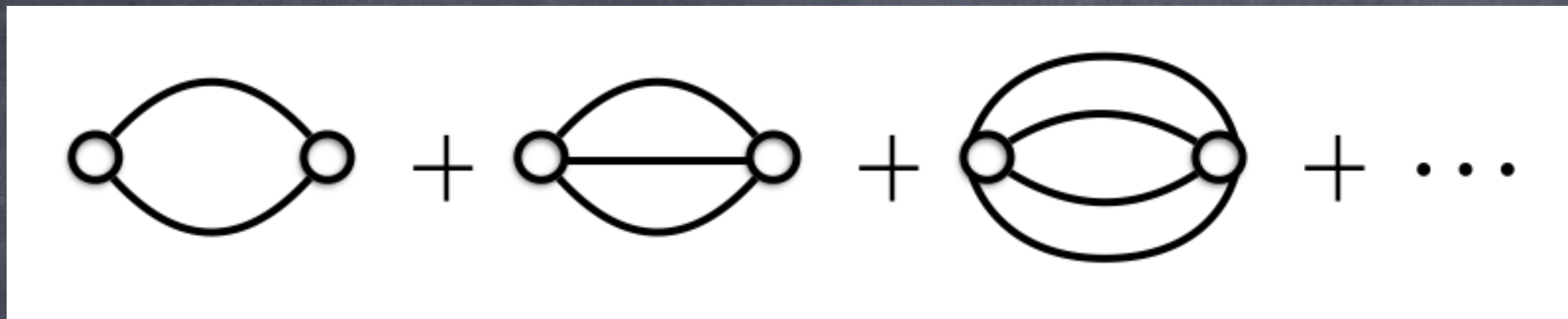


Second order in κ



$$\dot{\Gamma}_2 = \tfrac{1}{2}\,\mathrm{Str}\,\dot{\triangle}_\Lambda \Gamma_2^{(2)} - \tfrac{1}{2}\,\mathrm{Str}\,\dot{\triangle}_\Lambda \Gamma_1^{(2)} \triangle_\Lambda \Gamma_1^{(2)}$$

$$f^2_\Lambda(\varphi) = \sum_{n=0}^{\infty} g^2_n(\Lambda)\,\delta^{(n)}_\Lambda(\varphi) \qquad f^1_\Lambda(\varphi) = \sum_{n=n_\sigma}^{\infty} g^1_n \delta^{(n)}_\Lambda(\varphi)$$

$$0 = s_0\,\Gamma_2 + \tfrac{1}{2}\,(\Gamma_1, \Gamma_1) + \mathrm{Tr}\,C^\Lambda\,\Gamma^{(2)}_{1*} \triangle_\Lambda \Gamma_1^{(2)}$$

$$\delta^{(m)}_\Lambda(\varphi)\,\delta^{(n)}_\Lambda(\varphi) = \Lambda^{-1-m-n} \sum_{j=0}^{\infty} \Lambda^j\, \mathring{c}^{\,j}_{mn}\, \delta^{(j)}_\Lambda(\varphi)$$

$$\dot{g}^2_j(\Lambda) \sim \sum_{mn}^{\infty} \mathring{c}^{\,j}_{mn}\, g^1_m g^1_n \Lambda^{j-m-n-1}$$