Introduction to Supersymmetry

Lecture III : The Minimal Supersymmetric Standard Model (MSSM)

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Outline

Bibliography

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A Supersymmetry primer, [Ch. 5,6,8],
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I use $g^{\mu\nu} = (1, -1, -1, -1)$, but otherwise Martin's notation.

SUSY standard model building

Gauge vector supermultiplets in the MSSM

Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	λ	A_μ
$V_3^{(\alpha)}$	$(\mathbf{8}, \mathbf{1}, 0)$	$\tilde{g}$,	G_μ
$V_2^{(i)}$	$(\mathbf{1}, \mathbf{3}, 0)$	$\tilde{W}$,	W_μ
V_1	$(\mathbf{1}, \mathbf{1}, 0)$	$\tilde{B}^0$,	B_μ^0

SUSY standard model building

Chiral supermultiplets in the MSSM

Fields	$SU(3)_C \times SU(2)_L \times U(1)_Y$	$\phi \quad \psi$
Q	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$\begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L , \begin{pmatrix} u \\ d \end{pmatrix}_L$
$\bar{d}$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$	$\tilde{d}_R^* \ , \ d_R^\dagger$
$\bar{u}$	$(\bar{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$	$\tilde{u}_R^* \ , \ u_R^\dagger$
L	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$\begin{pmatrix} \tilde{\nu}_e \\ \tilde{e} \end{pmatrix}_L , \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$
$\bar{e}$	$(\mathbf{1}, \mathbf{1}, 1)$	$\tilde{e}_R^* \ , \ e_R^\dagger$
H_u	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$	$\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} , \begin{pmatrix} \tilde{H}_u^+ \\ \tilde{H}_u^0 \end{pmatrix}$
H_d	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} , \begin{pmatrix} \tilde{H}_d^0 \\ \tilde{H}_d^- \end{pmatrix}$

Remarks

1. The vector superfields interact with the quark and lepton superfields
2. There are two complex Higgs doublets superfields to cancel gauge anomalies
3. 2 Higgs doublets $=$ 8 d.o.f $=$ 3 are would-be Goldstone bosons, 2 CP-even neutral Higgs bosons h and H, 1 CP-Odd neutral Higgs boson A and 1 charged Higgs boson $H^{\pm}$.
4. The spin-$\frac{1}{2}$ higgsinos mix with winos and binos $=$ 4 neutral (Majorana) fermions, the neutralinos and 2 charged (Dirac) fermions, the charginos
5. ... plus squarks and sleptons

Outline

(Superfield) Kinetic terms

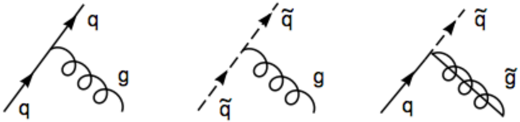
$$\mathcal{L}_{Kin} = \left[\Phi^* \exp\left(2g_1 V\, Y + 2g_2 V^{(i)} \frac{\tau^{(i)}}{2} + 2g_3 V^{(\alpha)} \frac{\lambda^{(\alpha)}}{2}\right) \Phi\right]_D$$

$$+ \left\{ \frac{1}{4}\left[\mathcal{W}\mathcal{W} + \mathcal{W}^{(i)}\mathcal{W}^{(i)} + \mathcal{W}^{(\alpha)}\mathcal{W}^{(\alpha)}\right]_F + c.c \right\}$$

$\Phi = (Q, \bar{u}, \bar{d}, L, \bar{e}, H_u, H_d)^T$

$(Y, \tau^{(i)}, \lambda^{(\alpha)})$ the generators of $U(1)_Y \times SU(2)_L \times SU(3)_C$

(Superfield) Kinetic terms



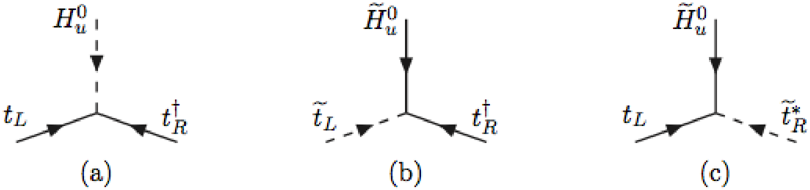
Questions:

From what term(s) in $\mathcal{L}_{Kin}$ these diagrams arise?
Are their couplings fixed?
What is the number of SUSY particles involved?
Draw the corresponding diagrams with Wino and Bino external fields. Which particles are involved?

Outline

The superpotential

$$W_{\rm MSSM} \ = \ \bar{u}\,\mathbf{y_u}\,Q\,H_u - \bar{d}\,\mathbf{y_d}\,Q\,H_d - \bar{e}\,\mathbf{y_e}\,L\,H_d + \mu H_u\,H_d$$

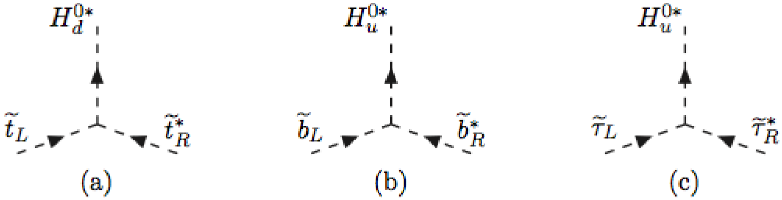


$$-\mathcal{L}_{SUSY\ \ Higgs\ \ mass} = |\mu|^2\,(|H_u^0|^2 + |H_d^0|^2 + |H_u^+|^2 + |H_d^-|^2) \qquad (1)$$

positive-definite term

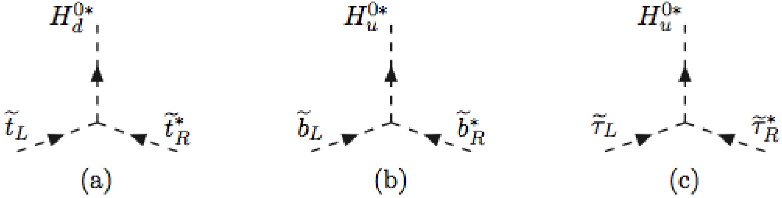
the μ-problem

The superpotential



(scalar)^3 : Diagrams mix left and right squarks

The superpotential



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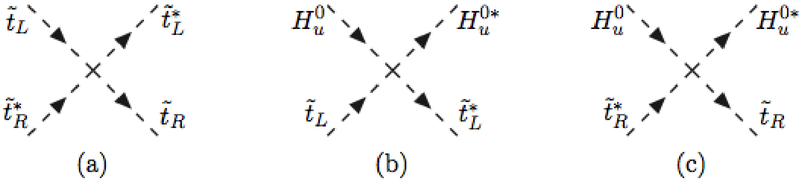

(scalar)^4 : Diagrams (b,c) contribute to the Higgs boson mass

R-parity (matter parity) violating superpotential

There are gauge invariant terms that violate either total L and B

$$W_{\Delta L=1} = \frac{1}{2}\lambda^{ijk}\, L_i\, L_j\, \bar{e}_k + \lambda'^{\,ijk}\, L_i\, Q_j\, \bar{d}_k + \mu'^{\,i}\, L_i\, H_u$$

$$W_{\Delta B=1} = \frac{1}{2}\lambda''^{\,ijk}\bar{u}_i\bar{d}_j\bar{d}_k$$

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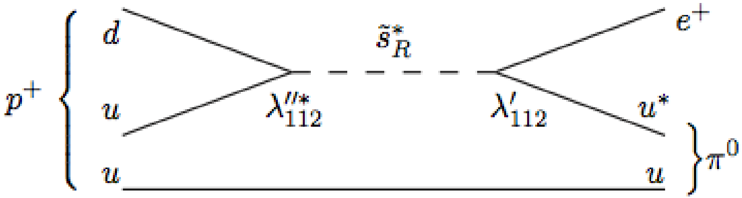
$$W_{\Delta B=1} = \frac{1}{2}\lambda''^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$$

Proton decays in a fraction of a sec if λ', λ'' are of order one



R-parity violating superpotential

R-parity is a discrete Z_2 symmetry

$$
\begin{aligned}
V &\rightarrow V \\
\theta &\rightarrow -\theta \\
\left\{\begin{array}{c} Q \\ \bar{u} \\ \bar{d} \\ L \\ \bar{e} \end{array}\right\} &\rightarrow -\left\{\begin{array}{c} Q \\ \bar{u} \\ \bar{d} \\ L \\ \bar{e} \end{array}\right\} \\
\left\{\begin{array}{c} H_u \\ H_d \end{array}\right\} &\rightarrow \left\{\begin{array}{c} H_u \\ H_d \end{array}\right\}
\end{aligned}
$$

$W_{\Delta L=1}$ and $W_{\Delta B=1}$ are both forbidden while $W_{\rm MSSM}$ is allowed.

Matter fermions (quarks and leptons) transform trivially but their scalar partners flip the sign $\Rightarrow$ **SUSY particles are pair-produced** $\Rightarrow$ **LSP is stable!**

The SUSY potential and EW breaking

$$\mathcal{V}(\phi^*, \phi) = F^{*i} F_i + \frac{1}{2} \sum_a D^{(a)} D^{(a)}$$

$$= \left| \frac{\partial W(\Phi)}{\partial \Phi^i} \right|^2 + \frac{1}{2} \sum_a g_a^2 \, (\phi^* T^{(a)} \phi)^2$$

By minimising the MSSM potential we find:

$$\langle Q \rangle = \langle \bar{u} \rangle = \langle \bar{d} \rangle = \langle L \rangle = \langle \bar{e} \rangle = 0$$

which leaves the vacuum invariant under $SU(3)_C \times U(1)_{EM}$.

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But also $\langle H_u \rangle = \langle H_d \rangle = 0$: Electroweak symmetry is not broken!

All masses so far are zero apart from Higgs multiplets that have a common mass μ. **It is time to break SUSY and see what happens!**

Soft SUSY breaking in MSSM

$$
\begin{aligned}
\mathcal{L}_{\text{soft}}^{\text{MSSM}} &= -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B} + \text{c.c}\right) \\
&- \left(\tilde{\bar{u}}\,\mathbf{a_u}\,\tilde{Q}H_u + \tilde{\bar{d}}\,\mathbf{a_d}\,\tilde{Q}H_d + \tilde{\bar{e}}\,\mathbf{a_e}\,\tilde{L}H_d + \text{c.c}\right) \\
&- \tilde{Q}^\dagger\,\mathbf{m_Q^2}\,\tilde{Q} - \tilde{\bar{u}}\,\mathbf{m_{\bar{u}}^2}\,\tilde{\bar{u}}^\dagger - \tilde{\bar{d}}\,\mathbf{m_{\bar{d}}^2}\,\tilde{\bar{d}}^\dagger - \tilde{L}^\dagger\,\mathbf{m_L^2}\,\tilde{L} - \tilde{\bar{e}}\,\mathbf{m_{\bar{e}}^2}\,\tilde{\bar{e}}^\dagger \\
&- m_{H_u}^2\,H_u^*H_u - m_{H_d}^2\,H_d^*H_d - \left(b\,H_uH_d + \text{c.c}\right) \qquad (2)
\end{aligned}
$$

$M_{1,2,3}$: soft SUSY breaking gaugino masses

$\mathbf{a_u}, \mathbf{a_d}, \mathbf{a_e}$: soft SUSY breaking trilinear couplings

$\mathbf{m^2_{Q,\bar{u},\bar{d},L,\bar{e}}}, m^2_{H_u,H_d}$: soft SUSY breaking masses

b : soft breaking bilinear mass

Outline

REWSB

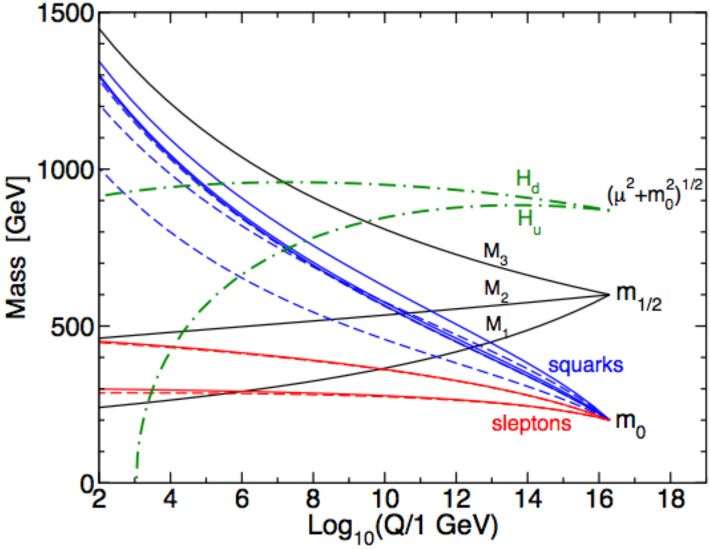
Motivated by the Unification of gauge couplings at M_{GUT}, we may assume universality of soft breaking terms at M_{GUT}

1. One common scalar mass, M_0
2. One common gauging mass, $M_{1/2}$
3. One (holomorphic) Higgs mass, $b = \mu B$
4. One trilinear coupling, $\mathbf{a}_F = A_0\,\mathbf{y_F}$

We then run the parameters down using RGEs

$$Q\frac{d}{dt}\begin{pmatrix} m^2_{H_u} \\ m^2_{\bar{u}_3} \\ m^2_{Q_3} \end{pmatrix} \approx \frac{|y_t|^2}{8\pi^2}\begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} m^2_{H_u} \\ m^2_{\bar{u}_3} \\ m^2_{Q_3} \end{pmatrix} + \ \ldots$$

REWSB



1500
1000
500
0
2
4
6
8
10
12
14
16
18
Mass [GeV]
Log$_{10}$(Q/1 GeV)
H_d
H_u
$(\mu^2+m_0^2)^{1/2}$
M_3
M_2
M_1
$m_{1/2}$
squarks
sleptons
m_0

Outline

FCNCs

We have 105 parameters and not so many nice SUSY breaking scenarios! The experiment however makes suggestions.

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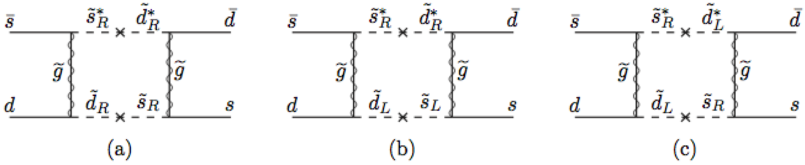

$$\frac{|\mathrm{Re}[m^2_{\tilde{s}^*_R \tilde{d}_R} m^2_{\tilde{s}^*_L \tilde{d}_L}]|^{1/2}}{m^2_{\tilde{q}}} < \left(\frac{m_{\tilde{q}}}{1000\ \mathrm{GeV}}\right) \times \begin{cases} 0.0016 & \text{for } m_{\tilde{g}} = 0.5 m_{\tilde{q}}, \\ 0.0020 & \text{for } m_{\tilde{g}} = m_{\tilde{q}}, \\ 0.0026 & \text{for } m_{\tilde{g}} = 2 m_{\tilde{q}}. \end{cases}$$

SUSY Flavor problem

Avoiding the SUSY flavour problem...

- Universality : squark and slepton masses are proportional to the identity matrix and the trilinear couplings proportional to the Yukawa ones at some high scale. Need RGEs to run down
- Hierarchy : first and second generation masses much heavier than the third
- Alignment : Flavour Symmetries to avoid large FCNCs

MSSM spectrum

Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates
Higgs bosons	0	+1	$H_u^0\ H_d^0\ H_u^+\ H_d^-$	$h^0\ H^0\ A^0\ H^\pm$
squarks	0	−1	$\widetilde{u}_L\ \widetilde{u}_R\ \widetilde{d}_L\ \widetilde{d}_R$	(same)
			$\widetilde{s}_L\ \widetilde{s}_R\ \widetilde{c}_L\ \widetilde{c}_R$	(same)
			$\widetilde{t}_L\ \widetilde{t}_R\ \widetilde{b}_L\ \widetilde{b}_R$	$\widetilde{t}_1\ \widetilde{t}_2\ \widetilde{b}_1\ \widetilde{b}_2$
sleptons	0	−1	$\widetilde{e}_L\ \widetilde{e}_R\ \widetilde{\nu}_e$	(same)
			$\widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{\nu}_\mu$	(same)
			$\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\nu}_\tau$	$\widetilde{\tau}_1\ \widetilde{\tau}_2\ \widetilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{B}^0\ \widetilde{W}^0\ \widetilde{H}_u^0\ \widetilde{H}_d^0$	$\widetilde{N}_1\ \widetilde{N}_2\ \widetilde{N}_3\ \widetilde{N}_4$
charginos	1/2	−1	$\widetilde{W}^\pm\ \widetilde{H}_u^+\ \widetilde{H}_d^-$	$\widetilde{C}_1^\pm\ \widetilde{C}_2^\pm$
gluino	1/2	−1	$\widetilde{g}$	(same)
goldstino (gravitino)	1/2 (3/2)	−1	$\widetilde{G}$	(same)

MSSM spectrum: an example

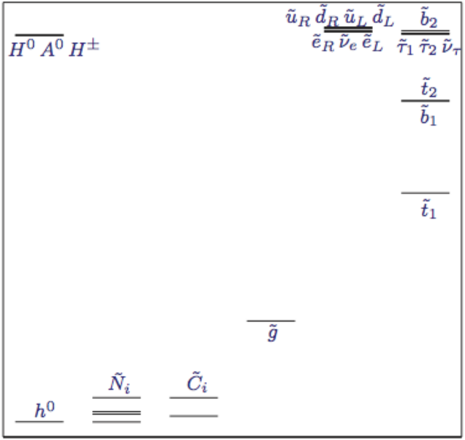

Figure: Assumes universality with $m_0^2 \gg m_{1/2}^2$

Examples

SUSY contributions to $\mu \to e\gamma$

Present a SUSY 1-loop Feynman diagram that triggers $\mu \to e\gamma$. Look at PDG and find the bound on $Br(\mu \to e\gamma)$. The SUSY flavour problem is evident also here like in the quark sector.

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A more minimal SUSY model?

Remove the two Higgs doublets from MSSM. Can you construct a realistic model out of the remaining fields? Present at least two serious problems associated with such a model.

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Next to Minimal MSSM

Throw a (gauge) singlet superfield in the MSSM spectrum. Write down the superpotential for such a model assuming R-parity conservation.

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Neutrino masses in MSSM

Neutrinos are massless in MSSM like in the minimal SM. Write down a superpotential analog for the see-saw mechanism for making (natural) non-zero and small neutrino masses.

Summary

1. We built the MSSM out of chiral and vector superfields

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What is the origin of the SUSY breaking terms?

For Further Reading I

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