Introduction to Supersymmetry

Lecture II : Building SUSY Lagrangians with Superfields

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Outline

Bibliography

J. Wess and J. Bagger,
Supersymmetry and supergravity, [Ch. IV - VII],
Princeton, Univ. Press (1992) 259 p.

Steve P. Martin,
A Supersymmetry primer, [Ch. 4],
hep-ph/9709356, v6 2011.

I use $g^{\mu\nu} = (1, -1, -1, -1)$, otherwise Martin's notation.
Please read [ch.4.1] from Martin's paper first.

Superspace

What is it? It is a space with coordinates:

$$z \equiv \{ x^\mu , \qquad \theta^\alpha , \qquad \theta^\dagger_{\dot\alpha} \} \tag{1}$$

θ and $\theta^\dagger$ are complex constant anticommuting spinors with mass dimension $[-\frac{1}{2}]$

Why do we need it? Because global SUSY transformations are infinitesimal translations in superspace

$$x^\mu \longrightarrow x^\mu + i\epsilon\sigma^\mu\theta^\dagger + i\epsilon^\dagger\bar\sigma^\mu\theta , \tag{2}$$

$$\theta \longrightarrow \theta + \epsilon , \tag{3}$$

$$\theta^\dagger \longrightarrow \theta^\dagger + \epsilon^\dagger . \tag{4}$$

Superfields

What are they? They are functions of superspace

$$S(x^\mu, \theta, \theta^\dagger) \tag{5}$$

Why do we need them? To construct SUSY invariant Lagrangians. Therefore, we need SUSY transformation of (local) superfields :

$$\begin{aligned} \sqrt{2}\delta_\epsilon S &= -i(\epsilon\hat{Q} + \epsilon^\dagger \hat{Q}^\dagger)S \\ &= S'(z) - S(z) \,. \end{aligned} \tag{6}$$

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where the differential operators that act on superfields are

$$\hat{Q}_\alpha = i\frac{\partial}{\partial \theta^\alpha} - (\sigma^\mu \theta^\dagger)_\alpha \partial_\mu \; , \qquad \hat{Q}^{\dagger\dot{\alpha}} = i\frac{\partial}{\partial \theta^\dagger_{\dot{\alpha}}} - (\bar{\sigma}^\mu \theta)^{\dot{\alpha}} \partial_\mu \; . \tag{7}$$

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Exercise 1:

Prove that $\hat{Q}$ and $\hat{Q}^\dagger$ close into SUSY algebra with $\hat{P}_\mu = i\partial_\mu$:
$\{\hat{Q}_\alpha, \hat{Q}^\dagger_{\dot{\alpha}}\} = 2\,\sigma^\mu_{\alpha\dot{\alpha}}\,\hat{P}_\mu \,, \quad \{\hat{Q}^\alpha, \hat{Q}^\beta\} = 0 \,, \quad \{\hat{Q}^\dagger_{\dot{\alpha}}, \hat{Q}^\dagger_{\dot{\beta}}\} = 0 \,.$

Superfields

To make contact with the real world we must extract the x-components of a given superfield. This can be done because of its finite term expansion in terms of θ's and $\theta^\dagger$'s

$$\begin{aligned} S(x,\theta,\theta^\dagger) &= a + \theta\xi + \theta^\dagger\chi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger c \\ &+ \theta^\dagger\bar{\sigma}^\mu\theta v_\mu + \theta^\dagger\theta^\dagger\theta\eta + \theta\theta\theta^\dagger\zeta^\dagger + \theta\theta\theta^\dagger\theta^\dagger d \end{aligned} \quad (8)$$

The expansion terminates because

$$\{\theta^\alpha, \theta^\beta\} = \{\theta^\dagger_{\dot{\alpha}}, \theta^\dagger_{\dot{\beta}}\} = \{\theta^\alpha, \theta^\dagger_{\dot{\beta}}\} = 0$$

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Exercise 2:

Starting from eqs.(6,7) prove the SUSY transformations for the various component fields of S in eq.(8). Show, for example, that $\sqrt{2}\delta_\epsilon a = \epsilon\xi + \epsilon^\dagger\xi^\dagger$ and $\sqrt{2}\delta_\epsilon\xi_\alpha = 2\epsilon_\alpha b + (\sigma^\mu\epsilon^\dagger)_\alpha(v_\mu - i\partial_\mu a)$, and $\sqrt{2}\delta_\epsilon b = \epsilon^\dagger\zeta^\dagger - \frac{i}{2}\epsilon^\dagger\bar{\sigma}^\mu\partial_\mu\xi$.

Covariant Derivatives

Action functionals depend on derivatives too, so we need covariant derivatives acting on superfields. The obvious choice $\partial/\partial\theta^\alpha$ does not work since it is not supersymmetric

$$\delta_\epsilon \frac{\partial S}{\partial\theta^\alpha} \neq \frac{\partial}{\partial\theta^\alpha}(\delta_\epsilon S) \tag{9}$$

We therefore need covariant derivatives that *all* (anti)commute with $\delta_\epsilon \sim \epsilon\hat{Q} + \epsilon^\dagger \hat{Q}^\dagger$.

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Exercise 3:

Prove that the following covariant derivatives

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu\theta^\dagger)_\alpha\partial_\mu \ , \qquad D^\dagger_{\dot\alpha} = -\frac{\partial}{\partial\theta^{\dagger\dot\alpha}} + i(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu \ . \tag{10}$$

anticommute with $\hat{Q}$ and $\hat{Q}^\dagger$. Therefore e.g., $\delta_\epsilon(D_\alpha S) = D_\alpha(\delta_\epsilon S)$.

Outline

Chiral Superfields

We now want to write the Wess-Zumino model in a manifestly covariant form. We need spin-0 and spin-1/2 fields. What superfield to use? The S has too many components! We need only a supermultiplet with ϕ, ψ and F fields!!

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is called anti-chiral (or right chiral) superfield.

Note that there is no need to apply e.o.m for component fields.

Chiral Superfields

We can solve the constraint (11) by writing Φ as a function of y and θ, where

$$y^\mu \;\equiv\; x^\mu \;+\; i\theta^\dagger\bar{\sigma}^\mu\theta \tag{13}$$

and note that $D^\dagger_{\dot\alpha} y^\mu = D^\dagger_{\dot\alpha}\theta^\beta = 0$. Then the superfield $\Phi(y,\theta)$ automatically satisfies the constraint $D^\dagger_{\dot\alpha}\Phi \;=\; 0$.

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Next, Taylor expand $\Phi(y,\theta)$ around x

$$\begin{aligned}\Phi(y,\theta) &= \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)\\ &= \phi(x) + i\theta^\dagger\bar{\sigma}^\mu\theta\partial_\mu\phi(x) - \frac{1}{4}\theta\theta\theta^\dagger\theta^\dagger\Box\phi(x) + \sqrt{2}\theta\psi(x)\\ &- \frac{i}{\sqrt{2}}\theta\theta\theta^\dagger\bar{\sigma}^\mu\partial_\mu\psi(x) + \theta\theta F(x)\end{aligned} \tag{14}$$

The Φ superfield contains the same fields needed for the Wess-Zumino model!

Chiral Superfields

Exercise 4:

Apply the results of Ex. 2 to a chiral superfield in order to confirm the SUSY transformations for ϕ, ψ and F fields in WZ-model given in eqs. (11-13, Lec. I).

Note that the "F-term"

$$\delta_\epsilon F \; = \; -i \, \epsilon^\dagger \, \bar{\sigma}^\mu \, \partial_\mu \psi \; , \tag{15}$$

transforms as a total derivative.

Outline

Vector Superfields

It is obtained from the constraint

$$V = V^* \tag{16}$$

This imposes certain constraints on S-component fields in (8) e.g., $v_\mu = v_\mu^* \equiv A_\mu$ and finally with redefinitions

$$\begin{aligned} V(x,\theta,\theta^\dagger) &= a + \theta\xi + \theta^\dagger\xi^\dagger + \theta\theta b + \theta^\dagger\theta^\dagger b^* \\ &+ \theta^\dagger\bar{\sigma}^\mu\theta A_\mu + \theta^\dagger\theta^\dagger\theta(\lambda - \frac{i}{2}\sigma^\mu\partial_\mu\xi^\dagger) + \theta\theta\theta^\dagger(\lambda^\dagger - \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\xi) \\ &+ \theta\theta\theta^\dagger\theta^\dagger(\frac{1}{2}D - \frac{1}{4}\Box a) \end{aligned} \tag{17}$$

If Φ is a chiral superfield then $\Phi + \Phi^*$, $i(\Phi - \Phi^*)$ and $\Phi^*\Phi$ are all real (vector) superfields

Vector Superfields

We need a gauge supermultiplet: a gauge boson (A^μ) $[\text{mass}]^1$, a gaugino (λ) $[\text{mass}]^{\frac{3}{2}}$ and an auxiliary field, (D) $[\text{mass}]^2$.

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V in eq.(17) has too many components. However, we still have gauge freedom to use. Suppose that V is a vector superfield for an Abelian gauge symmetry and consider the "gauge" transformation

$$V \to V \ + \ i\,(\Omega^* - \Omega) \tag{18}$$

where $\Omega = \phi + \sqrt{2}\theta\psi + \theta\theta F + ...$ is a chiral superfield. Then

$$a \to a + i(\phi^* - \phi) \tag{19}$$

$$\xi_\alpha \to \xi_\alpha - i\sqrt{2}\psi_\alpha \tag{20}$$

$$b \to b - iF \tag{21}$$

$$A_\mu \to A_\mu + \partial_\mu(\phi + \phi^*) \tag{22}$$

$$\lambda_\alpha \to \lambda_\alpha \tag{23}$$

$$D \to D \tag{24}$$

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Vector Superfields

Wess-Zumino gauge : Eliminate a, ξ_α, b from V.

$$V(x, \theta, \theta^\dagger)_{\mathrm{WZ}} = \theta^\dagger \bar{\sigma}^\mu \theta A_\mu + \theta^\dagger \theta^\dagger \theta \lambda + \theta\theta\theta^\dagger \lambda^\dagger + \frac{1}{2}\theta\theta\theta^\dagger\theta^\dagger D \quad (25)$$

This is not manifestly supersymmetric but by supergauge transformations [eqs.(19-24)] we can always restore $\delta_\epsilon V$ into WZ-gauge.

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Exercise 5:
Prove that D transforms as a total derivative,

$$\delta_\epsilon D = -i\epsilon\sigma^\mu \partial_\mu \lambda^\dagger - i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \lambda.$$

Outline

Building Lagrangians with Superfields

For a general superfield it is always valid that

$$\delta_\epsilon A = 0 \ , \quad \text{for} \quad A = \int d^4x \int d^2\theta \int d^2\theta^\dagger \, S(x, \theta, \theta^\dagger) \qquad (26)$$

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The D-term of a real superfield transforms as a total derivative, therefore

$$\begin{aligned} [V]_D \equiv \int d^2\theta \int d^2\theta^\dagger \; V(x, \theta, \theta^\dagger) &= V(x, \theta, \theta^\dagger)|_{\theta\theta\theta^\dagger\theta^\dagger} \\ &= \frac{1}{2} D - \frac{1}{4} \partial_\mu \partial^\mu a \qquad (27) \end{aligned}$$

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The F-term of a chiral superfield transforms also as total derivative

$$[\Phi]_F \equiv \int d^2\theta \Phi|_{\theta^\dagger=0} = \Phi|_{\theta\theta} = \int d^2\theta \int d^2\theta^\dagger \, \delta^{(2)}(\theta^\dagger)\Phi = F \quad (28)$$

The action should be real so we need $[\Phi]_F + \text{c.c.}$

Outline

Building the WZ-Lagrangian with chiral Superfields

Simple! Take a D-term and a F-term

$$S_{WZ} = \int d^4x\, [\Phi^{*i}\Phi_i]_D + ([W(\Phi_i)]_F + \mathrm{c.c}) \tag{29}$$

$W(\Phi_i)$ can be any holomorphic function of chiral superfields and is called superpotential. A simple choice is (also used in MSSM)

$$W(\Phi) = \frac{1}{2!} M^{ij}\Phi_i\Phi_j + \frac{1}{3!} y^{ijk}\Phi_i\Phi_j\Phi_k \tag{30}$$

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Exercise 6:
Start from the superfield WZ-action (29) and find the WZ-model written in terms of component fields given in eq.(10, Lec. I).

Outline

Gauge transformation of chiral superfields

We want now to describe superfield Lagrangians with spin-1 particles.

Global transformation of a chiral superfield:

$$\Phi \to e^{2i\alpha^{(a)}\, T^{(a)}}\Phi \tag{31}$$

$T^{(a)}$ hermitian generators of the group in rep R of an internal (unbroken) symmetry group.

Gauge this symmetry: $\alpha^{(a)} \to \Omega^{(a)}$. The full local symmetry transformation is

$$\Phi \to e^{2i\,\Omega^{(a)}\, T^{(a)}}\Phi \tag{32}$$

It must be $D^\dagger\Phi = 0 \Rightarrow D^\dagger\Omega^{(a)} = 0$, i.e., $\Omega =$ chiral superfield

Building $\mathcal{L}$s with chiral and vector superfields

Assume

$$S \;=\; \int d^4x\,[\Phi^*\Phi]_D + ([W(\Phi)]_F + \mathrm{c.c})$$

is invariant under global transformations (31). Now gauge this: $2\alpha^{(a)}T^{(a)} \to 2g^{(a)}\Omega^{(a)}T^{(a)} \equiv \Omega$. The kinetic term is not invariant since

$$\Phi^*\Phi \to \Phi^*\, e^{-i\Omega^*}\, e^{i\Omega}\,\Phi$$

To restore gauge invariance introduce a vector superfield $V^* = V$ such that $[V \equiv 2g^{(a)}V^{(a)}T^{(a)}]$

$$e^V \to e^{i\Omega^*}e^V e^{-i\Omega}$$

and write the action as

$$S \;=\; \int d^4x\,[\Phi^* e^V \Phi]_D + ([W(\Phi)]_F + \mathrm{c.c})$$

which is now gauge invariant! It admits the WZ-gauge option too...

Building $\mathcal{L}$s with chiral and vector superfields (cont'd)

Exercise 7:
By working in WZ-gauge expand the Lagrangian kinetic term in components to find:

$$
\begin{aligned}
[\Phi^{*i}(e^V)_i^j\Phi_j]_D &= F^{*i}\,F_i \;+\; \nabla_\mu\phi^{*i}\,\nabla^\mu\phi_i \;+\; i\psi^{\dagger i}\bar{\sigma}^\mu\nabla_\mu\psi_i \\
&- \sqrt{2}\,g_\alpha\,(\phi^*\,T^{(a)}\psi)\,\lambda^{(a)} - \sqrt{2}\,g_\alpha\,\lambda^{\dagger(a)}\,(\psi^\dagger\,T^{(a)}\phi) \\
&+ g_a\,(\phi^*\,T^{(a)}\phi)\,D^{(a)} \qquad (33)
\end{aligned}
$$

where the gauge covariant derivatives are

$$
\begin{aligned}
\nabla_\mu\phi_i &= \partial_\mu\phi_i - ig^aA_\mu^{(a)}(T^{(a)}\phi)_i \\
\nabla_\mu\phi^{*i} &= \partial_\mu\phi^{*i} + ig^aA_\mu^{(a)}(\phi^*\,T^{(a)})^i \\
\nabla_\mu\psi_i &= \partial_\mu\psi_i - ig^aA_\mu^{(a)}(T^{(a)}\psi)_i
\end{aligned}
$$

Building $\mathcal{L}$s with Abelian vector superfields

We also need a strength tensor superfield analog to construct gauge kinetic terms like $F^{\mu\nu}F_{\mu\nu}$. It is

$$\mathcal{W}_\alpha = -\frac{1}{4}D^\dagger D^\dagger D_\alpha V \, , \qquad \mathcal{W}^\dagger_{\dot\alpha} = -\frac{1}{4}DDD^\dagger_{\dot\alpha} V \, , \tag{34}$$

It has dimension $[\mathrm{mass}]^{3/2}$ and it is chiral superfield (with spinor index!)

In WZ-gauge and in y-space it is

$$\mathcal{W}_\alpha(y,\theta) = \lambda_\alpha + \theta_\alpha D + \frac{i}{2}(\sigma^\mu\bar\sigma^\nu\theta)_\alpha F_{\mu\nu} + i\theta\theta(\sigma^\mu\partial_\mu\lambda^\dagger)_\alpha \tag{35}$$

Therefore the action functional for the kinetic terms is

$$S = \int d^4x \frac{1}{4}[\mathcal{W}^\alpha\mathcal{W}_\alpha]_F + \mathrm{c.c} = \int d^4x \left[\frac{1}{2}D^2 + i\lambda^\dagger\bar\sigma^\mu\partial_\mu\lambda - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right]$$

Exercise 8:

In the Abelian case prove that $\mathcal{W}_\alpha$ is invariant under (super)gauge transformations, $V \to V + i(\Omega^* - \Omega)$.

Building $\mathcal{L}$s with Abelian vector superfields

A Fayet-Iliopoulos term

$$\mathcal{L}_{FI} = -2\kappa[V]_D = -\kappa D$$

is super gauge allowed. This term plays role in spontaneous SUSY breaking (see lecture IV).

Building $\mathcal{L}$s with non-Abelian vector superfields

In the non-Abelian case the field strength superfield is defined as

$$\mathcal{W}_\alpha = -\frac{1}{4} D^\dagger D^\dagger (e^{-V} D_\alpha e^V)$$

Under super gauge transformations it goes like

$$\mathcal{W}_\alpha \to e^{i\Omega} \mathcal{W}_\alpha e^{-i\Omega}$$

and

$$\begin{aligned} [\mathcal{W}^{(a)\,\alpha} \mathcal{W}^{(a)}_\alpha]_F &= D^{(a)} D^{(a)} + 2i\lambda^{(a)} \sigma^\mu \nabla_\mu \lambda^{\dagger(a)} - \frac{1}{2} F^{(a)\mu\nu} F^{(a)}_{\mu\nu} \\ &+ \frac{i}{4} \epsilon^{\mu\nu\rho\sigma} F^{(a)}_{\mu\nu} F^{(a)}_{\rho\sigma} \end{aligned} \tag{36}$$

and

$$S = \int d^4x \, \mathrm{Tr} \, [\mathcal{W}^\alpha \mathcal{W}_\alpha]_F \; + \; \mathrm{c.c}$$

Outline

The renormalizable SUSY Lagrangians

The most general renormalizable action with matter and gauge fields is

$$\mathcal{L} = \left(\frac{1}{4}\,[\mathcal{W}^{(a)\,\alpha}\,\mathcal{W}^{(a)}_{\alpha}]_F \;+\; \mathrm{c.c}\right) + \left[\Phi^{*i}\,(e^{2g_a T^{(a)} V^{(a)}})_i^j\,\Phi_j\right]_D$$
$$+ \quad \left([W(\Phi)]_F + \mathrm{c.c}\right) \tag{37}$$

where the super potential is gauge invariant and a holomorphic function of Φ's of at most the third power. All terms are fixed by symmetry apart from those in $W(\Phi)$.

In components just replace the relevant terms in eq.(37) with eqs.(36,33) and eq.(17, Lec. I).

Outline

The non-renormalizable SUSY Lagrangians

A non-renormalizable, gauge invariant theory with chiral and vector superfields

$$\mathcal{L} \;=\; [K(\Phi_i, \tilde{\Phi}^{*j}]_D + \left(\left[\frac{1}{4} f_{ab}(\Phi) \mathcal{W}^a \mathcal{W}^b + W(\Phi) \right]_F + \mathrm{c.c} \right) \quad (38)$$

where

$$\tilde{\Phi}^{*j} \equiv \Phi^{*k} (e^{2g_a T^{(a)} V^{(a)}})_k^j$$

can be constructed out of three functions:

Superpotential : $W(\Phi)$

Kahler potential : $K(\Phi, \Phi^*)$

Gauge kinetic superpotential : $f_{ab}(\Phi)$

Application : SUSY breaking

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(Super) Feynman Rules and Supergraphs

Following QFT for superfields, we can devise SFeynman Rules.

We can then use perturbation theory with Supergraphs

The "arrows" notation for Supergraphs is similar to that of normal Feynman diagrams with Weyl spinors, but the FRs for vertices and propagators involve superspace "D-algebra" technics that need certain familiarity.

Supergraphs are getting very complicated when SUSY and gauge symmetries are broken (I have never seen a paper calculating the mass of the Higgs boson in MSSM with supergraphs! Component diagrams are far easier!!)

If you insist on learning supergraphs, start with Wess and Bagger chapters IX-X and then read the classic masterpiece by Grisaru, Siegel and Rocek cited at the end of this lecture.

R-symmetries

$U(1)_R$ -symmetry is a continuous global symmetry that transforms the anti-commuting superspace coordinates

$$\theta \to e^{i\alpha}\,\theta\ , \qquad \theta^\dagger \to e^{-i\alpha}\,\theta^\dagger \tag{39}$$

Therefore if the theory is invariant under R-symmetry

$$S(x,\theta,\theta^\dagger) \to e^{ir_S\alpha}\,S(x,e^{-i\alpha}\theta,e^{i\alpha}\theta^\dagger) \tag{40}$$

For the components of a chiral superfield Φ this means

$$\phi \to e^{ir_\Phi\alpha}\phi\ , \quad \psi \to e^{i(r_\Phi-1)\alpha}\psi\ , \quad F \to e^{i(r_\Phi-2)\alpha}F \tag{41}$$

V-superfield has R-charge zero because it is real. In the WZ-gauge its components transform as

$$A^\mu \to A^\mu\ , \quad \lambda \to e^{i\alpha}\lambda\ , \quad D \to D \tag{42}$$

R-symmetry does not commute with supersymmetry

Few more on R-symmetries...

Other R-charges:

$$R[d^2\theta] = -2\ , \quad R[d^2\theta^\dagger] = +2\ , \quad R[D_\alpha] = -1\ , \quad R[D^\dagger_{\dot\alpha}] = 1$$

Therefore:

$$R[\mathcal{W}_\alpha] = +1\ , \quad R[W] = +2$$

Consider the superpotential

$$W = L\Phi + \frac{1}{2!}M\Phi^2 + \frac{y}{3!}\Phi^3$$

If we impose R-symmetry then only one of these terms survive!

R-symmetries play an important role in spontaneous symmetry breaking of global SUSY and in the non-renormalization theorem

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Let's construct an interesting model tomorrow morning...

For Further Reading I

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