Introduction to Supersymmetry

Lecture I : Motivation

Athanasios Dedes

Department of Physics
University of Ioannina, Greece

Pre-SUSY 2014
July 15-19, Univ. of Manchester, UK

Outline

Bibliography

J. Wess and J. Bagger,
Supersymmetry and supergravity, [Ch. III],
Princeton, Univ. Press (1992) 259 p.

Steve P. Martin,
A Supersymmetry primer, [Ch. 1-3],
hep-ph/9709356, v6 2011.

These assume a certain knowledge of QFT, e.g, Peskin and Schroeder's <u>or</u> Ramond's book.

I follow the notation of Martin's review but with $g^{\mu\nu} = (1, -1, -1, -1)$ instead

Motivation

Supersymmetry (SUSY) is a continuous symmetry that relates fermions and bosons. It has the virtue of allowing non-trivial interactions among particles.

In some sense, it answers the question: *why does Nature play with particles of different spin?*

Motivation

Supersymmetry (SUSY) is a continuous symmetry that relates fermions and bosons. It has the virtue of allowing non-trivial interactions among particles.

In some sense, it answers the question: *why does Nature play with particles of different spin?*

SUSY is motivated best by the solution it provides to the hierarchy problem. The latter is the instability of the Higgs mass under quadratically divergent radiative corrections.

Motivation

Supersymmetry (SUSY) is a continuous symmetry that relates fermions and bosons. It has the virtue of allowing non-trivial interactions among particles.

In some sense, it answers the question: *why does Nature play with particles of different spin?*

SUSY is motivated best by the solution it provides to the hierarchy problem. The latter is the instability of the Higgs mass under quadratically divergent radiative corrections.

Minimal Supersymmetric Standard Model (MSSM) includes:

- *Unification of gauge couplings*
- *Dark Matter*
- *Stability of the vacuum*
- *Radiative Electroweak Symmetry Breaking*

Motivation

Supersymmetry (SUSY) is a continuous symmetry that relates fermions and bosons. It has the virtue of allowing non-trivial interactions among particles.

In some sense, it answers the question: *why does Nature play with particles of different spin?*

SUSY is motivated best by the solution it provides to the hierarchy problem. The latter is the instability of the Higgs mass under quadratically divergent radiative corrections.

Minimal Supersymmetric Standard Model (MSSM) includes:

- *Unification of gauge couplings*
- *Dark Matter*
- *Stability of the vacuum*
- *Radiative Electroweak Symmetry Breaking*

Outline

Hierarchy problem

through a Renormalizable Toy Model

Complex Scalar field: ϕ

Weyl fermion: ψ

$$\begin{aligned}\mathcal{L} \;=\; & \partial^{\mu}\phi^{*}\partial_{\mu}\phi \;+\; i\psi^{\dagger}\bar{\sigma}^{\mu}\partial_{\mu}\psi \\ & -\; \frac{1}{2}M_F\,\psi\psi \;-\; \frac{1}{2}M_F\,\psi^{\dagger}\psi^{\dagger} \;-\; \lambda_F\,\phi\psi\psi \;-\; \lambda_F^{*}\,\phi^{*}\psi^{\dagger}\psi^{\dagger} \\ & -\, M_B^2\,\phi^{*}\phi \;-\; \lambda_B\,(\phi^{*}\phi)^2 \end{aligned} \tag{1}$$

Hierarchy problem

through a Renormalizable Toy Model

Complex Scalar field: ϕ

Weyl fermion: ψ

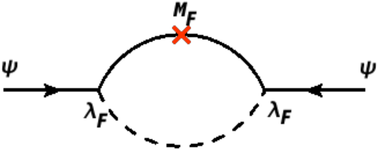
$$\begin{aligned}\mathcal{L} \;=\; & \partial^\mu \phi^* \partial_\mu \phi \;+\; i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi \\ & -\; \frac{1}{2} M_F\, \psi\psi \;-\; \frac{1}{2} M_F\, \psi^\dagger \psi^\dagger \;-\; \lambda_F\, \phi\psi\psi \;-\; \lambda_F^*\, \phi^* \psi^\dagger \psi^\dagger \\ & -\, M_B^2\, \phi^*\phi \;-\; \lambda_B\, (\phi^*\phi)^2 \end{aligned} \tag{1}$$

Symmetries: A chiral global $U(1)$ when $M_F = 0$

$$\phi \to e^{-2i\alpha}\phi\,, \qquad \psi \to e^{i\alpha}\psi$$

1-loop Fermion mass corrections

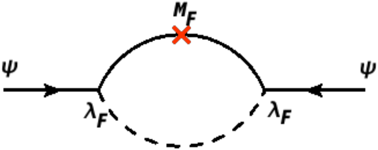
They must contain at least one M_F insertion e.g.,



$$\delta M_F \;\simeq\; \frac{\lambda_F^2}{16\pi^2}\, M_F \tag{2}$$

1-loop Fermion mass corrections

They must contain at least one M_F insertion e.g.,

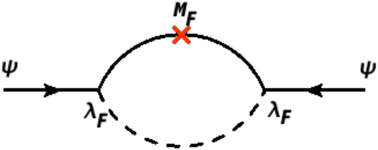


$$\delta M_F \simeq \frac{\lambda_F^2}{16\pi^2} M_F \tag{2}$$

Light Fermion masses are natural: they are stable under radiative corrections.

1-loop Fermion mass corrections

They must contain at least one M_F insertion e.g.,



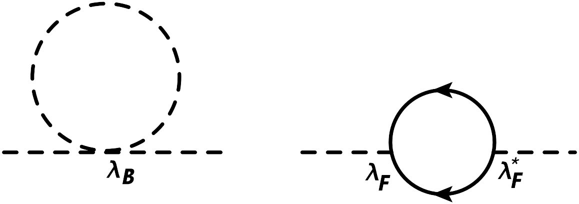
$$\delta M_F \simeq \frac{\lambda_F^2}{16\pi^2} M_F \tag{2}$$

Light Fermion masses are natural: they are stable under radiative corrections.

M_F is protected by the $U(1)$-symmetry.

1-loop Boson mass corrections

The boson mass is not protected by the chiral symmetry

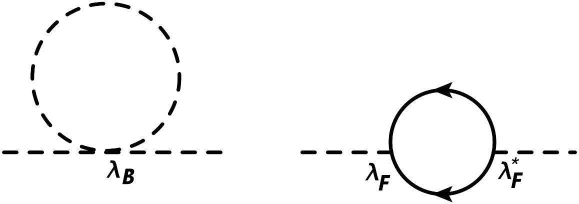


$$\delta M_B^2 \simeq \frac{\lambda_B}{16\pi^2}\Lambda^2 - \frac{\lambda_F^*\lambda_F}{16\pi^2}\Lambda^2 \tag{3}$$

where Λ is the UV cut-off.

1-loop Boson mass corrections

The boson mass is not protected by the chiral symmetry



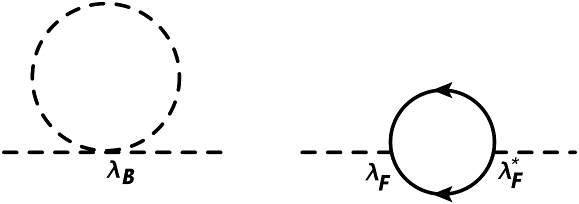
$$\delta M_B^2 \simeq \frac{\lambda_B}{16\pi^2}\Lambda^2 - \frac{\lambda_F^* \lambda_F}{16\pi^2}\Lambda^2 \tag{3}$$

where Λ is the UV cut-off.

Light Boson masses are not natural: they are *not* stable under radiative corrections.

1-loop Boson mass corrections

The boson mass is not protected by the chiral symmetry



$$\delta M_B^2 \simeq \frac{\lambda_B}{16\pi^2}\Lambda^2 \qquad - \qquad \frac{\lambda_F^* \lambda_F}{16\pi^2}\Lambda^2 \tag{3}$$

where Λ is the UV cut-off.

Light Boson masses are not natural: they are *not* stable under radiative corrections.

M_B receives large, quadratically divergent, radiative corrections, and so does the Higgs boson in the SM

- The cut-off Λ is presumably at M_{GUT} or M_{PLANCK}.

- The cut-off Λ is presumably at M_{GUT} or M_{PLANCK}.
- Cancellations in 1 part to 10^{15-17} between counterterms is needed: Hierarchy (or fine tuning) problem

- The cut-off Λ is presumably at M_{GUT} or M_{PLANCK}.
- Cancellations in 1 part to 10^{15-17} between counterterms is needed: Hierarchy (or fine tuning) problem
- Solution: there must be a relation between λ_F and λ_B but also a symmetry that guarantees this relation holds to all orders: this symmetry is called Supersymmetry.

- The cut-off Λ is presumably at M_{GUT} or M_{PLANCK}.
- Cancellations in 1 part to 10^{15-17} between counterterms is needed: Hierarchy (or fine tuning) problem
- Solution: there must be a relation between λ_F and λ_B but also a symmetry that guarantees this relation holds to all orders: this symmetry is called Supersymmetry.
- SUSY stabilises the ElectroWeak (EW) scale i.e., $M_W \ll M_P$, by cancelling all quadratic divergences

- The cut-off Λ is presumably at M_{GUT} or M_{PLANCK}.
- Cancellations in 1 part to 10^{15-17} between counterterms is needed: Hierarchy (or fine tuning) problem
- Solution: there must be a relation between λ_F and λ_B but also a symmetry that guarantees this relation holds to all orders: this symmetry is called Supersymmetry.
- SUSY stabilises the ElectroWeak (EW) scale i.e., $M_W \ll M_P$, by cancelling all quadratic divergences

We shall show how this works in the simplest SUSY model: the Wess-Zumino model

Outline

SUSY algebra

$$\{Q_\alpha,\, Q^\dagger_{\dot\alpha}\} \;=\; 2\,\sigma^\mu_{\alpha\dot\alpha}\,P_\mu \tag{4}$$

$$\{Q_\alpha,\, Q_\beta\} \;=\; \{Q^\dagger_{\dot\alpha},\, Q^\dagger_{\dot\beta}\} \;=\; 0 \tag{5}$$

$$[P_\mu,\, Q_\alpha] \;=\; [P_\mu,\, Q^\dagger_{\dot\alpha}] \;=\; 0 \tag{6}$$

$$[P_\mu,\, P_\nu] \;=\; 0\,. \tag{7}$$

Q_α : the SUSY generator

Theorem (Coleman-Mandula (1967))

If a QFT in $d > 2$ has a second conserved vector quantity other than the total energy-momentum, $P^\mu = (H, P^i)$, then $S = 1$, i.e., no scattering is allowed.

A consequence .....

In fact the most general possibility allowed by CM-theorem is

$$\{Q^I_\alpha,\, Q^{J\,\dagger}_{\dot\alpha}\} \;=\; 2\,\delta^{IJ}\,\sigma^\mu_{\alpha\dot\alpha}\,P_\mu \tag{8}$$

where $I, J = 1...N$. We shall only consider the simplest, $N = 1$, case, in these lectures.

Theorem (Noether (1918))

Symmetry of the Lagrangian $\leftrightarrow$ *conserved quantity*

$$\partial_\mu J^\mu_\alpha = 0\,, \qquad Q_\alpha = \int d^3x\, J^0_\alpha\,. \tag{9}$$

Outline

Wess-Zumino Model (1974)

Weyl fermion, ψ
Complex Scalar field, ϕ
Complex (auxiliary) field, F

$$\mathcal{L}_{WZ} = \partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^* F$$

$$+ \left\{ M\,(\phi\, F - \frac{1}{2}\,\psi\,\psi) + \lambda\,(\phi^2\, F - \phi\,\psi\,\psi) + \text{c.c.} \right\} \quad (10)$$

Wess-Zumino Model (1974)

Weyl fermion, ψ
Complex Scalar field, ϕ
Complex (auxiliary) field, F

$$\mathcal{L}_{WZ} = \partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^* F$$

$$+ \left\{ M\,(\phi\, F - \frac{1}{2}\,\psi\,\psi) + \lambda\,(\phi^2\, F - \phi\,\psi\,\psi) + \text{c.c.} \right\} \quad (10)$$

$\mathcal{L}_{WZ}$ is invariant under the supersymmetric transformations:

$$\delta_\epsilon \phi = \epsilon\,\psi\,, \quad (11)$$

$$\delta_\epsilon \psi_\alpha = -i\,(\sigma^\mu\,\epsilon^\dagger)_\alpha\,\partial_\mu \phi + \epsilon_\alpha\,F\,, \quad (12)$$

$$\delta_\epsilon F = -\,i\,\epsilon^\dagger\,\bar{\sigma}^\mu\,\partial_\mu \psi \quad (13)$$

fermion $\leftrightarrow$ boson

Wess-Zumino Model (1974)

Weyl fermion, ψ
Complex Scalar field, ϕ
Complex (auxiliary) field, F

$$\mathcal{L}_{WZ} = \partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi + F^* F$$

$$+ \left\{ M \left(\phi F - \frac{1}{2} \psi \psi\right) + \lambda \left(\phi^2 F - \phi \psi \psi\right) + \text{c.c.} \right\} \quad (10)$$

$\mathcal{L}_{WZ}$ is invariant under the supersymmetric transformations:

$$\delta_\epsilon \phi = \epsilon \psi , \quad (11)$$

$$\delta_\epsilon \psi_\alpha = -i \left(\sigma^\mu \epsilon^\dagger\right)_\alpha \partial_\mu \phi + \epsilon_\alpha F , \quad (12)$$

$$\delta_\epsilon F = - i \epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi \quad (13)$$

fermion $\leftrightarrow$ boson

Wess-Zumino Model

While ϕ and ψ are propagating fields, the F-field is not. It can be integrated out from $\mathcal{L}_{WZ}$ by using e.o.m for F and F^*, *i.e.*,

$$\frac{\partial \mathcal{L}_{WZ}}{\partial F} = 0 \Rightarrow F^* = -(M\,\phi + \lambda\,\phi^2) \tag{14}$$

The Lagrangian (10) now becomes on-shell:

$$\begin{aligned}\mathcal{L}'_{WZ} &= \partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi \\ &- \frac{1}{2}\,M\,\psi\psi - \frac{1}{2}\,M^*\,\psi^\dagger\psi^\dagger - \lambda\,\phi\,\psi\psi - \lambda^*\,\phi^*\,\psi^\dagger\psi^\dagger \\ &- \mathcal{V}(\phi,\phi^*)\,. \end{aligned} \tag{15}$$

where the scalar potential

$$\begin{aligned}\mathcal{V}(\phi,\phi^*) &= |M\phi + \lambda\phi^2|^2 \equiv |F|^2 \\ &= |M|^2\phi^*\phi + |\lambda|^2(\phi^*\phi)^2 + \lambda M^*\,\phi^*\phi\phi + \lambda^* M\,\phi\phi^*\phi^* \end{aligned} \tag{16}$$

is always positive definite.

Wess-Zumino Model (consequences)

Hierarchy problem solved

$\mathcal{L}'_{WZ}$ is the supersymmetric generalisation of the toy model of eq. (1) with $\lambda_F = \lambda$ and $\lambda_B = \lambda^* \lambda$. The hierarchy problem is technically solved: quadratic divergences cancel order by order and to all orders in perturbation theory!

Wess-Zumino Model (consequences)

Hierarchy problem solved

$\mathcal{L}'_{WZ}$ is the supersymmetric generalisation of the toy model of eq. (1) with $\lambda_F = \lambda$ and $\lambda_B = \lambda^* \lambda$. The hierarchy problem is technically solved: quadratic divergences cancel order by order and to all orders in perturbation theory!

Equality of Masses

$\mathcal{L}'_{WZ}$ contains a complex scalar field with mass M and a Weyl field with mass M. This is a general feature of SUSY because $P^2 = M^2$ is the Casimir operator of SUSY algebra (see eq. (6)).

Wess-Zumino Model (consequences)

Hierarchy problem solved

$\mathcal{L}'_{WZ}$ is the supersymmetric generalisation of the toy model of eq. (1) with $\lambda_F = \lambda$ and $\lambda_B = \lambda^* \lambda$. The hierarchy problem is technically solved: quadratic divergences cancel order by order and to all orders in perturbation theory!

Equality of Masses

$\mathcal{L}'_{WZ}$ contains a complex scalar field with mass M and a Weyl field with mass M. This is a general feature of SUSY because $P^2 = M^2$ is the Casimir operator of SUSY algebra (see eq. (6)).

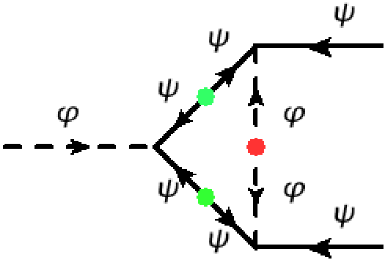
SUSY must be broken

The absence of SUSY partners (s-leptons, s-quarks, gauginos) of the observed particles (leptons, quarks, gauge bosons) means that SUSY must be broken in everyday life!

Outline

Non-renormalization Theorem

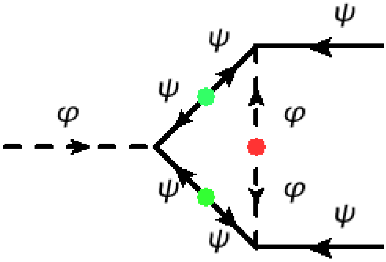
The Yukawa coupling, λ, is not renormalized to all orders in perturbation theory. Let's consider the following 1-loop, 3-point 1PI, diagram (possibly log-divergent by naive power counting)



This diagram does not exist: there is no $< \phi^* \phi^* >$ propagator (red blob) in WZ-model. (This diagram is also prop to $\lambda \cdot \lambda \cdot \lambda \cdot M \cdot M$ and violates an R-symmetry, see Lec. II)

Non-renormalization Theorem

The Yukawa coupling, λ, is not renormalized to all orders in perturbation theory. Let's consider the following 1-loop, 3-point 1PI, diagram (possibly log-divergent by naive power counting)



This diagram does not exist: there is no $< \phi^* \phi^* >$ propagator (red blob) in WZ-model. (This diagram is also prop to $\lambda \cdot \lambda \cdot \lambda \cdot M \cdot M$ and violates an R-symmetry, see Lec. II)

Of course the coupling λ does have a β-function because of wave function renormalization contributions.

Non-renormalization Theorem and tadpole cancellation

Exersice 1: Non-Renormalization at two-loops

Consider two-loop Feynman diagram contributions to the vertex $\phi\psi\psi$ in the WZ-model (on-shell). Take for simplicity zero mass, $M = 0$. Following the argument of the previous slide, prove that non of these diagrams exist, and the non-renormalization theorem stays at this level too.

Example: No tadpole contributions

Prove, for simplicity at one-loop, that scalar tadpole diagrams cancel each other in WZ-model. In fact this cancellation persists to all orders in perturbation theory.

Outline

Exercise 2: More general WZ-model

It is possible to add more complicated interactions inside the curly bracket of eq. (10) by the simple use of a general, analytic, function $W(\phi)$ of scalar fields, called superpotential. Prove that the general WZ-Lagrangian with $i = 1, ...n$ copies of (ϕ, ψ, F) fields

$$\mathcal{L}_{WZ} = \partial^\mu \phi^{*i} \partial_\mu \phi_i + i\psi^{\dagger i} \bar{\sigma}^\mu \partial_\mu \psi_i + F^{*i} F_i + \left\{ -\frac{1}{2} W^{ij} \psi_i \psi_j + W^i F_i + \mathrm{c.c.} \right\} \tag{17}$$

with

$$W^{ij} = \frac{\partial^2 W(\phi)}{\partial \phi_i \partial \phi_j}, \qquad W^i = \frac{\partial W(\phi)}{\partial \phi_i} \tag{18}$$

is invariant under the SUSY transformations (11-13). The simple WZ-model of eq.(10) is recovered for $W(\phi) = \frac{1}{2} M \phi^2 + \frac{\lambda}{3} \phi^3$.

Outline

Exercise 3: Soft SUSY breaking terms

Add to $\mathcal{L}_{WZ}$ a scalar mass terms or trilinear couplings of the form

$$\mathcal{L}_{SUSY\ \ breaking} = (m^2)^j_i \phi^{*i} \phi_j \quad (19)$$

$$+ (a^{ijk} \phi_i \phi_j \phi_k + c.c) \quad (20)$$

$$+ (b^{ij} \phi_i \phi_j + c.c) \quad (21)$$

Prove that, in general, these terms individually are not invariant under the SUSY transformations (11-13). These terms are called soft SUSY breaking terms because they do not destroy the cancellation of quadratic divergences.

Summary

- SUSY relates fermions and bosons non-trivially
- SUSY algebra is a mathematically consistent extension of the Poincare algebra
- In WZ-model quadratic divergences cancel
- The Yukawa coupling is not renormalized
- SUSY must be broken at low energies
- Remarks
  - More general interactions in WZ-model through the superpotential
  - SUSY breaking terms

For Further Reading I

J. Wess and B. Zumino,
"Supergauge Transformations in Four-Dimensions,"
....among the original SUSY papers
Nucl. Phys. B **70**, 39 (1974); Phys. Lett. B **49**, 52 (1974).

S. R. Coleman and J. Mandula,
"All Possible Symmetries of the S Matrix,"
Phys. Rev. **159**, 1251 (1967).

E. Witten,
"Dynamical Breaking of Supersymmetry,"
Nucl. Phys. B **188**, 513 (1981).

H. K. Dreiner, H. E. Haber and S. P. Martin,
Review on Weyl spinor technics (extremely useful!)
Phys. Rept. **494**, 1 (2010), arXiv:0812.1594