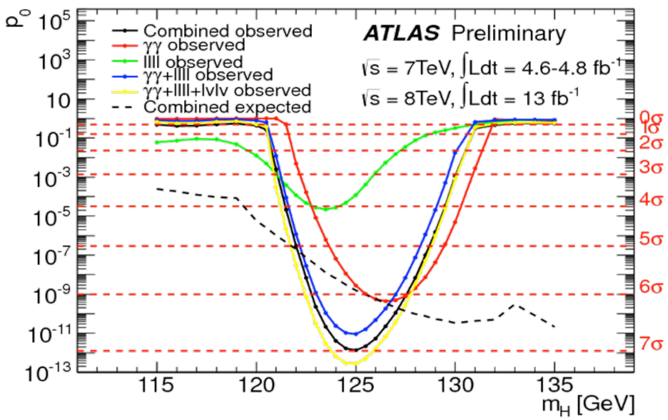
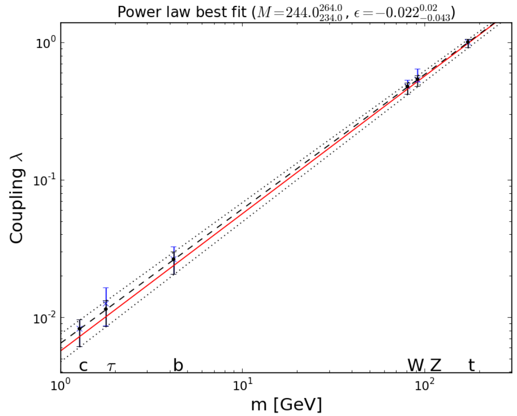
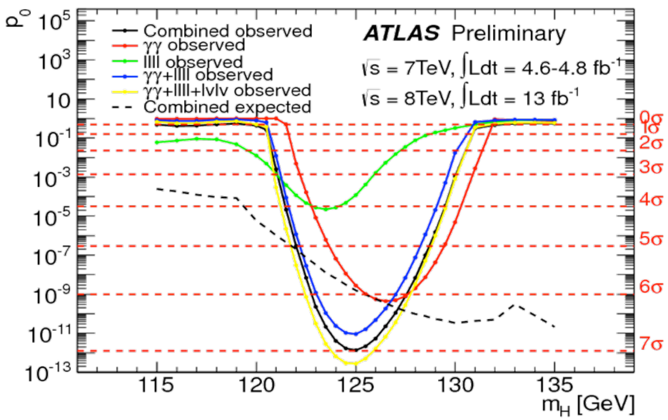
V. Phenomenology of SUGRA

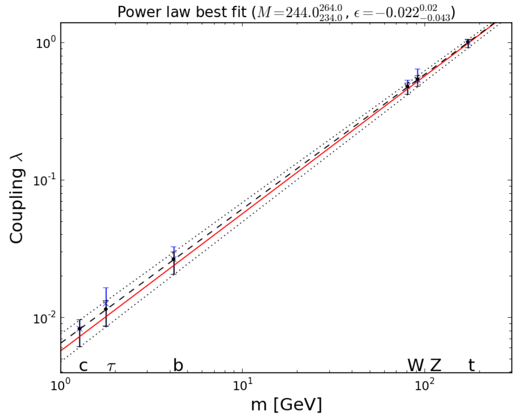
Higgs discovery! … completes the "Standard Model"





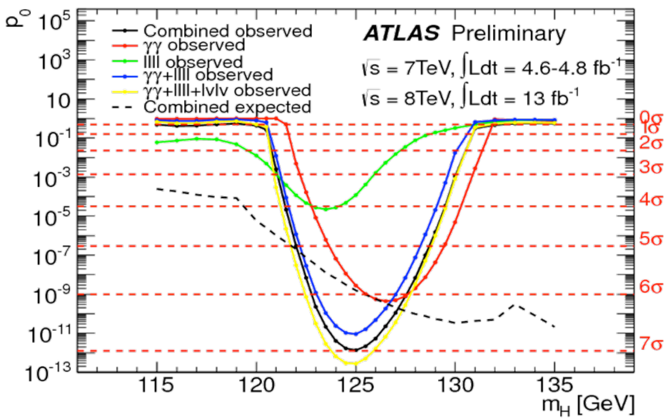
Higgs discovery! … completes the “Standard Model”

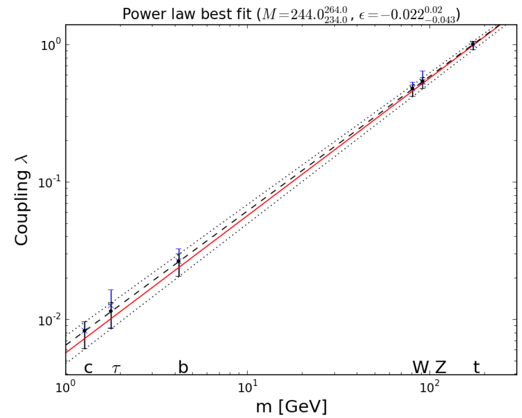




- “Light”, weakly interacting SUSY ✔

Higgs discovery! … completes the "Standard Model"





- "Light", weakly interacting SUSY ✔
- "Heavy", no evidence for sparticles SUSY ✗

$$m_h^2 = M_Z^2 + \frac{3m_t^2 h_t^2}{4\pi^2}\left(\ln\left(\frac{M_s^2}{m_t^2}\right)+\delta_t\right)+\ldots \simeq 126\,GeV$$

$$\delta m_{H_u}^2 \simeq -\frac{3y_t^2}{4\pi^2}\left(m_{stop}^2+\frac{g_s^2}{3\pi^2}m_{gluino}^2\log\left(\frac{\Lambda}{m_{gluino}}\right)\right)\log\left(\frac{\Lambda}{m_{stop}}\right) \; ?$$

SUSY under pressure "Little hierarchy problem"

Little hierarchy problem $\Rightarrow$ definite SUSY breaking structure

MSSM: 105 +(19) Parameters

$$M_Z^2 = \sum_{\tilde{q},\tilde{l}} a_i \widetilde{m}_i^2 + \sum_{\tilde{g},\widetilde{W},\widetilde{B}} b_i \widetilde{M}_i^2 + \ldots$$

$$m_{\tilde{q}} > 0.6 - 1 TeV \quad \Rightarrow \quad \Delta > a \frac{\widetilde{m}^2}{M_Z^2} \sim 100$$

(Unless light stop $m_{\tilde{t},LHC} > 250\ GeV$)

$\Rightarrow$ Correlations between SUSY breaking parameters and/or additional low-scale states

Little hierarchy problem $\Rightarrow$ definite SUSY breaking structure

MSSM: 105 +(19) Parameters

$$M_Z^2 = \sum_{\tilde{q},\tilde{l}} a_i \widetilde{m}_i^2 + \sum_{\tilde{g},\widetilde{W},\widetilde{B}} b_i \widetilde{M}_i^2 + \ldots$$

$$m_{\tilde{q}} > 0.6 - 1 TeV \quad \Rightarrow \quad \Delta > a \frac{\widetilde{m}^2}{M_Z^2} \sim 100$$

(Unless light stop $m_{\tilde{t},LHC} > 250\ GeV$)

$\Rightarrow$ Correlations between SUSY breaking parameters and/or additional low-scale states

Fine Tuning measure:

$$\Delta(a_i) = \left| \frac{a_i}{M_Z} \frac{\partial M_Z}{\partial a_i} \right|,$$

$$\Delta_m = Max_{a_i} \Delta(a_i), \quad \Delta_q = \left(\sum \Delta_{\gamma_i}^2 \right)^{1/2}$$

Ellis, Enquist, Nanopoulos, Zwirner

Barbieri, Giudice

Fine tuning from a likelihood fit:

"Nuisance" variable

$$L\left(\text{data} \mid \gamma_i\right) \propto \int d\text{v}\,\delta\left(m_Z - m_Z^0\right)\delta\left(\text{v-}\left(-\frac{m^2}{\lambda}\right)^{1/2}\right)L\left(\text{data} \mid \gamma_i;\text{v}\right)$$

$$= \frac{1}{\Delta_q}\delta\left(n_q(\ln\gamma_i - \ln\gamma_i^S)\right)L\left(\text{data} \mid \gamma_i;\text{v}_0\right)$$

Fine tuning not optional!

Probabilistic interpretation:

$$\chi^2_{new} = \chi^2_{old} + 2\ln\Delta_q \qquad \Delta_q \ll 100$$

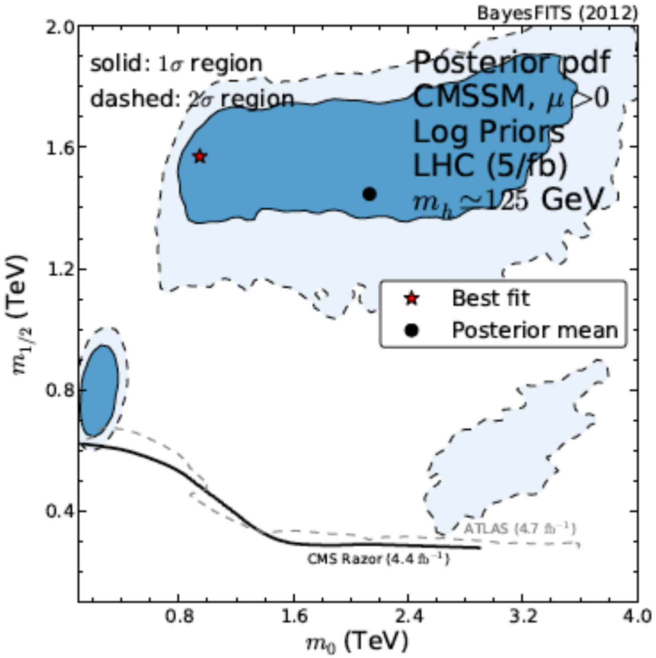
- The CMSSM

$$\gamma_i = \mu_0, m_0, m_{1/2}, A_0, B_0$$

assume correlation between SUSY breaking parameters

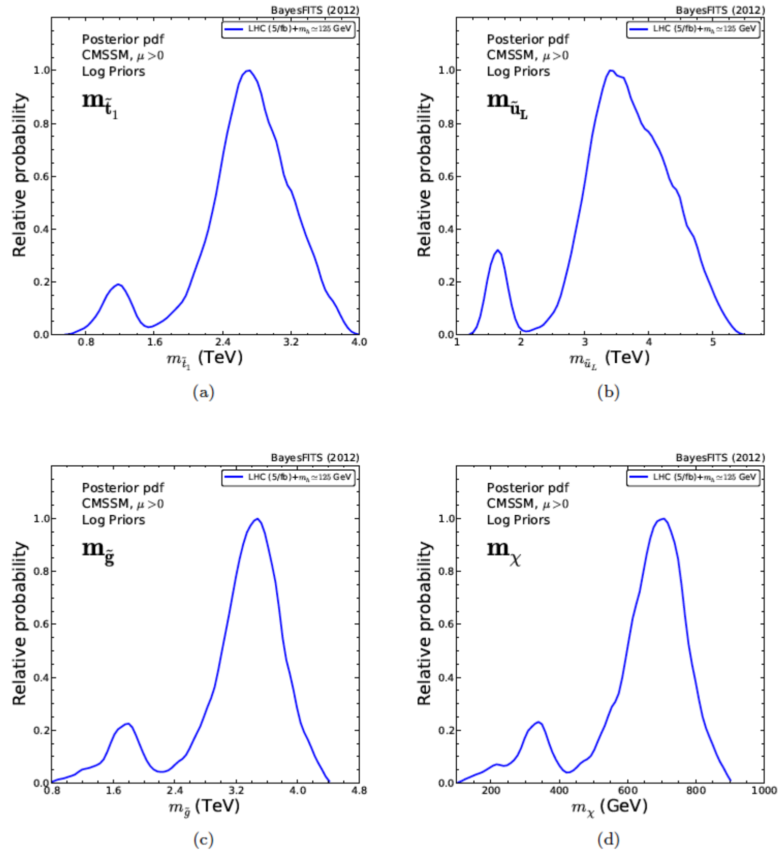
SUSY spectrum : CMSSM

$m_{1/2}$



m_0

$\tilde{t}$ $\tilde{u}_L$



$\tilde{g}$ $\tilde{\chi}$

Fowlie et al

Fine tuning in the CMSSM

$$
\begin{aligned}
V &= m_1^2\,|H_1|^2 + m_2^2\,|H_2|^2 - (m_3^2\,H_1\cdot H_2 + h.c.)\\
&+ \frac{1}{2}\lambda_1\,|H_1|^4 + \frac{1}{2}\lambda_2\,|H_2|^4 + \lambda_3\,|H_1|^2\,|H_2|^2 + \lambda_4\,|H_1\cdot H_2|^2\\
&+ \left[\frac{1}{2}\lambda_5\,(H_1\cdot H_2)^2 + \lambda_6\,|H_1|^2\,(H_1\cdot H_2) + \lambda_7\,|H_2|^2\,(H_1\cdot H_2) + h.c.\right]
\end{aligned}
$$

Minimisation conditions:

$$v^2 = -m^2/\lambda, \qquad 2\lambda\frac{\partial m^2}{\partial\beta} = m^2\frac{\partial\lambda}{\partial\beta}$$

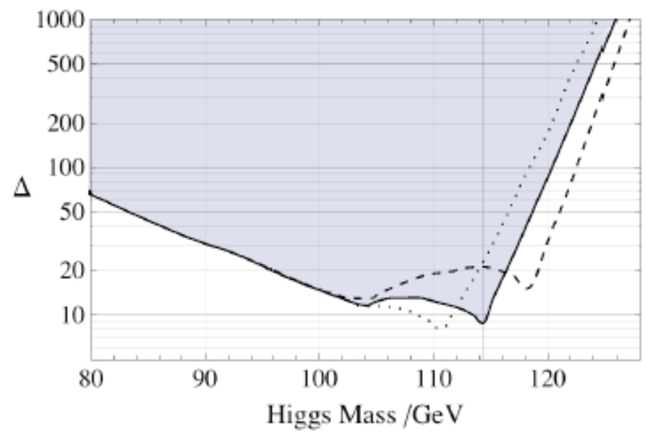
$$
\begin{aligned}
m^2 &= m_1^2\cos^2\beta + m_2^2\sin^2\beta - m_3^2\sin 2\beta\\
\lambda &= \frac{\lambda_1}{2}\cos^4\beta + \frac{\lambda_2}{2}\sin^4\beta + \frac{\lambda_{345}}{4}\sin^2 2\beta + \sin 2\beta\,(\lambda_6\cos^2\beta + \lambda_7\sin^2\beta)
\end{aligned}
$$

$$\Delta \equiv \max\left|\Delta_p\right|_{p=\{\mu_0^2, m_0^2, m_{1/2}^2, A_0^2, B_0^2\}}, \qquad \Delta_p \equiv \frac{\partial\ln v^2}{\partial\ln p}$$

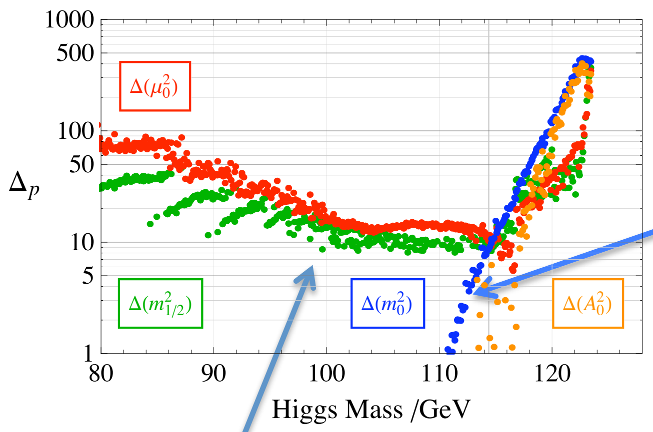
Couplings and masses evaluated to two loop (leading log) order

…enhanced sensitivity due to small tree-level $\lambda = \frac{1}{8}\left(g_1^2 + g_2^2\right)\cos^2 2\beta$

The CMSSM - before LHC



$$\Delta_i,\ i = \mu_0, m_0, m_{1/2}, A_0, B_0$$



Limit of focus point

λ increase with m_H

$$v^2 = -\frac{m^2}{\lambda}$$

Constraints

SUSY particle masses

$3.20 < 10^4\ \mathrm{Br}(b \to s\gamma) < 3.84$

$\mathrm{Br}(b \to \mu\mu) < 1.8 \times 10^{-8}$

$\delta a_\mu < 292 \times 10^{-11}$

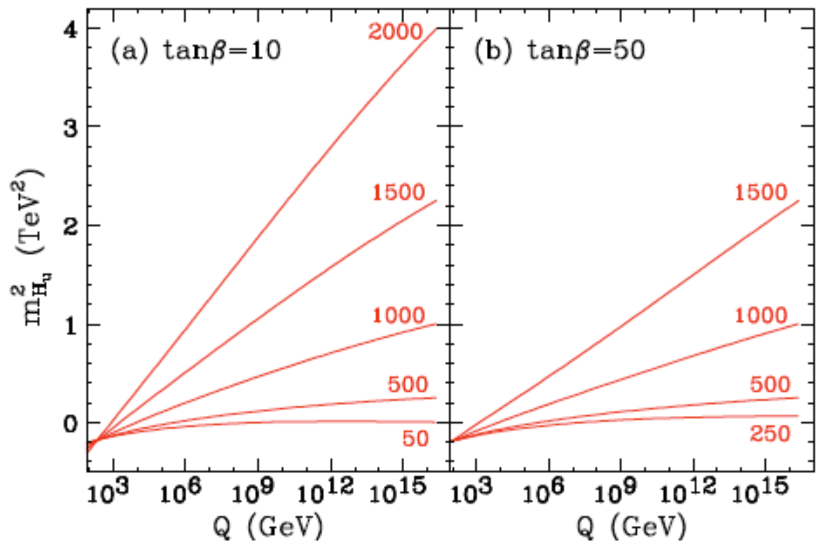
$-0.0007 < \delta\rho < 0.0012$

Radiative EW breaking

Relic density unrestricted

Guuge coupling unification

Focus Point



$2|y_t|^2(m^2_{H_u}+m^2_{Q_3}+m^2_{\bar{u}3})+2|a_t|^2$

$$16\pi^2\frac{d}{dt}m^2_{H_u}=3X_t-6g_2^2|M_2|^2-\frac{6}{5}g_1^2|M_1|^2$$

$$16\pi^2\frac{d}{dt}m^2_{Q_3}=X_t+X_b-\frac{32}{3}g_3^2|M_3|^2-6g_2^2|M_2|^2-\frac{2}{15}g_1^2|M_1|^2$$

$$16\pi^2\frac{d}{dt}m^2_{\bar{u}3}=2X_t-\frac{32}{3}g_3^2|M_3|^2-\frac{32}{15}g_1^2|M_1|^2$$

$$m^2_{H_u}\left(Q^2\right)=m^2_{H_u}\left(M_P^2\right)+\frac{1}{2}\left(m^2_{H_u}\left(M_P^2\right)+m^2_{Q_3}\left(M_P^2\right)+m^2_{\bar{u}3}\left(M_P^2\right)\right)\left[\left(\frac{Q^2}{M_P^2}\right)^{\frac{3y_t^2}{4\pi^2}}-1\right]$$

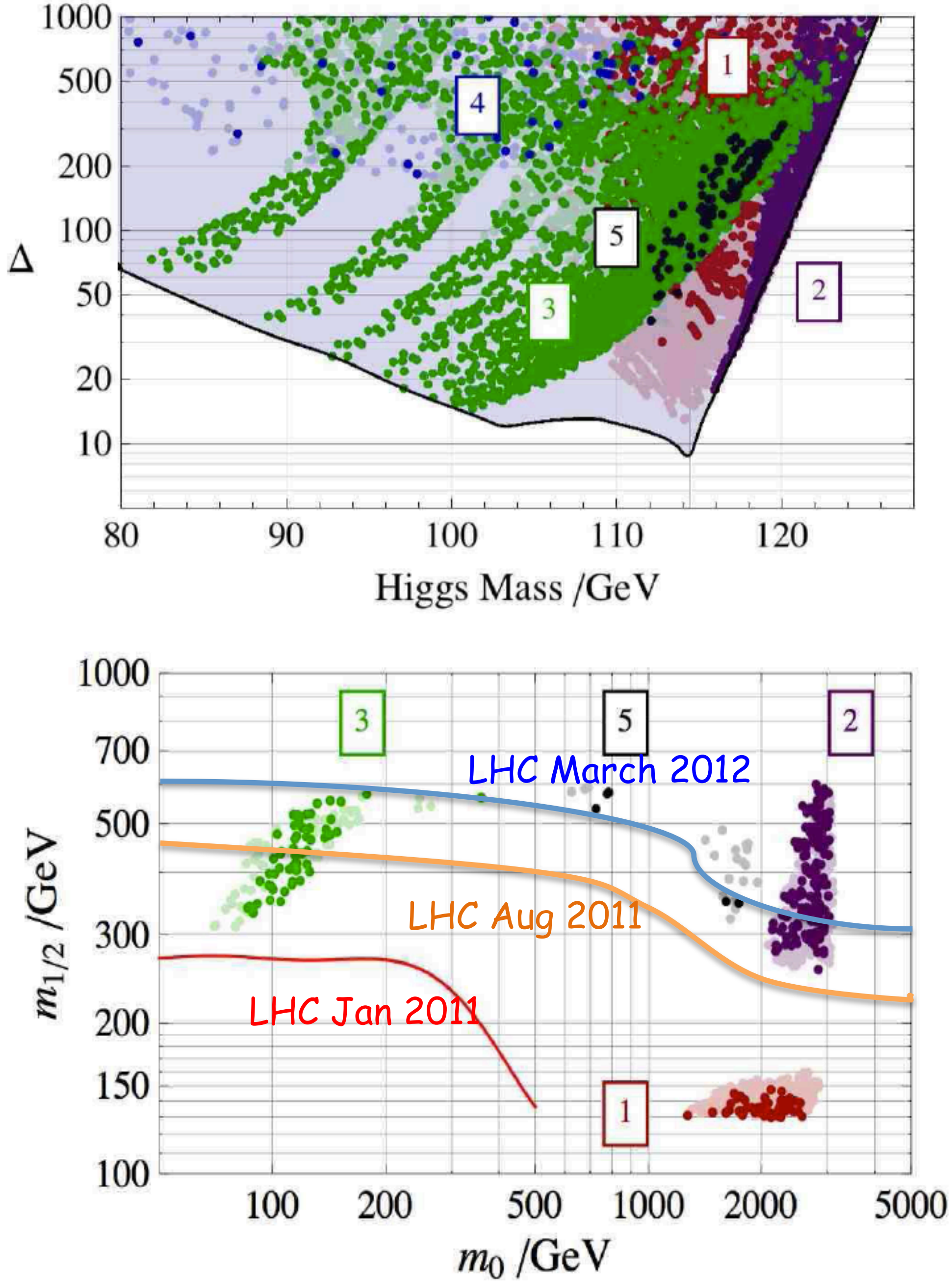
m_0^2 $\qquad$ $3m_0^2$ $\qquad$ $\simeq -\frac{2}{3},\ Q^2\simeq M_Z^2$

"Focus point": $m^2_{H_u}(0)=m^2_{Q_3}(0)=m^2_{\bar{u}3}(0)\equiv m^2$

$$m^2_{H_u}(t_0)=a_0\ m^2+..,\ a_0\le 0.1$$

Natural choice

$i.e.\ m^3_{Q_3}, m^2_{\bar{u}3}\gg M_Z^2$ possible



Relic density restricted

1 h^0 resonant annihilation

2 $\tilde{h}$ t-channel exchange

3 $\tilde{\tau}$ co-annihilation

4 $\tilde{t}$ co-annihilation

• 5 A^0 / H^0 resonant annihilation

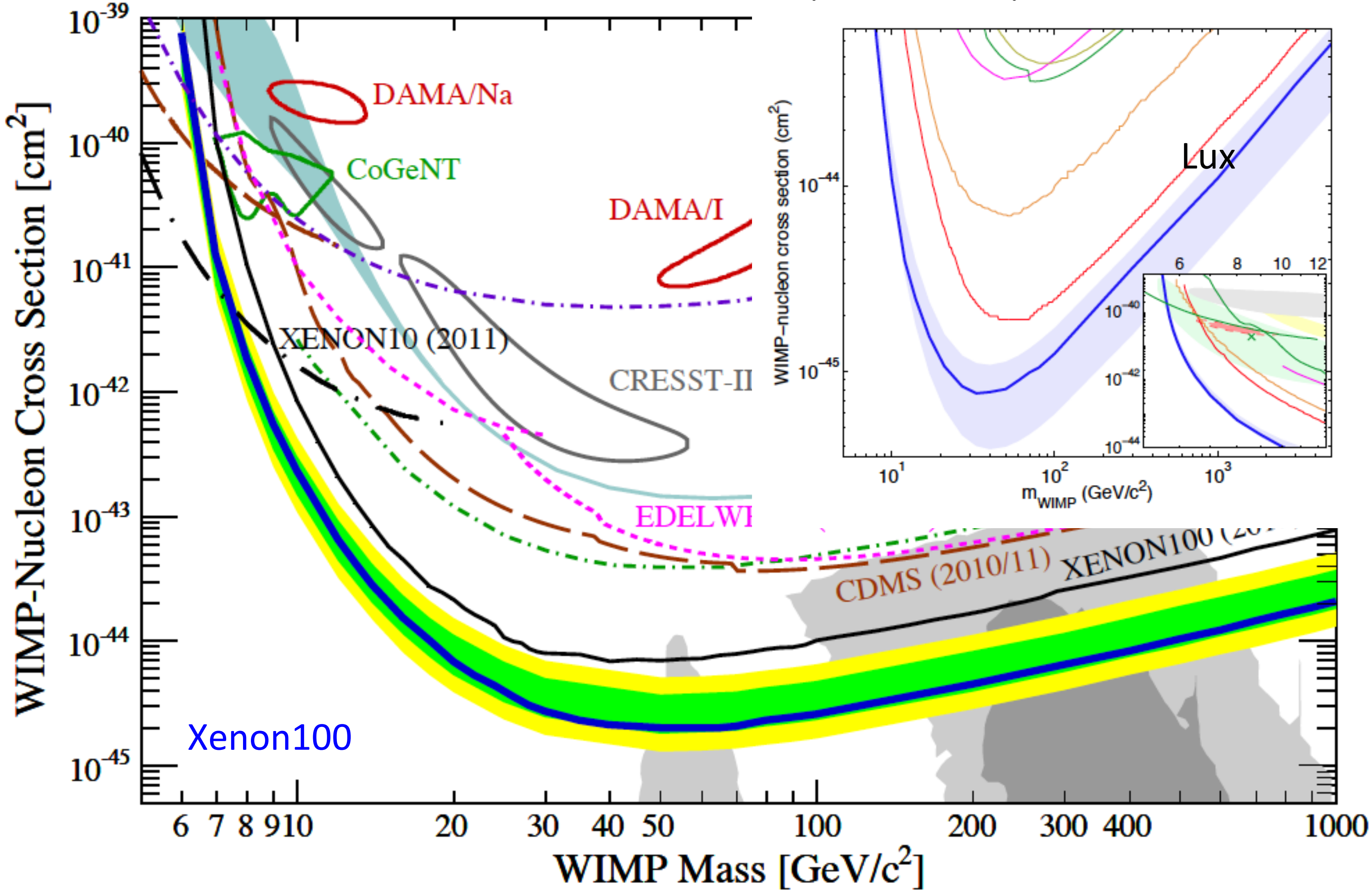
Within 3σ WMAP:

$\Delta_{Min} = 15, \quad m_h = 114.7 \pm 2 GeV$

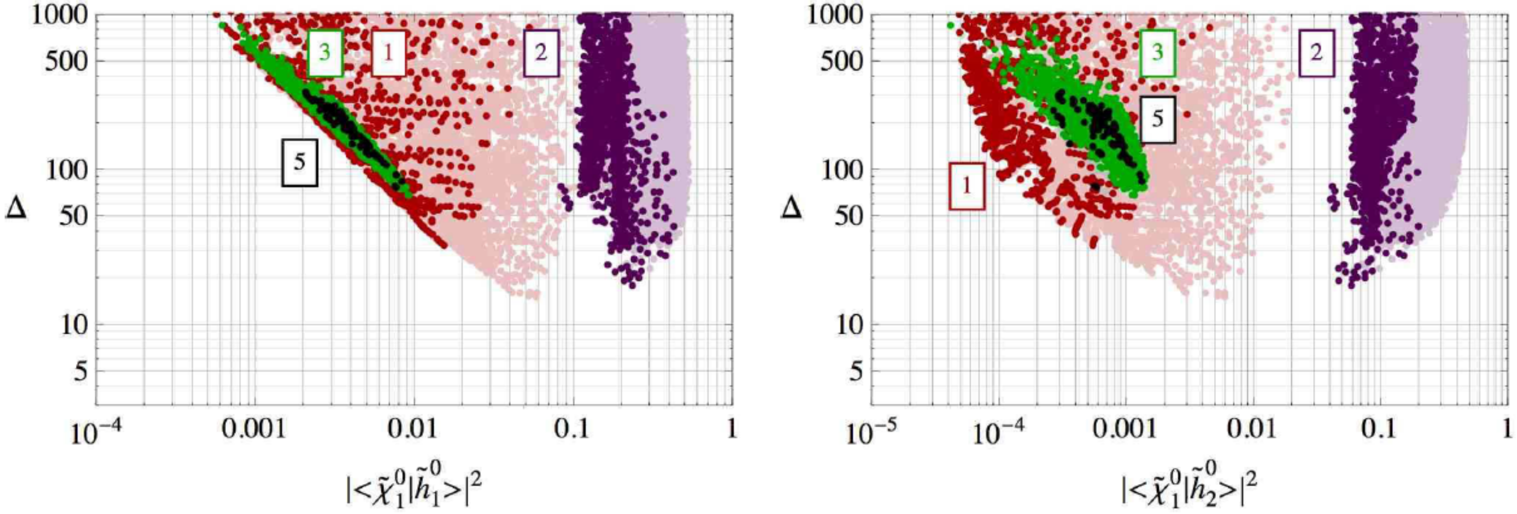
< 3σ WMAP:

$\Delta_{Min} = 18, \quad m_h = 115.9 \pm 2 GeV$

Direct dark matter searches: (spin independent)

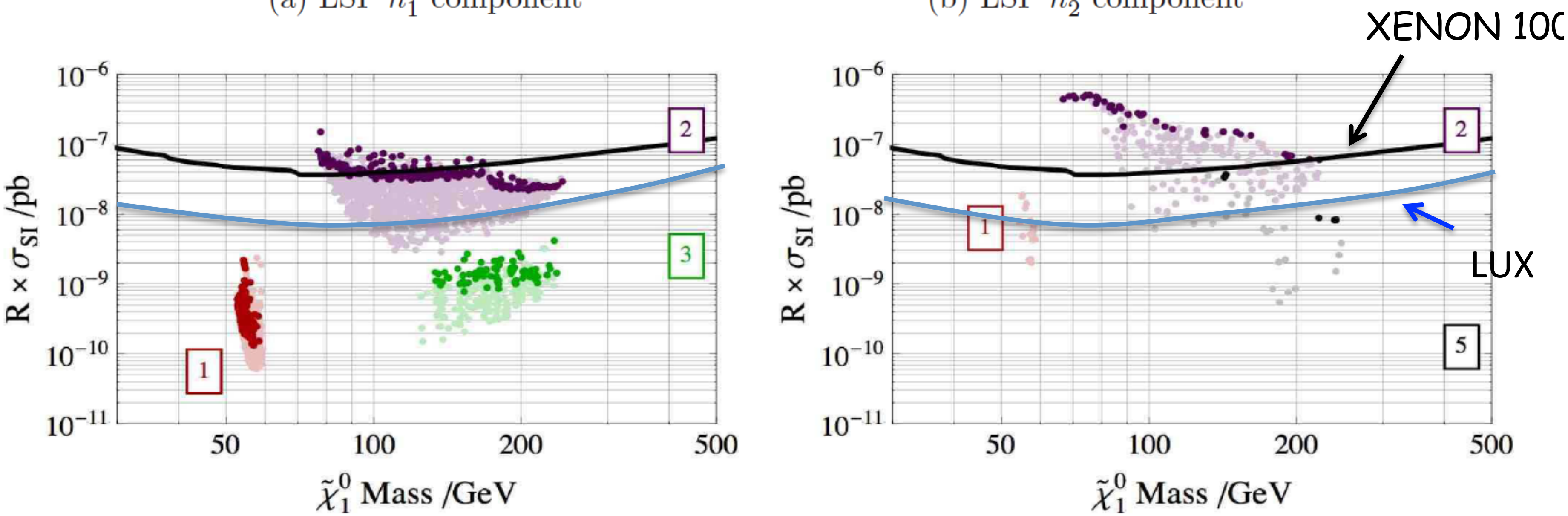


DM - Scaled spin independent cross section for LSP-proton scattering:



(a) LSP $\tilde{h}_1^0$ component

(b) LSP $\tilde{h}_2^0$ component



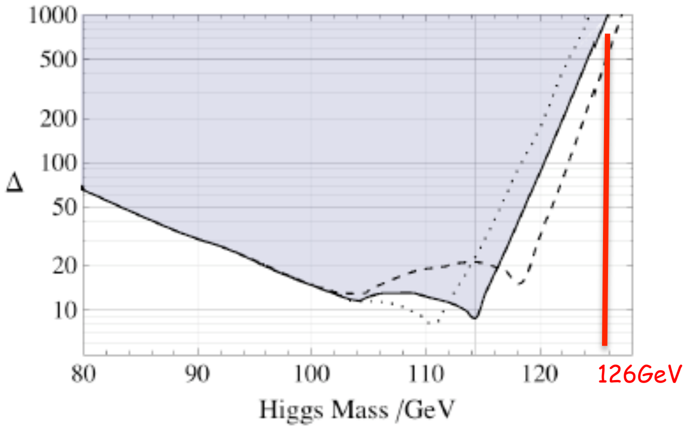
(a) $\tan\beta \leq 45$

$\Delta < 100$

(b) $50 \leq \tan\beta \leq 55$

$\Delta < 100$

The CMSSM - after Higgs discovery



$M_S^2 = m_{q_3} m_{U_3}$

$$M_{h^0}^2 = M_Z^2 \cos^2 2\beta + \frac{3M_t^2 h_t^2}{4\pi^2}\left(\ln(\frac{M_S^2}{M_t^2}) + \delta_t\right) + \cdots \qquad \simeq 126GeV$$

$$\Delta_{Min} > 350\,, \quad m_h = 125.6 \pm 3GeV$$

Reduced fine tuning (c.f. CMSSM)

- New focus points?

Gauginos: $M_{\tilde{g},\widetilde{W},\widetilde{B}}$ Non-universal gaugino correlations

- New degrees of freedom

Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2|a_t|^2\right) - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

Abe, Kobayashi, Omura
Horton, GGR

(Also improves precision of gauge coupling unification)

Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2|a_t|^2\right) - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

Natural ratios?

$$\int d^2\theta\, f_{ab}\, Tr\, W^{a\alpha} W_\alpha^b + h.c. \qquad f_{ab} = \delta_{ab}\left[\frac{1}{g_a^2} + \frac{f_X X}{M_P} + \ldots\right]$$

$$m_{1/2} = \frac{\sqrt{3}}{2} \mathrm{Re}(f_X) m_{3/2}$$

Nonuniversal masses if X non-singlet – classify by representation of X

Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2|a_t|^2\right) - 6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

Natural ratios? e.g.:

GUT: $SU(5): \quad \Phi^N \subset (24 \times 24)_{symm} = 1 + 24 + 75 + 200; \quad SO(10): \quad (45 \times 45)_{symm} = 1 + 54 + 210 + 770$

Representation	$M_3 : M_2 : M_1$ at M_{GUT} ($\eta_3 : 1 : \eta_1$)	$M_3 : M_2 : M_1$ at M_{EWSB} ($2.7\eta_3 : 1 : 0.5\eta_1$)
1	1:1:1	6:2:1
24	2:(-3):(-1)	12:(-6):(-1)
75	1:3:(-5)	6:6:(-5)
200	1:2:10	6:4:10

Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2|y_t|^2 (m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2|a_t|^2\right) - 6 g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

Natural ratios? e.g.:

GUT: $SU(5): \quad \Phi^N \subset (24\times 24)_{symm} = 1 + 24 + 75 + 200; \quad SO(10): \quad (45\times 45)_{symm} = 1 + 54 + 210 + 770$

$\eta_3 : 1 : \eta_1$ $\qquad$ $2.7\eta_3 : 1 : 0.5\eta_1$

Representation	$M_3 : M_2 : M_1$ at M_{GUT}	$M_3 : M_2 : M_1$ at M_{EWSB}
1	1:1:1	6:2:1
24	2:(-3):(-1)	12:(-6):(-1)
75	1:3:(-5)	6:6:(-5)
200	1:2:10	6:4:10

String: $\left(3+\delta_{GS}\right):\left(-1+\delta_{GS}\right):\left(-\frac{33}{5}+\delta_{GS}\right)$ (OII, also mixed moduli anomaly)

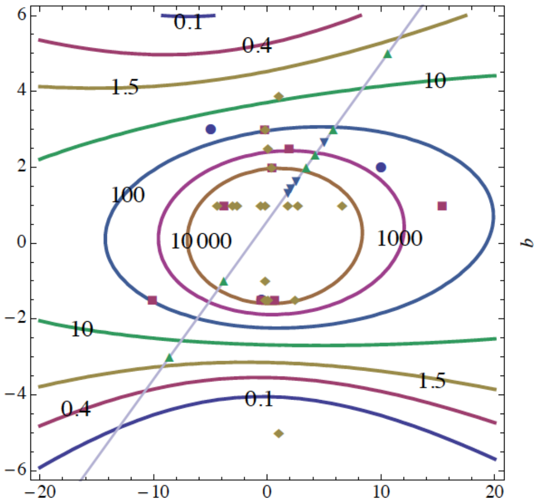
Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2\,|y_t|^2\,(m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2\,|a_t|^2\right) - 6 g_2^2\,|M_2|^2 - \frac{6}{5} g_1^2\,|M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

$$M_3 : M_2 : M_1 = 1 : b : a$$

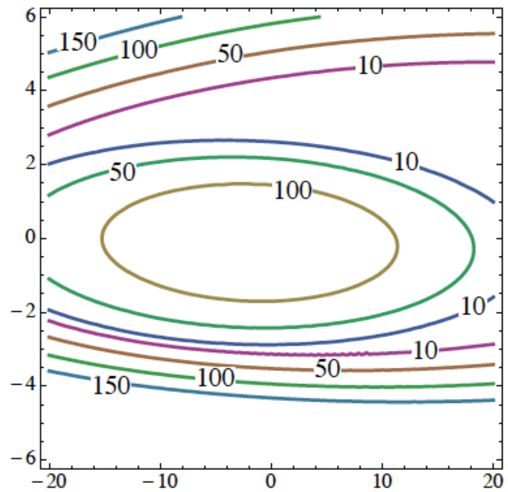
b



a

Focus point scale

b



a

Fine tuning measure

GGR, Kaminska, Schmidt-Hoberg

Reduced fine tuning : nonuniversal gaugino masses

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3\left(2\,|y_t|^2\,(m_{H_u}^2 + m_{Q_3}^2 + m_{\bar{u}_3}^2) + 2\,|a_t|^2\right) - 6 g_2^2\,|M_2|^2 - \frac{6}{5} g_1^2\,|M_1|^2$$

New focus point: cancellation between M_3 and M_2 contributions if $|M_2|^2 \simeq |M_3|^2$ at M_{SUSY}

$$\Delta_{Min}^{CMSSM} = 60\ (500), \quad m_h = 125.6 \pm 3 GeV$$

LHC8 SUSY bounds ✔

DM relic abundance ✔

DM searches ✗

Reduced fine tuning : Beyond the MSSM

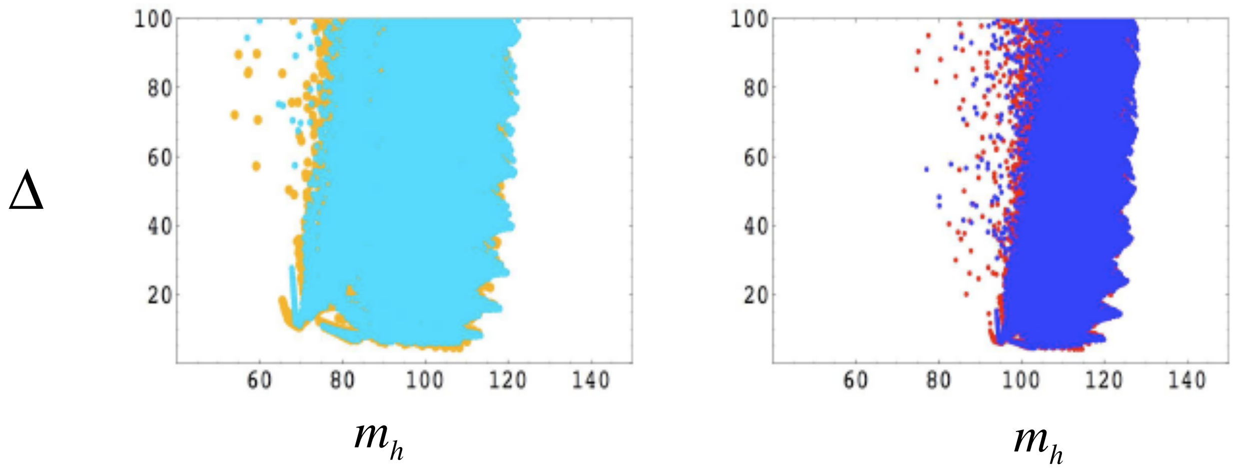
Reduced fine tuning : Beyond the MSSM

New heavy states – higher dimension operators

$$\delta L = \int d^2\theta \frac{1}{M_*}(\mu_0 + c_0 S)(H_u H_d)^2, \quad S = m_0 \theta\theta$$

Dimension 5

$$\delta V = \varsigma_1 \left(|h_u|^2 + |h_d|^2\right) h_u h_d + \varsigma_2 (h_u h_d)^2; \quad \varsigma_1 = \frac{\mu_0}{M_*}, \ \varsigma_2 = \frac{c_0 m_0}{M_*}$$



MSSM

+ dim 5 operators

Even for M_*=65 μ_0 a significant shift of m_h for constant Δ

...effect mainly comes from ς_1 term ... origin?

Reduced fine tuning : Beyond the MSSM

New heavy states – higher dimension operators

$$\delta L = \int d^2\theta \frac{1}{M_*}(\mu_0 + c_0 S)(H_u H_d)^2, \quad S = m_0\theta\theta$$ Dimension 5

$$\delta V = \varsigma_1\left(|h_u|^2 + |h_d|^2\right)h_u h_d + \varsigma_2 (h_u h_d)^2; \quad \varsigma_1 = \frac{\mu_0}{M_*}, \; \varsigma_2 = \frac{c_0 m_0}{M_*}$$

Singlet extensions

$$W = W_{\text{Yukawa}} + \lambda S H_u H_d + \frac{\kappa}{3} S^3$$ NMSSM

$$W = W_{\text{Yukawa}} + (\mu + \lambda S) H_u H_d + \frac{\mu_S}{2} S^2 + \frac{\kappa}{3} S^3 + \xi S$$ GNMSSM

$$\mu_S >> m_{3/2}: \quad W_{eff}^{GNMSSM} = (H_u H_d)^2/\mu_s + \mu H_u H_d$$

$$\delta V = \frac{\mu}{\mu_S}\left(|H_u|^2 + |H_d|^2\right) H_u H_d$$ ✔

Reduced fine tuning : Beyond the MSSM

New heavy states – higher dimension operators

$$\delta L = \int d^2\theta \frac{1}{M_*}(\mu_0 + c_0 S)(H_u H_d)^2, \quad S = m_0 \theta\theta$$

Dimension 5

$$\delta V = \varsigma_1\left(|h_u|^2 + |h_d|^2\right) h_u h_d + \varsigma_2 (h_u h_d)^2; \quad \varsigma_1 = \frac{\mu_0}{M_*}, \ \varsigma_2 = \frac{c_0 m_0}{M_*}$$

Z_N^R R-symmetry

N	q_{10}	$q_{\bar{5}}$	q_{H_u}	q_{H_d}	q_S
4	1	1	0	0	2
8	1	5	0	4	6

Singlet extensions

$$W = W_{\text{Yukawa}} + \lambda S H_u H_d + \frac{\kappa}{3} S^3$$

$$W = W_{\text{Yukawa}} + (\mu + \lambda S) H_u H_d + \frac{\mu_S}{2} S^2 + \frac{\kappa}{3} S^3 + \xi S$$

R-symmetry ensures singlets light

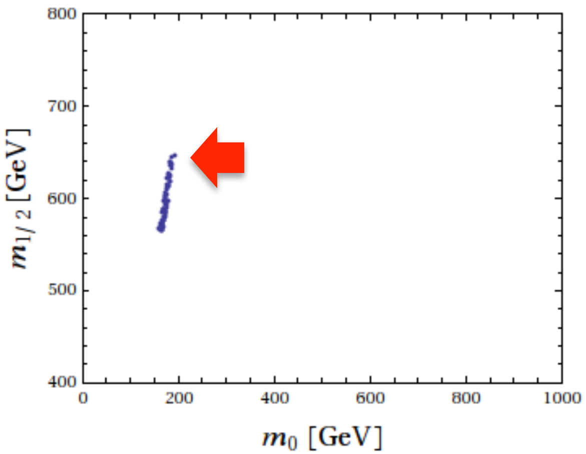
Fine tuning in the CGNMSSM $(\lambda \leq 0.7^{\dagger})$

$$\Delta_{Min} = 60\ (500), \quad m_h = 125.6 \pm 3GeV$$

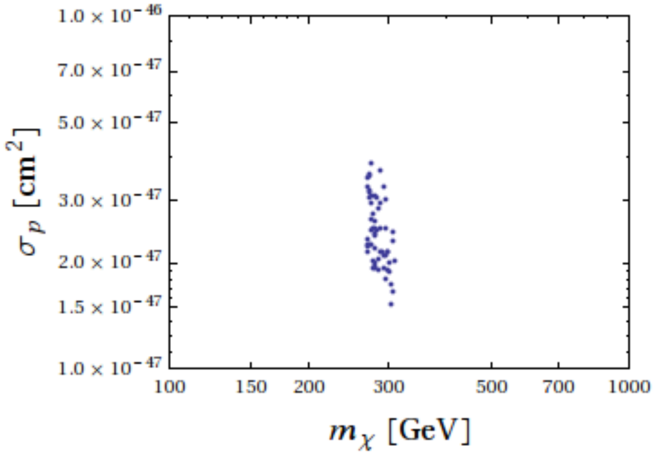
LHC8 SUSY bounds ✗

DM relic abundance ✔

DM searches ✔



Stau co-annihilation



DM searches insensitive

Fine tuning in the ©GNMSSM $(\lambda \leq 0.7^{\dagger})$

Non-unversal gaugino masses

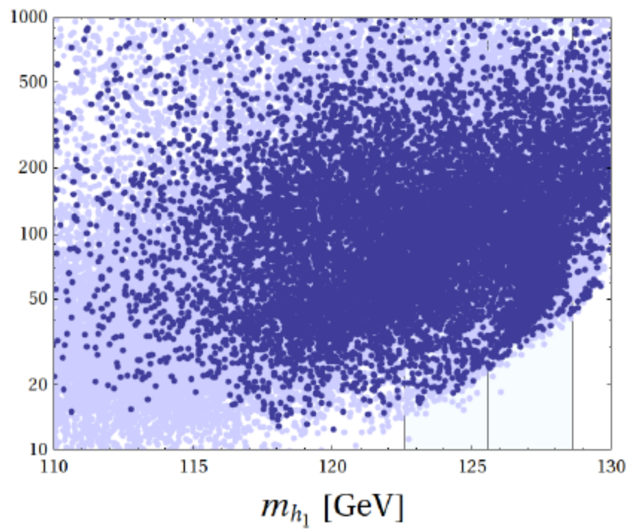
$$\Delta_{Min} = 20\,, \quad m_h = 125.6 \pm 3GeV$$

LHC8 SUSY bounds ✔

DM relic abundance ✔

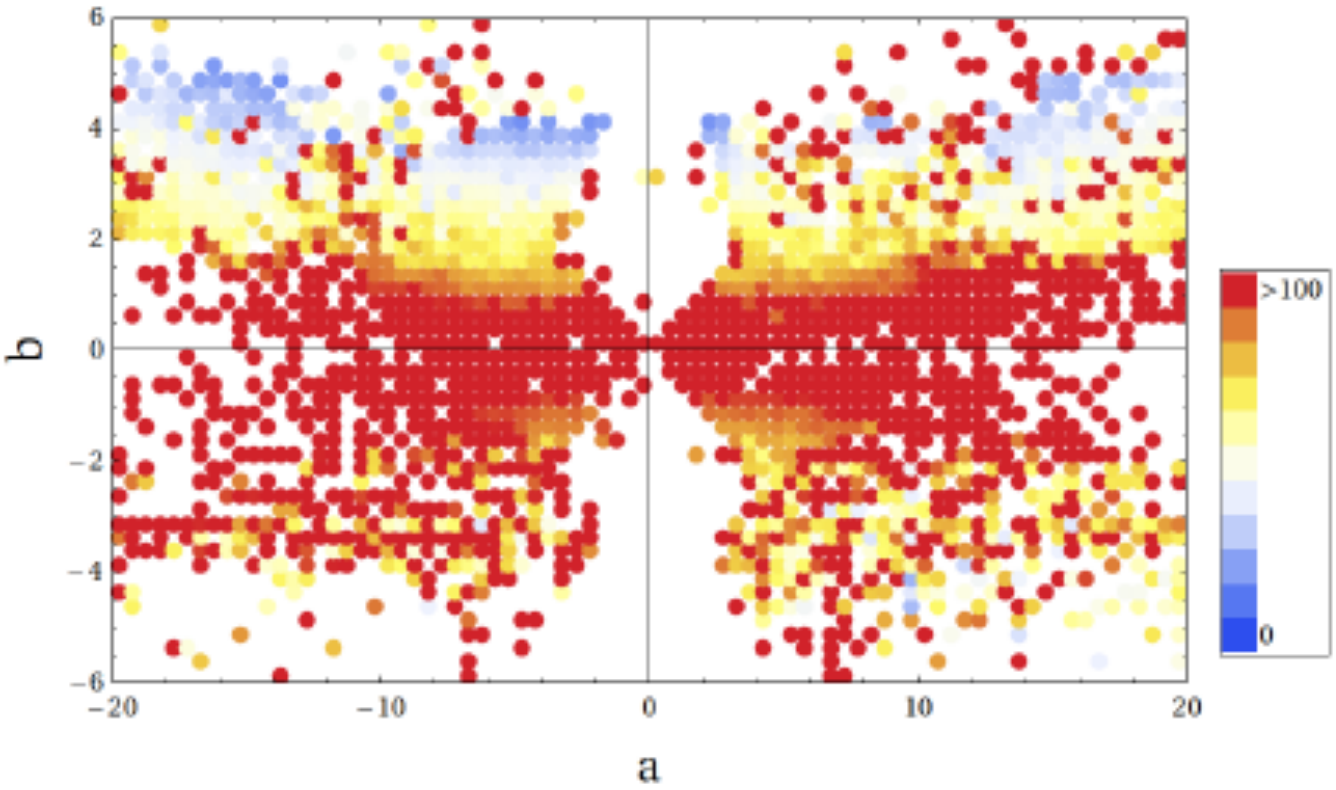
DM searches ✔

Δ



Fine tuning in the CGNMSSM

…fine tuning v/s gaugino mass ratios



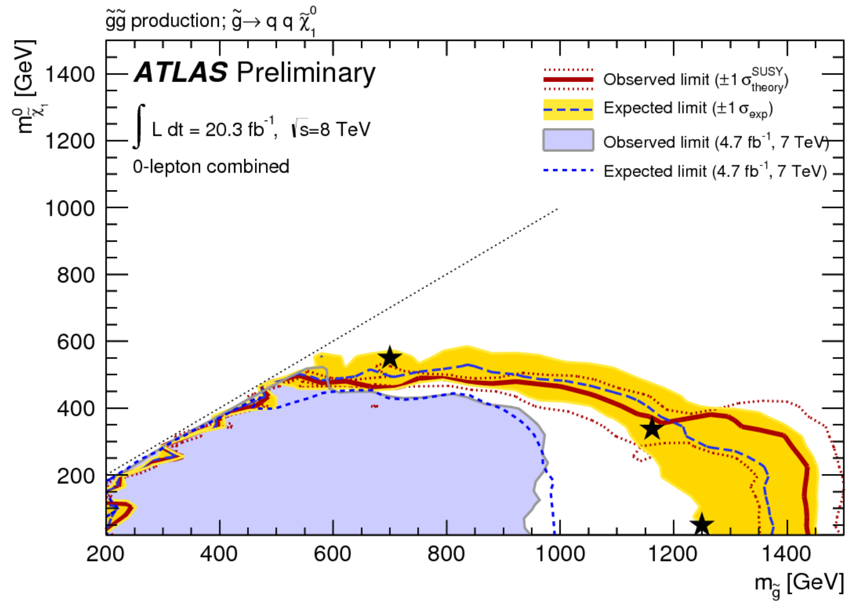
F.T.

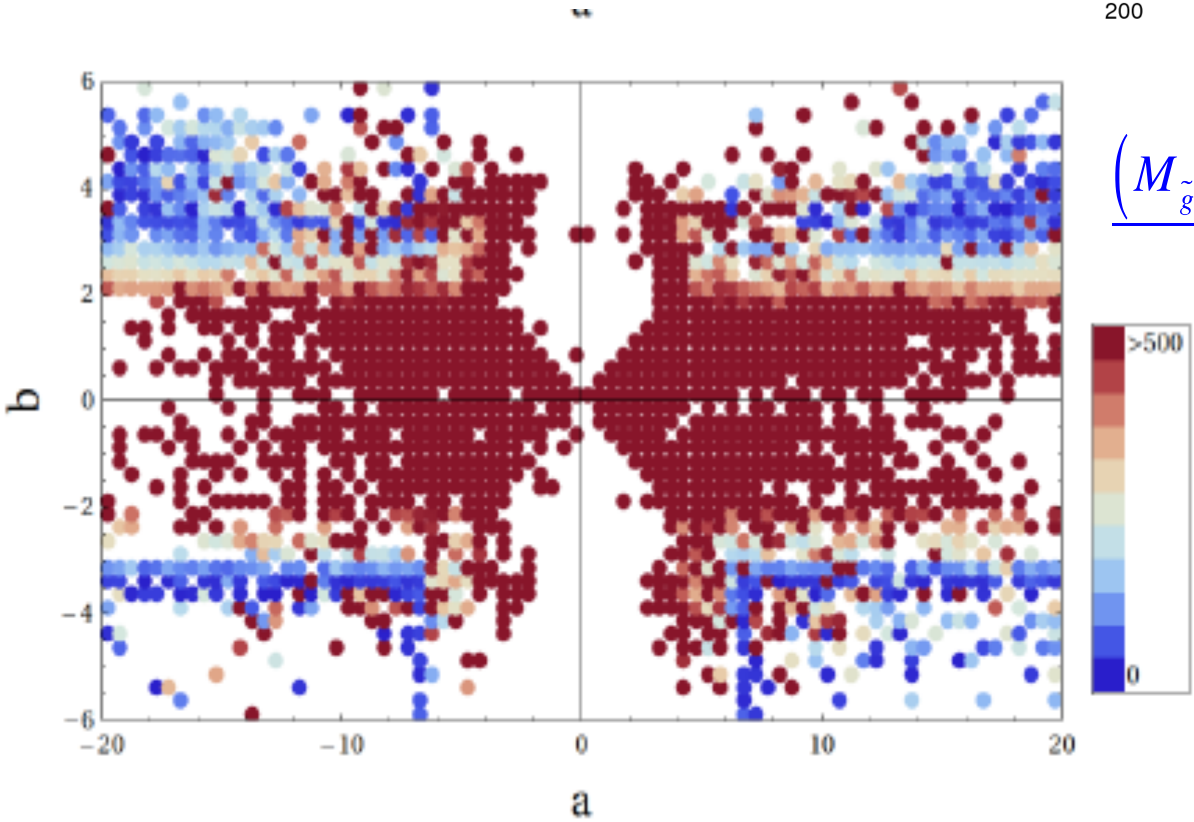
>100

0

$$M_3 = m_{1/2}, M_2 = b.m_{1/2}, M_1 = a.m_{1/2}$$

Compressed spectrum





$$\frac{\left(M_{\tilde{g}} - M_{LSP}^{neutralino}\right)}{GeV}$$

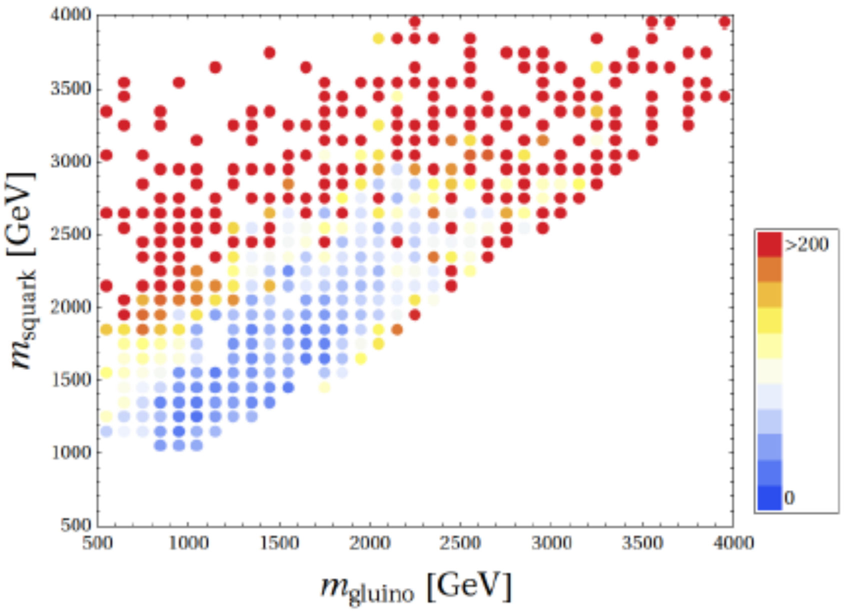
> 500

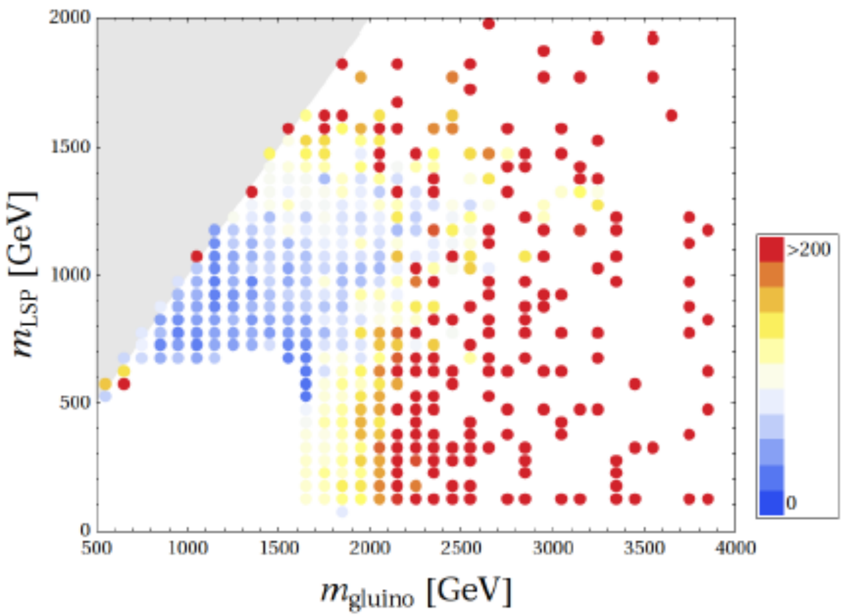
0

Masses v/s fine tuning

m_{squark}

m_{LSP}



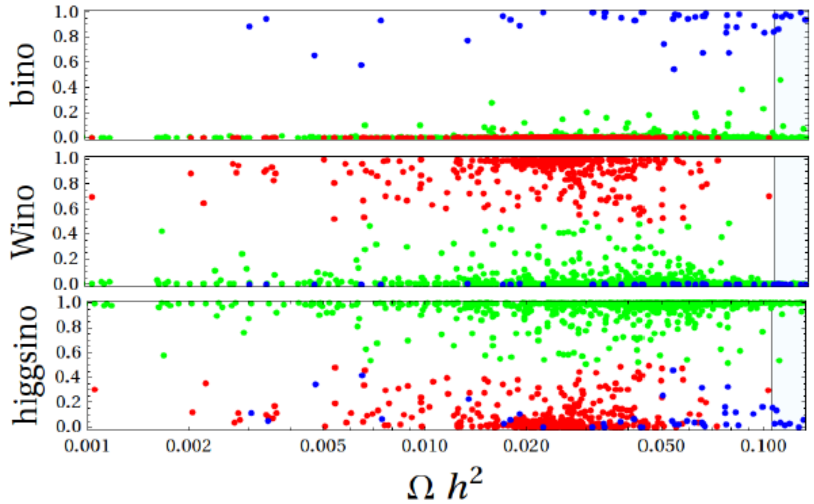


> 500

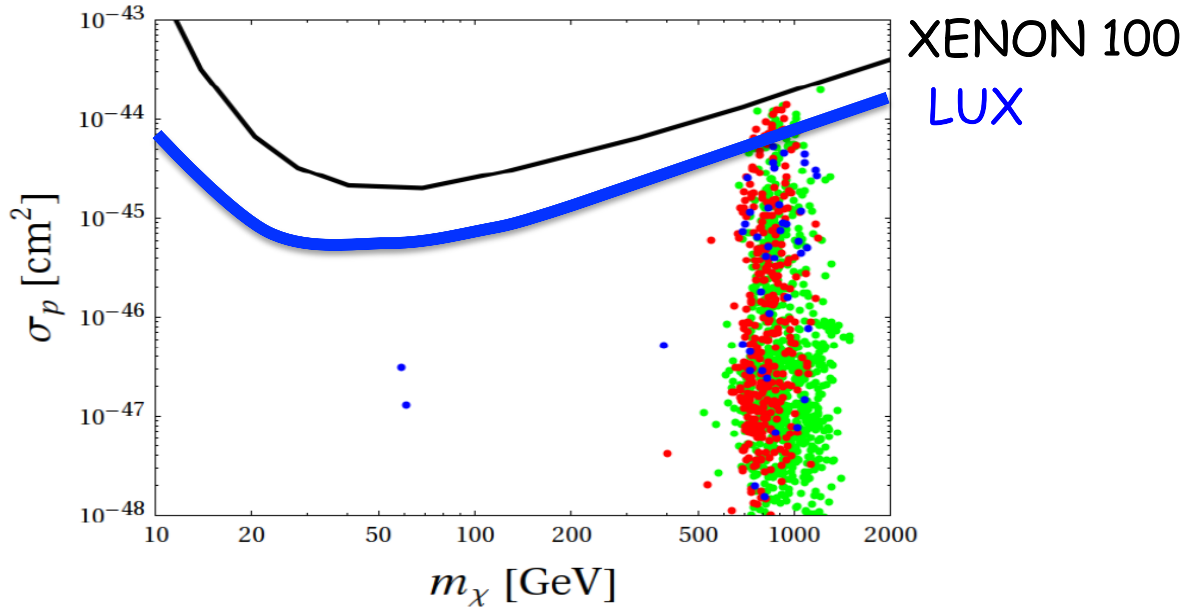
0

M_{gluino}

Dark matter



LSP composition



Direct DM searches

Summary

- GUTs $\Longrightarrow$ SUSY-GUTS (hierarchy problem)
- Low fine tuning not optional
- Fine tuning sensitive to SUSY spectrum

  ...scalar and gaugino focus points

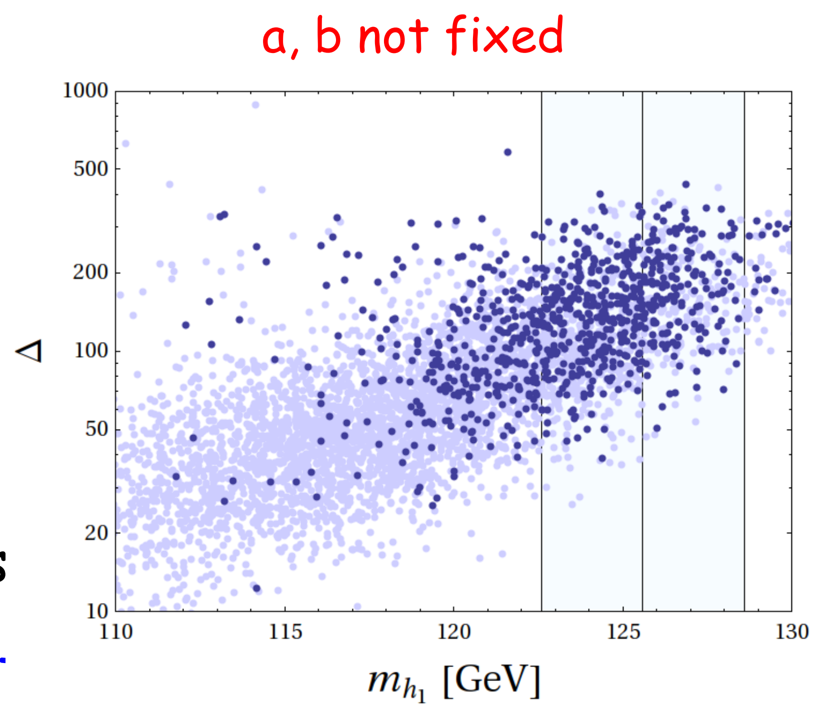
- $\Delta^{CMSSM} > 350$ ✗ $\Delta^{(C)MSSM} > 60$ ✗

  $\Delta^{CGMSSM} > 60$ ✗ $\Delta^{(C)GNMMS} > 20$ ✔

$$c.f. \quad \Delta^{CMSSM}_{Low\ scale} = (10-30), \quad m_{\tilde{t}} = (1-5)TeV$$

Barger et al

Summary



- GUTs ⟹ SUSY-GUTS
- Low fine tuning not optional
- Fine tuning sensitive to SUSY s

  ...scalar and gaugino focus poin

- $\Delta^{CMSSM} > 350$ ✗ $\Delta^{(C)MSSM} > 60$ ✗

  $\Delta^{CGMSSM} > 60$ ✗ $\Delta^{(C)GNMMS} > 20$ ✔

$$c.f. \quad \Delta^{CMSSM}_{Low\ scale} = (10-30), \quad m_{\tilde{t}} = (1-5)TeV$$

Barger et al

Summary

- GUTs $\Longrightarrow$ SUSY-GUTS (hierarchy problem)
- Low fine tuning not optional
- Fine tuning sensitive to SUSY spectrum

  ...scalar and gaugino focus points

- $\Delta^{CMSSM} > 350$ $\Delta^{(C)MSSM} > 60$

  $\Delta^{CGMSSM} > 60$ $\Delta^{(C)GNMMS} > 20$

- Well motivated SUSY models remain to be tested

  LHC14?

  Compressed spectra, TeV squarks and gluinos