IV. SUGRA (Supergravity)

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$$\left\{Q,\overline{Q}\right\} = 2\gamma^{\mu} P_{\mu}$$

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$$\{Q, \overline{Q}\} = 2\gamma^{\mu} P_{\mu}$$

Must go to local version of SUSY - SUGRA

$$\delta_1 B \sim \overline{\varepsilon}_1(x) F,$$
$$\delta_2 F \sim \varepsilon_2(x) \partial B$$

$$e.g.\ L = -\left(\partial^{\mu}\phi^{*}\right)\left(\partial_{\mu}\phi\right) - \frac{1}{2}\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi$$

Under global SUSY transformation

$$\left.\begin{aligned} \delta\phi &= \varepsilon\psi \\ \delta\psi &= -i\sigma^{\mu}\overline{\varepsilon}\,\partial_{\mu}\phi \end{aligned}\right\} \quad \begin{matrix} \delta L \equiv 0 \\ \text{(Total divergence)} \end{matrix}$$

Aitchison hep-ph/0505105

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$$\delta\psi = -i\sigma^{\mu}\overline{\varepsilon}\partial_{\mu}\phi$$

} $\delta L \equiv 0$ (Total divergence)

but, under local SUSY

$$\delta L = \partial_{\mu}\varepsilon^{\alpha}K_{\alpha}^{\mu} + h.c. \qquad \left(K_{\mu}^{\alpha} \equiv -\partial_{\mu}\phi^{*}\psi^{\alpha} - \frac{i}{2}\psi^{\beta}\left(\sigma_{\mu}\overline{\sigma}^{\nu}\right)_{\beta}^{\alpha}\partial_{\nu}\phi^{*}\right)$$

$$e.g.\ L = -\left(\partial^{\mu}\phi^{*}\right)\left(\partial_{\mu}\phi\right) - \frac{1}{2}\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi$$

Under global SUSY transformation

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Introduce spin 3/2 gravitino, Ψ, new gauge field

$$\Psi^{\mu}_{\alpha} \to \Psi^{\mu}_{\alpha} + \frac{1}{k}\partial^{\mu}\varepsilon_{\alpha}$$

$$L_{N} \equiv kK^{\alpha}_{\mu}\Psi^{\mu}_{\alpha}$$

δL_{N} cancels δL

$$L + L_N = -\left(\partial^\mu \phi^*\right)\left(\partial_\mu \phi\right) - \frac{1}{2}\overline{\psi}\gamma^\mu \partial_\mu \psi + k K_\mu^\alpha \Psi_\alpha^\mu$$

$$K_\mu^\alpha \equiv -\partial_\mu \phi^* \psi^\alpha - \frac{i}{2}\psi^\beta \left(\sigma_\mu \overline{\sigma}^\nu\right)_\beta^\alpha \partial_\nu \phi^*$$

$$L+L_N=-\left(\partial^\mu\phi^*\right)\left(\partial_\mu\phi\right)-\frac{1}{2}\overline{\psi}\gamma^\mu\partial_\mu\psi+kK_\mu^\alpha\Psi_\alpha^\mu$$

$$K_\mu^\alpha\equiv-\partial_\mu\phi^*\psi^\alpha-\frac{i}{2}\psi^\beta\left(\sigma_\mu\overline{\sigma}^\nu\right)_\beta^\alpha\partial_\nu\phi^*$$

But $\delta\left(L+L_N\right)=k\left(\delta K_\mu^\alpha\right)\Psi_\alpha^\mu=k\overline{\psi}_\mu\gamma_\nu\varepsilon T^{\mu\nu}$

cancelled by $\delta L_g\equiv\delta\left(-g_{\mu\nu}T^{\mu\nu}\right)$

provided $\delta g_{\mu\nu}=k\overline{\psi}_\mu\gamma_\nu\varepsilon$

i.e. necessary to include gravity

Gravity supermultiplet $\left(g_{\mu\nu},\Psi_\mu^\alpha\right)$

D=4, N=1 supergravity

$$L = \left\{ \int d^4\theta \mathbf{E} \left(-3M_P^2 \exp\left(-K(Q^\dagger, e^{-V}Q) / 3M_P^2 \right) \right) \right.$$

$$\left. + \int d^2\theta \, \Xi \left(W(Q) + f(Q) W_\alpha W^\alpha + h.c. + \frac{1}{2} M_P^2 e^{-K/3M_P^2} R + .. \right) \right\}$$

$\mathbf{E}, \Xi$ Vielbein superfields

D=4, N=1 supergravity

$$L = \sqrt{-g}\left\{\int d^4\theta\left(-3M_P^2 \exp\left(-K(Q^\dagger, e^{-V}Q)/3M_P^2\right)\right)\right.$$

$$\left.+\int d^2\theta\left(W(Q)+f(Q)W_\alpha W^\alpha\right)+h.c.+\frac{1}{2}M_P^2 e^{-K/3M_P^2}R+..\right\}$$

D=4, N=1 supergravity

$$L=\sqrt{-g}\left\{\int d^4\theta\left(-3M_P^2\exp\left(-K(Q^\dagger,e^{-V}Q)/3M_P^2\right)\right)\right.$$
$$\left.+\int d^2\theta\left(W(Q)+f(Q)W_\alpha W^\alpha\right)+h.c.+\frac{1}{2}M_P^2e^{-K/3M_P^2}R+..\right\}$$

Performing a Weyl transformation $g_{\mu\nu}\rightarrow e^{K/3M_P^2}g_{\mu\nu}$ and integrating out auxiliary fields one gets canonical form

$$L=\sqrt{-g}\left\{\frac{M_P^2}{2}R+K_{ij}\left(\tilde{q}^\dagger,\tilde{q}\right)D_\mu\tilde{q}^{i\dagger}D^\mu\tilde{q}^j-\mathrm{V}\left(\tilde{q}^\dagger,\tilde{q}\right)\right.$$
$$\left.-f(\tilde{q})\left(F_{\mu\nu}F^{\mu\nu}+iF_{\mu\nu}\tilde{F}^{\mu\nu}\right)+h.c.+\text{fermion terms}\right\}$$

Wess, Bagger

D=4, N=1 supergravity

i.e. defined by choice of the Kahler potential and superpotential

$$G = -K / M_P^2 - \ln\left(W / M_P^3\right) - \ln\left(W^* / M_P^3\right)$$

$$M_P = 2.4 \times 10^{18}\, GeV$$

D=4, N=1 supergravity

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$$G = -K / M_P^2 - \ln\left(W / M_P^3\right) - \ln\left(W^* / M_P^3\right)$$

Bosonic Lagrangian (M_P=1)

$K = \phi^* \phi$ - canonical kinetic term $\quad G_j^i = -\delta_j^i$

$$e^{-1} L_B = -\frac{1}{2} R - G_j^i D_\mu \phi_i D^\mu \phi^{*i} - e^{-G}\left[3 + G_k \left(G^{-1}\right)_l^k G^l\right]$$

$$e = \sqrt{-\det g_{\mu\nu}}, \quad G_j^i = \frac{\partial^2 G}{\partial \phi_i \partial \phi^{*j}}$$

D=4, N=1 supergravity

i.e. defined by choice of the Kahler potential and superpotential †

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$$D^\alpha = g G^i T_i^{\alpha j} \phi_j$$

† and gauge kinetic term $f(\phi)$

D=4, N=1 supergravity

i.e. defined by choice of the Kahler potential and superpotential

$$G = -K / M_P^2 - \ln\left(W / M_P^3\right) - \ln\left(W^* / M_P^3\right)$$

Bosonic Lagrangian (M_P=1)

$$e^{-1} L_B = -\frac{1}{2} R - G_j^i D_\mu \phi_i D^\mu \phi^{*i} - e^{-G}\left[3 + G_k \underbrace{\left(G^{-1}\right)_l^k G^l}_{F}\right] + \underbrace{\frac{1}{2} f_{\alpha\beta}^{-1} D^\alpha D^\beta}_{D}$$

$$D^\alpha = g G^i T_i^{\alpha j} \phi_j$$

SUSY breaking, F or D non-zero, i.e. $\langle G^i \rangle \neq 0$

For the case of canonical kinetic terms $G^i_j = -\delta^i_j, \quad f_{\alpha\beta} = \delta_{\alpha\beta}$

$$V_{local} = e^{\left(\frac{\phi_k^* \phi_k}{M_P^2}\right)} \left(\left[\left| \frac{\partial W}{\partial \phi_i} + \frac{\phi_i^* W}{M_P^2} \right|^2 \right] - \frac{3|W|^2}{M_P^2} \right) + \frac{1}{2} g \left(\phi^{i*} T_i^{\alpha j} \phi_j \right)^2$$

$$c.f. \quad V_{global} = \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \frac{1}{2} g \left(\phi^{i*} T_i^{\alpha j} \phi_j \right)^2$$

SUSY breaking:

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≠0…F breaking

≠0…D breaking

$$c.f. \quad V_{global} = \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \frac{1}{2} g \left(\phi^{i*} T_i^{\alpha j} \phi_j \right)^2 \geq 0$$

F≠0, V=0 possible in local case!

SUSY breaking: Goldstino/Gravitino

Global SUSY:

Fermion partner of non-vanishing auxiliary field massless - goldstino

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Fermion partner of non-vanishing auxiliary field massless - goldstino

Local SUSY:

$$e^{(-G/2)}\overline{\psi}_\mu \sigma^{\mu\nu}\psi_\nu - e^{(-G/2)}G^i\overline{\psi}_\mu\gamma^\mu\chi_i + e^{(-G/2)}\left[G^{ij} - G^iG^j - G^l\left(G^{-1}\right)_l^k G_k^{ij}\right]\chi_i\chi_j$$

"Super Higgs" effect

$\left\langle G^i \right\rangle \neq 0$, spin ½ Goldstone fermion mixes with gravitino, making massive state

$$m_{3/2} = M_P e^{-G/2}$$

SUSY breaking: Goldstino/Gravitino

Global SUSY:

Fermion partner of non-vanishing auxiliary field massless - goldstino

Local SUSY:

$$e^{-1}L_B = -\frac{1}{2}R - G^i_j D_\mu\phi_i D^\mu\phi^{*i} - e^{-G}\left[3 + G_k \left(G^{-1}\right)^k_l G^l\right] + \frac{1}{2} f^{-1}_{\alpha\beta} D^\alpha D^\beta$$

$$\left\langle G^i \right\rangle \neq 0, \quad m_{3/2} = M_P e^{-G/2}$$

If D=0 +canonical kinetic terms $\Rightarrow G^i_j = -\delta^i_j,\ f_{\alpha\beta} = \delta_{\alpha\beta}$

$$V = 0 \Rightarrow G_i G^i = 3$$

suppressed

$$M_S^2 = \sqrt{3} M_P^2 e^{-G/2}, \quad m_{3/2} = M_S^2 / \sqrt{3} M_P$$

(Soft) SUGRA breaking terms:

Hidden sector breaking:

$$W = W_{\mathrm{vis}}\left(\phi_i\right) + W_{hid}\left(X\right), \qquad K = \phi^{*i}\phi_i + X^* X$$

$$V_{local} = e^{\left(\frac{\phi^{*i}\phi_i + X^* X}{M_P^2}\right)}\left(\left[\left|\frac{\partial W}{\partial \phi_i} + \frac{\phi_i^* W}{M_P^2}\right|^2 + \left|\frac{\partial W}{\partial X} + \frac{X^* W}{M_P^2}\right|^2\right] - \frac{3\left|W\right|^2}{M_P^2}\right)$$

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$$\langle X \rangle = xM_P, \langle W_{hid} \rangle = wM_P^2, \langle \delta W_{hid} / \delta X \rangle = w' M_P$$

$$m_{3/2} = |F_X| / \sqrt{3} M_P = e^{|x|^2/2}|w|$$

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$$e^{-|x|^2/2} W_{\rm vis}$$

$$\begin{aligned} V &= (W^*_{\rm vis})_i (W_{\rm vis})^i + m^2_{3/2}\varphi^{*i}\varphi_i \\ &+ e^{|x|^2/2}\left[w^*\varphi_i (W_{\rm vis})^i + (x^* w'^* + |x|^2 w^* - 3w^*) W_{\rm vis} + {\rm c.c.}\right]. \end{aligned}$$

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$$e^{-|x|^2/2} W_{\text{vis}}$$

$$V = (W_{\text{vis}}^*)_i (W_{\text{vis}})^i + m_{3/2}^2 \varphi^{*i} \varphi_i + e^{|x|^2/2} \left[w^* \varphi_i (W_{\text{vis}})^i + (x^* w'^* + |x|^2 w^* - 3w^*) W_{\text{vis}} + \text{c.c.} \right].$$

$$m_{\phi_i}^2 = m_0^2 = m_{3/2}^2$$

$$A_0 = -x^* \langle F_X \rangle / M_P = -x^* \sqrt{3} m_{3/2}$$

$$B_0 = \left(\frac{1}{x + w'^* / w^*} - x^* \right) \sqrt{3} m_{3/2}$$

Gaugino masses

$$f_{ab} = \delta_{ab}\left[\frac{1}{g_a^2} + \frac{f_X X}{M_P} + \ldots\right]$$

$$\int d^2\theta\, f_{ab}\, Tr\, W^{a\alpha} W_\alpha^b + h.c.$$

$$W^\alpha W_\alpha = \lambda\lambda + \theta\left(\lambda\sigma_{\mu\nu}F^{\mu\nu} + \ldots\right) + \theta^2\left(F_{\mu\nu}F^{\mu\nu} + \ldots\right)$$

$$\Rightarrow \frac{f_X F_X}{2M_P}\lambda\lambda$$

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$$\Rightarrow \frac{f_X F_X}{2M_P}\lambda\lambda$$

$$m_{1/2} = \frac{\sqrt{3}}{2}\mathrm{Re}\left(f_X\right) m_{3/2}$$

Universal gaugino masses if f_X the same for the 3 gauge groups (X gauge singlet)

Dynamical origin of SUSY breaking (c.f. Technicolour)

Gaugino condensate

Global SUSY ✗

Only supermultiplet containing $\lambda\lambda$

$$W^{\alpha}W_{\alpha} = \lambda\lambda + \theta\left(\lambda\sigma_{\mu\nu}F^{\mu\nu} + \ldots\right) + \theta^{2}\left(F_{\mu\nu}F^{\mu\nu} + \ldots\right)$$

Not F-term

Dynamical origin of SUSY breaking (c.f. Technicolour)

Gaugino condensate

SUGRA ✔

Fermions also appear in F (and D) terms:

$$F_i = \exp(-G/2)(G^{-1})_i^j G_j + \tfrac{1}{4} f_{\alpha\beta k}(G^{-1})_i^k \lambda^\alpha \lambda^\beta - (G^{-1})_i^k G_k^{jl} \chi_j \chi_l - \tfrac{1}{2}\chi_i (G_j \chi^j) .$$

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$$M_S^2 = \frac{\mu^3}{M_P} = \frac{\langle \lambda\lambda \rangle}{M_P}, \qquad \mu \approx M_P e^{-\frac{1}{b_0 g^2(M_P)}}$$

Natural hierarchy generation

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Natural hierarchy generation

$$m_{3/2} = \frac{M_S^2}{M_P} = \frac{\mu^3}{M_P^2}$$

Anomaly mediation

Classical (tree level) SUSY is superconformal (scale) invariant

… broken by RG logarithmic terms – superconformal anomaly

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$$L = e\left\{\int d^4\theta\left(-3M_P^2 e^{-K/3M_P^2}\right)\Phi^\dagger\Phi + \int d^2\theta\left(\Phi^3 W + fW_\alpha W^\alpha\right)\right\} + ..$$

Φ chiral superfield, part of supergravity supermultiplet

$$\Xi = e\Phi^3, \ \mathrm{E} \propto \Phi^\dagger\Phi$$

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Φ chiral superfield, part of supergravity supermultiplet

Consider "sequestered" sector

$$K = -3M_P^2 \ln\left(1 - \frac{k_{vis}}{3M_P^2} - \frac{k_{hid}}{3M_P^2}\right), \quad W = W_{vis} + W_{hid}, \; fW^2 = f_{vis}W_{vis}^2 + f_{hid}W_{hid}^2$$

$$L = e\left\{\int d^4\theta\left(k_{vis} + k_{hid}\right)\Phi^\dagger\Phi + \int d^2\theta\left(\Phi^3\left[W_{vis} + W_{hid}\right] + ..\right)\right\} +$$

SUSY breaking in hidden sector communicated only by common gravitational supermultiplet

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Hidden sector SUSY breaking induces F-term $\Phi = 1 + F_\Phi\theta\theta$

$$m_{3/2} = F_\Phi$$

At tree level, visible sector supersymmetric

$\Phi^3 Q_{vis}^3 \rightarrow \widetilde{Q}_{vis}^3, etc$ (tree level scale invariance – super Weyl transformation)

but scale invariance broken at quantum level through cut-off

Anomaly mediation

$$e.g. \int d^2\theta \frac{1}{4g^2(\mu/\Lambda_{UV}\Phi)} W_\alpha W^\alpha = \int d^2\theta \left(\frac{1}{4g^2(\mu/\Lambda_{UV})} - \frac{b_i}{32\pi^2} \ln \Phi \right) W_\alpha W^\alpha$$

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Expanding the log and performing the superpace integral gives

$$m_\lambda = \frac{\beta(g^2)}{2g^2} m_{3/2}$$

Anomaly mediation

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$$m_\lambda = \frac{\beta(g^2)}{2g^2} m_{3/2}$$

Note $\dfrac{\beta(g^2)}{2g^2 m_\lambda}$ is RG invariant, so result is true at any scale!

Anomaly mediation

$$W(\Phi) = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

$$L_{\rm SB} = (m^2)^j_i \phi^i \phi_j + \left(\frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} M \lambda\lambda + {\rm h.c.} \right)$$

$$M = M_0 \frac{\beta_g}{g},$$

$$h^{ijk} = -M g \frac{dY^{ijk}}{dg},$$

$$b^{ij} = -M g \frac{d\mu^{ij}}{dg},$$

$$(m^2)^i{}_j = \frac{g^2}{2\beta_g} |M|^2 \frac{d\gamma^i{}_j}{dg}.$$

$$M_0 \equiv m_{3/2}$$

Relations apply at any scale

Anomaly mediation

BUT, dominant contribution to masses is

$$
\begin{aligned}
m_{\tilde{q}}^2 &= \frac{|F_\phi|^2}{(16\pi^2)^2}\left(8g_3^4 + \ldots\right), \\
m_{\tilde{e}_L}^2 &= -\frac{|F_\phi|^2}{(16\pi^2)^2}\left(\frac{3}{2}g_2^4 + \frac{99}{50}g_1^4\right) \\
m_{\tilde{e}_R}^2 &= -\frac{|F_\phi|^2}{(16\pi^2)^2}\frac{198}{25}g_1^4
\end{aligned}
$$

Need additional contribution e.g. "mirage mediation"

Gravitinos

If gravitinos once in thermal equilibrium then $m_{3/2} < 1keV$

Otherwise require inflation and bound on reheat temperature …

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otherwise inflation and bound on reheat temperature ...

Goldstino

$$\psi_\mu \sim i\sqrt{\frac{2}{3}}\frac{1}{m_{3/2}}\partial_\mu \psi \quad if\ E \gg m_{3/2}$$

$\begin{pmatrix} A_\mu \\ \lambda \end{pmatrix}$ $\begin{pmatrix} \phi \\ \psi \end{pmatrix}$

$$L = \frac{im_\lambda}{8\sqrt{6}m_{3/2}M_P}\overline{\psi}\left[\gamma_\mu,\gamma_\nu\right]\lambda F_{\mu\nu} + \frac{m_\chi^2 - m_\phi^2}{\sqrt{3}m_{3/2}M_P}\left(\overline{\psi}\chi_L\right)\phi^* + h.c.$$

D=5..dominates at high temperature

Gravitinos

If gravitinos once in thermal equilibrium then $m_{3/2} < 1keV$

otherwise inflation and bound on reheat temperature …

Goldstino

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$\begin{pmatrix} A_\mu \\ \lambda \end{pmatrix}$ $\begin{pmatrix} \phi \\ \psi \end{pmatrix}$

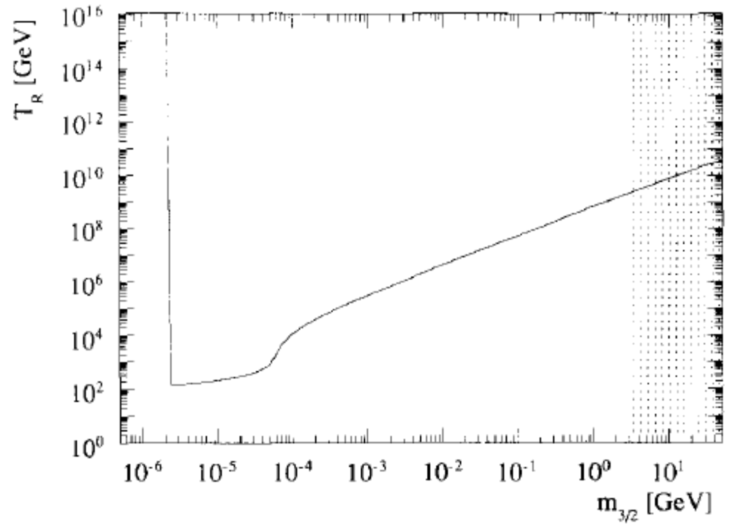
D=5..dominates at high temperature

$$\sigma_{A^a + A^b \to \psi + \lambda^c,\ \ldots} = \left(g^2 m_G^2 / 24\pi M_P^2 m_{3/2}^2\right) \times \text{(group theory factor)}$$

Boltzmann equation:

$$\frac{dn_{3/2}}{dt} + 3Hn_{3/2} = \langle \sigma_{tot} v_{rel} \rangle n_{rad}^2 + \sum_i n_i \langle \Gamma_i \rangle$$

$\propto T^3$



Bound on reheat temperature if is LSP

More generally:

$$\Omega_{3/2}h^2 \simeq 0.2 \left(\frac{T_{\rm rh}}{10^{10}\,{\rm GeV}} \right) \left(\frac{100\,{\rm GeV}}{m_{3/2}} \right) \left(\frac{m_{\tilde{g}}(\mu)}{1\,{\rm TeV}} \right)^2$$

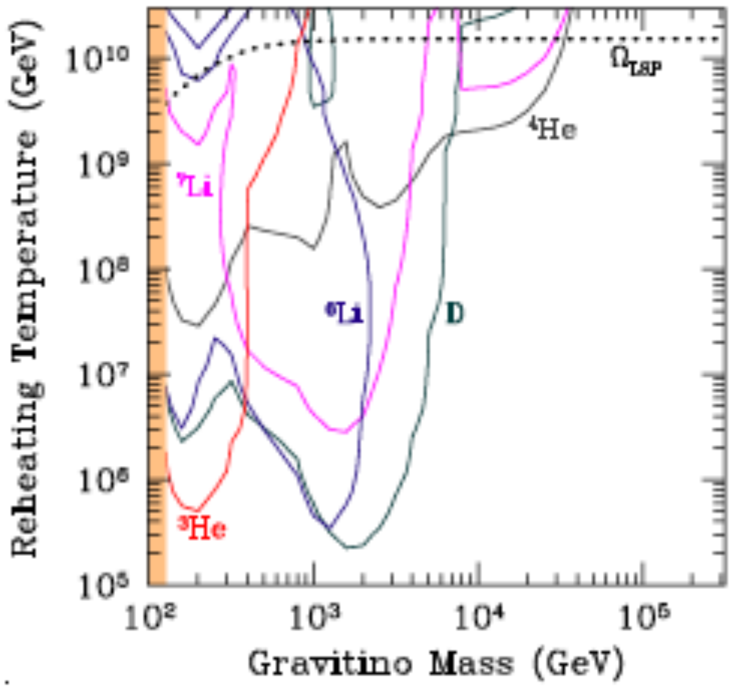


Figure 2: BBN constraints for the Case 1 at 95 % C.L. Each solid line shows upper bound on the reheating temperature from D, ^{3}He, ^{4}He, ^{6}Li, or ^{7}Li. The dotted line is the upper bound on the reheating temperature from the overclosure of the universe.

$m_{3/2}$	Case 1	Case 2	Case 3	Case 4
300 GeV	1×10^{6} (^{3}He)	4×10^{5} (^{3}He)	1×10^{6} (^{3}He)	–
1 TeV	5×10^{5} (^{6}Li)	9×10^{5} (^{6}Li)	3×10^{5} (^{6}Li)	3×10^{6} (^{6}Li)
3 TeV	5×10^{5} (D)	4×10^{5} (D)	2×10^{5} (D)	5×10^{5} (D)
10 TeV	2×10^{9} (^{4}He)	2×10^{9} (^{4}He)	2×10^{9} (^{4}He)	2×10^{9} (^{4}He)
30 TeV	9×10^{9} (^{4}He)	8×10^{9} (^{4}He)	7×10^{9} (^{4}He)	8×10^{9} (^{4}He)

Table 2: Upper bound on the reheating temperature (in units of GeV) from BBN for Cases 1 – 4. The light element which gives the most stringent bound is indicated in the parenthesis.

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R-symmetry restoration

If gravitinos once in thermal equilibrium then $m_{3/2} < 1keV$

otherwise inflation and bound on reheat temperature …

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If R-symmetry restored at high temperature gaugino masses vanish

$$\Omega_{3/2} h^2 = 0.2 \frac{16.6}{\sqrt{N}}\left(\frac{10^{10} GeV}{\lambda \Lambda_*}\right)\left(\frac{m_{\tilde{g}}(\mu)}{1 TeV}\right)^2$$

independent of reheat temperature!

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$\Longrightarrow$ Baryogenesis $\left(\Delta\left(B-L\right)=0\right)$

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→ Resonant leptogenesis

→ Sneutrino oscillation

→ R-symmetry restoration