Lectures on Particle Cosmology- III+IV

Pre-SUSY 2014, Manchester

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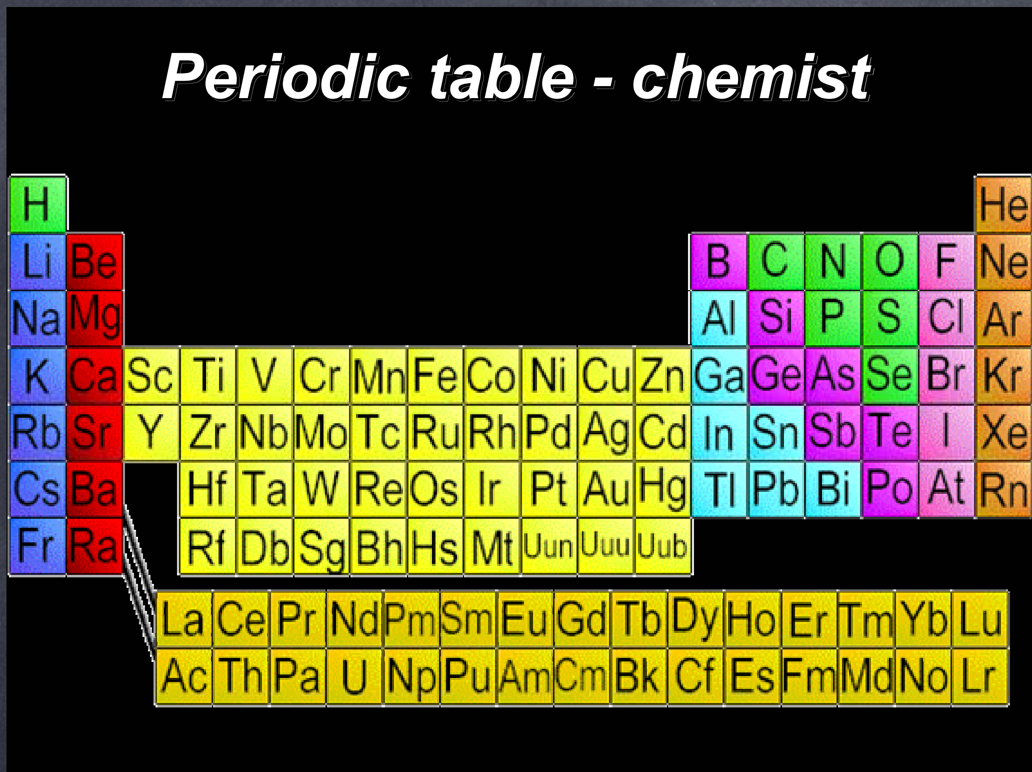
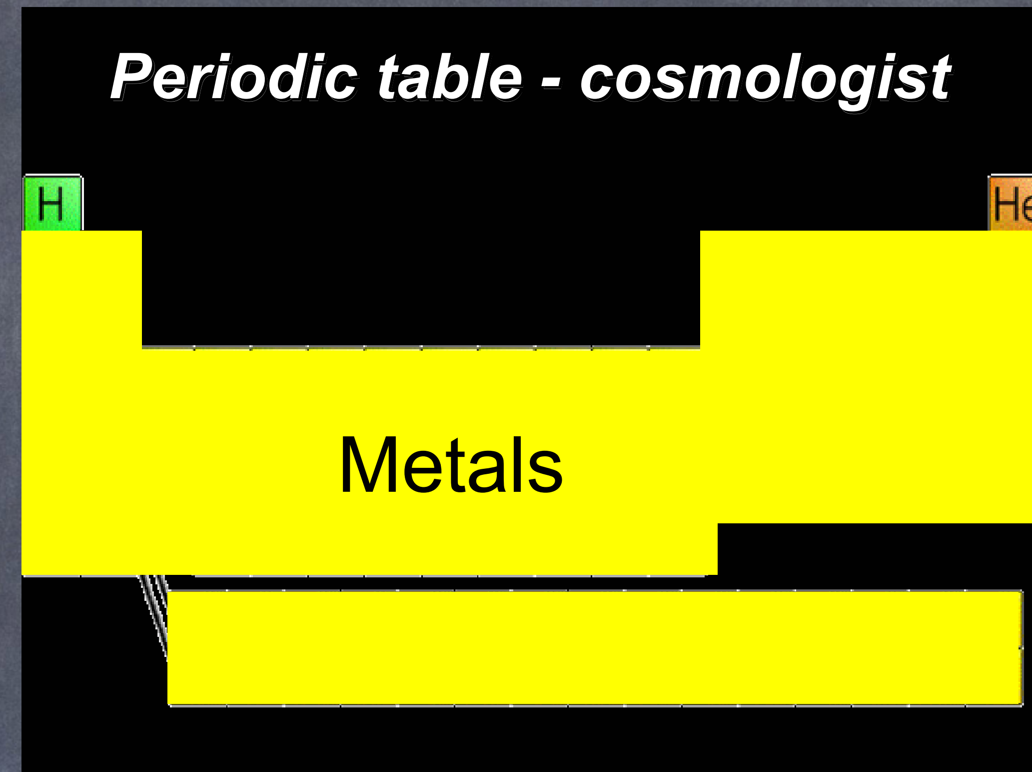
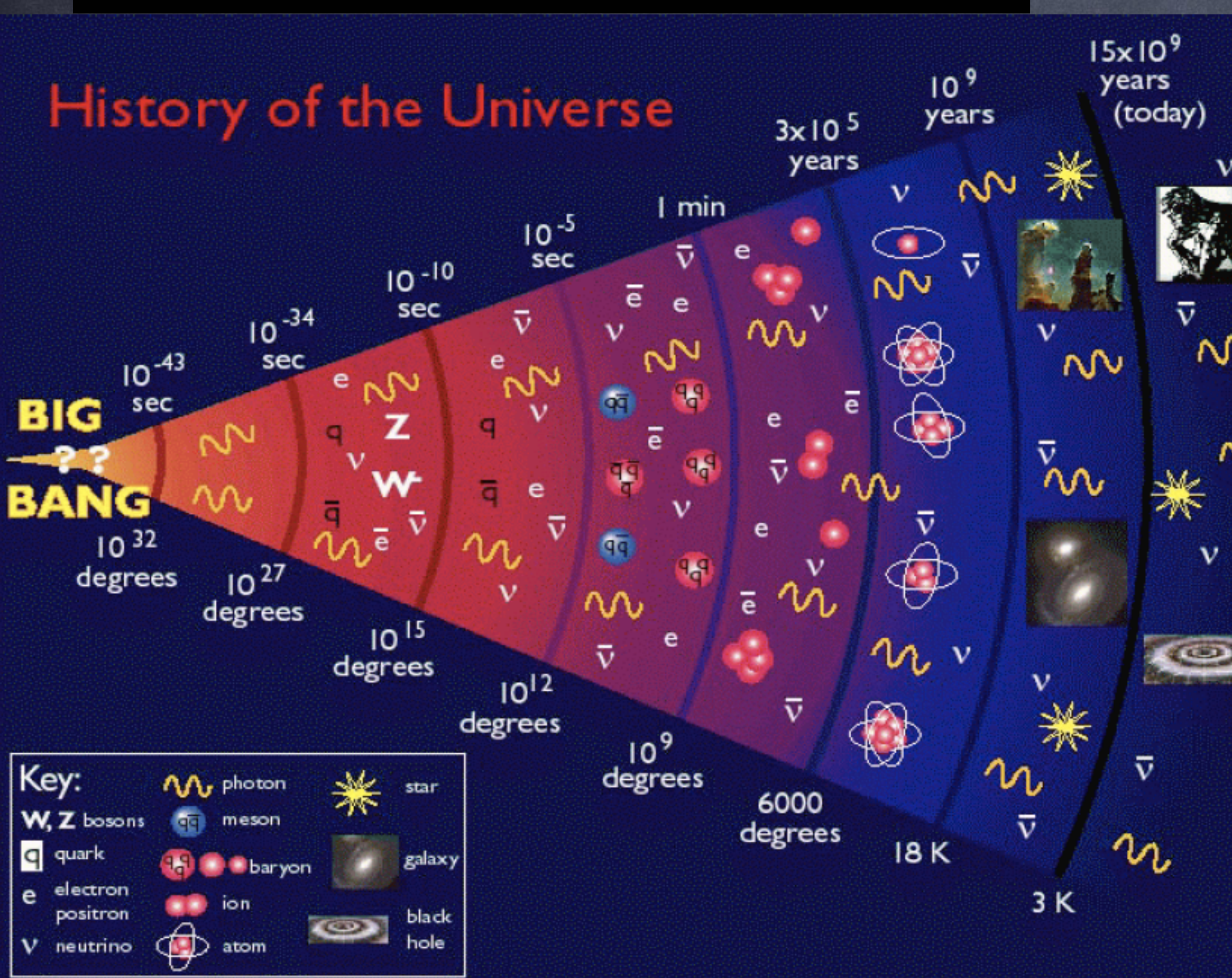
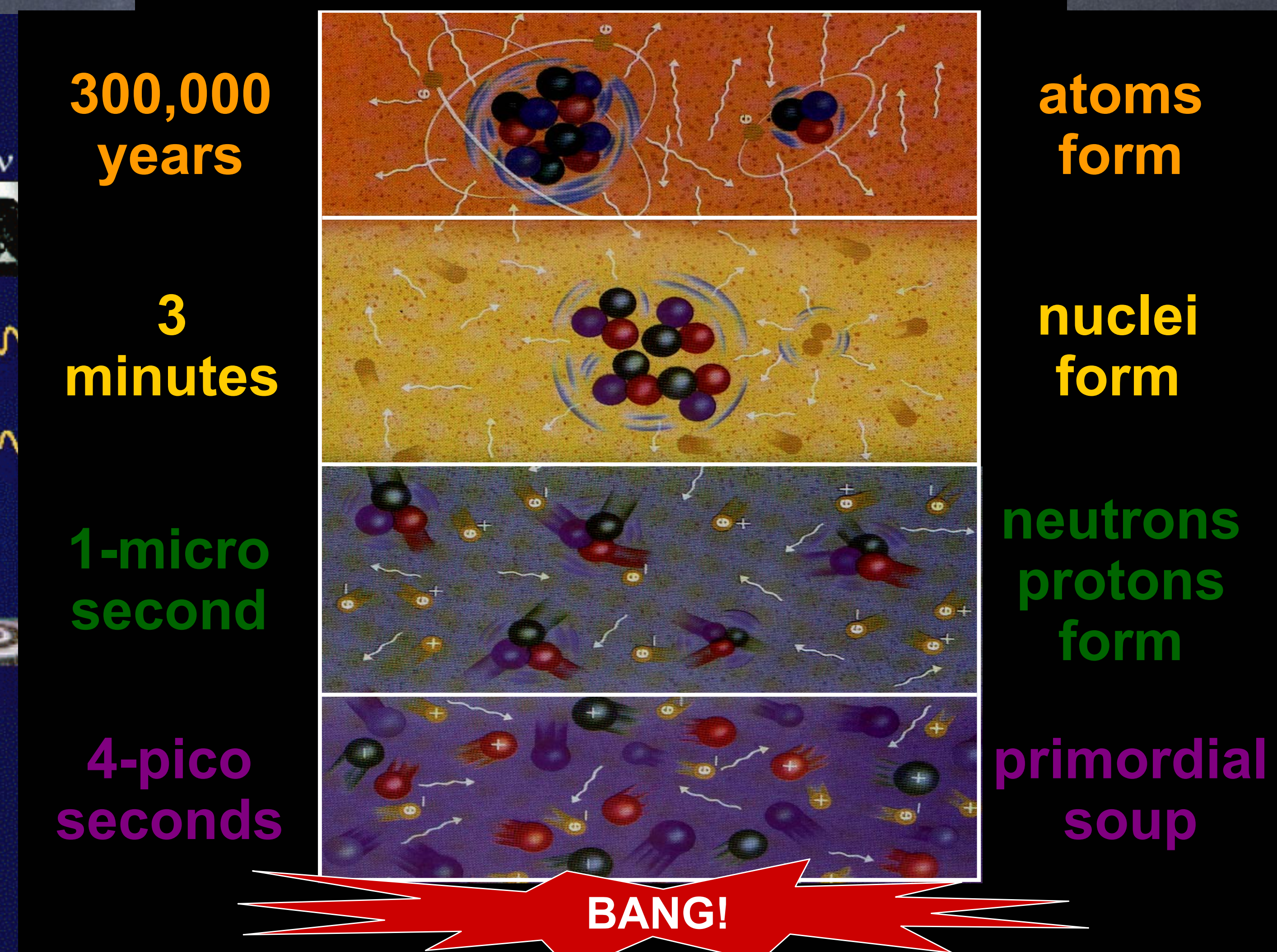
Synthesis of Light Elements (BBN)

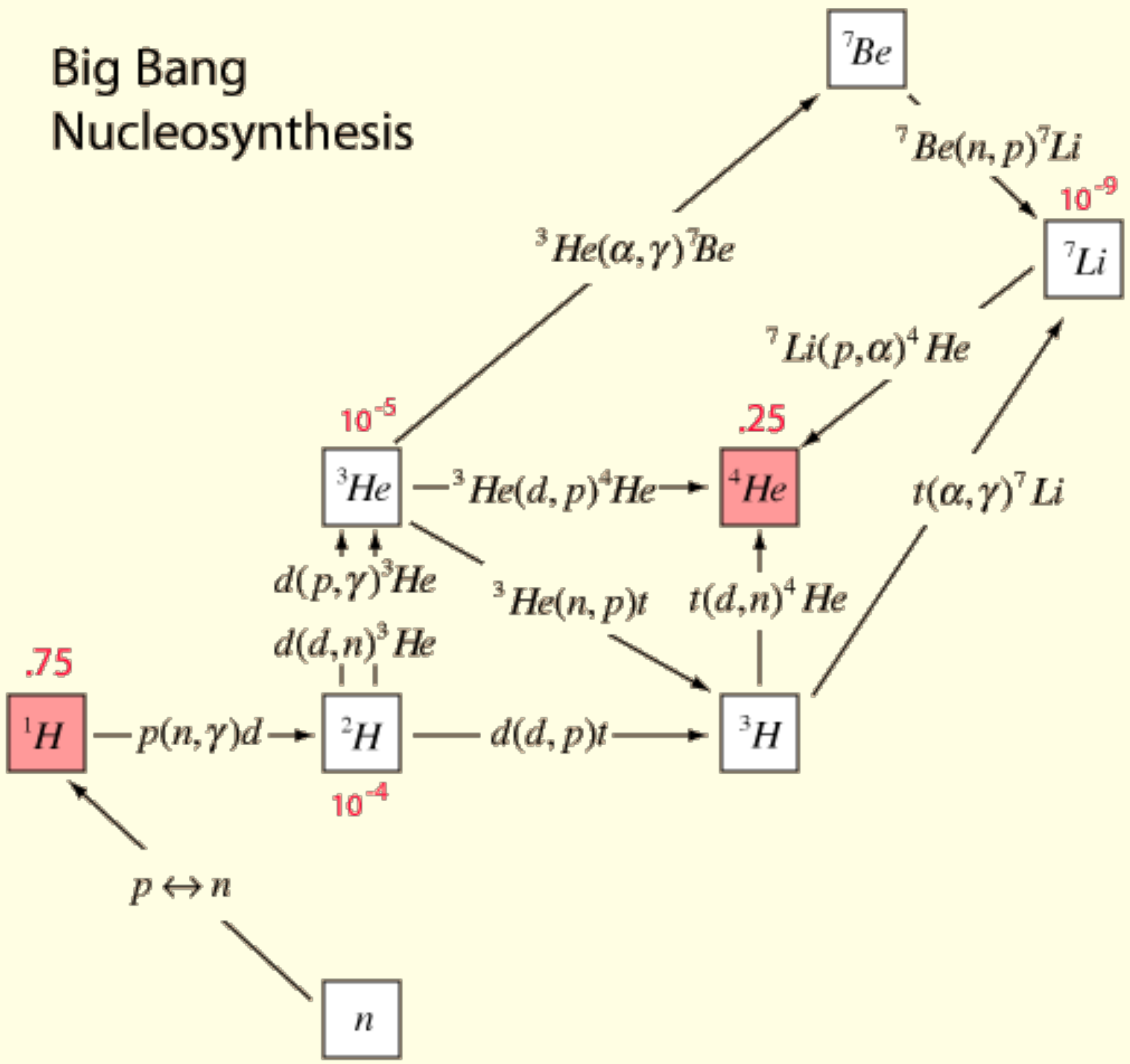








Big Bang Nucleosynthesis



$$\eta \equiv \frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma} \ : \ \text{Baryon Asymmetry}$$

$$X_n \to X_{n,eq} \equiv \frac{n_n}{n_n + n_p}$$

$$\sim (1/6) \ ; \ X_n \sim \frac{1}{6} \exp[-t/\tau_{n \to pe\nu}] \sim 0.15 \times 0.74 \sim 0.11$$

$$Y_P(^4\text{He}) \approx 2X_n \sim 0.22$$

$$T \gg 1\text{MeV} : \quad n + \nu_e \leftrightarrow p + e^-,$$
$$n_n \simeq n_p$$

$$T = T_F \simeq 1\text{MeV} : \quad \Gamma_{n\leftrightarrow p} \simeq H(T)$$
$$\frac{n_n}{n_p} \simeq e^{(m_n - m_p)/T_F} \simeq \frac{1}{6}$$

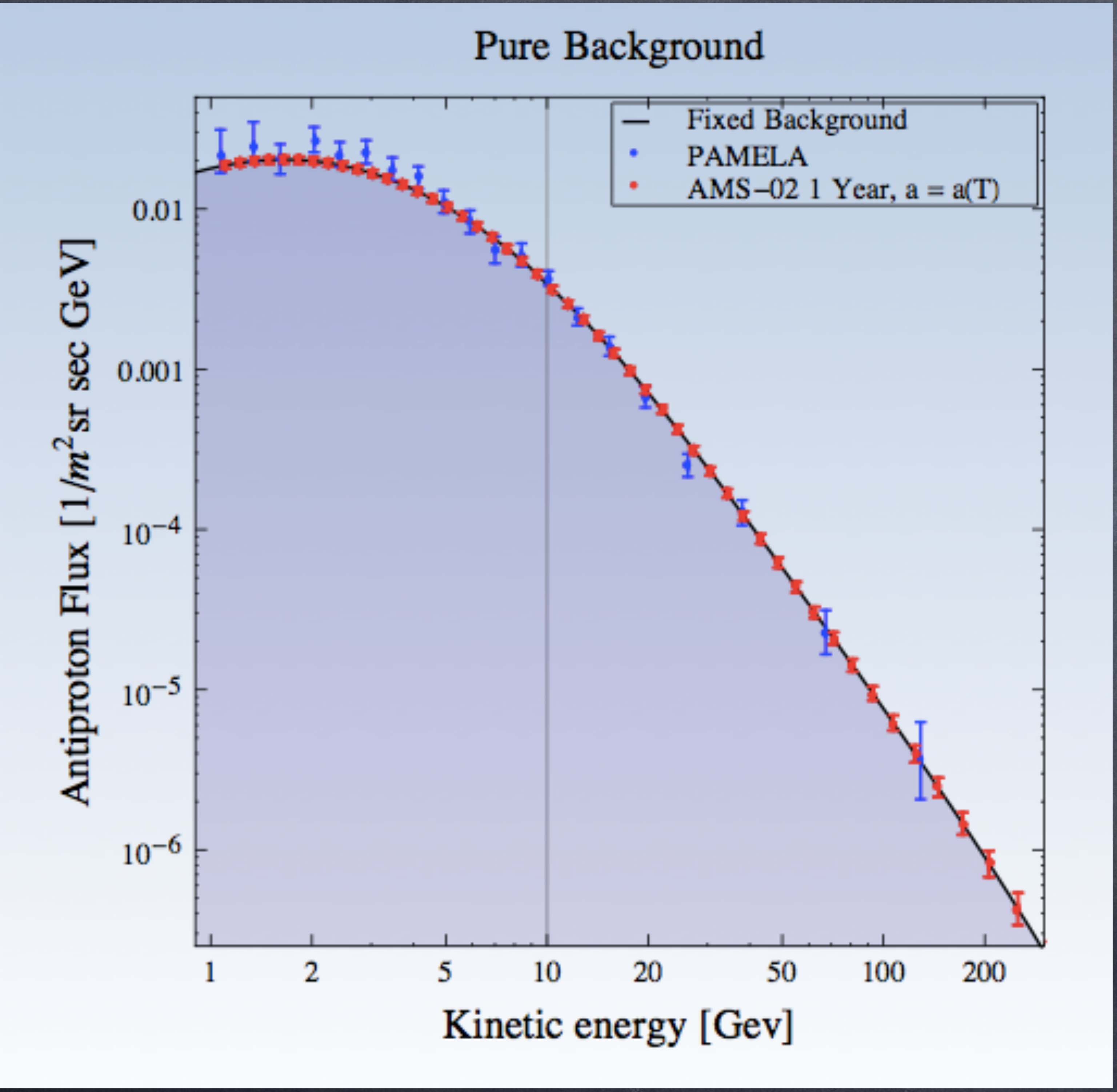
$$T = 0.3 - 0.1 \text{ MeV}$$
$$\text{Depending on } \eta = \frac{n_B{}'}{n_\gamma} \quad \frac{n_n}{n_p} = \frac{1}{7} - \frac{1}{6}$$

$$^4He \text{ mass fraction :}$$
$$Y \equiv \frac{2n_n}{n_n + n_p} \sim 0.22 \leftrightarrow \frac{n_n}{n_p} \sim \frac{1}{6}$$

$$X_A \propto \eta^{A-1} \qquad A = p + n$$

Rest of the elements have been synthesised in stars

Constraints on anti-protons



anti-protons are produced in secondaries

$$pp \to 3p + \bar{p}$$

$$\mathrm{DM} + \mathrm{DM} \;\to\; \mathrm{prim}\ \mathrm{prim}\ \cdots \;\to \bar{p}$$

Annihilating/decaying DM into quarks and gauge bosons => hadroniation leads to anti-protons

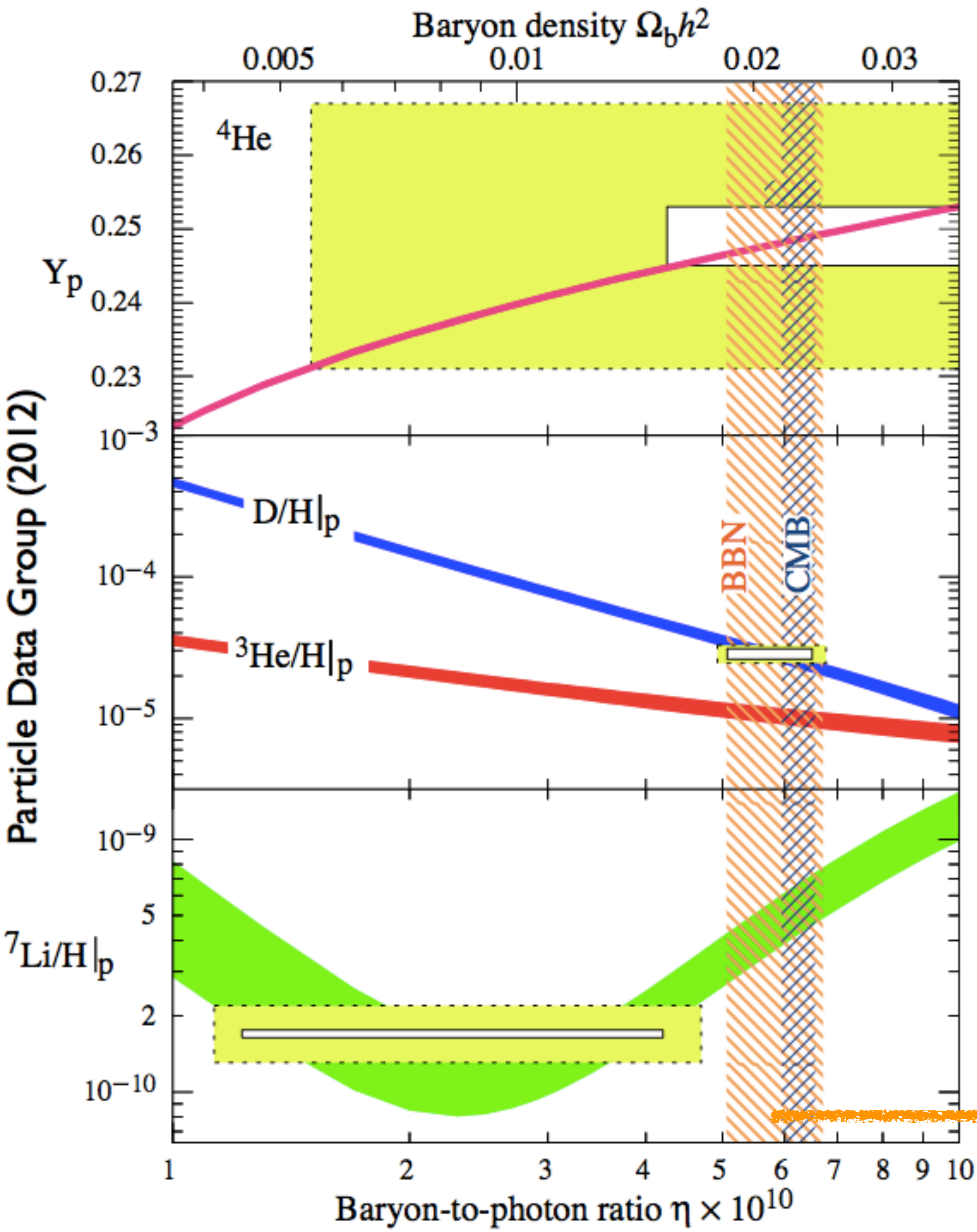
Annihilating/decaying DM into leptons => electroweak corrections yield anti-protons

all observed anti-protons are our cosmic background

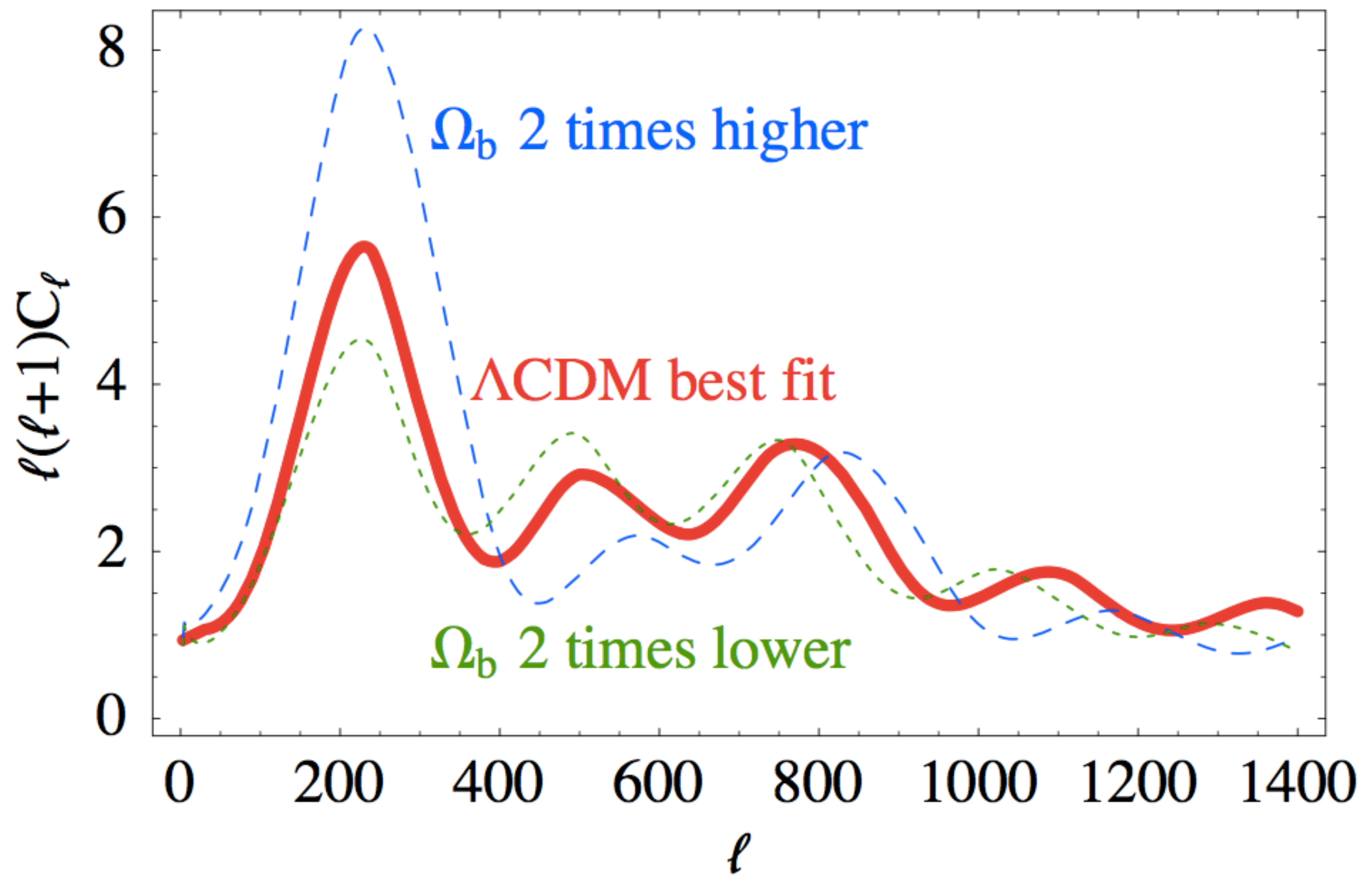
How do we generate Baryon Asymmetry ?

$$\eta \equiv \frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma} \ : \ \text{Baryon Asymmetry}$$

CMB & BBN



bands = 95% C.L.
smaller boxes = ±2σ statistics
larger boxes = ±2σ statistics
and systematics



$$\eta = (6.04 \pm 0.08) \times 10^{-10} \text{ (Planck)}$$

$$\eta = (5.1 - 6.5) \times 10^{-10}$$

$$\eta \equiv \frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma} \; : \; \text{Baryon Asymmetry}$$

How to create baryon asymmetry in the universe?

(1) Baryon number violation (particle physics)

(2) C & CP violation (particle physics)

(3) Departure from thermal equilibrium (cosmology)

$$\text{Suppose } \eta = 0\,,\ \text{at } T \le 1\ \text{GeV}:$$

$$\frac{n_b}{n_\gamma} \approx \frac{n_{\bar{b}}}{n_\gamma} \approx \left(\frac{m_p}{T}\right)^{3/2} e^{-m_p/T} \sim 10^{-18} \quad (\textbf{for T} \sim \textbf{20MeV})$$

$$\Gamma_{ann} \approx n_b \langle \sigma\ v \rangle \ge H(T) \sim 1.66 g_*^{1/2} (T^2/M_p)$$

C & CP violation

Charge conjugation symmetry, and the product of charge conjugation and parity are not exact symmetries of nature

Exact C symmetry: $i \to f \quad \Longrightarrow \bar{i} \to \bar{f} \;\; (|B_f| = |B_{\bar{f}}|)$

$$\mathrm{particle} \leftrightarrow \mathrm{anti-particle}$$

Exact CP symmetry:

$$\mathrm{CP\ invariance\ means\ time\ invariance\ due\ to\ CPT-theorem}$$

$$i(r_i, p_i, s_i) \to f(r_j, p_j, s_j) = f(r_j, -p_j, -s_j) \to i(r_i, -p_i, -s_i)$$

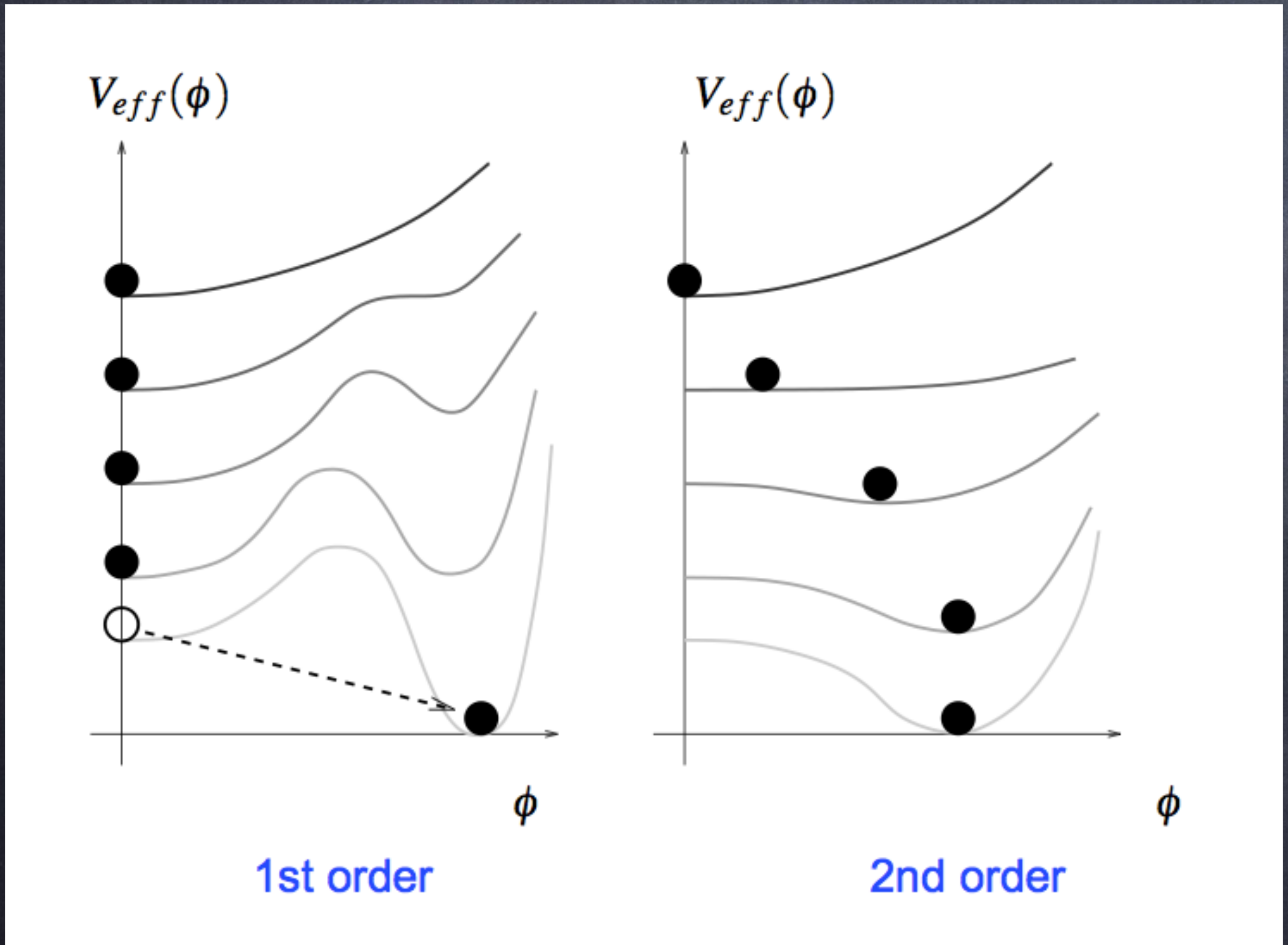
Weak interactions break C and CP both, e.g. neutrino sector, kaon sector

Out of Equilibrium Condition

(1) Expansion of the Universe $\Gamma_{process} < H(T) \sim 1.66 g_*^{1/2} (T^2/M_p)$

(2) First order Phase Transition

(3) Decay of any particle: Inflaton Reheating/Preheating/ Moduli late Decay, etc.



During reheating or preheating one can excite heavy states (out of equilibrium):

$$m_X \gg m_\phi, \; T_{rh}$$

Production of baryon asymmetry

$$r = \frac{\Gamma(X \to a)}{\Gamma_X} \quad (\mathrm{Baryon\ number\ B_a})\,, \quad 1 - \mathrm{r} = \frac{\Gamma(\mathrm{X} \to \mathrm{b})}{\Gamma_\mathrm{X}} \quad (\mathrm{Baryon\ number\ B_b})$$

$$\bar{r} = \frac{\Gamma(\bar{X} \to \bar{a})}{\Gamma_X} \quad (\mathrm{Baryon\ number\ } -\mathrm{B_a})\,, \quad 1 - \bar{\mathrm{r}} = \frac{\Gamma(\bar{\mathrm{X}} \to \bar{\mathrm{b}})}{\Gamma_\mathrm{X}} \quad (\mathrm{Baryon\ number\ } -\mathrm{B_b})$$

$$\Delta B = (r - \bar{r})B_a + [(1-r) - (1-\bar{r})]B_b = (r - \bar{r})(B_a - B_b)$$

$$\Delta B = \frac{1}{\Gamma_X}\sum_n B_n \left[\Gamma(X \to f_n) - \Gamma(\bar{X} \to \bar{f}_n)\right] \qquad B \equiv \frac{n_B}{s} \approx \frac{\Delta B n_X}{n_\gamma}\,, \Rightarrow B \equiv \frac{n_B}{s} \approx \frac{\Delta B n_\gamma}{g_* n_\gamma} \approx \frac{\Delta B}{g_*}$$

Out of equilibrium decay:

$$\rho_X \simeq M_X n_X \qquad \rho_X \approx \rho_r = \frac{\pi^2}{30} g_* T_{rh}^4 \qquad s = \frac{2\pi^2}{45} g_* T^3$$

$$B \equiv \frac{n_B}{s} \sim \frac{n_X}{s} \simeq \frac{3}{4}\frac{T_{rh}}{M_X}\Delta B$$

$$B \simeq \left(\frac{g_*^{-1/2}\alpha_X M_p}{M_X}\right)^{1/2} \Delta B$$

$$\Gamma_X \sim H(T_d) \sim \frac{8\pi\rho_X}{3M_p^2}\,, \quad \Gamma_\phi = \alpha_\phi M_\phi$$

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Unified Gauge Theories and the Baryon Number of the Universe

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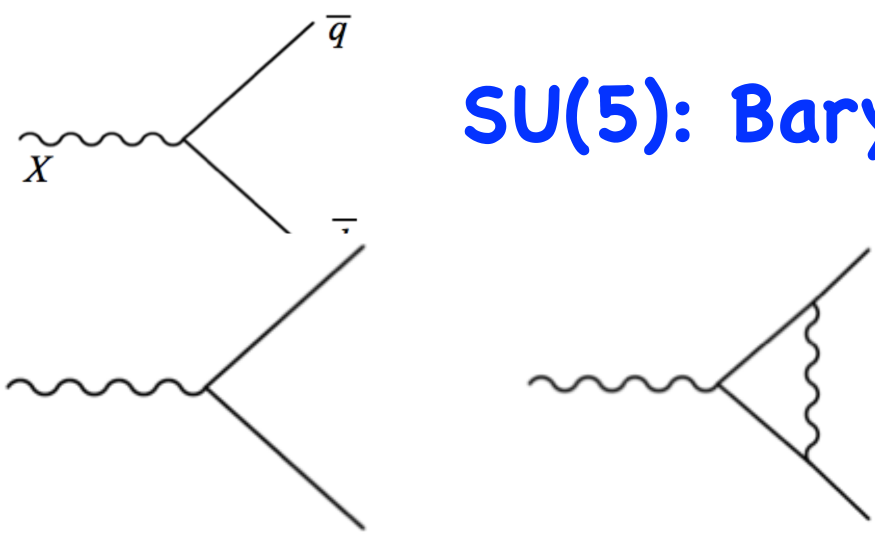
(Received 27 April 1978)

I suggest that the dominance of matter over antimatter in the present universe is a consequence of baryon-number–nonconserving reactions in the very early fireball. Unified guage theories of weak, electromagnetic, and strong interactions provide a basis for such a conjecture and a computation in specific SU(5) models gives a small ratio of baryon- to photon-number density in rough agreement with observation.



SU(5): Baryons and leptons are in same generation

C and CP are violated at one loop

Baryon numbers, B_a, B_b

process	branching ratio	B
$X \to q\,q$	r	$2/3$
$X \to \bar{q}\,\bar{l}$	$1-r$	$-1/3$
$\bar{X} \to \bar{q}\,\bar{q}$	$\bar{r}$	$-2/3$
$\bar{X} \to q\,l$	$1-\bar{r}$	$1/3$

Unfavored: Requires heavy gauge bosons :

$$T_{rh} \sim 10^{15}\sqrt{\alpha_\phi}\ \text{GeV}$$

Sphalerons

In the Standard Model, baryon and lepton number conservation are accidental symmetries. It was discovered by 't Hooft that non-perturbative effects can violate B and L: via instantons.

Baryon and lepton number are defined as: $B = \int d^3x J_0^B(x), \qquad L = \int d^3x J_0^L(x)$

Four current associated to B & L are

$$J_\mu^B = \frac{1}{3}\sum_i \left(\overline{q}_{L_i}\gamma_\mu q_{L_i} - \overline{u}^c_{L_i}\gamma_\mu u^c_{L_i} - \overline{d}^c_{L_i}\gamma_\mu d^c_{L_i}\right)$$

$$J_\mu^L = \sum_i \left(\overline{\ell}_{L_i}\gamma_\mu \ell_{L_i} - \overline{e}^c_{L_i}\gamma_\mu e^c_{L_i}\right) .$$

At the classical level, B and L are conserved:

$$\partial^\mu J_\mu^B = 0, \qquad \partial^\mu J_\mu^L = 0$$

For these two currents, the Adler-Bell-Jackiw triangular anomalies do not cancel : **B and L are anomalous at the quantum level**

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{N_f}{32\pi^2}\left[g_2^2 F_{\mu\nu}\tilde{F}^{\mu\nu} - g_1^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right]$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

N_f: Number of fermion generation

Sphalerons

$\Delta(B+L) \neq 0$ $\Delta(B-L) = 0$

$$\partial_\mu j^\mu_{B+L} = \frac{N_f}{16\pi^2}[g_2^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g_1^2 B_{\mu\nu}\tilde{B}^{\mu\nu}]$$

$$\partial_\mu j^\mu_{B-L} = 0$$

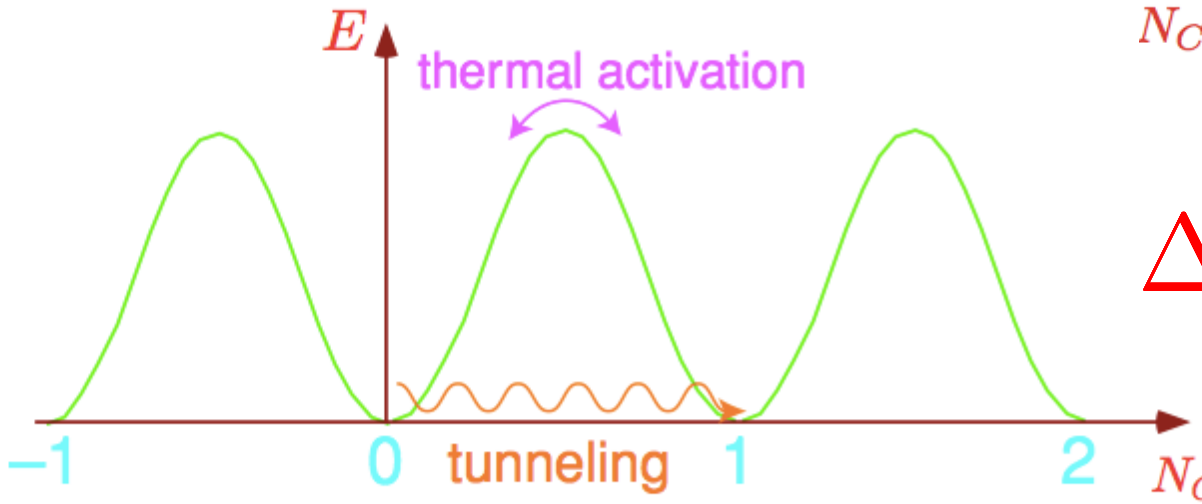
Chiral $\mathrm{U(1)_{B+L}}$ Anomaly

Change in B and L are related to change in Topological charge: Chern Simons number

$$B(t_f) - B(t_i) = \frac{N_f}{32\pi^2}\int_{t_i}^{t_f} d^4x \left[g_2^2 \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu}) - g_1^2 B_{\mu\nu}\tilde{B}^{\mu\nu}\right]$$

$$= N_f\left[N_{CS}(t_f) - N_{CS}(t_i)\right]$$

$$N_{CS}(t) = \frac{1}{32\pi^2}\int d^3x\, \epsilon_{ijk}\left[g_2^2 \mathrm{Tr}\left(F_{ij}A_k - \frac{2}{3}gA_iA_jA_k\right) - g_1^2 B_{ij}B_k\right]_t$$



$N_{CS} \in$ config. space(∞-dim)

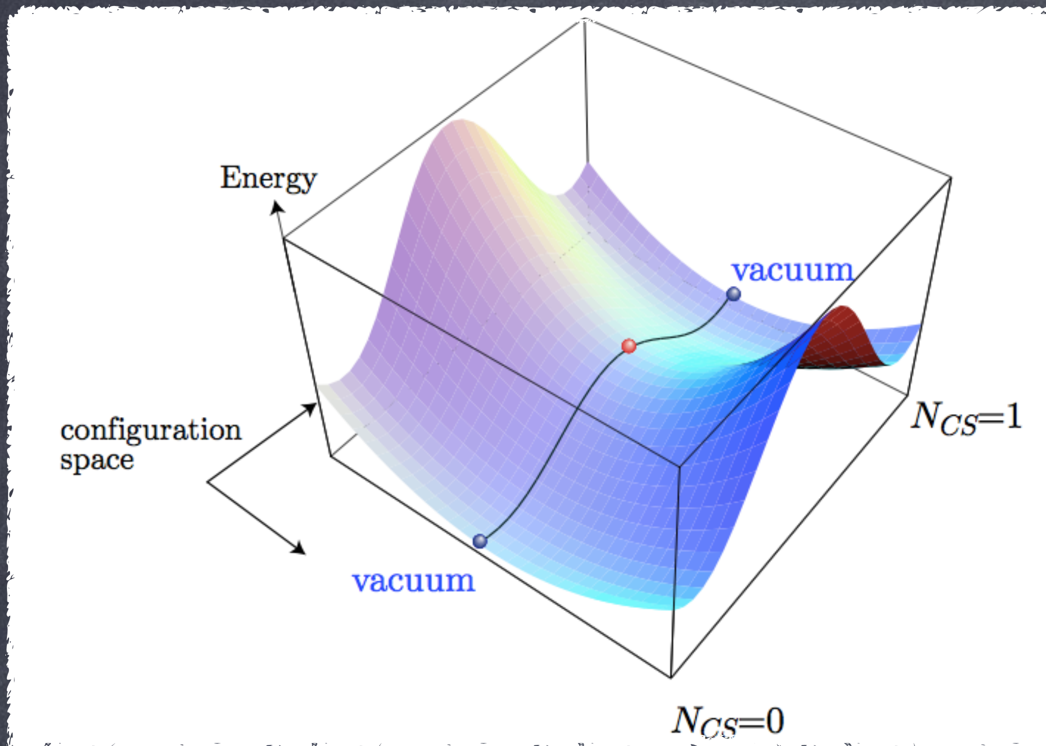
$$\Delta(B+L) = 2N_f N_{CS} = 6$$

$\Delta(B+L) \neq 0$ process
- ▷ **Quantum Tunneling** low-T
- ▷ **Thermal Activation** high-T

tunnel. prob. $\sim e^{-2S_{\mathrm{instanton}}} = e^{-8\pi^2/g_2^2} \simeq e^{-164} \ll 1$

$\therefore$ no proton-decay problem $(\Delta(B+L) = 2)$

Sphaleron: High temperature effects



$$T < T_c \implies \Gamma \sim E e^{-E_{sph}/T}$$

$$T > T_c \implies \Gamma \sim \kappa(\alpha_W T)^4$$

$$10^{-4} \leq \kappa \leq 10^{-1}$$

$$\frac{\Gamma_{sph}}{T^3} \geq H \implies T \leq \alpha_W^4 \frac{M_p}{g_*^{1/2}} \sim 10^{12} \text{ GeV}$$

Sphaleron transitions are active:

$$T_c \sim 100 \text{ GeV} < T < 10^{12} \text{GeV}$$

ON ANOMALOUS ELECTROWEAK BARYON-NUMBER NON-CONSERVATION IN THE EARLY UNIVERSE

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Received 8 February 1985

this means that if the primordial baryon asymmetry is generated by the $(B-L)$ conserving processes (which is the case in the minimal SU(5) model [22]), it is completely washed out by the moment of the electroweak phase transition.

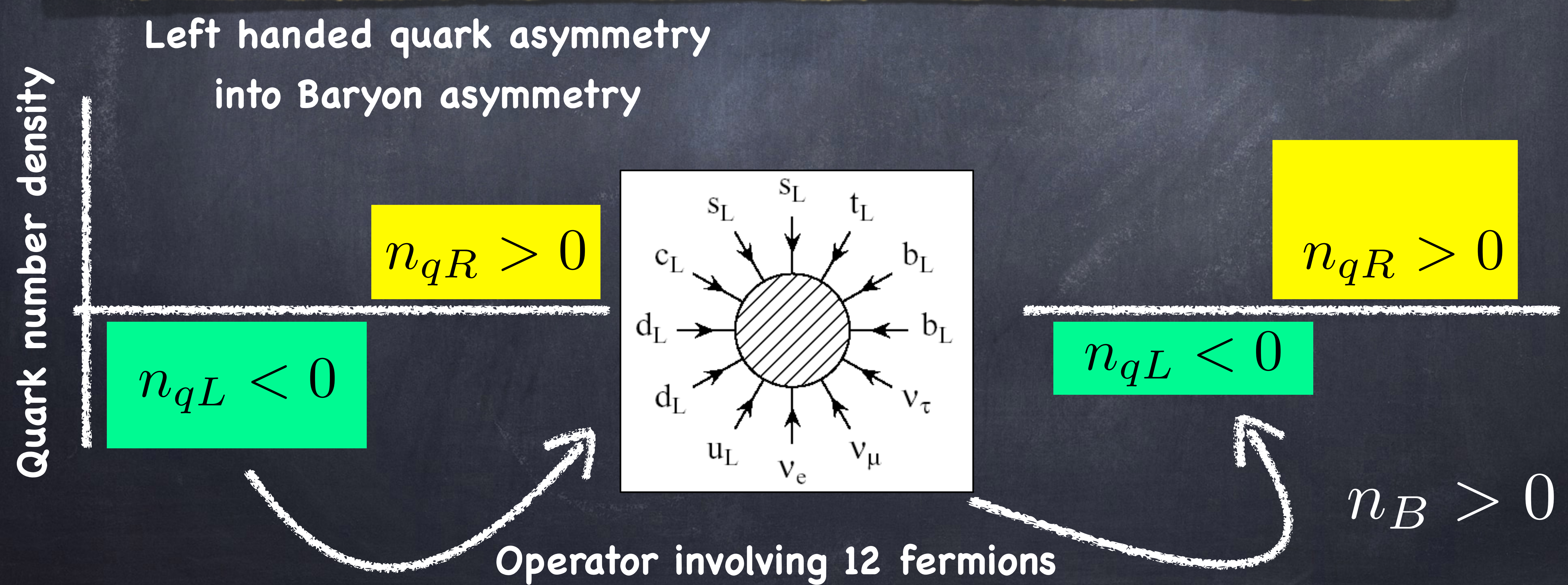
We estimate the rate of the anomalous electroweak baryon-number non-conserving processes in the cosmic plasma and find that it exceeds the expansion rate of the universe at $T >$ (a few)$\times 10^2$ GeV. We study whether these processes wash out the

Sphalerons

$$B = \frac{B-L}{2} + \frac{B+L}{2}$$

$$B \propto (B-L)_{primordial} \neq 0 \quad T \leq T_c$$

$$(B+L) \neq 0 \text{ at EW Transition}$$



Sphalerons: conversion of B-L to B asymmetry

(1) Thermal equilibrium, weak plasma, Chemical potential:

$$n_B - n_{\bar{B}} = \frac{1}{6}\, g\, T^2 \sum_{i=1}^{N_f} (2\mu_{Q_i} + \mu_{u_i} + \mu_{d_i})$$

$$n_L - n_{\bar{L}} = \frac{1}{6}\, g\, T^2 \sum_{i=1}^{N_f} (2\mu_{\ell_i} + \mu_{e_i})$$

(2) 12 fermion interactions induced by sphalerons:

$$\sum_i (3\mu_{q_i} + \mu_{\ell_i}) = 0$$

(3) SU(3) instantons lead to interactions between LH and RH quarks:

$$\sum_i (2\mu_{q_i} - \mu_{u_i} - \mu_{d_i}) = 0$$

(4) Total hypercharge must vanish in the plasma:

$$\sum_i (\mu_{q_i} + 2\mu_{u_i} - \mu_{d_i} - \mu_{\ell_i} - \mu_{e_i} + \frac{2}{N_f}\mu_H) = 0$$

(5) Yukawa interactions are in thermal equilibrium:

$$\mu_{q_i} - \mu_H - \mu_{d_j} = 0$$

$$\mu_{q_i} + \mu_H - \mu_{u_j} = 0$$

$$\mu_{\ell_i} - \mu_H - \mu_{e_j} = 0$$

Sphalerons: conversion of B-L to B asymmetry

All chemical potentials can be expressed in terms of μ_l

$$\mu_e = \frac{2N_f+3}{6N_f+3}\mu_\ell, \quad \mu_d = -\frac{6N_f+1}{6N_f+3}\mu_\ell, \quad \mu_u = \frac{2N_f-1}{6N_f+3}\mu_\ell$$

$$\mu_q = -\frac{1}{3}\mu_\ell, \quad \mu_H = \frac{4N_f}{6N_f+3}\mu_\ell$$

$$B = -\frac{4}{3}N_f\mu_\ell \,, \; L = \frac{14N_f^2+9N_f}{6N_f+3}\mu_\ell$$

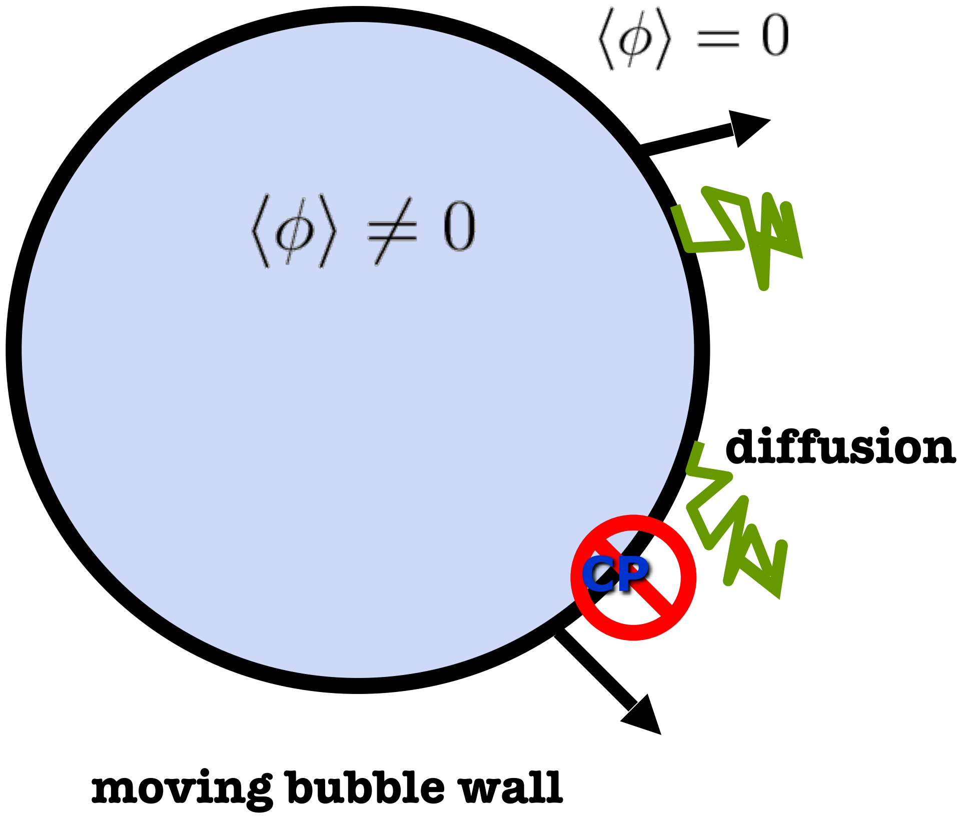
$$B = c\,(B-L)$$

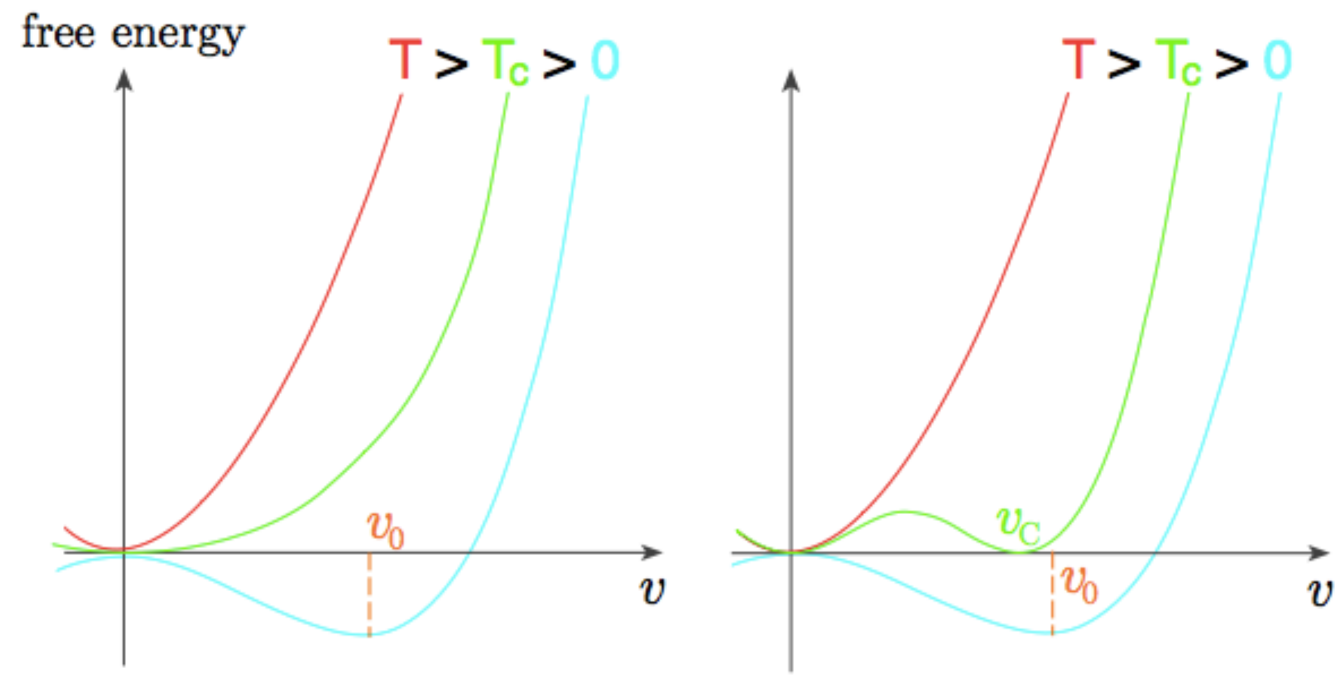
$$L = (c-1)\,(B-L)$$

$$c = \frac{8N_f+4N_H}{22N_f+13N_H}$$

c = 28/79 in Standard Model with 3 generations

First order phase transition at the EW scale within Standard Model



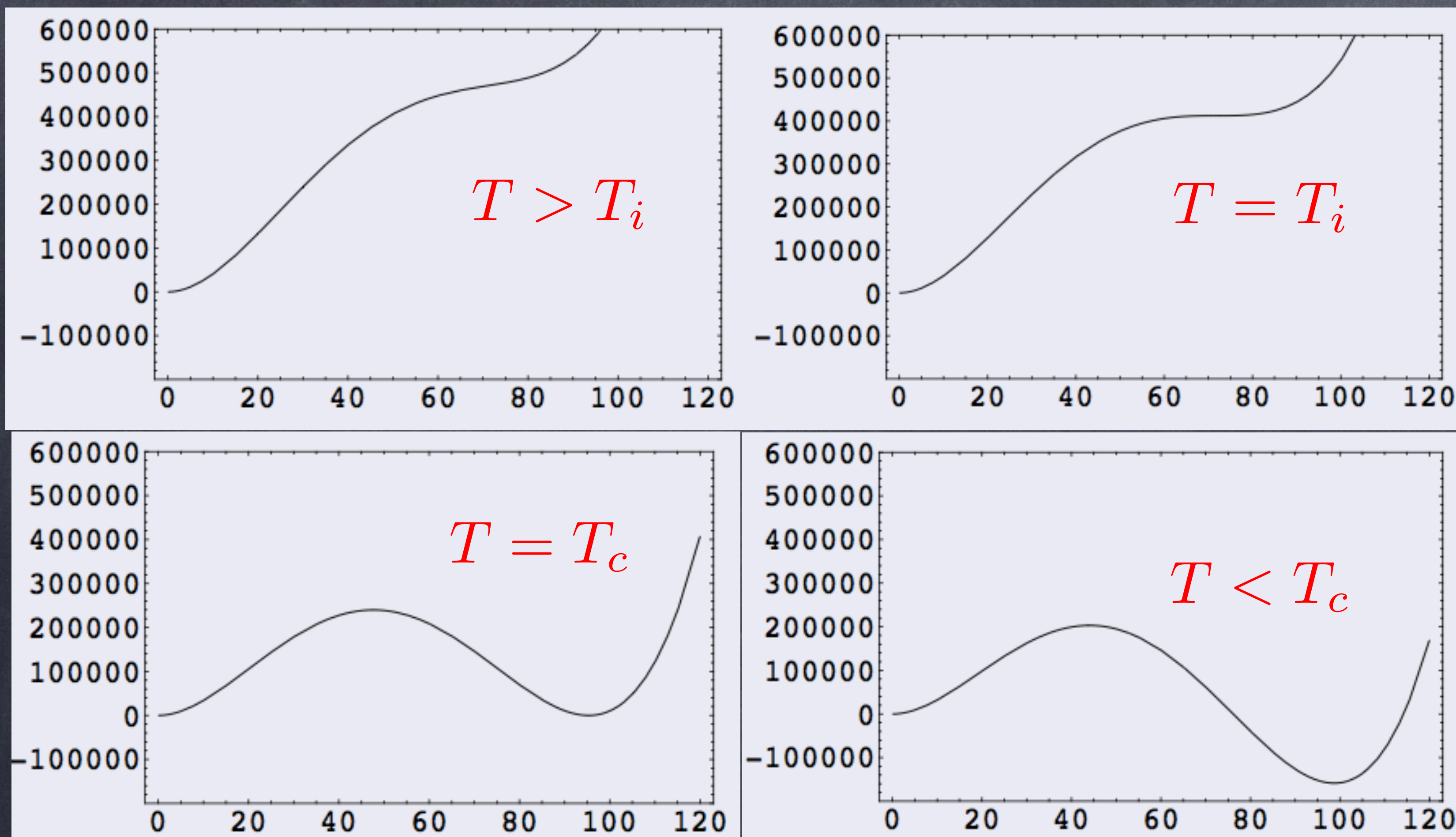


* **Sufficient CP violation**
* **Strong 1st order phase transition**

$$V_{\text{eff}}(\varphi;T) \simeq D(T^2 - T_0^2)\varphi^2 - E\,T\varphi^3 + \frac{\lambda_T}{4}\varphi^4$$

At T_C, $^{\exists}$degenerate minima: $\varphi_C = \dfrac{2E\,T_C}{\lambda_{T_C}}$

First order phase transition at the EW scale



$$T_i^2 = \frac{8\lambda_T(T_i) D T_0^2}{8\lambda_T(T_i) D - 9E^2}$$

$$\langle \phi(T_i) \rangle = \frac{3ET_i}{2\lambda_T(T_i)}$$

$$T_c^2 = \frac{\lambda_T(T_c) D T_0^2}{\lambda(T_c) D - E^2}$$

$$\phi_{max,\ min}(T) = \frac{3ET}{2\lambda_T(T)} \pm \frac{1}{2\lambda_T(T)} \sqrt{9E^2T^2 - 8\lambda_T(T) D (T - T_0^2)}$$

$$\phi_{max}(T_c) = \frac{ET_c}{\lambda_T(T_c)}$$

$$\phi_{min}(T_c) = \frac{2ET_c}{\lambda_T(T_c)}$$

$@\ T = T_0$ *the barrier disappears*

$$\phi_{min}(T_0) = \frac{3ET_0}{\lambda_T(T_0)}$$

$$\text{Strong } 1^{\text{st}} \text{ order phase transition : } \frac{ET_c}{\lambda_T(T_c)} > 1$$

$$\Phi = \begin{pmatrix} \chi_1 + i\chi_2 \\ \frac{\phi_c + h + i\chi_3}{\sqrt{2}} \end{pmatrix}$$

$$V_0(\phi_c) = -\frac{m^2}{2}\phi_c^2 + \frac{\lambda}{4}\phi_c^4$$

$$v^2 = \frac{m^2}{\lambda}$$

$$m_h^2(\phi_c) = 3\lambda\phi_c^2 - m^2$$

$$m_\chi^2(\phi_c) = \lambda\phi_c^2 - m^2$$

$$m_h^2(v) = 2\lambda v^2 = 2m^2,\ m_\chi^2(v) = 0$$

$$m_W^2(\phi_c) = \frac{g^2}{4}\phi_c^2$$

$$m_Z^2(\phi_c) = \frac{g^2 + g'^2}{4}\phi_c^2$$

$$m_t^2(\phi_c) = \frac{h_t^2}{2}\phi_c^2$$

$$D = \frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v^2}$$

$$T_0 = \frac{2m_h^2 - 8Bv^2}{4D}$$

$$E = \frac{2m_W^3 + m_Z^3}{4\pi v^3}$$

$$B = \frac{3}{64\pi^2 v^4}\left(2m_W^4 + m_Z^4 - 4m_t^4\right)$$

$$\lambda_T(T) = \lambda - \frac{3}{64\pi^2 v^4}\left(2m_W^4 \log\frac{m_W^2}{A_B T^2} + m_Z^4 \log\frac{m_Z^2}{A_B T^2} - 4m_t^4 \log\frac{m_t^2}{A_F T^2}\right)$$

$$\log A_B = \log a_b - (3/2),\ \ \log A_F = \log a_F - (3/2)$$

$$\log a_b = 5.4076,\ \ \log a_f = 2.6351$$

Current Status

$$\Gamma_{\rm sph}^{\rm (br)} < H(T_C) \quad \Longleftrightarrow \quad \frac{\varphi_C}{T_C} \gtrsim 1$$

SM

$$\frac{2ET_c}{\lambda_T(T_c)} \simeq \frac{4Ev^2}{m_h^2} \geq 1$$

$$m_H = \sqrt{2}\lambda v_0 \leq 46 \text{ GeV}$$

$m_h > 73\text{GeV} \Rightarrow$ PT disappears (crossover)

CP asymmetry is too small (CKM)

MSSM

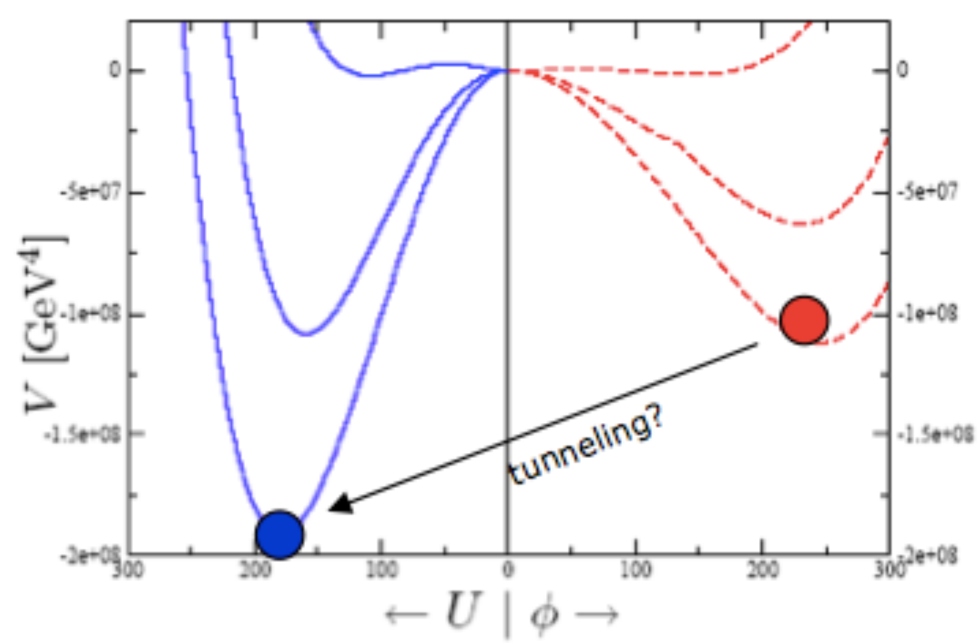
Many Bosons, especially Stops

Large cubic coefficient, Strong 1st order phase transition, many sources of CP violations

BUT

RH stop < 125 GeV, LH stop > 6.5 TeV (to avoid color-breaking phase transition) in MSSM

Carena, Nardini, Quiros, Wagner, 2008



NMSSM ?

Some of the conditions can be relaxed, specially 1st order transition is relatively easy

ELECTROWEAK BARYON NUMBER NON-CONSERVATION IN THE EARLY UNIVERSE AND IN HIGH ENERGY COLLISIONS

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Abstract

We review recent progress in the study of the anomalous baryon number non-conservation at high temperatures and in high energy collisions. Recent results on high temperature phase transitions are described, and applications to electroweak baryogenesis are considered. The current status of the problem of electroweak instanton-like processes at high energies is outlined.
This paper is written on the occasion of Sakharov's 75th anniversary and will
memorial volume of Uspekhi (Usp. Fiz. Nauk, volume 166, No 5, May 1996).

MSSM ELECTROWEA http://arxiv.org/abs/1207.6330v2 LHC DATA

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ABSTRACT

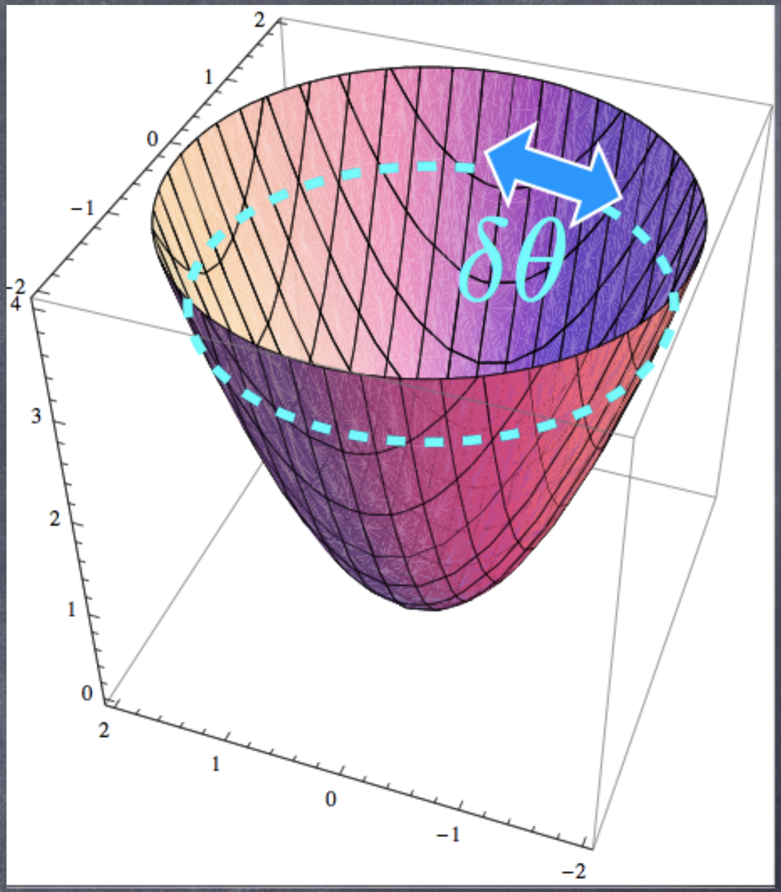
Electroweak baryogenesis is an attractive scenario for the generation of the baryon asymmetry of the universe as its realization depends on the presence at the weak scale of new particles which may be searched for at high energy colliders. In the MSSM it may only be realized in the presence of light stops, and with moderate or small mixing between the left- and right-handed components. Consistency with the observed Higgs mass around 125 GeV demands the heavier stop mass to be much larger than the weak scale. Moreover the lighter stop leads to an increase of the gluon-gluon fusion Higgs production cross section which seems to be in contradiction with indications from current LHC data. We show that this tension may be considerably relaxed in the presence of a light neutralino with a mass lower than about 60 GeV, satisfying all present experimental constraints. In such a case the Higgs may have a significant invisible decay width and the stop decays through a three or four body decay channel, including a bottom quark and the lightest neutralino in the final state. All these properties make this scenario testable at a high luminosity LHC.

Reviews & recent activities, and references therein

Dynamical generation of baryon number

$$V(\phi) = m^2|\phi|^2 + \lambda(\phi^4 + \phi^{*\ 4}) + \frac{|\phi|^6}{M^2} + \cdots$$

CP is invariant: $\phi \leftrightarrow \phi^*$



Initial condition can break CP

$$\phi = i\phi_0, \quad \dot{\phi} = 0$$

$$\phi_k = \frac{A_k}{mt}\sin(mt + \delta_k). \qquad k = I, R$$

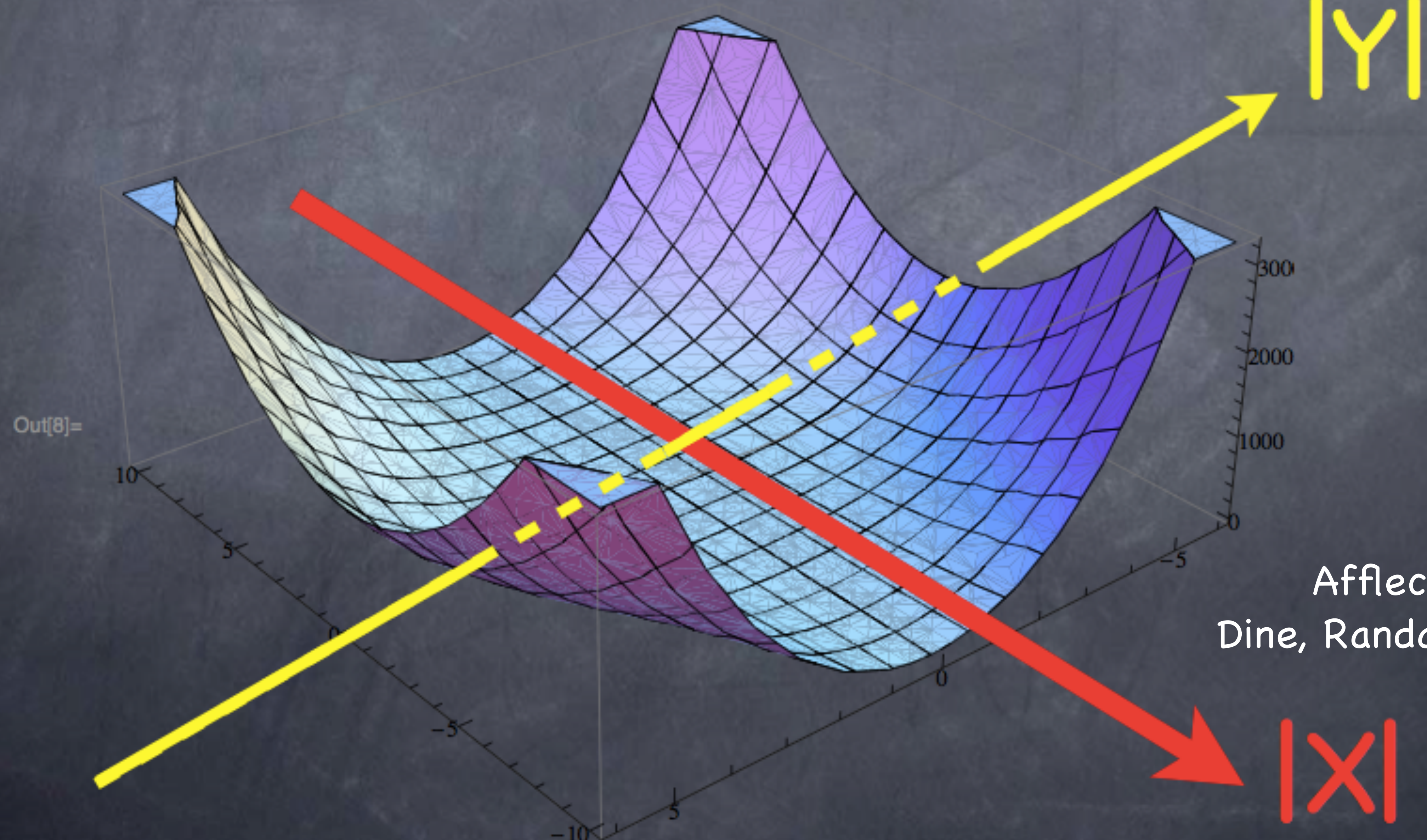
$$n_B = 2(\phi_I\dot{\phi}_R - \phi_R\dot{\phi}_I) = \frac{2A_IA_R}{mt^2}\sin(\delta_I - \delta_R)$$

Affleck + Dine (84)

Affleck-Dine Baryogenesis

$W = XY^2 \Rightarrow V = |Y|^4 + 4|X|^2|Y|^2$

The scalar potential vanishes along $|Y|=0$.



Affleck + Dine (84)
Dine, Randall, Thomas (95,96)

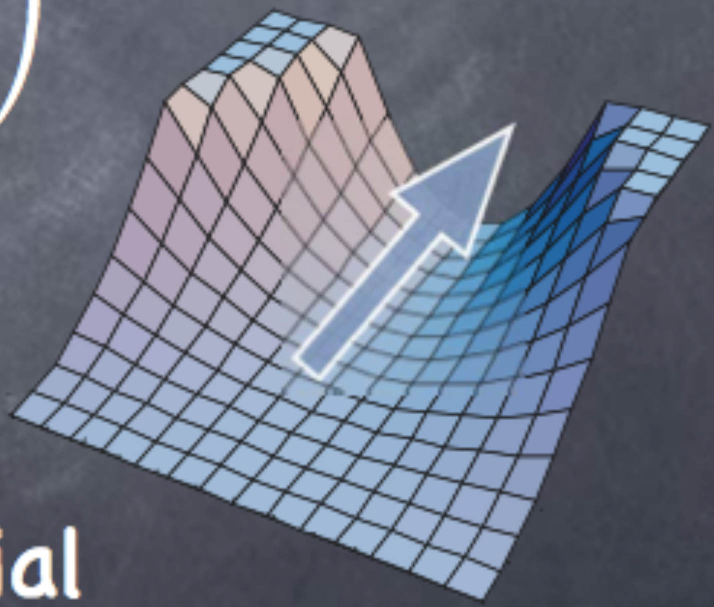
Quantum fluctuations can take large VEVs along flat direction : it does not cost in energy

Affleck-Dine Baryogenesis/Leptogenesis

In the SSM, there are many flat directions composed of squarks and/or sleptons. (e.g. udd, eLL)

$$V(\phi) = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \frac{1}{2} \sum_A \left(\sum_{ij} \phi_i^* (t_A)_{ij} \phi_j \right)^2$$

(F-term) (D-term)



Along a flat directions the scalar potential vanishes in the global SUSY limit at the level of renormalizable operators.

Affleck-Dine Baryogenesis

$$W_{\rm NR} = \frac{\Phi^n}{M^{n-3}} \quad (n = 4-9)$$

e.g.) $W = (LH_u)^2, (udd)^2, (eLL)^2$

U(1)-preserving terms

U(1)-breaking terms

$$V(\Phi) = \left(m_\Phi^2 - cH^2\right)|\Phi|^2 + \frac{|\Phi|^{2n-2}}{M^{2n-6}} + \left(a\frac{m_{3/2}}{nM^{n-3}}\Phi^n + \text{h.c.}\right)$$

soft mass

due to inflaton

non-ren. operator

A-term: NR operator and ~~U(1)~~R

Stabilises the potential

Affleck-Dine Baryogenesis

$$V(\Phi) = \left(m_\Phi^2 - cH^2\right)|\Phi|^2 + \frac{|\Phi|^{2n-2}}{M^{2n-6}} + \left(a\frac{m_{3/2}}{nM^{n-3}}\Phi^n + \text{h.c.}\right)$$

soft mass

due to inflaton

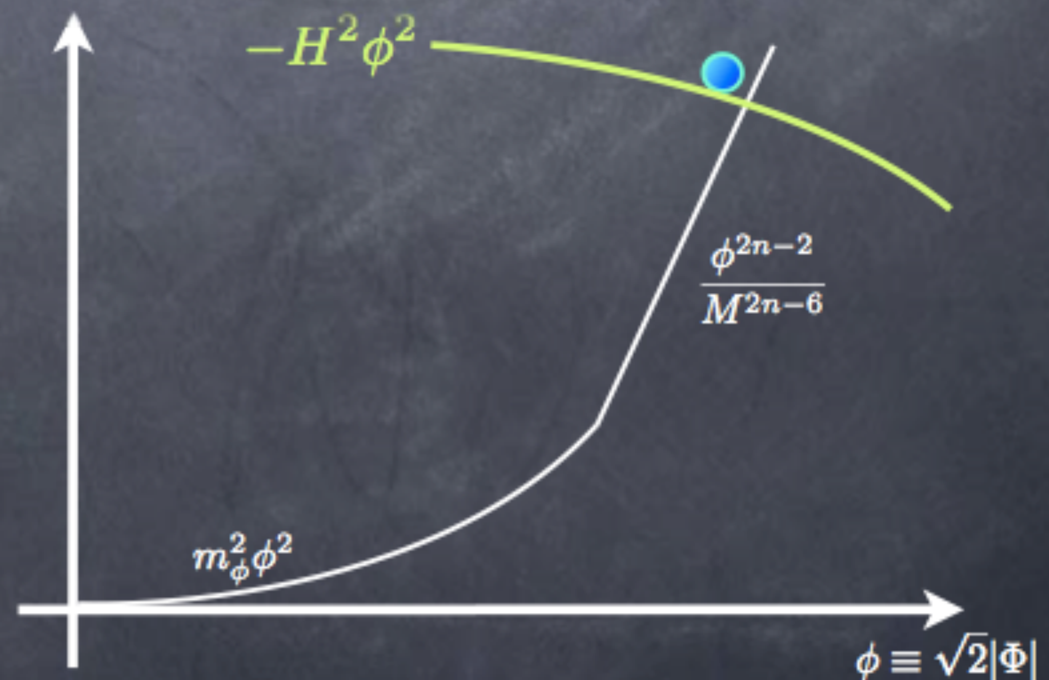
non-ren. operator

A-term: NR operator and ~~U(1)~~R

$$\phi \sim \left(H_I M^{n-3}\right)^{1/(n-2)}$$

e.g.) $\phi \sim 10^{15}\,\text{GeV}$ for

$n = 4,\ H_I = 10^{12}\,\text{GeV},\ M = M_P$



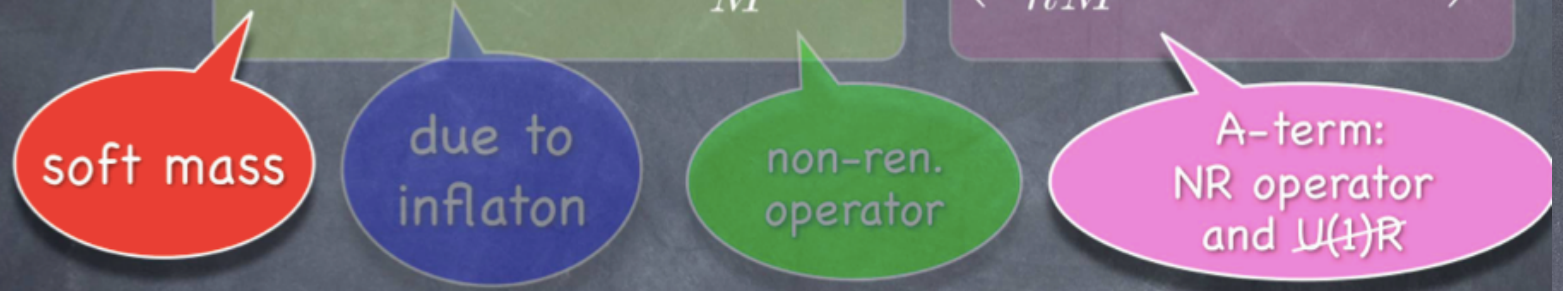
$$V(\Phi) = \left(m_\Phi^2 - cH^2\right)|\Phi|^2 + \frac{|\Phi|^{2n-2}}{M^{2n-6}} + \left(a\frac{m_{3/2}}{nM^{n-3}}\Phi^n + \mathrm{h.c.}\right)$$

soft mass

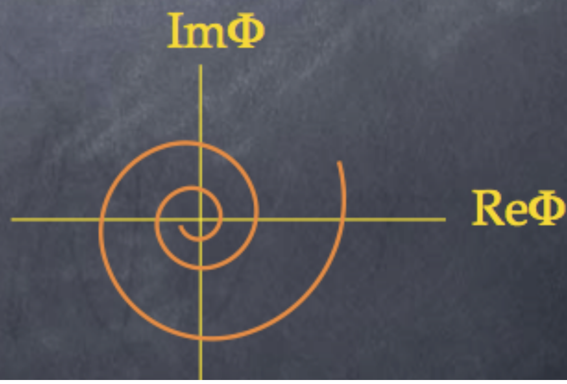
due to inflaton

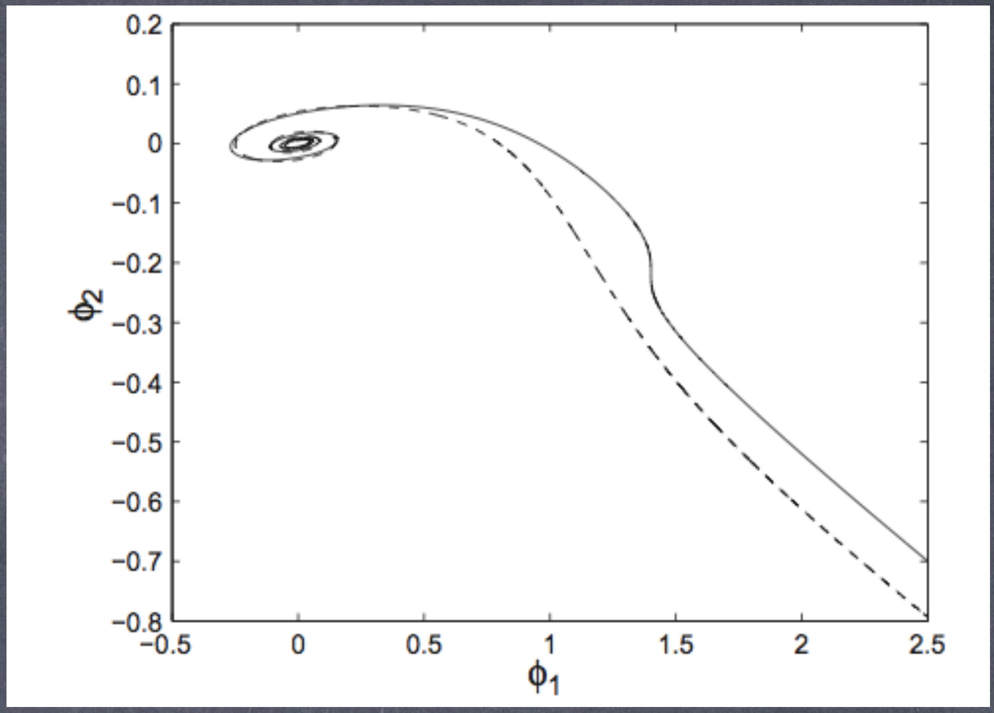
non-ren. operator

A-term: NR operator and ~~U(1)R~~



The AD field starts to oscillate.
Kicked into the phase direction due to the B-number violating term.





$$n_{\mathrm{B,L}} = \beta \mathrm{i}(\dot{\phi}^*\phi - \phi^*\dot{\phi})$$

$$\dot{n}_{\mathrm{B,L}} + 3Hn_{\mathrm{B,L}} = 2\beta\,\mathrm{Im}\left[\frac{\partial V(\phi)}{\partial\phi}\phi\right] = 2\beta\frac{1}{M_{\mathrm{P}}^{n-3}}\mathrm{Im}\left[(a_{\mathrm{H}}H + A)\phi^n\right]$$

$$\frac{n_{\mathrm{B,L}}}{s} = \frac{1}{4}\frac{T_{\mathrm{R}}}{M_{\mathrm{P}}^2 H(t_{\mathrm{osc}})^2} n_{\mathrm{B,L}}(t_{\mathrm{osc}}) \approx \beta\frac{n-2}{6(n-3)}\frac{T_{\mathrm{R}}}{M_{\mathrm{P}}^2 m_\phi}\left(m_\phi M_{\mathrm{P}}^{n-3}\right)^{2/(n-2)}$$

$$\frac{n_{\mathrm{B,L}}}{s} \approx 10^{-10} \times \beta\left(\frac{1\,\mathrm{TeV}}{m_\phi}\right)\left(\frac{T_{\mathrm{R}}}{10^9\,\mathrm{GeV}}\right)$$

$$\frac{n_{\mathrm{B,L}}}{s} \approx 10^{-10} \times \beta\left(\frac{1\,\mathrm{TeV}}{m_\phi}\right)^{1/2}\left(\frac{T_{\mathrm{R}}}{100\,\mathrm{GeV}}\right)$$

A mini review on Affleck–Dine baryogenesis

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Abstract. The Affleck–Dine mechanism is an attractive scenario for generating the observed baryon asymmetry of the universe utilizing flat directions in the scalar potential of supersymmetric theories. In this mini review, we describe this mechanism in its original version, its explicit realization within the minimal supersymmetric standard model and its variants. We discuss the formation of a condensate along the flat directions in the inflationary era, its post-inflationary evolution leading to baryogenesis and its fate. In some cases the condensate may fragment into non-topological solitons, known as Q-balls, during its evolution. In models of gravity-mediated supersymmetry breaking, the Q-balls can be long-lived, in which case their decay will be the source of all baryons and dark matter in the form of the lightest supersymmetric particle. In models of gauge-mediated supersymmetry breaking, the Q-balls can be absolutely stable and form dark matter that can be searched for directly.

REVIEWS OF MODERN PHYSICS, VOLUME 76, JANUARY 2004

Origin of the matter-antimatter asymmetry

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(Published 16 December 2003)

Although the origin of matter-antimatter asymmetry remains unknown, continuing advances in theory and improved experimental limits have ruled out some scenarios for baryogenesis, for example, sphaleron baryogenesis at the electroweak phase transition in the Standard Model. At the same time, the success of cosmological inflation and the prospects for discovering supersymmetry at the Large Hadron Collider have put some other models in sharper focus. We review the current state of our understanding of baryogenesis with emphasis on those scenarios that we consider most plausible.

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Cosmological consequences of MSSM flat directions

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Abstract

We review the cosmological implications of the flat directions of the minimally supersymmetric standard model (MSSM). We describe how field condensates are created along the flat directions because of inflationary fluctuations. The post-inflationary dynamical evolution of the field condensate can charge up the condensate with B or L in a process known as Affleck–Dine baryogenesis. Condensate fluctuations can give rise to both adiabatic and isocurvature density perturbations and could be observable in future cosmic microwave experiments. In many cases the condensate is however not the state of lowest energy but fragments, with many interesting cosmological consequences. Fragmentation is triggered by inflation-induced perturbations and the condensate lumps will eventually form non-topological solitons, known as Q-balls. Their properties depend on how supersymmetry breaking is transmitted to the MSSM; if by gravity, then the Q-balls are semi-stable but long-lived and can be the source of all the baryons and LSP dark matter; if by gauge interactions, the

Reviews &
recent activities,
and references therein

Thermal Leptogensis

"Baryogenesis Without Grand Unification", Phys.Lett.B174:45,1986, by Fukugita and Yanagida.

Connecting Neutrino masses and Lepton asymmetry

Dirac vs Majorana

$$\bar{\psi}\psi \quad \text{vs.} \quad \psi^T C^{-1} \psi$$

$\psi \to e^{i\alpha}\psi$: $U(1)$ symmetry : $U(1)_{em}$

$e,\ \mu,\ q, \cdots$ *are Dirac,*
because $Q_e \neq 0$

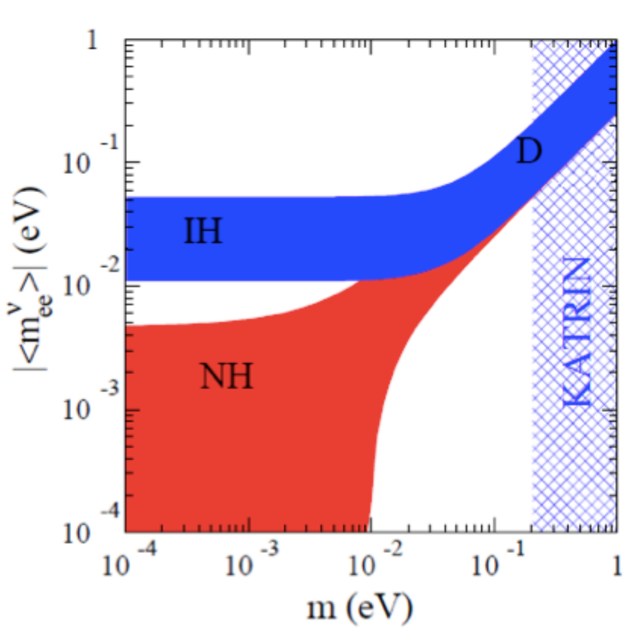
Dirac ν : $\nu \neq \bar{\nu}$
(Exact lepton number symmetry)

$Q(\nu) = 0,\ U(1)_{em}$ *does not impose* ν *to be Dirac*

Majorana ν : $\nu = \bar{\nu}$
(Lepton number violated)

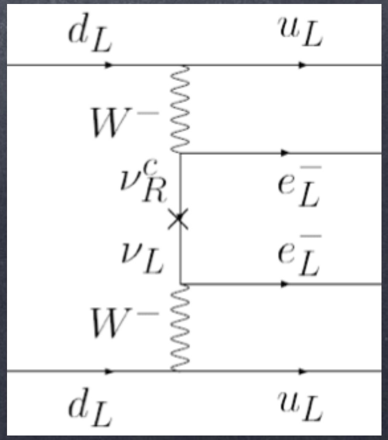
Neutrinoless double beta decay : Majorana

Neutrino: $(A, Z) \to (A, Z+2) + e^- + e^-$ (allowed)



The rate of $0\nu2\beta$ depends crucially on the spectrum. If neutrinos are degenerate or inverse hierarchical, $0\nu2\beta$ could be observed in the next generation of experiments (CUORE, GERDA...)

Bahcall, Murayama
Peña-Garay



$\tau_d \sim 10^{24} - 10^{25}$ Years

Neutrino mass constraints

Solar
$\nu_e \to \nu_\mu, \nu_\tau$
(SNO, BOREXino, Super-Kamiokande, GALLEX/GNO, SAGE, Homestake, Kamiokande)

VLBL Reactor
$\bar{\nu}_e$ disappearance
(KamLAND)

$\to \left\{ \begin{array}{l} \Delta m^2_S \simeq 7.6 \times 10^{-5}\ \mathrm{eV}^2 \\ \sin^2\vartheta_S \simeq 0.30 \end{array} \right.$

Atmospheric
$\nu_\mu \to \nu_\tau$
(Super-Kamiokande, Kamiokande, IMB, MACRO, Soudan-2)

LBL Accelerator
ν_μ disappearance
(K2K, MINOS, T2K)

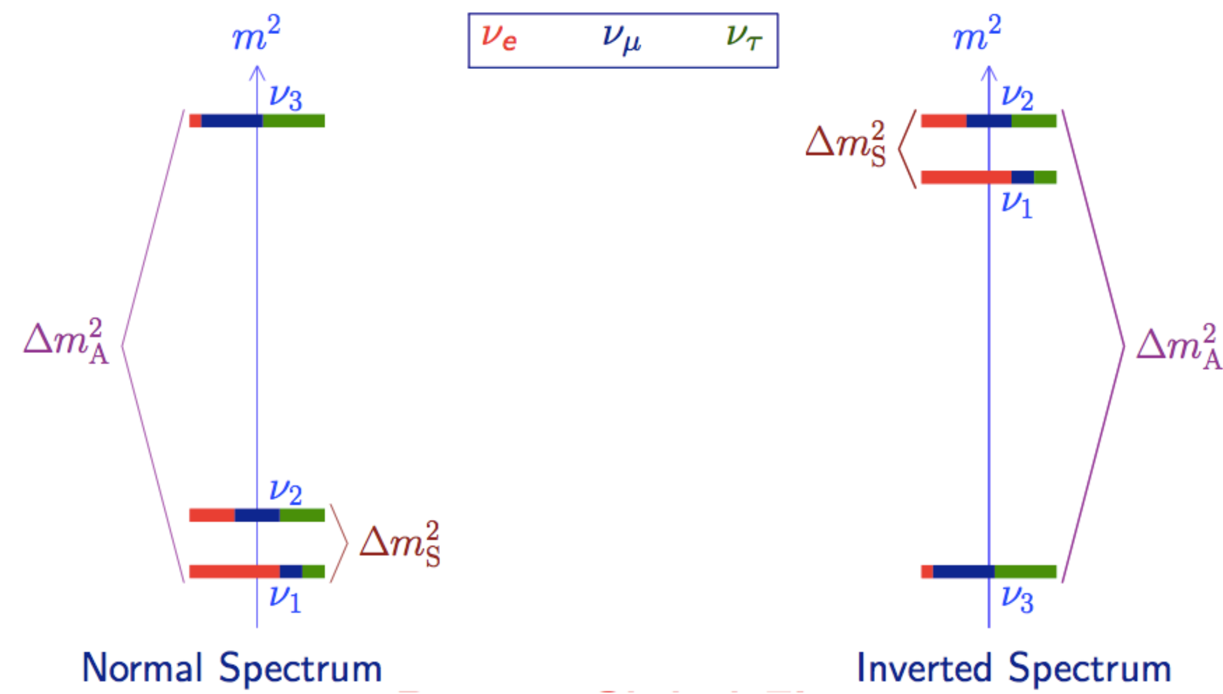
LBL Accelerator
$\nu_\mu \to \nu_\tau$
(Opera)

$\to \left\{ \begin{array}{l} \Delta m^2_A \simeq 2.4 \times 10^{-3}\ \mathrm{eV}^2 \\ \sin^2\vartheta_A \simeq 0.50 \end{array} \right.$

LBL Accelerator
$\nu_\mu \to \nu_e$
(T2K, MINOS)

LBL Reactor
$\bar{\nu}_e$ disappearance
(Daya Bay, RENO, Double Chooz)

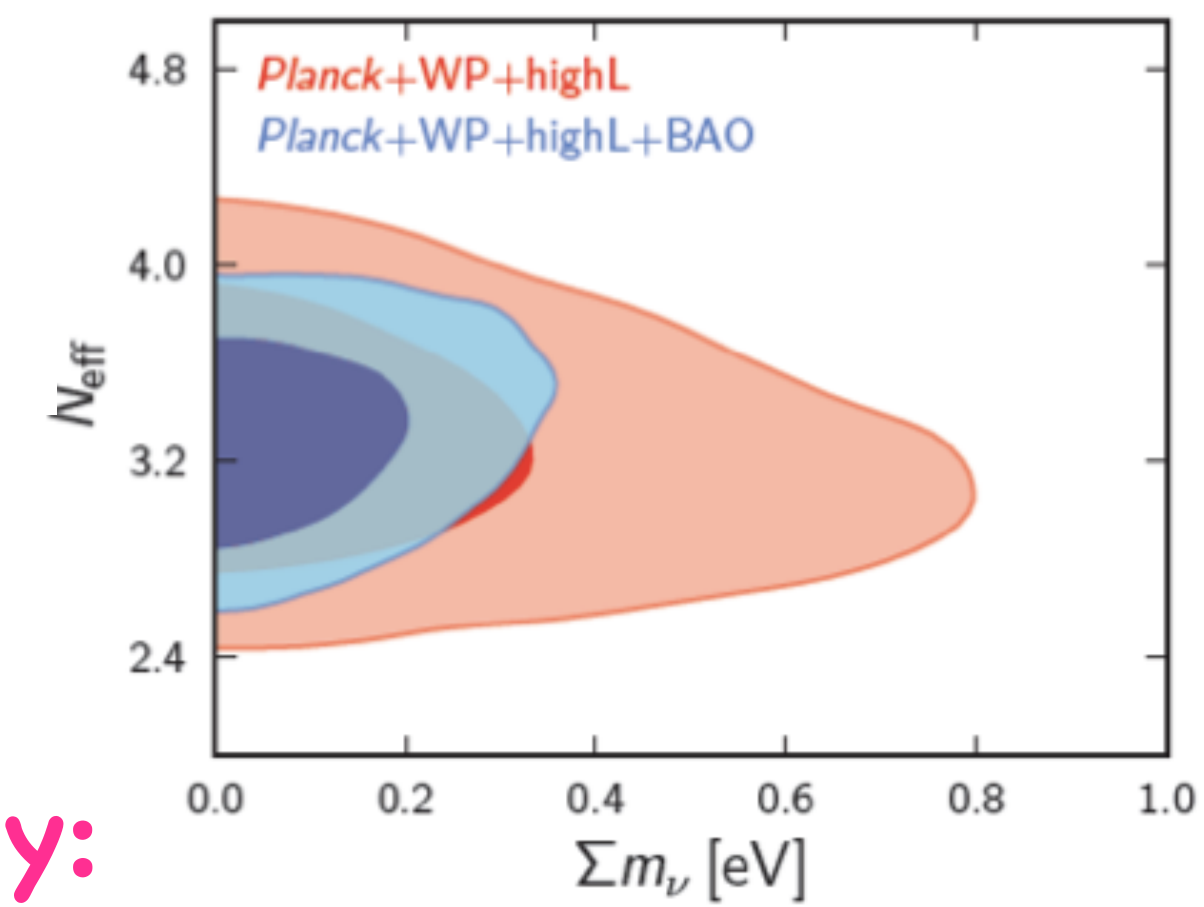
$\to \left\{ \begin{array}{l} \Delta m^2_A \\ \sin^2\vartheta_{13} \simeq 0.023 \end{array} \right.$



Tritium beta decay : $m_{\nu_e} \leq 2$ eV (95%C.L.)

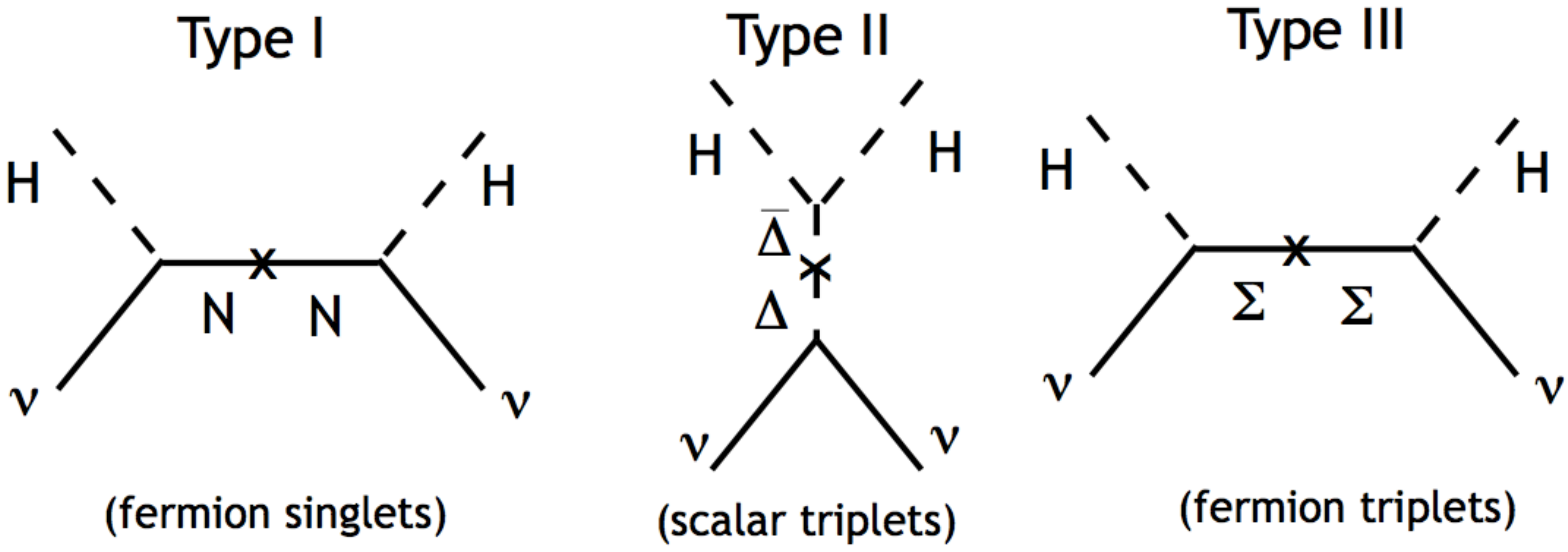
$(0\nu\beta\beta)$ experiments : $m_{\nu_e} \leq 0.34 - 0.78$ (95%C.L.)

Cosmology:



Origin of Neutrino masses

See-Saw mechanism



Type-1 See-Saw mechanism

[Gell-Mann, Ramond, Slansky; Yanagida; Glashow; Mohapatra, Senjanović

$$\nu_e, \ \nu_\mu, \ \nu_\tau, \ \nu_{s,1}, \ \nu_{s,2}, \ \cdots, \ \nu_{s,N}$$

$$\mathcal{L} = \mathcal{L}_{SM} + \bar{\nu}_{s,a}(i\partial_\mu \gamma^\mu)\nu_{s,a} - y_{\alpha a} H \bar{L}_\alpha \nu_{s,a} - \frac{M_{ab}}{2}\bar{\nu}_{s,a}\nu_{s,b} + h.c.$$

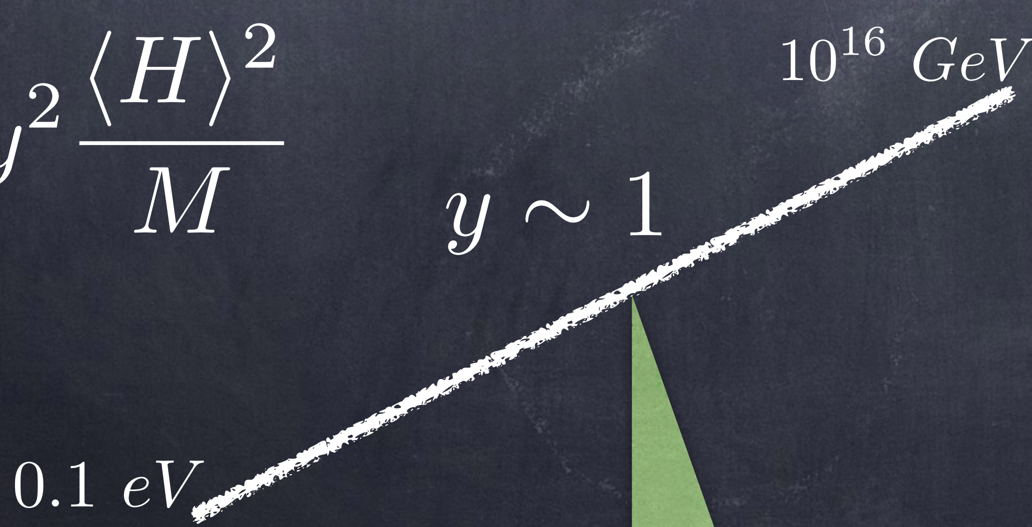
$$\alpha = e, \ \mu, \ \tau$$

$$M = \begin{pmatrix} 0 & D_{3\times N} \\ D^T_{N\times 3} & M_{N\times 3} \end{pmatrix}$$

$$D_{ij} = y_{ij}\langle H \rangle$$ **(in the SM)**

Small neutrino mass may not always imply small Yukawa

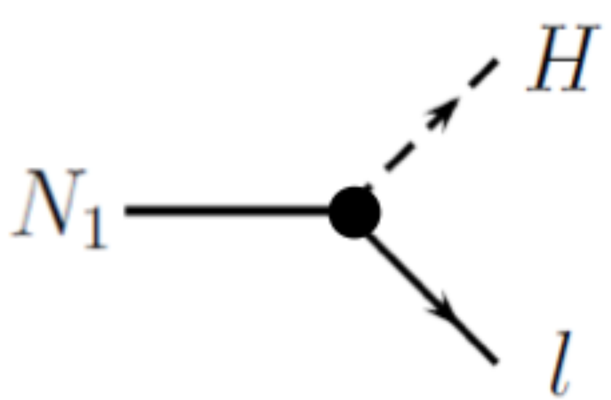
$$m_\nu \sim y^2 \frac{\langle H \rangle^2}{M}$$

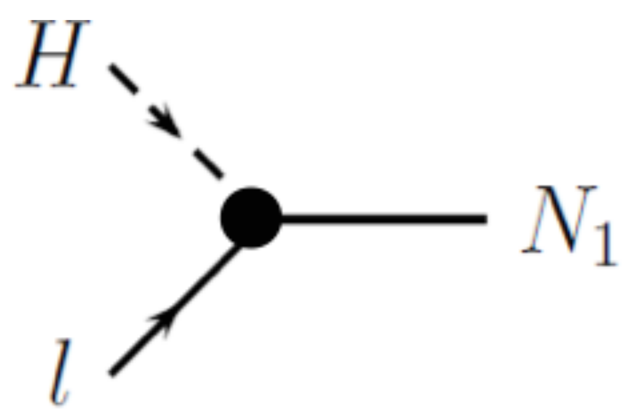


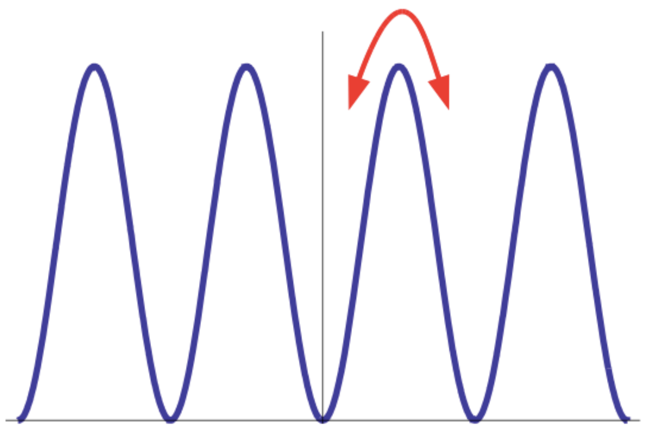
Thermal Leptogensis

$$\mathcal{L}_N = Y N H L + M N N$$

Decay **Wash out** **Sphalerons**





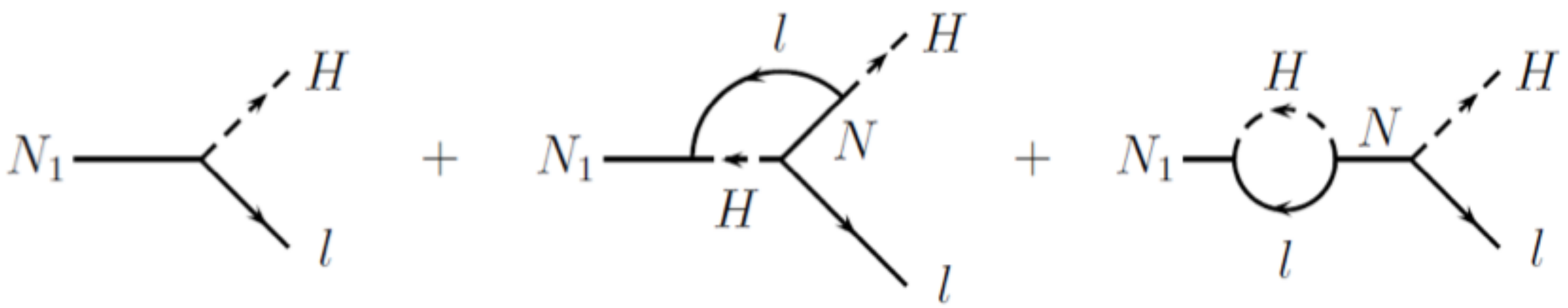


$$\Gamma_{\rm tot} = \Gamma(N_1 \to l H) + \Gamma(N_1 \to l^c H^c) = \frac{1}{8\pi}(h_\nu h_\nu^\dagger)_{11} M_1$$

$$\epsilon_1 = \frac{\Gamma(N_1 \to l H) - \Gamma(N_1 \to l^c H^c)}{\Gamma(N_1 \to l H) + \Gamma(N_1 \to l^c H^c)} = 0$$

No CP asymmetry

Thermal Leptogensis



$$\epsilon_1 = \frac{\Gamma(N_1 \to lH) - \Gamma(N_1 \to l^c H^c)}{\Gamma(N_1 \to lH) + \Gamma(N_1 \to l^c H^c)}$$
$$\simeq \frac{1}{8\pi} \frac{1}{(h_\nu h_\nu^\dagger)_{11}} \sum_{i=2,3} \mathrm{Im}\left[(h_\nu h_\nu^\dagger)^2_{1i}\right] \left[f\left(\frac{M_i^2}{M_1^2}\right) + g\left(\frac{M_i^2}{M_1^2}\right)\right]$$

Yukawa has to be complex

Vertex correction :

$$f(x) = \sqrt{x}\left[1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right]$$

Wavefunction renormalisation :

$$g(x) = \frac{\sqrt{x}}{1-x}$$

calculable only when

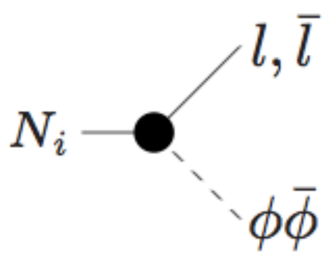
$$|M_i - M_1| \gg |\Gamma_i - \Gamma_1|$$

Resonant enhancement if the masses are degenerate !

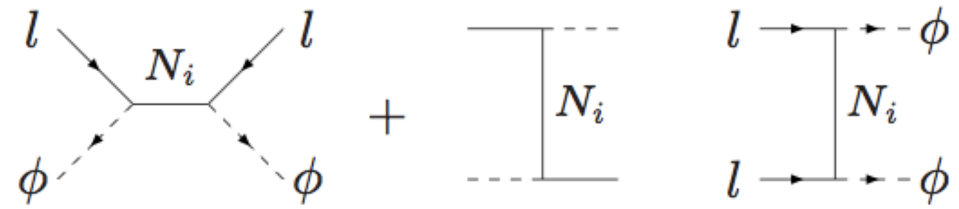
Departure from thermal equilibrium

decays (D), inverse decays (ID)

$N_i \leftrightarrow l\,\phi\,,\ \bar{l}\,\bar{\phi}$



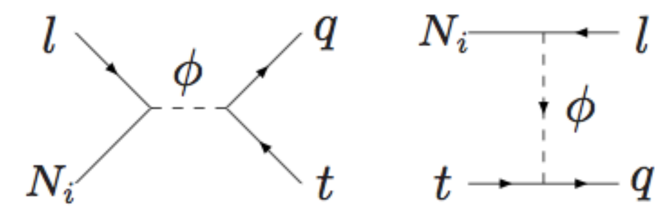
$\Delta L = 2$ processes (N_i virtual)



$l\,\bar{\phi} \leftrightarrow \bar{l}\,\phi$ (N)

$l\,l \leftrightarrow \phi\,\phi$
$\bar{l}\,\bar{l} \leftrightarrow \bar{\phi}\,\bar{\phi}$ $(N,\, t)$

$\Delta L = 1$ processes (N_i real)



$N_i\,l \leftrightarrow \bar{t}\,q$ $(\phi,\, s)$

$N_i\,t \leftrightarrow \bar{l}\,q$ $(\phi,\, t)$

$$\frac{dN_1}{dz} = -(D+S)(N_{N_1} - N_{N_1}^{eq})$$

Evolve the number density in a co-moving volume

$$z = \frac{M_1}{T}, \quad D = \frac{\Gamma_D}{Hz} \ \text{(Decay rate)}, \quad S = \frac{\Gamma_S}{Hz} \ \text{(Scattering rate)}$$

$$\frac{dN_{B-L}}{dz} = -\epsilon_1 D(N_{N_1} - N_{N_1}^{eq}) - W N_{B-L}$$

$$W = \frac{\Gamma_W}{Hz} \ \text{(Washout rate)}$$

Competition between decay, inverse decay, scattering and washout effects

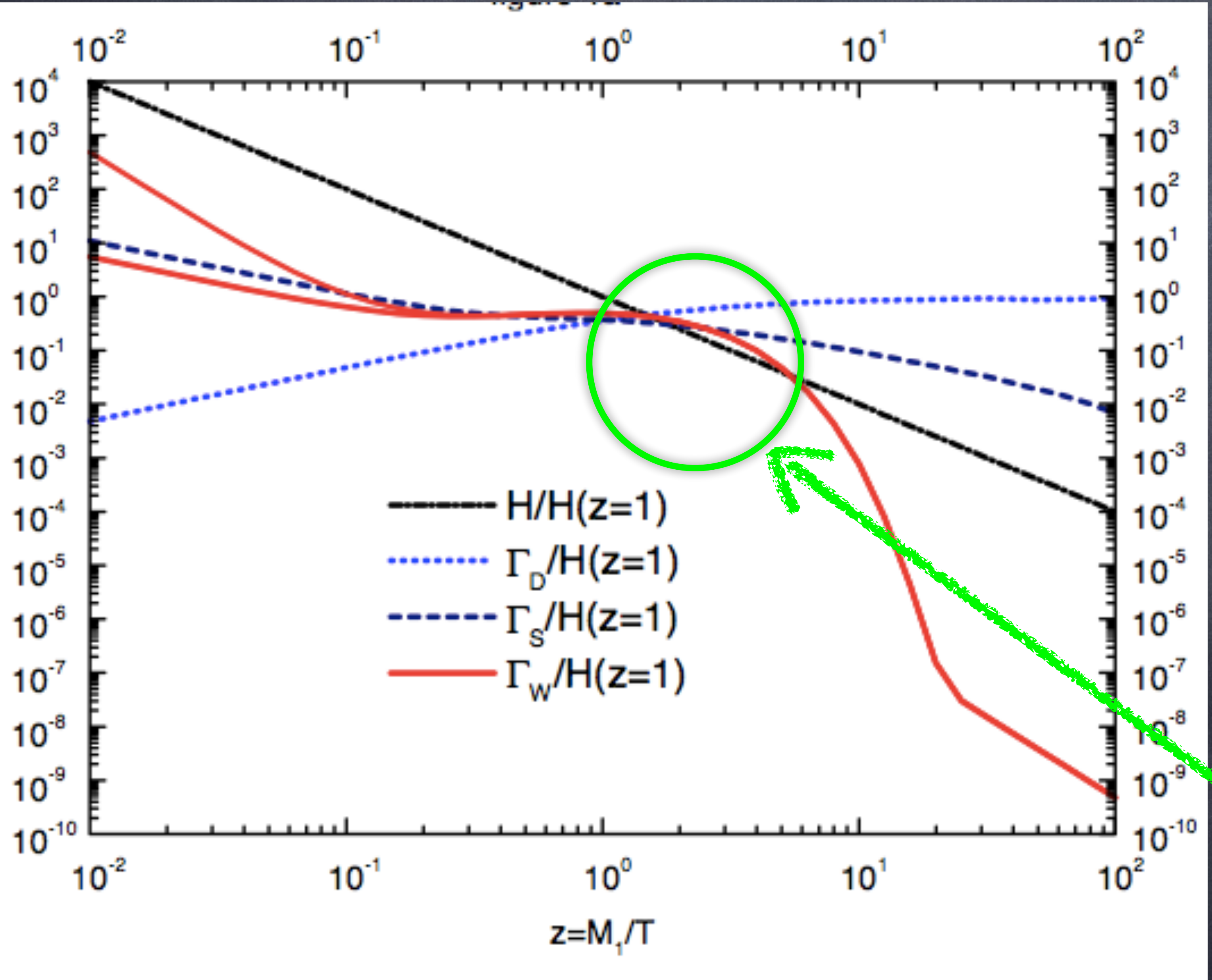
Decay parameter

$$K = \frac{\Gamma_{D_1}|_{T=0}}{H|_{T=M_1}} = \frac{\frac{1}{8\pi}(hh^\dagger)_{11} M_1}{1.66 g_*^{1/2}\frac{M_1^2}{M_P}} = \frac{(hh^\dagger)_{11}\frac{v^2}{M_1}}{8\pi\, 1.66 g_*^{1/2}\frac{v^2}{M_P}} = \frac{\widetilde{m}_1}{m_*}$$

Effective neutrino mass ($\widetilde{m}_1$)

Equilibrium neutrino mass (m_*)

Reaction rates of various parameters



Buchmuller, Bari, Plumacher (2004)

$$m_* \simeq 10^{-3}\ \text{eV}$$

$$H(T) = 1.66 g_*^{1/2}\frac{T^2}{M_p}$$

$$M_1 = 10^{10}\ \text{GeV},\ \widetilde{m}_1 = 10^{-3}\ \text{eV},$$

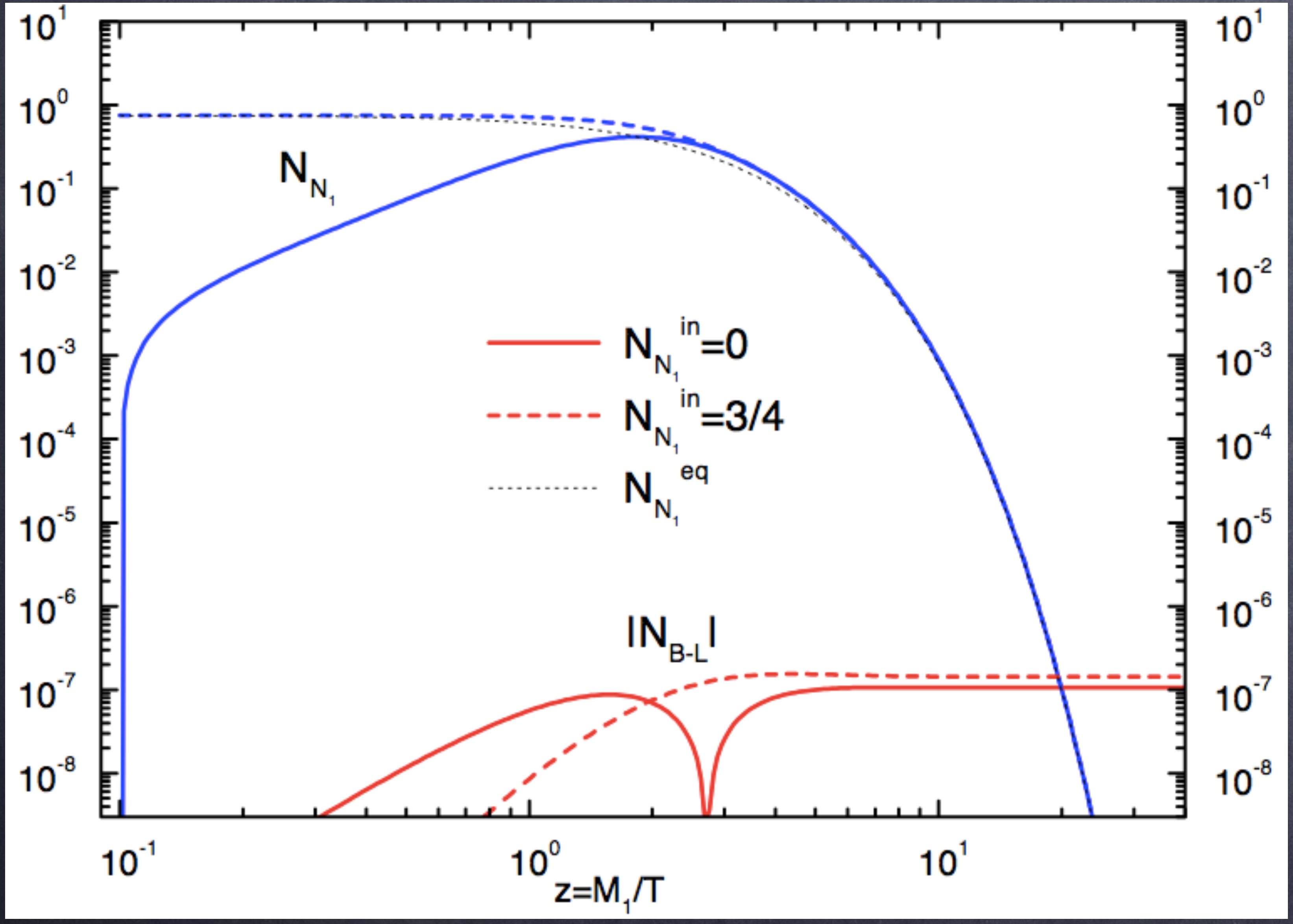
$$\bar{m} = \sqrt{m_{\nu 1}^2 + m_{\nu 2}^2 + m_{\nu 3}^2} = 0.05\ \text{eV}$$

Out of Equilibrium :

$$T \simeq M_1, \quad \widetilde{m}_1 \simeq m_*$$

Evolution of B-L

For different initial conditions : zero initial abundance, and thermal abundance



Yukawa interactions are strong enough to bring the heavy neutrinos in thermal equilibrium

$$M_1 = 10^{10}\ \mathrm{GeV},\ \widetilde{m}_1 = 10^{-3}\ \mathrm{eV},$$

$$\bar{m} = \sqrt{m_{\nu 1}^2 + m_{\nu 2}^2 + m_{\nu 3}^2} = 0.05\ \mathrm{eV}$$

$$|\epsilon_1| = 10^{-6}$$

Observed asymmetry: $\eta_B \simeq 10^{-3} \times N_{B-L} \sim 10^{-10}$

Buchmuller, Bari, Plumacher (2004

Key conditions

(1) Initial abundance of right handed neutrinos: $N_{N_1}^{in} = 0, \quad \text{or} \quad N_{N_1}^{in} = 3/4$

(2) Decay parameter: $K = \frac{\Gamma_D(z=\infty)}{H(z=1)} = \frac{\widetilde{m}}{m_*}$

(3) CP asymmetry: ϵ

$K \ll 1$

Far out of equilibrium, z >> 1

Weak washout regime : initial B-L asymmetry is not washed out

$K \gg 1$

Strong washout regime: final B-L asymmetry is independent of the initial condition

Buchmuller, Bari, Plumacher (2004)

Beyond Vanilla Thermal Leptogensis

- Flavour effects in heavy RH neutrino sector, and in lepton sector
- Resonant Enhancements of CP phase: One can realize leptogenesis close to TeV scale
- Dirac leptogenesis
- Non-thermal leptogenesis

S. Davidson, E. Nardi and Y.Nir, Phys. Rept. 466, 105 (2008)[arXiv: 0802.2962 [hep-ph]]

A. Pilaftsis and T.E.J.Underwood,Electroweak-scale resonant leptogenesis,''Phys.Rev.D 72, 113001 (2005)[hep-ph/0506107].

W. Buchmuller, P. Di Bari and M. Plumacher,``Leptogenesis for pedestrians,'' Annals Phys.315, 305 (2005)[hep-ph/0401240].

Thermal Leptogensis

Davidson+Ibarra (02), Buchmuller, et.al (03), Hambye , et.al. (03)

CP Asymmetry

For $M_{2,3} \gg M_1$: *Upper bound on* ϵ

$$\epsilon = \frac{3}{16}\frac{M_1 m_{\nu 3}}{\langle H\rangle^2} f(m_{\nu i}, \tilde{m}_1)$$

Hierarchical: $m_{\nu 1} \to 0 \implies \epsilon = \frac{3}{16\pi}\frac{M_1 m_{\nu 3}}{\langle H\rangle^2}$

Degenerate: $m_{\nu 3} = m_{\nu 1} \implies \epsilon = 0$

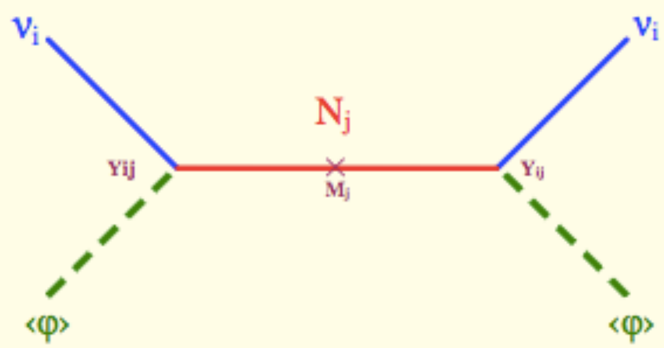
Efficiency parameter

$$\eta = \frac{\Gamma_{N1}}{H(T{=}M_1)} \quad \leq 1$$

$$\eta \sim \frac{10^{-3} eV}{\tilde{m}_1}$$

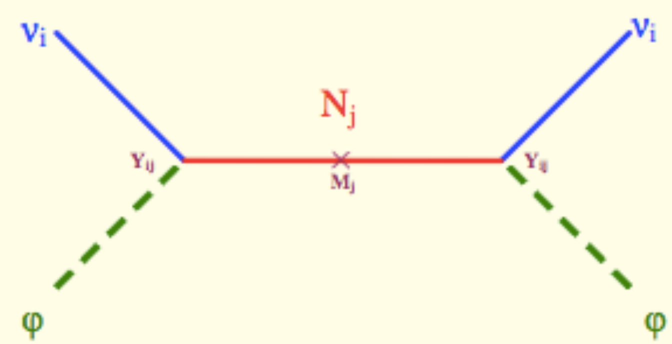
$$\tilde{m}_1 \equiv \frac{(Y^\dagger Y)_{11}\langle H\rangle^2}{M_1} \leq 10^{-3} eV$$

$M_1 \gtrsim 10^9$ GeV ($\implies T_{RH} \gtrsim 10^9$ GeV)



Light Neutrino Masses

$m_i \propto \sum_j Y_{ij}^2/M_j$



$L-$changing $2\to 2$ Scattering

$\sigma \propto \sum_i |\sum_j Y_{ij}^2/M_j|^2$

- Require that $\Delta L = 2$ washout effects are not too strong:

$$m_1^2 + m_2^2 + m_3^2 \lesssim (0.15\ eV)^2$$

Neutrino decoupling

$$e^{+} + e^{-} \leftrightarrow \nu_e + \bar{\nu}_e, \quad e^{\pm} + \nu_e \to^{\pm} + \nu_e, \; e^{\pm} + \bar{\nu}_e \to e^{\pm} + \bar{\nu}_e$$

$$\sigma_{e\nu} \approx \mathcal{O}(1)\frac{\alpha_w^2}{M_{W,Z}^4}T^2 \qquad \text{Neutrinos decouple} : \Gamma_{e\nu} \sim (\sigma_{e\nu} n_e) \sim H(T)$$

$$T_{\nu_e} \sim \mathcal{O}(1)\alpha_w^{-2/3} M_W^{4/3} \sim 1.5\text{MeV}$$

ν_μ, & ν_τ decouple earlier

$$\text{Note} : \; T \sim m_e \sim 0.5 \text{ MeV}$$

$$\left(\frac{T_\gamma}{T_\nu}\right)^3 \left(\frac{g_\gamma + g_{e^\pm}}{g_\nu}\right) = \text{const.} = \frac{11}{4}$$

$$T_\nu \sim (2.73 \text{ K}/1.4) \sim 1.95 \text{ K}$$

photons: $g_\gamma = 2$ (2 spin states);

neutrinos: $g_\nu = 6$ (6 flavors);

electrons: $g_{e^-} = 2$ (2 spin states);

positrons: $g_{e^+} = 2$ (2 spin states);

$$\rho = g_\star \frac{\pi^2}{30} T^4 \equiv \frac{\pi^2}{30} T^4 \left[\sum_{i=\text{bosons}} g_i + \frac{7}{8} \sum_{i=\text{fermions}} g_i \right]$$

$$g_\star = 2 + \frac{7}{8}(6 + 2 + 2) = 2 + \frac{70}{8} = 2 + 8.75 = 10.75.$$

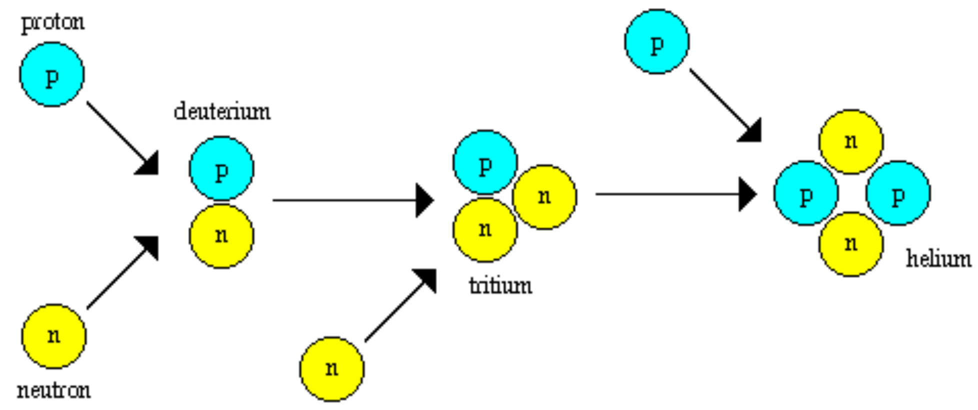
$$\rho \;\; = \;\; \tilde{g}_\star \frac{\pi^2}{30} T^4 \equiv \frac{\pi^2}{30} \left[T_\gamma^4 \sum_{i=\text{bosons}} g_i + T_\nu^4 \frac{7}{8} \sum_{i=\text{fermions}} g_i \right] = \frac{\pi^2}{30} T_\gamma^4 \left[2 + \left(\frac{4}{11}\right)^{4/3} \frac{21}{4} \right] = \frac{\pi^2}{30} T_\gamma^4 [3.36]$$

Big Bang Nucleosynthesis

Weak interactions & nuclear reactions in an expanding universe

Nucleosynthesis

as the Universe cools, protons and neutrons can fuse to form heavier atomic nuclei



$$\frac{n_p^{(0)}}{n_n^{(0)}} = \frac{g_p \left(\frac{m_p T}{2\pi}\right)^{3/2} e^{-m_p/T}}{g_n \left(\frac{m_n T}{2\pi}\right)^{3/2} e^{-m_n/T}} = \left(\frac{m_p}{m_n}\right)^{3/2} e^{(m_n - m_p)/T} \approx e^{Q/T}$$

$$\begin{aligned} p + e^- &\leftrightarrow n + \nu_e \\ p + \bar{\nu}_e &\leftrightarrow n + e^+ \\ n &\leftrightarrow p + e^- + \bar{\nu}. \end{aligned}$$

$$\Gamma \sim < \sigma |v| > n_A(\eta, T)$$

$$\eta \equiv \frac{n_B}{n_\gamma} = \frac{n_b - n_{\bar{b}}}{n_\gamma} \ : \ \text{Baryon Asymmetry}$$

$$\eta = 2.68 \times 10^{-5} (\Omega_B h^2)$$

$$\Gamma_{pe\to\nu n} = \begin{cases} \Gamma_{n\to pe\nu} (T/m_e)^3 \exp(-Q/T) & if\ T \le Q,\ m_e; \\ G_F^2 T^5 & if\ T \ge Q,\ m_e. \end{cases}$$

$$\Gamma_F / H(T) \sim (T/0.8\ \text{MeV})^3$$

$$\tau_{n\to pe\nu} \sim 886.7\ \text{s}$$

$$Q \sim 1.293\ \text{MeV}$$

$$X_n \to X_{n,EQ} \equiv \frac{n_n^{(0)}}{n_n^{(0)} + n_p^{(0)}} = \frac{1}{1 + (n_p^{(0)}/n_n^{(0)})} \sim (1/6)\ ; \ X_n \sim \frac{1}{6} \exp[-t/\tau_{n\to pe\nu}] \sim 0.15 \times 0.74 \sim 0.11$$

$$Y_P(^4\text{He}) \approx 2X_n \sim 0.22$$

$$X_A \propto \eta^{A-1}$$

Sources of Baryogenesis

$$B = \frac{B-L}{2} + \frac{B+L}{2}$$

Asymmetry in baryonic sector

Asymmetry in both

Asymmetry in leptonic sector

(Leptogenesis)

10^{12} GeV

100 GeV

1 MeV

Conversion into baryonic asymmetry

~~EW Accidental Symmetry~~

Observed baryonic abundance

Thermal history of the universe

Thermodynamics of ultra-relativistic plasma:

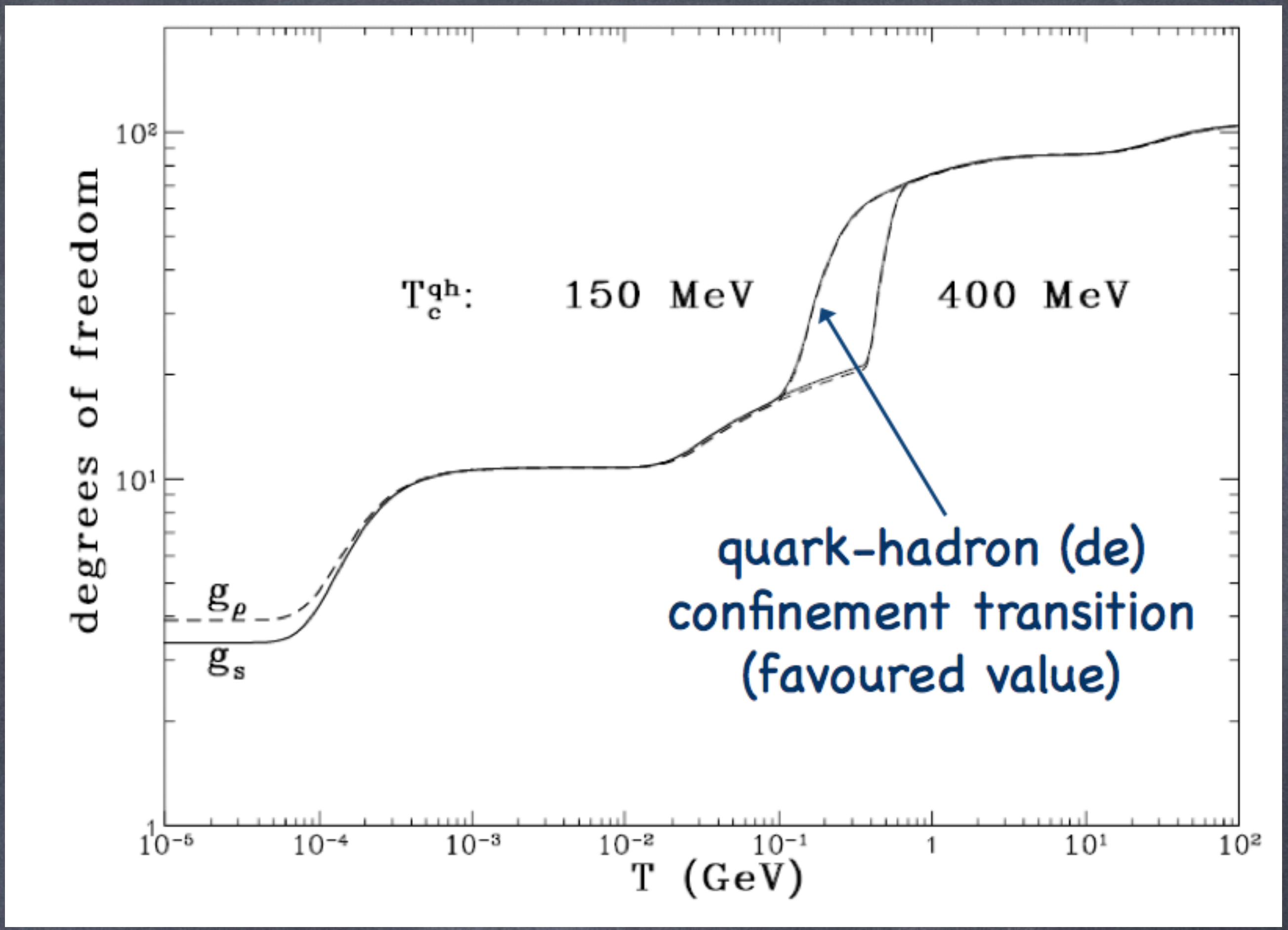
Number density: $n = \frac{\xi(3)}{\pi^2} g'(T) T^3$

Energy density: $\rho = 3p = \frac{\pi^2}{30} g(T) T^4$

Entropy density: $s \equiv \frac{p+\rho}{T} = \frac{2\pi^2}{45} g(T) T^3$

$$g'(T) = g_b(T) + \frac{3}{4} g_f(T)$$

$$g(T) = g_b(T) + \frac{7}{8} g_f(T)$$



$$H = \left(\frac{\dot{a}}{a}\right) = \sqrt{\frac{\rho}{3M_p^2}} = 1.66 \times g^{1/2} \frac{T^2}{M_p}$$

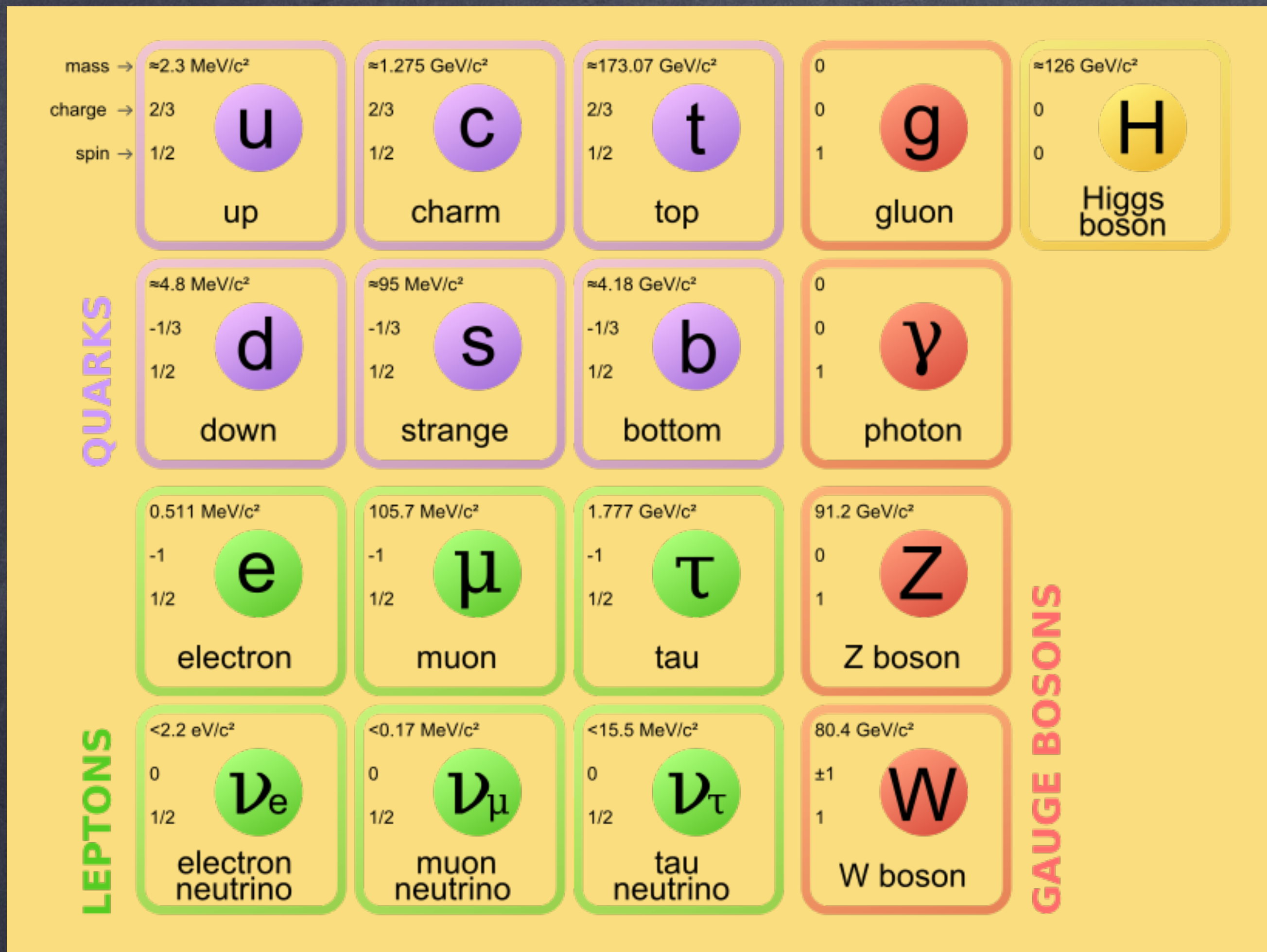
$$\text{Entropy Conservation}: \ \frac{d}{dt}(sa^3) = 0\,, \quad => s \propto a^{-3}\,, \ a(T) \propto \frac{1}{T}$$

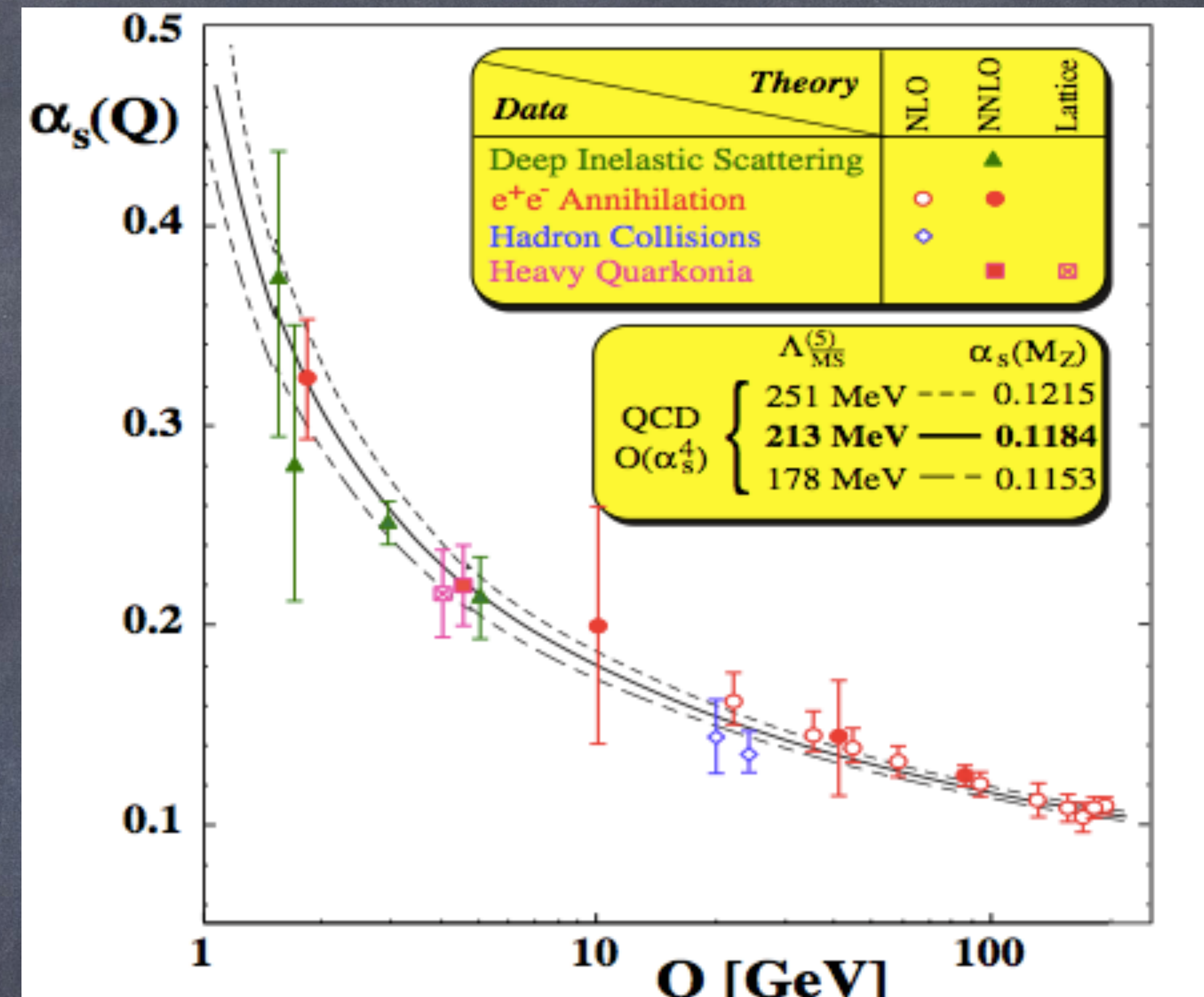
History of $g\ (T)$

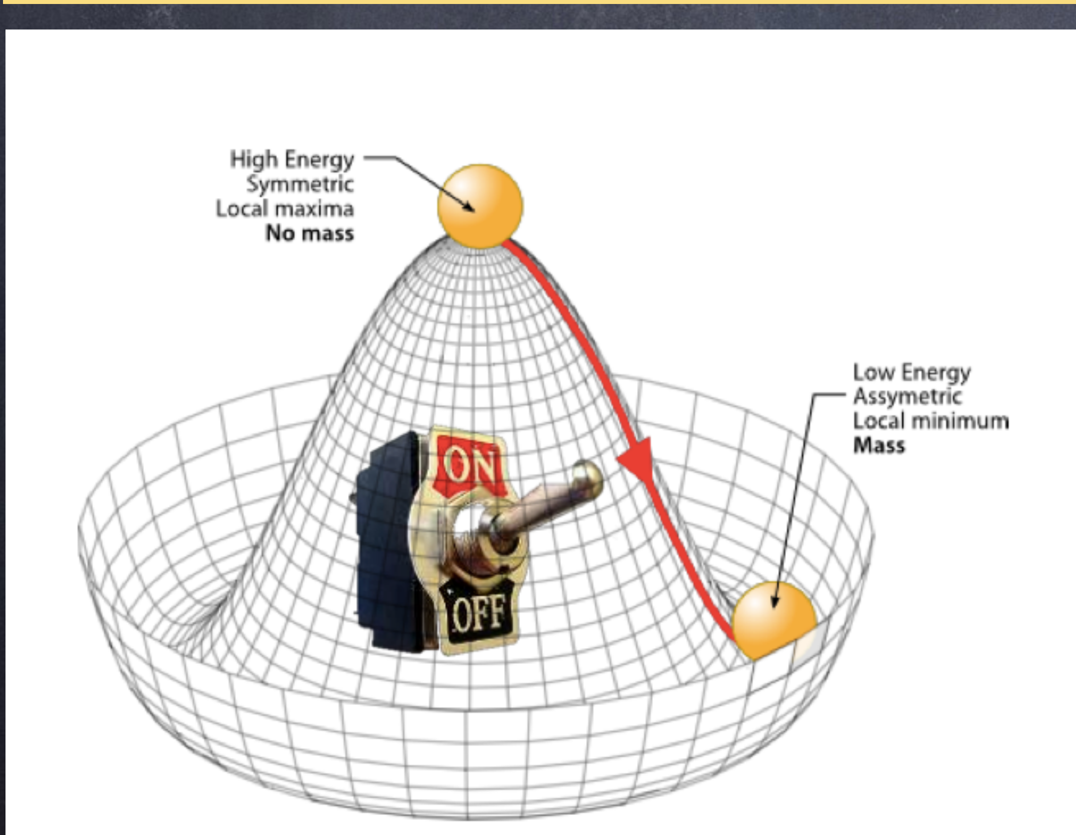
$T \sim 200$ GeV	all present	106.75	
$T \sim 100$ GeV	EW transition	(no effect)	
$T < 170$ GeV	top-annihilation	96.25	
$T < 80$ GeV	$W^\pm$, Z^0, H^0	86.25	
$T < 4$ GeV	bottom	75.75	
$T < 1$ GeV	charm, τ^-	61.75	
$T \sim 150$ MeV	QCD transition	17.25	(u,d,g$\rightarrow \pi^{\pm,0}$, $37 \rightarrow 3$)
$T < 100$ MeV	$\pi^\pm$, π^0, μ^-	10.75	$e^\pm$, ν, $\bar{\nu}$, γ left
$T < 500$ keV	e^- annihilation	(7.25)	$2 + 5.25(4/11)^{4/3} = 3.36$

$$\frac{t}{1s} \approx 2.42 \times g^{-1/2} \left(\frac{1\ \text{MeV}}{T}\right)^2$$

Standard Model







$$\mu^2 \frac{\partial \alpha_s(\mu)}{\partial \mu^2} = \beta(\alpha_s(\mu))$$

$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + \alpha_s(\mu_0^2)\beta_0 \ln \frac{\mu^2}{\mu_0^2}}$$

If $\beta_0 > 0$ we have $\alpha_s(\mu^2) \to 0$ for $\mu^2 \to \infty \Leftrightarrow$ Asymptotic Freedom