#### The Standard model as an effective field theory...

A renormalisable, spontaneously broken, local gauge quantum field theory

$$L_{eff}\left(\phi_{light}, \psi_{heavy}, M_{X}, E\right) \xrightarrow{E \ll M_{X}} L_{eff}\left(\phi_{light}, E\right) + O\left(\frac{1}{M_{X}}\right)$$

1



#### The Standard model as an effective field theory...

A renormalisable, spontaneously broken, local gauge quantum field theory

$$L_{eff}\left(\phi_{light}, \psi_{heavy}, M, E\right) \xrightarrow{E \ll M_X} L_{eff}\left(\phi_{light}, E\right) + O\left(\frac{1}{M}\right)$$
  
• Renormalisable
  
• Vectors gauge bosons
  
•  $A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\theta$ 
  
• Fermions chiral
  
• Massless gauge bosons - vectorlike couplings
  
• Massive gauge bosons - chiral couplings
  
•

#### The Standard model as an effective field theory...

A renormalisable, spontaneously broken, local gauge quantum field theory

$$L_{eff}\left(\phi_{light},\psi_{heavy},M,E\right) \xrightarrow{E \ll M_X} L_{eff}\left(\phi_{light},E\right) + O\left(\frac{1}{M}\right)$$

- Renormalisable
- Vectors gauge bosons
- Fermions chiral
- Massless gauge bosons vectorlike couplings
- Massive gauge bosons chiral couplings
- Light Higgs X The hierarchy problem





Field theory:  $\delta m^2$  not measureable ...only  $m^2 = m_0^2 + \delta m^2$  "physical"

Hierarchy problem?  

$$h \begin{pmatrix} t, W, \\ Z, h \end{pmatrix} h$$

$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2} \left(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2\right) \Lambda^2 = \left(\frac{\Lambda}{500 GeV}\right)^2$$

**Field theory:**  $\delta m^2$  not measureable

...only 
$$m^2 = m_0^2 + \delta m^2$$
 "physical"

#### **GUTS**:

$$\delta m_h^2 \propto M_X^2 \ln \left( \frac{Q^2 + M_X^2}{\Lambda^2} \right)$$

h(Q) (X) h(Q)

- "real hierarchy problem"

Hierarchy problem?  

$$\delta m_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2} \left(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2\right) \Lambda^2 = \left(\frac{\Lambda}{500 GeV}\right)^2$$
Field theory:  $\delta m^2$  not measureable

...only 
$$m^2 = m_0^2 + \delta m^2$$
 "physical"

#### GUTS:

$$\delta m_h^2 \propto M_X^2 \ln \left( \frac{Q^2 + M_X^2}{\Lambda^2} \right)$$

- "real hierarchy problem"



### ${\rm GUTS} \Longrightarrow {\rm SUSYGUTS}$

$$\delta m_h^2 \propto \left( M_{\tilde{X}}^2 - M_X^2 \right)$$

# III. SUSY GUTS



### Supermultiplets

SO(10):  $V_{45}$  Vector +3  $\varphi_{16}$  chiral +  $H_{10}$  chiral +...

### SUSY gauge coupling unification

$$\alpha_i^{-1}(\mu) = \alpha^{-1}(M_x) + \frac{1}{2\pi}b_i \ln\left(\frac{M_x}{\mu}\right) + \dots$$

$$b_i^{SM} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_g \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + H \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix}$$

$$b_i^{MSSM} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_g \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + H \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$



#### SUSY gauge coupling unification



 $\sin^2 \theta_W = 0.2334 \pm 0.0025 - 0.25(\alpha_s - 0.119) = 0.2311 \pm 0.0007 \quad (Expt)$  $\alpha_s = 0.134 \pm 0.01 - 4(\sin^2 \theta_W - 0.2334) = 0.119 \pm 0.01 \quad (Expt)$ 



 $\sin^2 \theta_W = 0.23116(12) \quad (Expt)$ 

$$\alpha_s = 0.134 \pm 0.01 - 4(\sin^2 \theta_W - 0.23116)$$
 c.f. 0.1184(7) (Expt)

#### **Gauge unification - Heterotic String**

$$L_{eff}^{HS} = \int d^{10}x \sqrt{g} e^{-\phi} \left(\frac{4}{\alpha'^4}R + \frac{k_i}{\alpha'^3}TrF_i^2 + ...\right)$$

$$\int d^4x V \qquad \alpha_{10}^{-1} \qquad \alpha' = 1/M_{string}^2 \text{ only scale}$$

$$G_N = \frac{\alpha_{10}\alpha'^4}{64\pi V}, \quad \alpha_{string} = \frac{\alpha_{10}\alpha'^3}{16\pi V} \qquad \Theta_N = \frac{\alpha_{string}\alpha'}{4}$$

$$\frac{1}{g_i^2(M_Z)} = \frac{k_i}{g_{string}^2} + b_i \ln\left(\frac{M_{string}}{M_Z}\right) + \Delta_i$$

 $M_{string} = g_{string} M_{Planck} = 3.6 \times 10^{17} GeV \quad c.f.M_{U}^{"expt"} = (2.6 \pm 2).10^{16} GeV$ 

#### **Gauge unification - Heterotic String**

$$L_{eff}^{HS} = \int d^{10}x \sqrt{g} e^{-\phi} \left(\frac{4}{\alpha'^4}R + \frac{k_i}{\alpha'^3}TrF_i^2 + ...\right)$$

$$\int d^4x V \qquad \widehat{\alpha_{10}}^{-1} \qquad \alpha' = 1/M_{string}^2 \text{ only scale}$$

$$G_N = \frac{\alpha_{10}\alpha'^4}{64\pi V}, \quad \alpha_{String} = \frac{\alpha_{10}\alpha'^3}{16\pi V} \qquad \Theta_N = \frac{\alpha_{String}\alpha'}{4}$$

$$\frac{1}{g_i^2(M_Z)} = \frac{k_i}{g_{string}}^2 + b_i \ln\left(\frac{M_{string}}{M_Z}\right) + \Delta_i$$

 $M_{string} = g_{string} M_{Planck} = 3.6 \times 10^{17} GeV \dots close \dots but not close enough!$ ...string threshold corrections,  $\Delta_i$ ?

 $SU(5) \xrightarrow{M_X}{\Sigma_{24}} SU(3) \times SU(2) \times U(1) \xrightarrow{M_W}{H_{\overline{5}}} SU(3) \times U(1)$ 

$$SU(5) \xrightarrow{M_X} SU(3) \times SU(2) \times U(1) \xrightarrow{M_W} SU(3) \times U(1)$$

$$P = \frac{\beta_2}{2} M Tr(\Sigma^2) + \frac{\beta_3}{3} Tr(\Sigma^3)$$

superpotential

 $\langle \Sigma \rangle = v_3 \text{Diagonal}(2,2,2,-3,-3)$ 

corrections

 $SU(5) \xrightarrow{M_X} SU(3) \times SU(2) \times U(1) \xrightarrow{M_W} SU(3) \times U(1)$ 

$$SU(5) \xrightarrow{M_{X}}{\Sigma_{24}} SU(3) \times SU(2) \times U(1) \xrightarrow{M_{W}}{H_{5}} SU(3) \times U(1)$$
$$P_{5_{M}} = -\frac{1}{\sqrt{2}} M_{ij}^{d} \psi_{i\alpha} \chi_{j}^{\alpha\beta} H_{d\beta} - \frac{1}{4} M_{ij}^{u} \varepsilon_{\alpha\beta\gamma\delta\rho} \chi_{i}^{\alpha\beta} \chi_{j}^{\gamma\delta} H_{u}^{\rho}$$

$$SU(5) \xrightarrow{M_{X}}{\Sigma_{24}} SU(3) \times SU(2) \times U(1) \xrightarrow{M_{W}}{H_{5}} SU(3) \times U(1)$$
$$P_{5_{M}} = -\frac{1}{\sqrt{2}} M_{ij}^{d} \psi_{i\alpha} \chi_{j}^{\alpha\beta} H_{d\beta} - \frac{1}{4} M_{ij}^{u} \varepsilon_{\alpha\beta\gamma\delta\rho} \chi_{i}^{\alpha\beta} \chi_{j}^{\gamma\delta} H_{u}^{\rho}$$
$$P_{Higgs} = \mu H_{u} H_{d} + \lambda H_{u} \Sigma H_{d}$$
$$V = \left( \left| \mu H_{u} + \lambda H_{u} \Sigma \right|^{2} + \left| \mu H_{d} + \lambda \Sigma H_{d} \right|^{2} \right) + \left| H_{u} H_{d} - \frac{1}{5} (H_{u} H_{d}) \right|^{2}$$



$$SU(5) \xrightarrow{M_{\chi}}{\Sigma_{24}} SU(3) \times SU(2) \times U(1) \xrightarrow{M_{W}}{H_{5}} SU(3) \times U(1)$$

$$P_{5_{M}} = -\frac{1}{\sqrt{2}} M_{ij}^{d} \psi_{i\alpha} \chi_{j}^{\alpha\beta} H_{d\beta} - \frac{1}{4} M_{ij}^{u} \varepsilon_{\alpha\beta\gamma\delta\rho} \chi_{i}^{\alpha\beta} \chi_{j}^{\gamma\delta} H_{u}^{\rho}$$

$$P_{Higgs} = \mu H_{u} H_{d} + \lambda H_{u} \Sigma H_{d}$$

$$V = \left( \left| \mu H_{u} + \lambda H_{u} \Sigma \right|^{2} + \left| \mu H_{d} + \lambda \Sigma H_{d} \right|^{2} \right) + \left| H_{u} H_{d} - \frac{1}{5} (H_{u} H_{d}) \right|^{2}$$

Must forbid these terms by symmetry

$$SU(5) \xrightarrow{M_{X}}{\Sigma_{24}} SU(3) \times SU(2) \times U(1) \xrightarrow{M_{W}}{H_{5}} SU(3) \times U(1)$$

$$P_{5_{M}} = -\frac{1}{\sqrt{2}} M_{ij}^{d} \psi_{i\alpha} \chi_{j}^{\alpha\beta} H_{d\beta} - \frac{1}{4} M_{ij}^{u} \varepsilon_{\alpha\beta\gamma\delta\rho} \chi_{i}^{\alpha\beta} \chi_{j}^{\gamma\delta} H_{u}^{\rho}$$

$$P_{Higgs} = \mu H_{u} H_{d} + \lambda H_{u} \Sigma H_{d}$$

$$V = \left( \left| \mu H_{u} + \lambda H_{u} \Sigma \right|^{2} + \left| \mu H_{d} + \lambda \Sigma H_{d} \right|^{2} \right) + \left| H_{u} H_{d} - \frac{1}{5} (H_{u} H_{d}) \right|^{2}$$

Must forbid these terms by symmetry + doublet- triplet splitting



D=5 proton decay amplitude

Missing doublet mechanism

No (1,2) component

 $\Theta_{50} = (8,2) + (6,3) + (\overline{6},1) + (3,2) + (\overline{3},1) + (1,1)$ 

Missing doublet mechanism

No (1,2) component

 $\Theta_{50} = (8,2) + (6,3) + (\overline{6},1) + (3,2) + (\overline{3},1) + (1,1)$ 

$$P_{MD} = b \Theta \Sigma_{75} H_u + b' \overline{\Theta} \Sigma_{75} H_d + \widetilde{M} \overline{\Theta} \Theta$$

$$ig \langle \Sigma_{75}ig 
angle \propto M$$
 breaks SU(5) to SM

Missing doublet mechanism

No (1,2) component

 $\Theta_{50} = (8,2) + (6,3) + (\overline{6},1) + (3,2) + (\overline{3},1) + (1,1)$ 

$$P_{MD} = b \Theta \Sigma_{75} H_u + b' \overline{\Theta} \Sigma_{75} H_d + \widetilde{M} \overline{\Theta} \Theta$$

$$\left< \Sigma_{75} \right> \propto M$$
 breaks SU(5) to SM

$$P_{MD} \supset bM \Theta_3 H_{uT} + b' M \overline{\Theta}_3 H_{dT} + \widetilde{M} \overline{\Theta}_3 \Theta_3$$

Triplets get mass  $\frac{M^2}{\widetilde{M}}$  (Still need to drive SSB - later)

### Higher dimensions (String unification)



![](_page_25_Figure_3.jpeg)

### Higher dimensions (String unification)

![](_page_26_Figure_2.jpeg)

Breit, Ovrut, Segre

*e.g.* 
$$SU(5): H = Z_3, \overline{H} = Diag(\alpha, \alpha, \alpha, 1, 1), \alpha = e^{2i\pi/3}$$
  
 $\left(R \otimes \overline{R}\right): (1 \otimes \overline{5}) \rightarrow \left(\begin{array}{c} H^-\\ \overline{H}^0 \end{array}\right)_1, (3, \overline{5}) \rightarrow \left(\begin{array}{c} e\\ v_e \end{array}\right)_1 \oplus \left(\begin{array}{c} d^c\\ d^c\\ d^c \end{array}\right)_{\alpha^2}, \text{ Matter } \rightarrow (3, \overline{5} + 10)$ 

### SUSY GUTS - Nucleon decay

![](_page_28_Figure_0.jpeg)

![](_page_29_Figure_0.jpeg)

![](_page_30_Figure_0.jpeg)

$$\Delta(B-L)=0$$

![](_page_30_Figure_2.jpeg)

D=5 proton decay amplitude

![](_page_31_Figure_0.jpeg)

$$\Delta(B-L)=0$$

![](_page_31_Figure_2.jpeg)

D=5 proton decay amplitude

 $W = h^{E} L H_{d} \overline{E} + h^{D} Q H_{d} \overline{D} + h^{U} Q H_{u} \overline{U} + \mu H_{d} H_{u}$ +  $\lambda L L \overline{E} + \lambda' L Q \overline{D} + \kappa L H_{u} + \lambda'' \overline{U} \overline{D} \overline{D}$ +  $\frac{1}{M} (Q Q Q L + Q Q Q H_{d} + Q \overline{U} \overline{E} H_{d} + ...(L))$ 

 $W = h^{E} LH_{d} \overline{E} + h^{D} QH_{d} \overline{D} + h^{U} QH_{u} \overline{U} + \mu H_{d} H_{u}$  $+ \lambda LL\overline{E} + \lambda' LQ\overline{D} + \kappa LH_{u} + \lambda'' \overline{U}\overline{D}\overline{D}$  $+ \frac{1}{M} (QQQL + QQQH_{d} + Q\overline{U}\overline{E}H_{d} + ...(L))^{e.g.(LH_{u})^{2}}$ 

$$W = h^{E}LH_{d}\overline{E} + h^{D}QH_{d}\overline{D} + h^{U}QH_{u}\overline{U} + \mu H_{d}H_{u} + \lambda LL\overline{E} + \lambda'LQ\overline{D} + \kappa LH_{u} + \lambda''\overline{U}D\overline{D} + \frac{1}{M} \left( QQQL + QQQH_{d} + Q\overline{U}\overline{E}H_{d} + ...(\cancel{L}) \right)$$

$$\frac{\mathbf{R}-\text{parity:}}{L,\overline{E},Q,\overline{D},\overline{U}, \ \theta \ -1}$$
SUSY states odd Weinberg, Sakai

Discrete gauge symmetry -anomaly free

Ibanez, GGR

 $W = h^{E} L H_{d} \overline{E} + h^{D} Q H_{d} \overline{D} + h^{U} Q H_{u} \overline{U} + \mu H_{d} H_{u}$  $+ \lambda L L \overline{E} + \lambda' L Q \overline{D} + \kappa L H_{u} + \lambda'' \overline{U} \overline{D} \overline{D}$  $+ \frac{1}{M} \left( Q Q Q L + Q Q Q H_{d} + Q \overline{U} \overline{E} H_{d} + \dots (L') \right)$ 

R-parity: Z<sub>2</sub> SUSY states odd

Baryon "parity":  $Z_3$ 

LSP unstable

**Proton hexality:**  $Z_6 = Z_2^R \times Z_3^B$ 

LSP stable  $\frac{1}{M}LLH_{u}H_{u}$ 

Dreiner, Luhn, Thormeier

	$R_2$	$R_3L_3$	$R_3$	$L_3$	$R_3^2L_3$	$R_{6}^{5}L_{6}^{2}$	$R_6$	$R_{6}^{3}L_{6}^{2}$	$R_6L_6^2$	all $\boldsymbol{Z}_9\&\boldsymbol{Z}_{18}$
$H_d H_u$	$\checkmark$	<b>~</b>	✓	✓	✓	<ul> <li>Image: A start of the start of</li></ul>	✓	~	✓	
$LH_u$		~								
$LL\bar{E}$		<ul> <li>Image: A start of the start of</li></ul>								
$LQ\bar{D}$		<ul> <li>Image: A start of the start of</li></ul>								
$\bar{U}\bar{D}\bar{D}$				~						
QQQL	✓		✓				✓			
$\bar{U}\bar{U}\bar{D}\bar{E}$	$\checkmark$		✓				✓			
$QQQH_d$				✓						
$Q\bar{U}\bar{E}H_d$		~								
$LH_uLH_u$	$\checkmark$	<b>~</b>				<ul> <li>Image: A start of the start of</li></ul>				
$LH_uH_dH_u$		<b>√</b>								
$\bar{U}\bar{D}^*\bar{E}$		✓								
$H_u^* H_d \bar{E}$		<ul> <li>Image: A start of the start of</li></ul>								
$Q\bar{U}L^*$		~								
$QQ\bar{D}^*$				✓						

 $W = h^{E} L H_{d} \overline{E} + h^{D} Q H_{d} \overline{D} + h^{U} Q H_{u} \overline{U} + \mu H_{d} H_{u}$  $+ \lambda L L \overline{E} + \lambda' L Q \overline{D} + \kappa L H_{u} + \lambda'' \overline{U} \overline{D} \overline{D}$  $+ \frac{1}{M} \left( Q Q Q L + Q Q Q H_{d} + Q \overline{U} \overline{E} H_{d} + \dots (L') \right)$ 

![](_page_38_Figure_2.jpeg)

**R-parity:**  $Z_2$ 

Baryon "parity": Z<sub>3</sub>

LSP unstable

SUSY states odd

**Proton hexality:**  $Z_6 = Z_2^R \times Z_3^B$ 

LSP stable  $\frac{1}{M}LLH_{u}H_{u}$ 

Dreiner, Luhn, Thormeier

$$W = h^{E} LH_{d} \overline{E} + h^{D} QH_{d} \overline{D} + h^{U} QH_{u} \overline{U} + \mu H_{d} H_{u} + \lambda LL\overline{E} + \lambda' LQ\overline{D} + \kappa LH_{u} + \lambda'' \overline{U} \overline{D} \overline{D} + \frac{1}{M} (QQQL + QQQH_{d} + Q\overline{U}\overline{E}H_{d} + ...(\mathcal{L}))$$

$$R-parity: Z_{2} \qquad \qquad SUSY \text{ states odd}$$

$$Baryon "parity": Z_{3} \qquad \qquad LSP \text{ unstable}$$

$$Proton hexality: Z_{6} = Z_{2}^{R} \times Z_{3}^{B} \qquad \qquad LSP \text{ stable}$$

$$Z_{N}^{R} R-symmetry \qquad N=4,6,8,12,24 \qquad \qquad LSP \text{ stable}$$

N=4,6,8,12,24

LSP stable  $\frac{1}{M}LLH_{u}H_{u}$ 

Lee, Raby, Ratz, Ross, Schieren, Schmidt-Hoberg, Vaudrevange Babu, Gogoladze, Wang

### A unique solution : $Z_4^R$ discrete **R** symmetry

MSSM spectrum No perturbative  $\mu$  term Commutes with SO(10) Anomaly cancellation

N	$q_{10}$	$q_{\overline{5}}$	$q_{H_u}$	$q_{H_d}$	$q_{\scriptscriptstyle N}$
4	1	1	0	0	2

#### A unique solution : $Z_4^R$ discrete **R** symmetry

MSSM spectrum No perturbative  $\mu$  term Commutes with SO(10) Anomaly cancellation

N	$q_{10}$	$q_{\overline{5}}$	$q_{H_u}$	$q_{H_d}$	$q_{\scriptscriptstyle N}$
4	1	1	0	0	2

![](_page_41_Figure_3.jpeg)

$$A_{\mathrm{SU}(3)-\mathrm{SU}(3)-\mathbb{Z}_{N}} = \frac{1}{2} \sum_{i} \left[ 3 \cdot q_{\mathbf{10}_{i}} + q_{\overline{\mathbf{5}}_{i}} - 4R \right] + 3R$$
  

$$A_{\mathrm{SU}(2)-\mathrm{SU}(2)-\mathbb{Z}_{N}} = \frac{1}{2} \sum_{i} \left[ 3 \cdot q_{\mathbf{10}_{i}} + q_{\overline{\mathbf{5}}_{i}} - 4R \right] + 2R + \frac{1}{2} \left( q_{H} + q_{\bar{H}} - 2R \right)$$
  

$$A_{\mathrm{U}(1)_{Y}-\mathrm{U}(1)_{Y}-\mathbb{Z}_{N}^{R}} = \frac{1}{2} \sum_{g=1}^{3} \left( 3q_{\mathbf{10}}^{g} + q_{\overline{\mathbf{5}}}^{g} \right) + \frac{3}{5} \left[ \frac{1}{2} \left( q_{H_{u}} + q_{H_{d}} \right) - 11 \right] \qquad (R = 1)$$

 $\Rightarrow N = 3, 4, 6, 8, 12, 24$ 

### A unique solution : $Z_4^R$ discrete R symmetry

MSSM spectrum No perturbative  $\mu$  term Commutes with SO(10) Anomaly cancellation

N	$q_{10}$	$q_{\overline{5}}$	$q_{H_u}$	$q_{H_d}$	$q_{\scriptscriptstyle N}$
4	1	1	0	0	2

D=5 operators

$$\frac{1}{M}QQQL \qquad \frac{1}{M}LLH_{u}H_{u}$$

SUSY breaking

 $\langle W \rangle, \langle \lambda \lambda \rangle$  R=2 non-perturbative breaking

Domain walls safe

 $Z_{4R} \rightarrow Z_2^R \quad R-parity$  $\mu \sim m_{3/2}, \ O(\frac{m_{3/2}}{M^2}QQQL)$ 

 $M_{higgs} \approx M_{SUSY}$ 

 $\mu, \mathcal{B}, \mathcal{L}$ 

### Nucleon decay outlook

• Nucleon decay D=6 operators  

$$\tau(p \to \pi^{0}e^{+}) = \left(\frac{M_{\rm GUT}}{10^{16} \text{ GeV}}\right)^{4} \left(\frac{1/35}{\alpha_{\rm GUT}}\right)^{2} \left(\frac{0.015 \text{ GeV}^{3}}{\alpha_{N}}\right)^{2} \left(\frac{5}{A_{L}}\right)^{2} 4.4 \times 10^{34} \text{ yr.}$$
Hadronic matrix element  

$$\tau_{p \to e^{+}\pi^{0}}^{SuperK} > 1 \times 10^{34} \text{ yrs}$$
Giudice, Romanino

$$M_{\rm GUT} > \left(\frac{\alpha_{\rm GUT}}{1/35}\right)^{1/2} \left(\frac{\alpha_N}{0.015 \ {\rm GeV^3}}\right)^{1/2} \left(\frac{A_L}{5}\right)^{1/2} \ 6 \times 10^{15} \ {\rm GeV}$$

 $c.f.M_U = (2.5 \pm 2).10^{16} GeV$