Grand Unified Theories

G. Ross, Pre-SUSY 2014,
Manchester, July 2014

I. Motivation

The Standard Model:

Local Gauge Symmetry: $SU(3)\times SU(2)\times U(1)$

$$L_{SM} = L_{YM} + L_{WD} + L_{Yu} + L_{H}$$

$$L_{YM} = L_{QCD} + L_{I_W} + L_{Y}$$

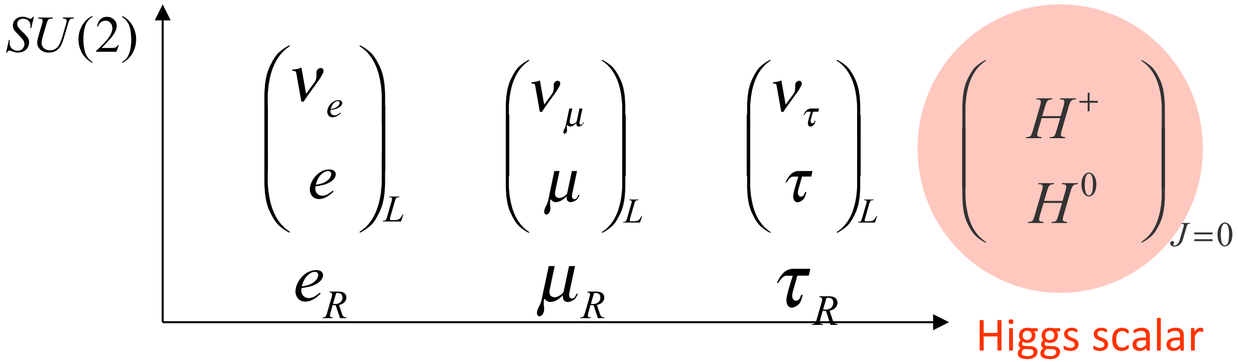
$$= -\frac{1}{4g_3^2}\sum_{A=1}^{8}{}^{\dagger} G^A_{\mu\nu}G^{\mu\nu A} - \frac{1}{4g_2^2}\sum_{a=1}^{3} F^a_{\mu\nu}F^{\mu\nu a} - \frac{1}{4g_1^2}B_{\mu\nu}B^{\mu\nu}$$

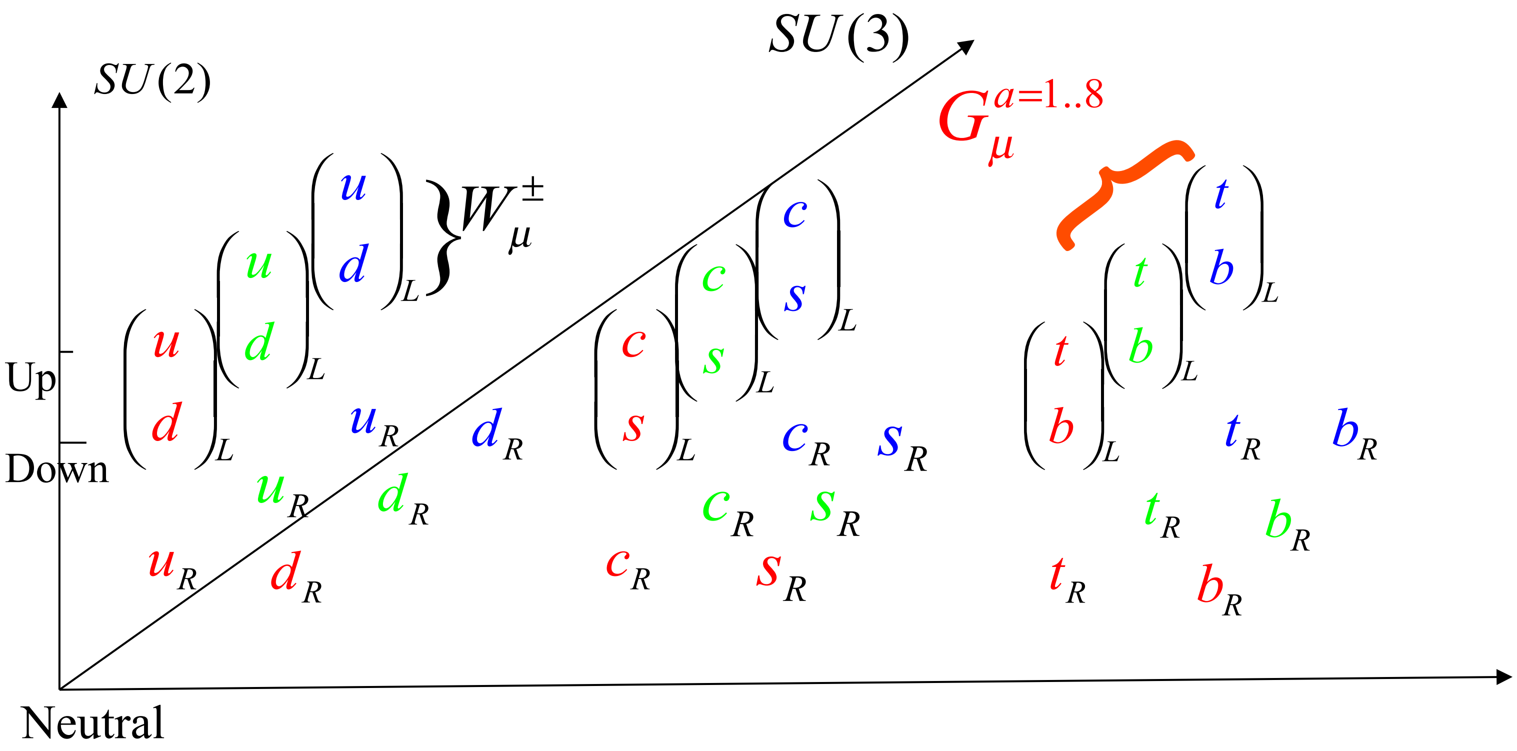
† Adjoint representation: Dimension $SU(n):\ n^2-1$

Multiplet structure –'chiral'

$SU(3)\otimes SU(2)\otimes U(1)$

$$L^{K}_{fermion} = \overline{\psi}\gamma_{\mu}D^{\mu}\psi$$





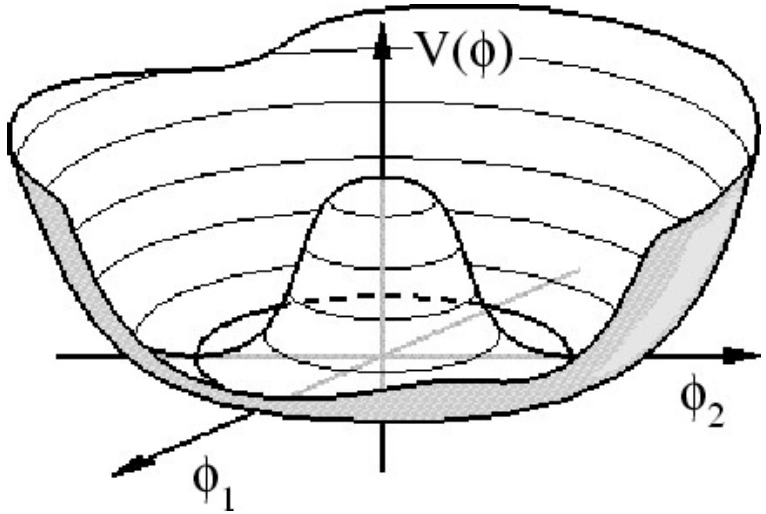
Spontaneously broken

$$\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}_{J=0}$$

Higgs scalar

$$L_H = \left(D_\mu H\right)^\dagger \left(D^\mu H\right) - V(H)$$

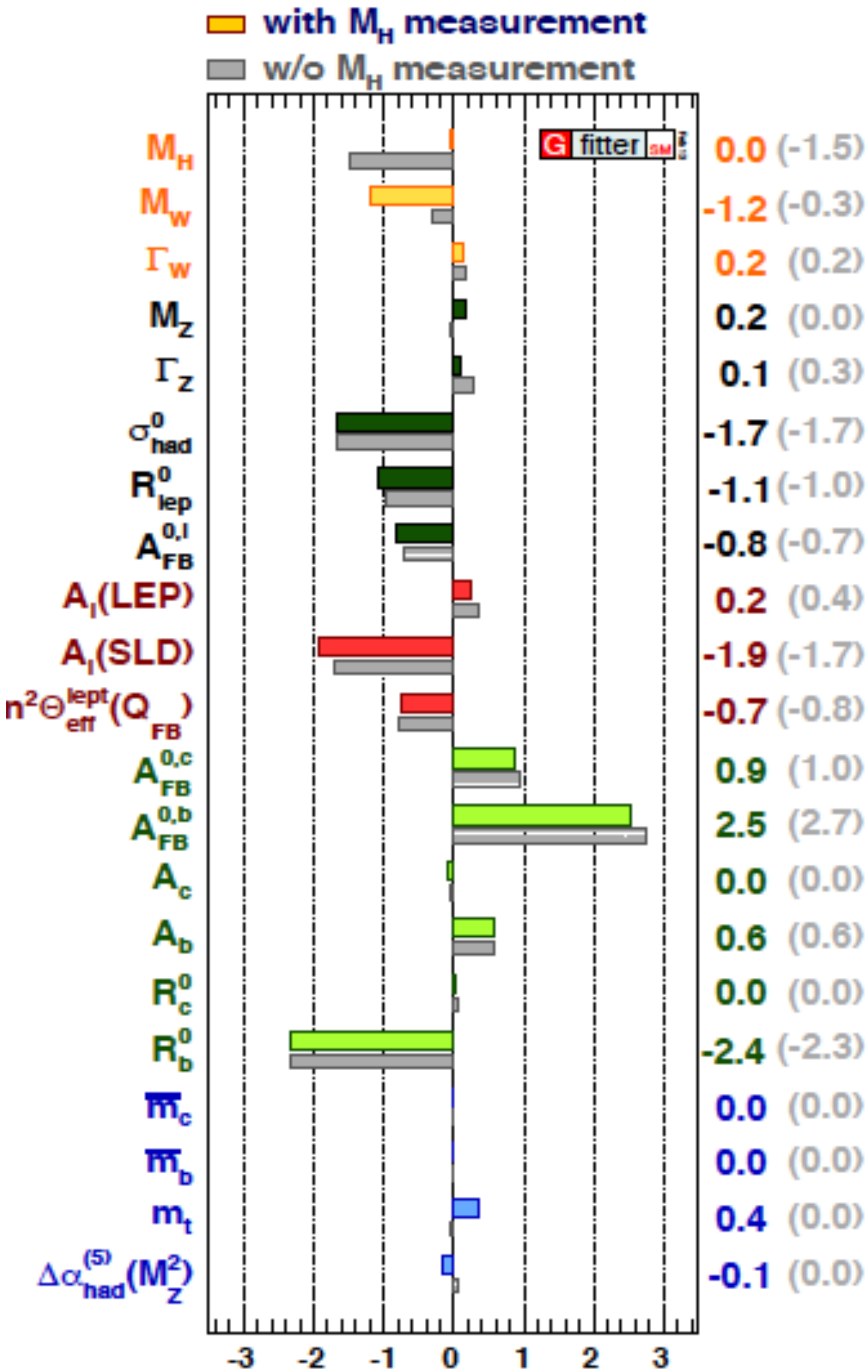
$$V = -m^2 |H|^2 + \lambda |H|^4$$

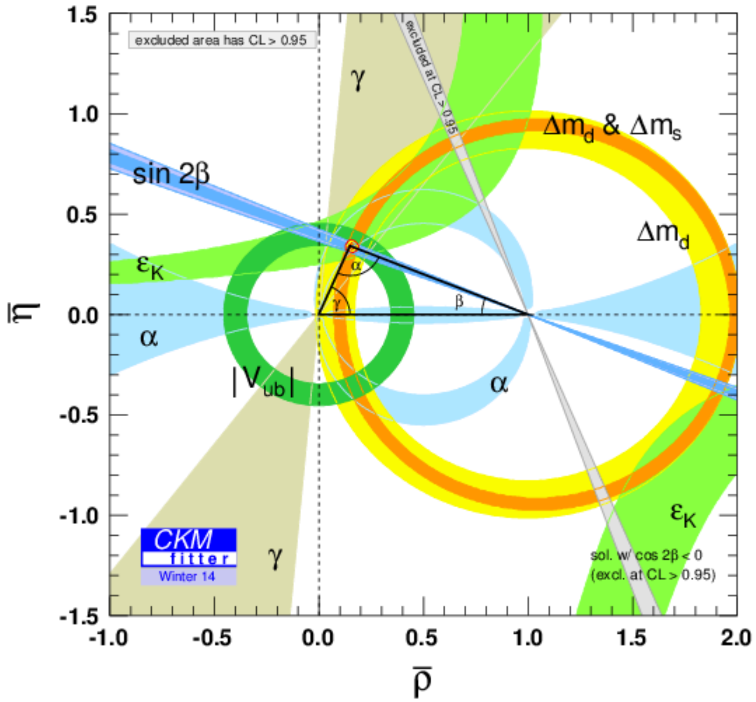


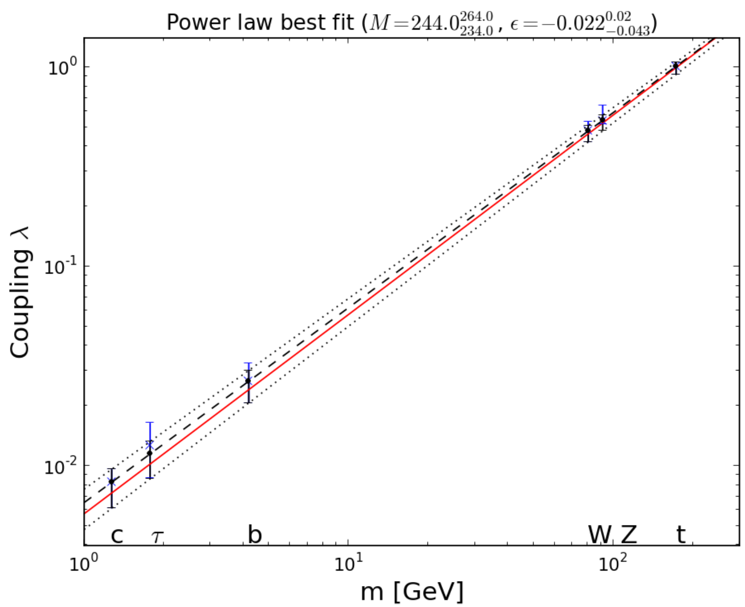
$$H = e^{i\theta.\sigma} \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$SU(2)$: adjoint rep 3 dimensional

$\Rightarrow$ 3 Goldstone modes, θ_i (in absence of gauge interactions)


with M_H measurement
w/o M_H measurement
Gfitter SM
M_H 0.0 (-1.5)
M_W -1.2 (-0.3)
Γ_W 0.2 (0.2)
M_Z 0.2 (0.0)
Γ_Z 0.1 (0.3)
σ⁰_had -1.7 (-1.7)
R⁰_lep -1.1 (-1.0)
A^0,l_FB -0.8 (-0.7)
A_l(LEP) 0.2 (0.4)
A_l(SLD) -1.9 (-1.7)
sin²Θ^lept_eff(Q_FB) -0.7 (-0.8)
A^0,c_FB 0.9 (1.0)
A^0,b_FB 2.5 (2.7)
A_c 0.0 (0.0)
A_b 0.6 (0.6)
R⁰_c 0.0 (0.0)
R⁰_b -2.4 (-2.3)
m̄_c 0.0 (0.0)
m̄_b 0.0 (0.0)
m_t 0.4 (0.0)
Δα^(5)_had(M²_Z) -0.1 (0.0)
-3 -2 -1 0 1 2 3


excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
β
|V_ub|
CKM fitter
Winter 14
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
η̄
ρ̄
1.5 1.0 0.5 0.0 -0.5 -1.0 -1.5
-1.0 -0.5 0.0 0.5 1.0 1.5 2.0


Power law best fit ($M = 244.0^{264.0}_{234.0}$, $\epsilon = -0.022^{0.02}_{-0.043}$)
Coupling λ
m [GeV]
c τ b W Z t
10⁰ 10⁻¹ 10⁻²
10⁰ 10¹ 10²

The Standard Model – unanswered questions

- Complicated choice of multiplets
- Fractional and integral charges?
- Neutrino masses?
- Many parameters 16 (25)
- Only partial unification $A_\mu^\gamma = \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu$
- The hierarchy problem
- ...

II. Grand Unification

H. Georgi, Lie Algebras in Particle Physics, Harvard University Press, 1992

R. Slansky, Group Theory for Unified Model Building, Phys. Rep. 79 (1981), 1

P. Langacker, Grand Unified Theories, Phys. Rep. 72 (1981), 1985

G. Ross, Grand Unified Theories, Benjamin/Cummings, 1984

R. Mohapatra, Unification and Supersymmetry, Springer, 1986

...

II. Grand Unification

$$SU(3)\otimes SU(2)\otimes U(1)\overset{?}{\subset}\mathbf{G}$$

$$(3,2)+2.(3,1)+(1,2)+(1,1)\overset{?}{\subset}\mathbf{R}$$

II. Grand Unification

$$SU(3)\otimes SU(2)\otimes U(1) \overset{?}{\subset} \mathbf{G}$$

$$(3,2)+2.(3,1)+(1,2)+(1,1) \overset{?}{\subset} \mathbf{R}$$

$\mathbf{G} \geq \text{Rank } 4$ (# diagonal generators)

$SU(5)$... unique viable rank 4 possibility

Georgi Glashow

$SU(5)$: Group of 5×5 complex unitary matrices with determinant 1

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$50 - 25 - 1 = 24$ independent matrices - adjoint representation

$$U = \exp\left(-i\sum_{i=1}^{24}\beta^i L^i\right), \qquad U^\dagger U = 1 \quad \Rightarrow \quad L^i \text{ Hermitian generators}$$

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$$SU(3)\times SU(2)\times U(1) \subset SU(5)$$

$$L^{a=1..8} = \begin{bmatrix} & & & 0 & 0 \\ & \lambda^a & & 0 & 0 \\ & & & 0\,L^a & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad SU(3)$$

$$L^{a=9,10} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \multicolumn{2}{c}{\sigma_{1,2}} \\ 0 & 0 & 0 & & \end{bmatrix}$$

$L^{11} =$ Diagonal $(0,0,0,1,-1)$

$\Big\}\ SU(2)$

$L^{12} =$ Diagonal $(-2,-2,-2,3,3)$ $\qquad U(1)$

Fermions

Convenient to use Weyl notation for fermions

The Lorentz group

Rotations J_i Boosts K_i

$$[J_i, J_j] = i\varepsilon_{ijk} J_k$$
$$[J_i, K_j] = i\varepsilon_{ijk} K_k$$
$$[K_i, K_j] = -i\varepsilon_{ijk} J_k$$

} Generate the group SO(3,1)

$$\left(M_{\rho\sigma} = i\left(x_\rho \frac{\partial}{\partial x^\sigma} - x_\sigma \frac{\partial}{\partial x^\rho}\right) \qquad J_i = \tfrac{1}{2}\varepsilon_{ijk} M_{jk} \qquad K_i = M_{0i}\right)$$

To construct representations a more convenient (non-Hermitian) basis is

$$N_i = \tfrac{1}{2}(J_i + iK_i)$$

$$[N_i, N_j] = i\varepsilon_{ijk} N_k$$
$$[N_i^\dagger, N_j^\dagger] = i\varepsilon_{ijk} N_k^\dagger$$
$$[N_i, N_j^\dagger] = 0$$

} $SU(2) \otimes SU(2)$ *representation* (n, m)

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$$(M_{\rho\sigma} = i(x_\rho \tfrac{\partial}{\partial x^\sigma} - x_\sigma \tfrac{\partial}{\partial x^\rho}) \qquad J_i = \tfrac{1}{2}\varepsilon_{ijk} M_{jk} \qquad K_i = M_{0i})$$

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$$[N_i, N_j^\dagger] = 0$$

Representations $J_i = N_i + N_i^\dagger$

$(n, m) \quad J = n + m$

$(0,0)$	scalar	$J=0$
$(\frac{1}{2},0), (0,\frac{1}{2})$	LH and RH spinors	$J=\frac{1}{2}$
$(\frac{1}{2},\frac{1}{2})$	vector	$J=1$, etc

Weyl spinors

$(\tfrac{1}{2},0) \quad (0,\tfrac{1}{2})$

$\psi_L \qquad \psi_R$

2-component spinors of SU(2)

Rotations and Boosts

$$\psi_{L(R)} \to S_{L(R)}\,\psi_{L(R)}$$

$$S_{L(R)} = e^{i\frac{\alpha}{2}.\sigma} \quad : \ \text{Rotations}$$

$$S_{L(R)} = e^{\mp\frac{\nu}{2}.\sigma} \quad : \ \text{Boosts}$$

†

Dirac spinor

Can combine $\psi_L, \ \psi_R$ to form a 4-component "Dirac" spinor $\psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}$

Lorentz transformations $\psi \to e^{i\omega\sigma}\psi, \qquad \omega\sigma = \omega^{\mu\nu}\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu,\gamma_\nu\right]\omega^{\mu\nu}$

$$\gamma_0 = \begin{bmatrix} 0 & -I \\ -I & 0 \end{bmatrix}, \quad \gamma_i = \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & o \end{bmatrix}, \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix}$$

†

$$\omega^{0i} \to \ boosts, \quad \omega^{ij} \to rotations \qquad i,j = 1,2,3$$

Weyl basis

$$\psi_{L(R)} = \tfrac{1}{2}(1 \mp \gamma_5)\psi$$

The Lagrangian

$$\mathfrak{L} = i\overline{\psi}\,\gamma_{\mu}\partial^{\mu}\,\psi - m\overline{\psi}\,\psi$$

- Masses

$$\psi^{\dagger}\gamma^{0}\psi \equiv \overline{\psi}\psi = \psi_{L}{}^{\dagger}\psi_{R} + \psi_{R}{}^{\dagger}\psi_{L}$$

- Kinetic term

$$\overline{\psi}\,\gamma_{\mu}\partial^{\mu}\,\psi = \psi^{\dagger}\gamma^{0}(\gamma^{0}\partial_{0} - \gamma^{i}\partial_{i})\psi = \psi^{\dagger}\left(\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}\partial_{0} - \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}\begin{pmatrix} 0 & -\sigma_{i} \\ \sigma_{i} & 0 \end{pmatrix}\partial_{i}\right)\psi$$

$$= \psi^{\dagger}\left(\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}\partial_{0} - \begin{pmatrix} \sigma_{i} & 0 \\ 0 & -\sigma_{i} \end{pmatrix}\partial_{i}\right)\psi$$

$$\equiv \psi_{L}^{\dagger}\sigma^{\mu}\,\partial_{\mu}\psi_{L} + \psi_{R}^{\dagger}\overline{\sigma}^{\mu}\,\partial_{\mu}\psi_{R} \qquad \left(\sigma^{\mu} = \left(1,\sigma^{i}\right),\ \ \overline{\sigma}^{\mu} = \left(1,-\sigma^{i}\right)\right)$$

Can construct LH spinors out of RH antispinors and vice-versa

$$\psi_L^c \equiv \sigma_2 \psi_R^* \sim (\frac{1}{2}, 0)$$

$$\text{Proof :} \quad \psi_L^c \to e^{\frac{\vec{\sigma}}{2}.\bar{v}} \psi_L^c \quad ?$$

$$\psi_R \to e^{-\frac{\vec{\sigma}}{2}.\bar{v}} \psi_R$$

$$\sigma_2 \psi_R^* \to \sigma_2 e^{-\frac{\vec{\sigma}^*}{2}.\vec{v}} \psi_R^* = \sigma_2 e^{-\frac{\vec{\sigma}^*}{2}.\vec{v}} \sigma_2 \sigma_2 \psi_R^* = e^{\frac{\vec{\sigma}}{2}.\vec{v}} \sigma_2 \psi_R^*$$

using $\sigma_2 \sigma_i^* \sigma_2 = -\sigma_i$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$SU(5):$ Group of 5×5 complex unitary matrices with determinant 1

$50 - 25 - 1 = 24$ independent matrices - adjoint representation

$$U = \exp\left(-i\sum_{i=1}^{24}\beta^i L^i\right), \qquad U^\dagger U = 1 \quad \Rightarrow \quad L^i \text{ Hermitian generators}$$

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Fermions:

$$L^K_{fermion} = \psi^\dagger_R \sigma_\mu D^\mu \psi_R$$

Fundamental representation $\psi_{5R} \equiv \begin{bmatrix} n^1 \\ n^2 \\ n^3 \\ n^4 \\ n^5 \end{bmatrix}_R$

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Covariant derivative:

Gauge bosons V_μ^a

$(3,1)+(1,2)$

Define $\frac{1}{\sqrt{2}}V_\mu \equiv \frac{1}{2}\sum_{a=1}^{24}V_\mu^a L^a,\qquad \left(D_\mu\psi_5\right)^i=\left[\delta^i_j\partial_\mu-\frac{ig}{2}\sum_{a=1}^{24}V_\mu^a\left(L^a\right)^i_j\right]\psi_5^j$

$$V_\mu=\begin{bmatrix} G_1^1-\frac{2B}{\sqrt{30}} & G_2^1 & G_2^1 & \bar{X}_1 & \bar{Y}_1 \\ G_1^2 & G_2^2-\frac{2B}{\sqrt{30}} & G_3^2 & \bar{X}_2 & \bar{Y}_2 \\ G_1^3 & G_2^3 & G_3^3-\frac{2B}{\sqrt{30}} & \bar{X}_3 & \bar{Y}_3 \\ X_1 & X_2 & X_3 & \frac{W_\mu^3}{\sqrt{2}}+\frac{3B}{\sqrt{30}} & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & -\frac{W_\mu^3}{\sqrt{2}}+\frac{3B}{\sqrt{30}} \end{bmatrix},$$

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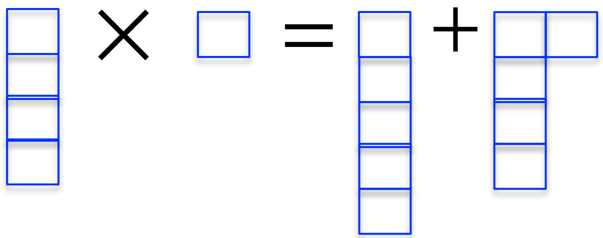
In terms of fundamental representation, $(\psi_5)^a = n^a$

$$L^K_{fermion} = \psi^\dagger_R \sigma_\mu D^\mu \psi_R$$

$$\bar{5} \times 5 = 1 + 24$$



Young Tableau

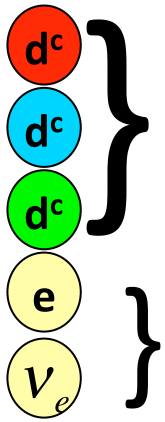
$$T^\beta_\alpha = \varepsilon_{\alpha abcd} n^a n^b n^c n^d \times n^\beta$$

$$24 = \frac{5 \times 4 \times 3 \times 2 \times 6}{1 \times 2 \times 3 \times 5}$$

Grand Unification

$$SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$$

$$\left(\bar{5}\right)_{La} :$$



$SU(3)$

$SU(2)$

$$(\bar{3},1)+(1,2)$$

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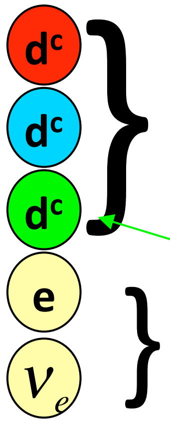

$SU(3)$

$SU(2)$

$$3Q_{d^c} + Q_{e^-} = 0$$

$$Q_{d^c} = 1/3$$

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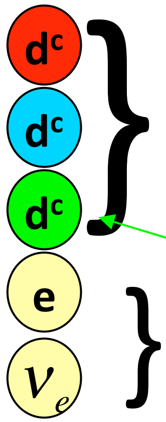


$SU(3)$

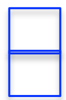
$SU(2)$

$$3Q_{d^c} + Q_{e^-} = 0$$

$$Q_{d^c} = 1/3$$

Remaining 10 states?

$T^{[\alpha,\beta]}$



$$\frac{n(n-1)}{1\times 2} = 10$$

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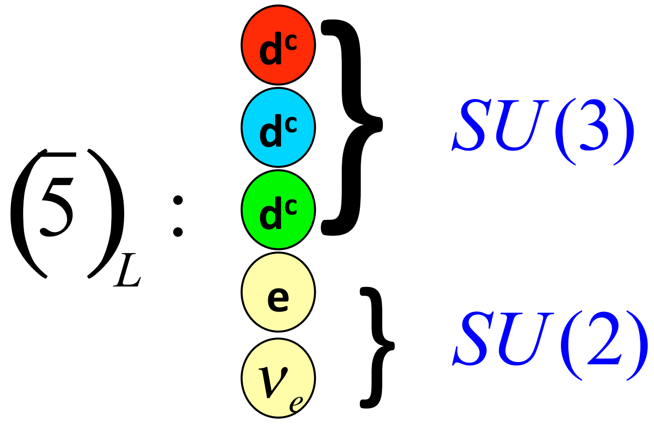

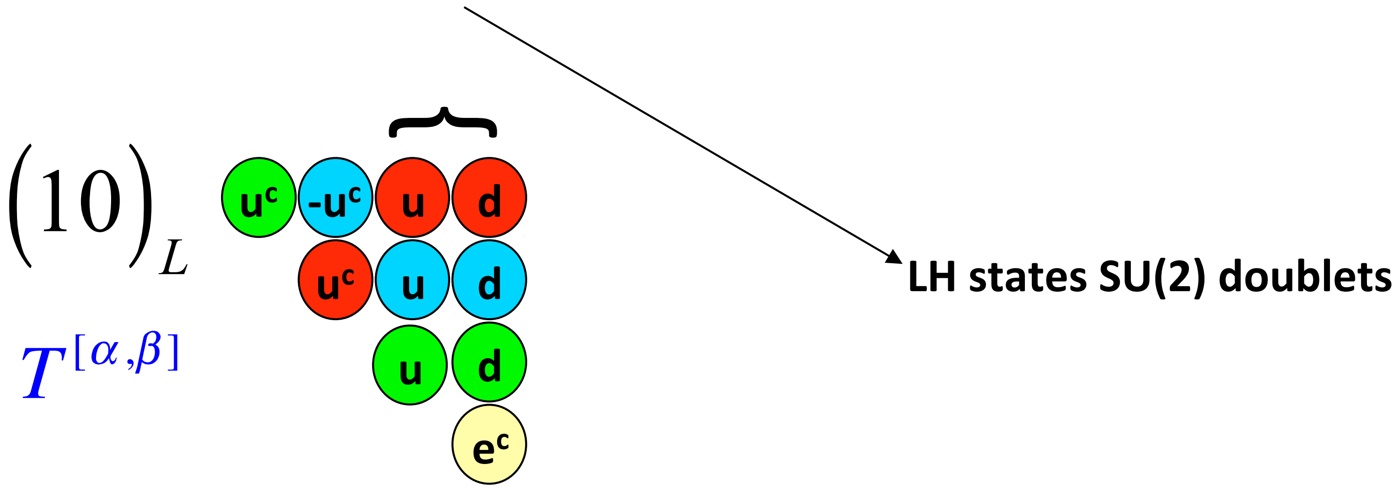

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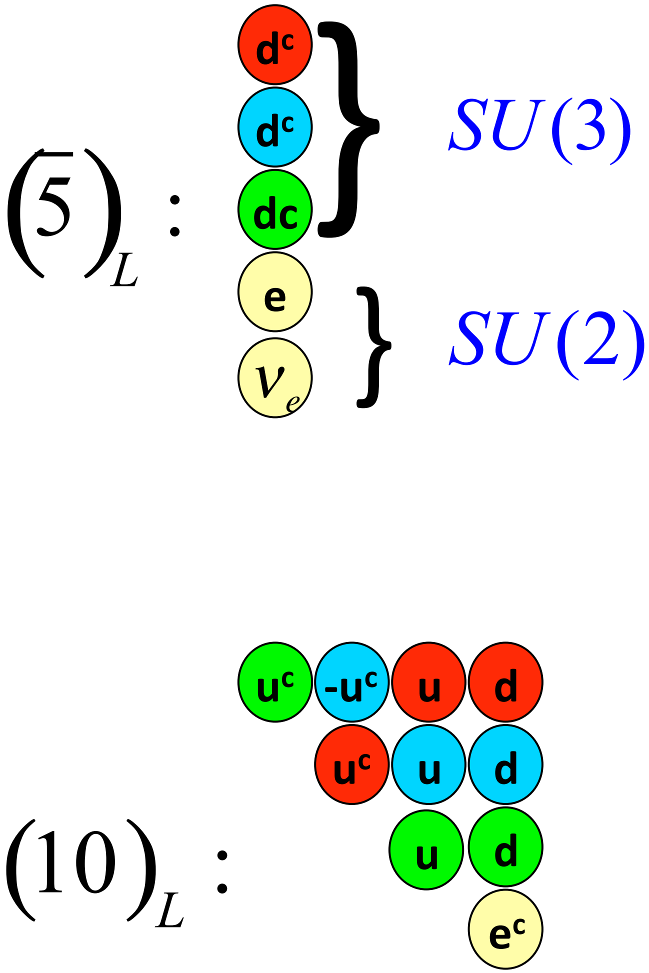
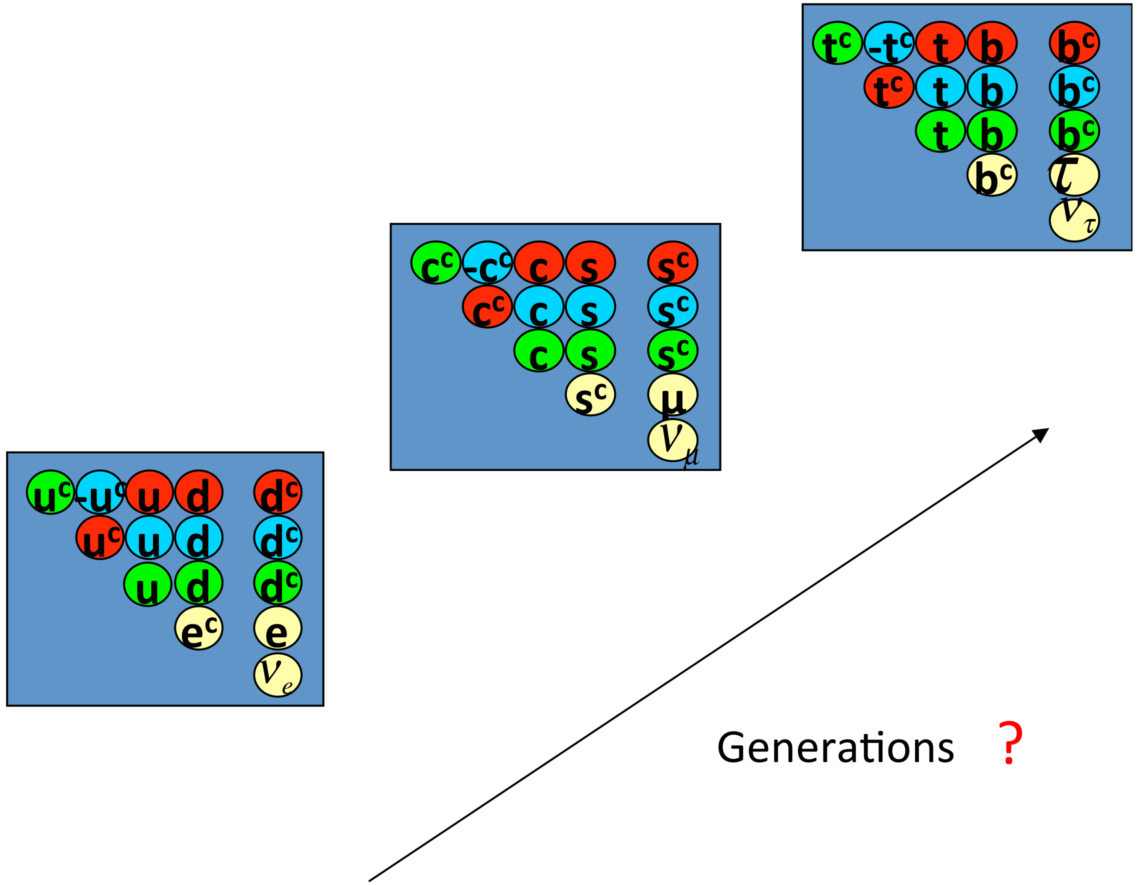


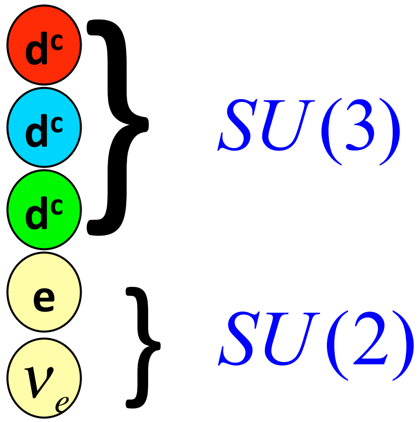


Grand Unification

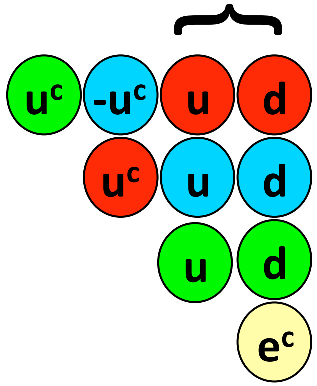
$$SO(10) \supset SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$$

Anomaly free

$(\bar{5})_L$:



$(10)_L$:



$$\nu_{e,L}{}^{c} \equiv \nu_{e,R}$$

$$(16)_L = (10)_L + (\bar{5})_L + (1)_L$$

$SO(10)$: Group of matrices R† that leave invariant length of 10-dim vector

$$R^T R = RR^T = 1$$

†Adjoint representation $SO(n): \; n^2 - (n^2 + n)/2 = n(n-1)/2$

$SO(10)$ 45 gauge bosons

$SO(10)$ **:** Group of matrices R† that leave invariant length of 10-dim vector

$$R^T R = RR^T = 1$$

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$SO(10)$ 45 gauge bosons

Rank 5

$$SO(10) \nearrow SU(5)\times U(1) \quad 45 = 24+1+10+\overline{10}$$

$$SO(10) \searrow SU(4)\times SU(2)_L \times SU(2)_R$$

$SO(10)$: Group of matrices R† that leave invariant length of 10-dim vector

$$R^T R = RR^T = 1$$

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$SO(10)$ 45 gauge bosons

Spinorial (16 dim) representation:

$$c.f.\ SO(3) \sim SU(2) \qquad \psi_{\alpha=1,2},\quad R = e^{i\omega^{ab}\sigma_{ab}},\quad \sigma_{ab} = \tfrac{1}{2}\varepsilon_{abc}\sigma_c \equiv \tfrac{i}{2}[\sigma_a, \sigma_b]$$

$$SO(10) \qquad \chi_{16}^{\pm} = \psi_1 \times \psi_2 \times \psi_3 \times \psi_4 \times \psi_5 \quad \text{with} \quad \sum_{i=1}^{5} \sigma_3^i = \pm 1$$

↖ 2^4

$SO(10)$: Group of matrices R† that leave invariant length of 10-dim vector

$$R^T R = RR^T = 1$$

Det R=1

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Standard Model embedding:

$$SO(10) \supset SO(6) \times SO(4) \sim SU(4) \times SU(2) \times SU(2)$$

$$\supset SU(3) \times U(1)_{B-L} \times SU(2)_L \times SU(2)_R$$

$SO(10)$: Group of matrices R† that leave invariant length of 10-dim vector

$$R^T R = RR^T = 1$$

Det R=1

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$SO(10)$ 45 gauge bosons

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Standard Model embedding:

$$SO(10)\supset SO(6)\times SO(4)\sim SU(4)\times SU(2)\times SU(2)$$

$$\supset SU(3)\times U(1)_{B-L}\times SU(2)_L\times SU(2)_R \qquad \begin{cases}\tau^L{}_3=-\sigma_3\times 1+1\times\sigma_3\\ \tau^L{}_{\pm}=\sqrt{2}\sigma_{\mp}\times\sigma_{\pm}\end{cases}$$

Identification of states of χ_{16}^{+} $\quad 16^{+} = 10 + \bar{5} + 1$

$$\begin{bmatrix} u^{i} \\ d^{i} \end{bmatrix}_L = \begin{bmatrix} |++--+\rangle, |+-+-+\rangle, |-++-+\rangle \\ |++-+-\rangle, |+-++-\rangle, |-+++-\rangle \end{bmatrix} \qquad (3,2)$$

$$u_{iL}^{c} = \left(|+--++\rangle, |-+-++\rangle, |--+++\rangle\right) \qquad (\bar{3},1)$$

$$d_{iL}^{c} = \left(|+----\rangle, |-+---\rangle, |--+--\rangle\right) \qquad (\bar{3},1)$$

$$\begin{bmatrix} \nu \\ e^{-} \end{bmatrix}_L = \begin{bmatrix} |----+\rangle \\ |---+-\rangle \end{bmatrix} \qquad (1,2)$$

$$e_L^{+} = |+++--\rangle \qquad (1,1)$$

$$N_L = |+++++\rangle \qquad (1,1)$$

$$(\tau^{L}{}_{3} = -\sigma_3 \times 1 + 1 \times \sigma_3, \ \tau^{L}{}_{\pm} = \sqrt{2}\sigma_{\mp} \times \sigma_{\pm})$$

Alternative structures

$SU(5)$

$$SO(10) \xrightarrow[16]{M_X^{'}} SU(5) \xrightarrow[45]{M_X} SU(3)\times SU(2)\times U(1) \xrightarrow[10]{M_W} SU(3)\times SU(2)\times U(1)$$

$$10_{SO(10)} \equiv \left(5+\bar{5}\right)_{SU(5)}$$

Alternative structures

$SU(5)$

$$Q = T_3 + \frac{Y}{2}$$

$$SO(10) \xrightarrow[16]{M_X^{'}} SU(5) \xrightarrow[45]{M_X} SU(3)\times SU(2)\times U(1) \xrightarrow[10]{M_W} SU(3)\times U(1)$$

Flipped SU(5)

$$Q = T_3 - \frac{1}{5} Y_Z + \frac{2}{5}\tilde{Y}_\chi$$

$$SO(10) \xrightarrow[16]{M_X^{'}} SU(5)\times U(1)_\chi \xrightarrow{M_X} SU(3)\times SU(2)\times U(1)_Z \times U(1)\chi$$

$$(\bar{5})_L : \begin{pmatrix} u^c \\ u^c \\ u^c \\ \nu_e \\ e \end{pmatrix} \qquad (10)_L : \begin{matrix} d^c & -d^c & d & d \\ & d^c & u & d \\ & & u & d \\ & & & \nu_e^c \end{matrix} \qquad (1)_L : \{e^c\}$$

Alternative structures

$SU(5)$

$$Q = T_3 + \frac{Y}{2}$$

$$SO(10) \xrightarrow[16]{M_X'} SU(5) \xrightarrow[45]{M_X} SU(3) \times SU(2) \times U(1) \xrightarrow[10]{M_W} SU(3) \times U(1)$$

Flipped SU(5)

$$Q = T_3 - \frac{1}{5} Y_Z + \frac{2}{5} \tilde{Y}_\chi$$

$$SO(10) \xrightarrow[16]{M_X'} SU(5) \times U(1)_\chi \xrightarrow{M_X} SU(3) \times SU(2) \times U(1)_Z \times U(1)\chi$$

$10_H . 10_H . 5_h \rightarrow \langle \nu_H^c \rangle d_H^c D$ - simple doublet –triplet splitting

$10_f . \overline{10}_H . \phi \rightarrow \langle \nu_{\overline{H}}^c \rangle \nu^c \phi$ - right handed neutrino masses

Alternative structures

$SU(5)$

$$SO(10)\xrightarrow[16]{M_X'}SU(5)\xrightarrow[45]{M_X}SU(3)\times SU(2)\times U(1)\xrightarrow[10]{M_W}SU(3)\times U(1)$$

Pati-Salam

leptons - 4th colour $\begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix}$

$$SO(10)\xrightarrow[54]{M_U}SU(4)\times SU(2)_L\times SU(2)_R\xrightarrow[45]{M_C}SU(3)\times U(1)_{B-L}\times SU(2)_L\times SU(2)_R$$

Parity restored

$$\xrightarrow[16]{M_R}SU(3)\times SU(2)\times U(1)\xrightarrow[10]{M_W}SU(3)\times U(1)$$

Spontaneous symmetry breaking

$$SU(5) \xrightarrow[\Sigma_{24}]{M_X} SU(3) \times SU(2) \times U(1) \xrightarrow[H_{\bar{5}}]{M_W} SU(3) \times U(1)$$

Spontaneous symmetry breaking

$$SU(5)\xrightarrow[\Sigma_{24}]{M_X} SU(3)\times SU(2)\times U(1)\xrightarrow[H_{\bar{5}}]{M_W} SU(3)\times U(1)$$

$$V(\Sigma_{24}) = -\mu^2 Tr(\Sigma^2) + \frac{1}{4}a\left[Tr(\Sigma^2)\right]^2 + \frac{1}{2}b\,Tr(\Sigma^4)$$

$$\langle\Sigma\rangle = V \text{ Diagonal}\left(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2}\right) \; if \; b,\mu^2 > 0,\, a > \tfrac{7}{5}b$$

$$M_X^2 = M_Y^2 = \frac{25}{8}g^2 V^2$$

Spontaneous symmetry breaking

$$SU(5)\xrightarrow[\Sigma_{24}]{M_X} SU(3)\times SU(2)\times U(1)\xrightarrow[H_5]{M_W} SU(3)\times U(1)$$

$$H_5=\begin{bmatrix} h^1 \\ h^2 \\ h^3 \\ h^+ \\ -h^0 \end{bmatrix} \quad (3,1)+(1,2)$$

$$V(H)=-\tfrac{1}{2}v^2|H|^2+\tfrac{1}{4}\lambda|H|^4+\alpha|H|^2 Tr\left(\Sigma^2\right)+\beta\overline{H}\Sigma^2 H$$

$$m_{h^i}^2=-\tfrac{5}{2}\beta V^2, \qquad M_W^2=M_Z^2\cos^2\theta_W=\tfrac{1}{4}g_2^2\left\langle h^0\right\rangle^2$$

$$\tfrac{1}{2}\lambda\left\langle h^0\right\rangle^2=v^2-\left(15\alpha+\tfrac{9}{2}\beta\right)V^2=O\left(10^{-24}\right)V^2$$

Hierarchy problem

Grand Unification – The Classic Predictions

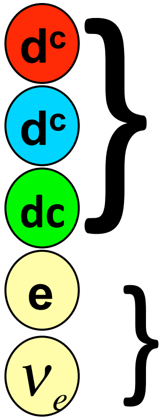
Gauge Couplings

$$SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$$

$$M_X$$

$$g_5 \quad \supset \quad g_3 \qquad g_2 \qquad g_1$$

$\left(\bar{5}\right)_L$:

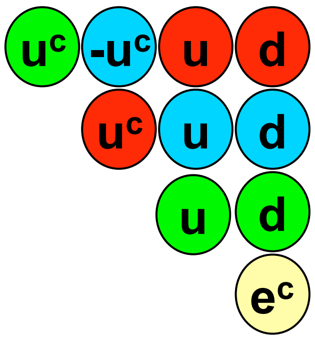


$SU(3)$

$SU(2)$

$$g_1(M_X) = g_2(M_X) = g_3(M_X) = g_5$$

$(10)_L$:

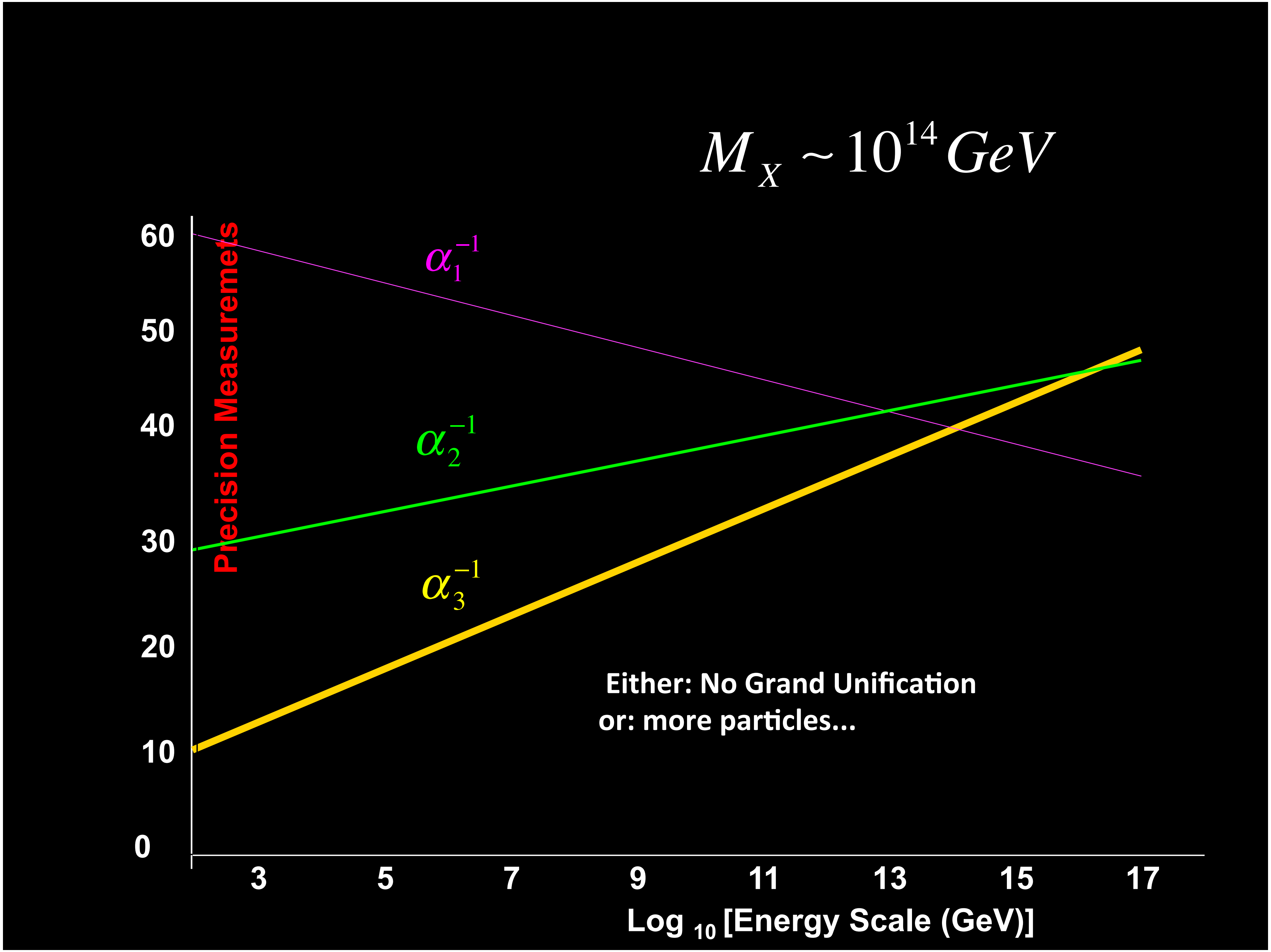


Gauge Couplings

SM evolution of gauge couplings

$$\alpha_i^{-1}(\mu) = \alpha^{-1}(M_X) + \frac{1}{2\pi} b_i \ln\left(\frac{M_X}{\mu}\right) + ..$$

$$b_i^{SM} = \begin{pmatrix} 0 \\ -\frac{22}{3} \\ -11 \end{pmatrix} + N_g \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} + H \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix}$$



$M_X \sim 10^{14}\,GeV$
α_1^{-1}
α_2^{-1}
α_3^{-1}
Precision Measuremets
60
50
40
30
20
10
0
3
5
7
9
11
13
15
17
Log 10 [Energy Scale (GeV)]
Either: No Grand Unification
or: more particles...

Fermion masses

$$\overline{5} \times 10 = 5 + \overline{45}$$

$$10 \times 10 = \overline{5} + 45 + 50$$

$$\overline{5} \times \overline{5} = \overline{10} + \overline{15}$$

$$L^{5}_{Yukawa} = \left(\psi^{\dagger}_{Ri\alpha}\right) m^{D}_{ij} \chi^{\alpha\beta}_{Lj} H^{\dagger}_{\beta} - \tfrac{1}{4} \varepsilon_{\alpha\beta\gamma\delta\varepsilon} \left(\chi^{T}\right)^{\alpha\beta}_{Li} \sigma^{2} m^{U}_{ij} \chi^{\gamma\delta}_{Lj} H^{\varepsilon} + h.c.$$

Fermion masses

$$\bar{5} \times 10 = 5 + \overline{45}$$

$$10 \times 10 = \bar{5} + 45 + 50$$

$$\bar{5} \times \bar{5} = \overline{10} + \overline{15}$$

$$L^{5}_{Yukawa} = \left(\psi^{\dagger}_{Ri\alpha}\right) m^{D}_{ij} \chi^{\alpha\beta}_{Lj} H^{\dagger}_{\beta} - \tfrac{1}{4}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\left(\chi^{T}\right)^{\alpha\beta}_{Li} \sigma^{2} m^{U}_{ij} \chi^{\gamma\delta}_{Lj} H^{\varepsilon} + h.c.$$

After diagonalising down quark mass matrix:

$m_d = m_e$ ✗

$m_s = m_\mu$ ✗

$m_b = m_\tau$ ✔?

Fermion masses

$$\bar{5}\times 10 = 5 + \overline{45}$$
$$10\times 10 = \bar{5} + 45 + 50$$
$$\bar{5}\times\bar{5} = \overline{10} + \overline{15}$$

$$L^5_{Yukawa} = \left(\psi^\dagger_{Ri\alpha}\right) m^D_{ij}\chi^{\alpha\beta}_{Lj} H^\dagger_\beta - \tfrac{1}{4}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\left(\chi^T\right)^{\alpha\beta}_{Li}\sigma^2 m^U_{ij}\chi^{\gamma\delta}_{Lj} H^\varepsilon + h.c.$$

$$L^{45}_Y = \left(\psi^\dagger_{Ri\alpha}\right) m^d_{ij}\chi^{\beta\gamma}_{Lj} H^{\dagger\alpha}_{\beta\gamma} + \varepsilon_{\alpha\beta\gamma\rho\tau}\left(\chi^T\right)^{\alpha\beta}_{Li}\sigma^2 m^u_{ij}\psi^{\gamma\delta}_{Lj} H^{\rho\tau}_\delta + h.c.$$

$$-3m_d = m_e \;✗$$
$$-3m_s = m_\mu \;✓$$
$$-3m_b = m_\tau \;✗$$

$$\left\langle H^{b5}_a\right\rangle = \mathrm{v}_{45}\left(\delta^b_a - 4\delta^4_a\delta^b_4\right), a,b = 1..4$$

SU(2)XU(1) invariant component

Fermion masses

Georgi-Jarlskog $\left(L^5\right)_{33+12+21} + \left(L^{45}\right)_{22}$

$$\frac{m_s}{m_\mu}(M_X) = \frac{1}{3}$$

$$Det(M^l) = Det(M^d)|_{M_X}$$

$$\frac{m_b}{m_\tau}(M_X) = 1$$

$$\frac{M^{d,l}}{m_3} = \begin{pmatrix} 0 & \varepsilon^3 & 0 \\ \varepsilon^3 & a\varepsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\varepsilon^d = 0.15, \quad a_{45}^s = 1$$

$$\varepsilon^l = 0.15, \quad a_{45}^\mu = -3$$

$$m_b = 3 m_\tau \; ✔$$

$$m_s = 3.\frac{1}{3}.m_\mu \; ✔$$

$$m_d = 3.3.m_e \; ✔$$

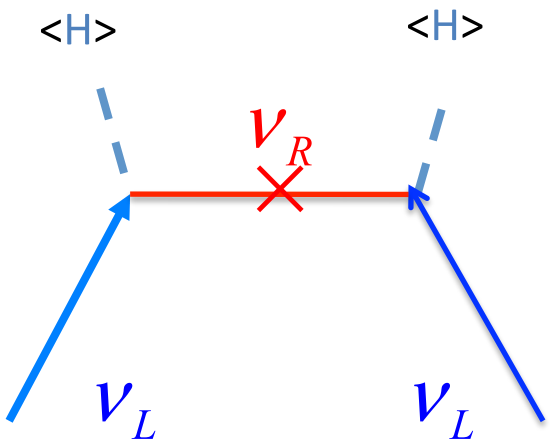
Neutrino mass

$$L_{Yukawa} = \left(\psi^{\dagger}_{Ri\alpha}\right) m^{D}_{ij} \chi^{\alpha\beta}_{Lj} H^{\dagger}_{\beta} - \tfrac{1}{4}\varepsilon_{\alpha\beta\gamma\delta\varepsilon}\left(\chi^{T}\right)^{\alpha\beta}_{Li} \sigma^{2} m^{U}_{ij} \chi^{\gamma\delta}_{Lj} H^{\varepsilon}$$
$$+\left(\psi^{\dagger}_{Ri\alpha}\right) m^{\nu}_{ij} \mathrm{N}^{c}_{Lj} H^{\alpha} + N^{cT}_{Li} \sigma^{2} M^{ij}_{M} N^{c}_{Lj} + h.c.$$

SO(10): $H_{10} \quad = 5 + \bar{5} \quad (SU(5))$

$$m^{d}_{i} = m^{l}_{i}$$
$$m^{u}_{i} = m^{\nu}_{i}$$

Neutrino mass



$$m_{\nu_L} \propto \frac{\lambda^2 < H >^2}{M_M} \sim \frac{m_t^2}{M_M}$$

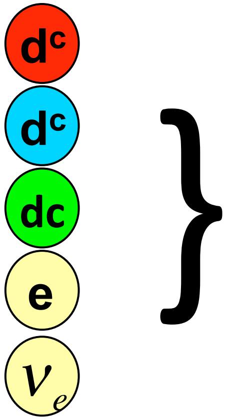
$$M_M \sim 10^{14}\, GeV$$

Nucleon decay

$$SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$$

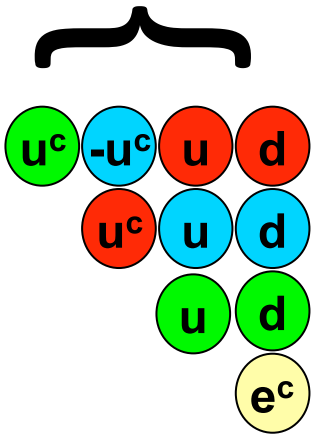
$$M_X$$

$(\bar{5})_L$:



X_μ : New lepto-quark gauge interactions

$(10)_L$:



Gauge couplings

$$L^{5}_{Kin} = i\overline{\psi}_{5i}\gamma^{\mu}\left(D_{\mu}\psi_{5}\right)^{i} = i\overline{\psi}_{5i}\gamma^{\mu}\left(\partial_{\mu}\delta^{i}_{i} - i\frac{g}{\sqrt{2}}\left(V_{\mu}\right)^{i}_{j}\right)\psi^{j}_{5}$$

$$L^{10}_{Kin} = \frac{i}{2}\left(\overline{\chi}\right)_{ac}\left(\partial_{\mu}\delta^{a}_{b} - \frac{2ig}{\sqrt{2}}\left(V_{\mu}\right)^{a}_{b}\right)\gamma^{\mu}\chi^{bc}$$

$$\frac{g}{\sqrt{2}}\overline{X}^{i}_{\mu}\left(\overline{d}_{iR}\gamma^{\mu}e^{+}_{R} + \varepsilon_{ijk}\overline{u}^{cj}_{L}\gamma^{\mu}u^{k}_{L} + \overline{d}_{iL}\gamma^{\mu}e^{+}_{L}\right) +$$
$$\frac{g}{\sqrt{2}}\overline{Y}^{i}_{\mu}\left(-\overline{d}_{iR}\gamma^{\mu}\nu^{c}_{R} + \varepsilon_{ijk}\overline{u}^{cj}_{L}\gamma^{\mu}d^{k}_{L} - \overline{u}_{iL}\gamma^{\mu}e^{+}_{L}\right) + h.c.$$

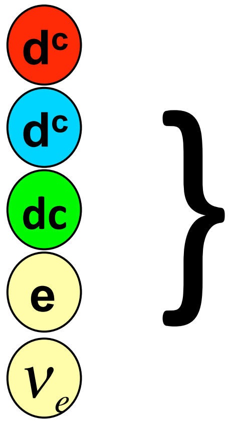
$$Q^{X,Y}_{B-L} = 2/3 \quad \Rightarrow \quad \Delta\left(B-L\right) = 0$$

Nucleon decay

$$SU(5) \supset SU(3) \otimes SU(2) \otimes U(1)$$

$$M_X$$

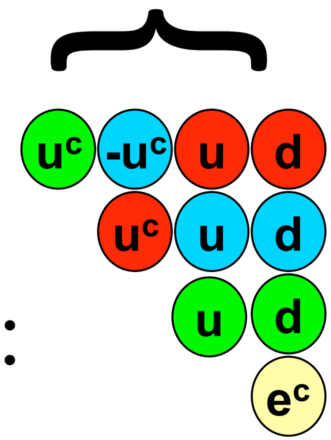
$\left(\bar{5}\right)_L$:

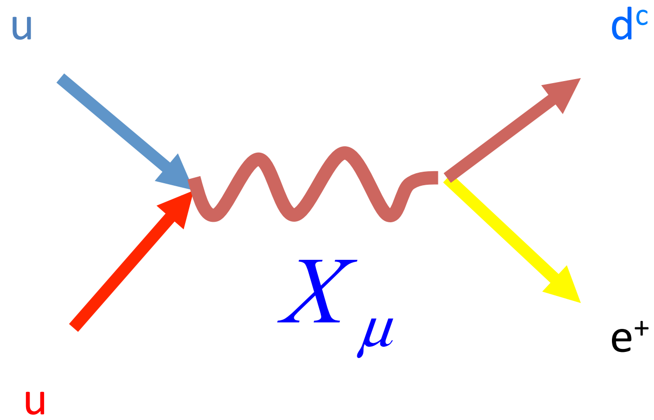


X_μ : New lepto-quark gauge interactions

$$p \to \pi^0 e^+$$

$\left(10\right)_L$:





$$O_{D=6} = \left(\varepsilon_{ijk} \overline{u}^{ck} \gamma_\mu u_L^j\right)\left(2\overline{e}_L^+ \gamma^\mu d_L^i + \overline{e}_R^+ \gamma^\mu d_R^i\right) + ..$$

$$\tau \propto M_X{}^4 \qquad \tau > 10^{32}\,\text{yrs}, \quad M_X > 10^{16}\,GeV$$