

Lectures on Higgs Physics

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Standard Model

- Gauge Theory based on the gauge group

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

- The subscript L denote the fact that only left-handed quark and leptons transform in a non-trivial way under the weak gauge group

- This imply that Dirac mass terms for fermions are forbidden by the gauge symmetry

$$\mathcal{L}_M = -m_D \bar{\psi}_L \psi_R + h.c.$$

- Also forbidden are explicit masses for the gauge bosons. The inclusion of such terms would lead to **non-renormalizability** and the breakdown of perturbation theory at energies of the order of 1 TeV (unitarity violation)
- The so-called Higgs mechanism, proposed by several authors, including **Brout, Englert and Higgs**, leads to a solution of these problems and to the appearance of a new, neutral scalar degree of freedom : the Higgs Boson particle. A particle with the expected Higgs properties, with mass 125 GeV, was recently discovered at the LHC.

Standard Model Particles. Quantum Numbers

- There are three generations of quarks and leptons. The only difference between generations are their masses, provided by the Higgs field, as we shall discuss.
- Quarks transform in the fundamental representation of SU(3).
- Left-handed quarks Q_L transform in the fundamental representation of SU(2) and carry hypercharge 1/6.
- Right-handed quarks u_R and d_R are singlet under SU(2) and carry hypercharge 2/3 and -1/3, respectively
- Left-handed leptons L_L transform in the fundamental representation of SU(2) and carry hypercharge -1/2
- Right-handed leptons l_R and ν_R are singlets under SU(2) and carry hypercharge -1 and 0, respectively
- There are SU(3) gauge bosons, named gluons, and a massive charge gauge boson, W_μ^\pm and a massive neutral gauge boson, Z_μ .
- There is a scalar field, carrying hypercharge 1/2 transforming in the fundamental representation of SU(2). Only one of the 4 degrees of freedom is physical, the neutral **Higgs Boson**.

Standard Model Particles and quantum numbers

At low energies only electromagnetic gauge symmetry is manifest. Fermions, with the possible exception of neutrinos, form Dirac particles, with equal charges for left and right chiralities.

	mass → $\approx 2.3 \text{ MeV}/c^2$ charge → $2/3$ spin → $1/2$	mass → $\approx 1.275 \text{ GeV}/c^2$ charge → $2/3$ spin → $1/2$	mass → $\approx 173.07 \text{ GeV}/c^2$ charge → $2/3$ spin → $1/2$	mass → 0 charge → 0 spin → 1	mass → $\approx 126 \text{ GeV}/c^2$ charge → 0 spin → 0
	u up	c charm	t top	g gluon	H Higgs boson
QUARKS	mass → $\approx 4.8 \text{ MeV}/c^2$ charge → $-1/3$ spin → $1/2$	mass → $\approx 95 \text{ MeV}/c^2$ charge → $-1/3$ spin → $1/2$	mass → $\approx 4.18 \text{ GeV}/c^2$ charge → $-1/3$ spin → $1/2$	mass → 0 charge → 0 spin → 1	
	d down	s strange	b bottom	γ photon	
	mass → $0.511 \text{ MeV}/c^2$ charge → -1 spin → $1/2$	mass → $105.7 \text{ MeV}/c^2$ charge → -1 spin → $1/2$	mass → $1.777 \text{ GeV}/c^2$ charge → -1 spin → $1/2$	mass → $91.2 \text{ GeV}/c^2$ charge → 0 spin → 1	
	e electron	μ muon	τ tau	Z Z boson	
LEPTONS	mass → $< 2.2 \text{ eV}/c^2$ charge → 0 spin → $1/2$	mass → $< 0.17 \text{ MeV}/c^2$ charge → 0 spin → $1/2$	mass → $< 15.5 \text{ MeV}/c^2$ charge → 0 spin → $1/2$	mass → $80.4 \text{ GeV}/c^2$ charge → ± 1 spin → 1	GAUGE BOSONS
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	

How do the fermions and gauge bosons acquire mass ?
 In the SM is via the “Higgs” mechanism. What is it and
 how can we test it ? Is it unique ?

Spontaneous Symmetry Breaking of Continuous Symmetries

- Occurs when the vacuum state is not invariant under a symmetry of the Hamiltonian

$$[S, H] = 0; \quad S |\Omega\rangle \neq |\Omega\rangle$$

- Let's take as an example a set of scalar fields transforming in some representation of a group G , where the dimension of the representation $d(G) = n$,

$$\phi_i(x) \rightarrow \phi_i(x) + i \epsilon^a T_{ij}^a \phi_j(x)$$

- We consider that the scalar fields acquire vacuum expectation value (ground state)

$$\langle \phi_i \rangle = v_i$$

- Since the potential is invariant under the transformations, one obtains for all fields

$$\delta V = \frac{\partial V}{\partial \phi_i} \delta \phi_i = 0;$$
$$\frac{\partial^2 V}{\partial \phi_i \partial \phi_k} T_{ij}^a \phi_i + \frac{\partial V}{\partial \phi_i} T_{ik}^a = 0$$

- At the minimum, the second term vanishes and the first term is proportional to the mass matrix.

Goldstone Theorem

- Hence, if the theory is invariant under a continuous symmetry the condition

$$M_{ki}^2 T_{ij}^a v_j = 0$$

must be fulfilled

- Now, if the vacuum state is invariant under the action of the group, no information may be obtained from here. However, if the state is not invariant the above define massless (Nambu Goldstone) modes.

- More specifically, assume that

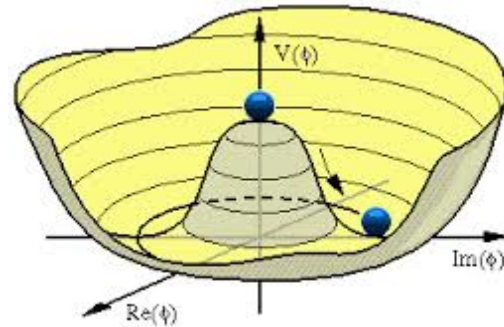
$$T_{ij}^b v_j = 0 \quad \text{for } b = 1, 2, \dots, n'$$

and

$$T_{ij}^c v_j \neq 0 \quad \text{for } c = n' + 1, \dots, n$$

- There will be $n'-n$ massless Nambu Goldstone Bosons associated with the broken generators of the group G.

Higgs Potential



Representation of a Scalar potential invariant under $U(1)$ rotations

Ground state is not invariant under $U(1)$ transformations

No potential curvature in the direction of the field transformation:
Massless Mode associated with fluctuations in this direction

Gauge Theories

- Theorem no longer valid if there is a gauge Symmetry.
- The reason is that in this case, the gauge symmetry defines the equivalency of all vacua related by gauge transformations. One can always fix the gauge, eliminating the massless Goldstone modes from the theory.
- But something else happens : The gauge bosons associated with the broken generators acquire a mass proportional to the gauge couplings and the vacuum expectation values
- Formally, lets take again a scalar field transforming under some general representation of the group G, of dimension n, and again lets take a field that has a nontrivial v.e.v.

$$(\mathcal{D}_\mu \phi)^\dagger \mathcal{D}^\mu \phi \rightarrow g^2 \phi^\dagger A_\mu^a T^a T^b A_\mu^b \phi$$

- Now, take for simplicity real v.e.v.'s, the above expression may be rewritten as

$$\frac{1}{2} A_\mu^a A^{\mu,b} \mathcal{M}_{ab}^2$$

- The mass matrix elements are non-trivial when the ground state fields are non-invariant

$$\mathcal{M}_{ab}^2 = g^2 (T_{ij}^a v_i) (T_{jk}^b v_k) \quad \phi_i = \frac{v_i}{\sqrt{2}}$$

- There is precisely **one massive gauge boson per “broken” generator** ! The Goldstone modes are replaced by the new, longitudinal degrees of freedom of the massive gauge fields.

Non-Simple Groups :The Standard Model

- The Standard Model is an example of a theory invariant under a non-simple group, namely $SU(3) \times SU(2) \times U(1)$. The $SU(3)$ generators are not broken and therefore the gluons remain massless. The expressions above can be simply generalized associating to each generator the corresponding gauge coupling. Using symmetry properties, now the mass Matrix may be rewritten as

$$\mathcal{M}_{ab}^2 = g_a g_b v_i \frac{\{T^a, T^b\}_{ik}}{2} v_k$$

- Let's take the case of a field in the fundamental representation of $SU(2)$, with hypercharge $1/2$ and

$$\langle \phi \rangle^T = \frac{1}{\sqrt{2}} (0, v) \quad \text{and for } SU(2) \quad T^a = \frac{\sigma^a}{2}$$

$$\mathcal{D}_\mu \phi = (\partial_\mu - ig A_\mu^a T^a - ig' Y B_\mu) \phi$$

- Using the fact that $\{\sigma_a, \sigma_b\} = \delta_{ab}$, $\{\sigma_a, I\} = 2\sigma_a$, $\{I, I\} = 2$

- We get the following mass matrix

$$\mathcal{M}_{ab}^2 = \frac{g^2 v^2}{4} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -g'/g \\ 0 & 0 & -g'/g & 1 \end{bmatrix}$$

Mass Eigenstates and Couplings

- The mass terms in the Lagrangian can then be written as

$$\mathcal{L}_M = \frac{1}{2} \frac{v^2}{4} \left[g^2 (A_\mu^1)^2 + (A_\mu^2)^2 + (gA_\mu^3 - g'B_\mu)^2 \right]$$

- Defining the states

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (A_\mu^1 \mp iA_\mu^2), \quad Z_\mu = \frac{gA_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}}$$

the Lagrangian can now be rewritten as

$$\mathcal{L}_M = \frac{g^2 v^2}{4} W_\mu^+ W^{\mu,-} + \frac{1}{2} \frac{(g^2 + (g')^2) v^2}{4} Z_\mu Z^\mu$$

- A massless mode, the photon, remains in the spectrum, $A_\mu = \frac{g'A_\mu^3 + gB_\mu}{\sqrt{g^2 + (g')^2}}$

- One can now replace the original fields in terms of the mass eigenstates and obtain the following couplings :

$$D_\mu = \partial_\mu - i \frac{g}{\sqrt{2}} (W_\mu^+ T^+ + W_\mu^- T^-) - i \frac{(g^2 T_3 - (g')^2 Y)}{\sqrt{g^2 + (g')^2}} Z_\mu - i \frac{gg'}{\sqrt{g^2 + (g')^2}} A_\mu (T_3 + Y)$$

$T^\pm = T_1 \pm iT_2$ that mediate transitions between states of different isospin

- From here one can identify the charge operator and the electromagnetic coupling with

$$e = \frac{gg'}{\sqrt{g^2 + (g')^2}} \text{ and } Q = T_3 + Y$$

Comments

The W bosons carry electromagnetic charge and mediate transitions between states of different charge, what makes these massive gauge bosons very different from the photon.

The ratio of the tree-level Z and W boson masses is governed by the weak gauge couplings

$$\frac{M_W}{M_Z} = \cos \theta_W, \text{ with } \cos \theta_W = \frac{g}{\sqrt{g^2 + (g')^2}}$$

Using this definition one can now rewrite the couplings in the following way

$$\mathcal{D}_\mu = \partial_\mu - i \frac{g}{\sqrt{2}} (W_\mu^+ T^+ + W_\mu^- T^-) - i \sqrt{g^2 + (g')^2} Z^\mu (T_3 - Q \sin^2 \theta_W) - ieQ A_\mu$$

Observe that all fields with non-trivial charge or isospin are coupled to the Z boson. The W bosons only couple to left-handed SM fermions, and the photon obviously only to the electromagnetically charged fields.

Fermion Masses

Since all left-handed quark and leptons transform in the fundamental representation of SU(2) and all right-handed ones transform as singlets, the scalar field that led to the generation of gauge boson masses can also lead to the generation of fermion ones.

Using for instance the generic coupling between the down-quarks and the Higgs (observe that is also invariant under hypercharge), where i and j are generation indices

$$-h_d^{ij} \bar{Q}_L^i \Phi d_R^j + h.c.$$

Once the scalar acquires vacuum expectation value one obtains a mass matrix for the down-quark fields (it is similar for charged leptons)

$$\mathcal{M}_d^{ij} = h_d^{ij} \frac{v}{\sqrt{2}}$$

For the up-quarks the hypercharge quantum numbers do not allow such a coupling. However, one can use the complex conjugate Higgs field and write the gauge invariant term (it would be similar for neutrinos, but right-handed neutrinos admit Majorana masses)

$$-h_u^{ij} \bar{Q}_L^i (-i\sigma_2 \Phi^*) u_R^j + h.c. \rightarrow \mathcal{M}_u^{ij} = h_u^{ij} \frac{v}{\sqrt{2}}$$

Heavier fermions then correspond to fields more strongly coupled to the Higgs.

Comments

- Fermion mass matrices are arbitrary complex matrices. They are therefore diagonalized by bi-unitary transformations,

$$U_L \mathcal{M}^u U_R^\dagger \rightarrow \text{Diagonal } \mathcal{M}^u, \quad D_L \mathcal{M}^d D_R^\dagger \rightarrow \text{Diagonal } \mathcal{M}^d$$

- They may be obtained by diagonalizing the hermitian matrices obtained by multiplying the mass matrix by its hermitian conjugate (and vice versa)

- Mass eigenstates are then defined by transforming the original weak eigenstates by

$$U_{L,R} u_{L,R} \rightarrow \text{up quark mass eigenstates}$$

$$D_{L,R} d_{L,R} \rightarrow \text{down quark mass eigenstates}$$

- Neutral gauge interactions are established between fields of the same chirality and charge quantum numbers and remain diagonal after this unitarity rotation. No FCNC

$$\bar{u}_{L,R} \gamma_\mu u_{L,R} \rightarrow \bar{u}_{L,R} \gamma_\mu U_{L,R} U_{L,R}^\dagger u_{L,R} \equiv \bar{u}_{L,R} \gamma_\mu u_{L,R}$$

- Charge gauge interactions, instead

$$\bar{u}_L \gamma_\mu d_L \rightarrow \bar{u}_L \gamma_\mu U_L D_L^\dagger d_L \equiv \bar{u}_L V_{CKM} \gamma_\mu d_L$$

- Non-trivial intergeneration structure. Close to diagonal in quark sector. Large mixings in lepton sector.

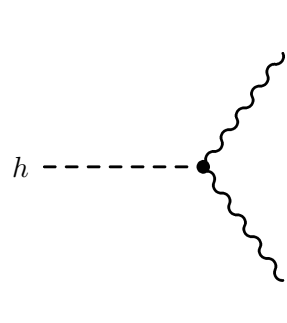
Where is the Higgs ?

- So far, we have discussed the mechanism of mass generation, but we have not identified the Higgs degree of freedom.
- Of the four degrees of freedom of the fundamental scalar doublet we introduced, three are the Goldstone modes associated with the directions of the non-trivial transformations of the v.e.v. The additional one, is a massive mode, the Higgs boson. Due to the real components of the T3 and hypercharge generators,

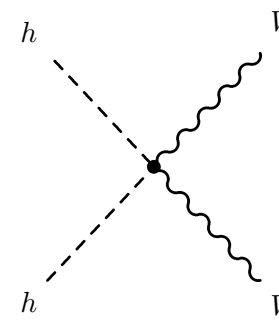
$$\Phi = \begin{pmatrix} G^+ \\ \frac{h+v}{\sqrt{2}} + i\frac{G^0}{\sqrt{2}} \end{pmatrix}$$

- This implies that the couplings of the Higgs h will be associated with the ones leading to mass generation.
- This means that couplings of the physical Higgs will be proportional to fermion masses, and that trilinear couplings with gauge bosons will be proportional to the square of gauge boson masses.

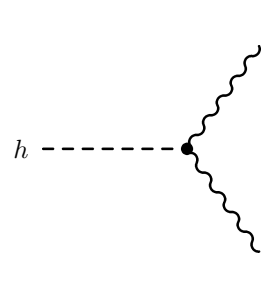
Higgs Couplings



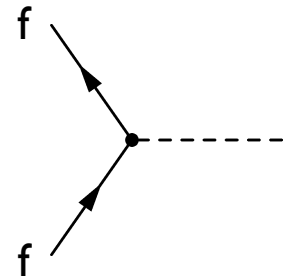
$$= i \frac{g^2 v}{2} g_{\mu\nu} = 2i \frac{M_W^2}{v} g_{\mu\nu}$$



$$= i \frac{g^2}{4} \cdot 2g_{\mu\nu} = 2i \frac{M_W^2}{v^2} g_{\mu\nu}$$



$$= i \frac{(g^2 + g'^2)v}{4} \cdot 2g_{\mu\nu} = 2i \frac{M_Z^2}{v} g_{\mu\nu}$$



$$= -i \frac{h_f}{\sqrt{2}} = -i \frac{m_f}{v}$$

These couplings govern the Higgs production rates and Branching Ratios, and we have increasing evidence of their approximate realization in nature for a Higgs mass of approximately 125 GeV. Relevant deviations are still possible, though.

Higgs Self Couplings

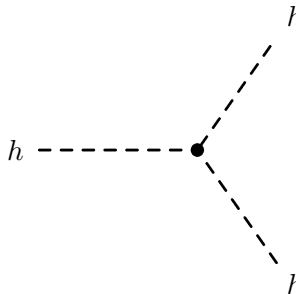
- In order to define the Higgs self couplings, we should give a mathematical representation of the potential. If one restrict oneself to renormalizable couplings,

$$V = -m^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

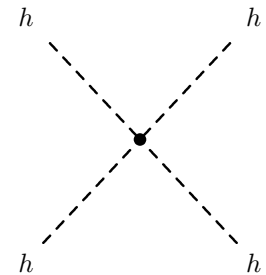
- Keeping terms that depend on the physical Higgs field, $v^2 = m^2/\lambda$, $\phi^\dagger \phi = \frac{(h+v)^2}{2}$

$$V = \lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4$$

- Hence, we get $m_h^2 = 2\lambda v^2$



$$= -i\lambda v \cdot 3! = -6i\lambda v = -3i \frac{m_h^2}{v}$$



$$= -i \frac{\lambda}{4} \cdot 4! = -6i\lambda = -3i \frac{m_h^2}{v^2}$$

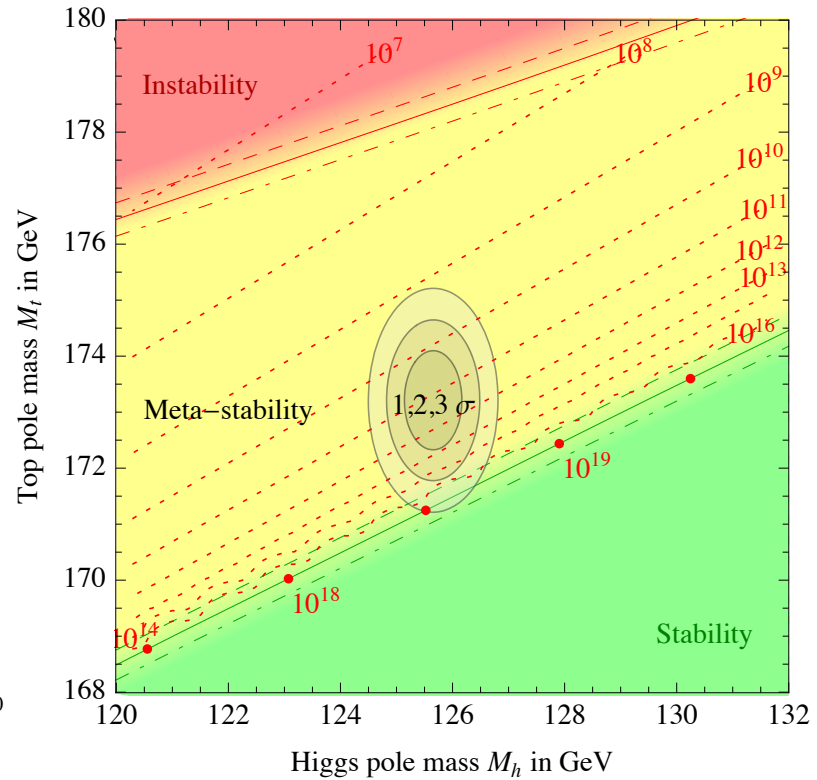
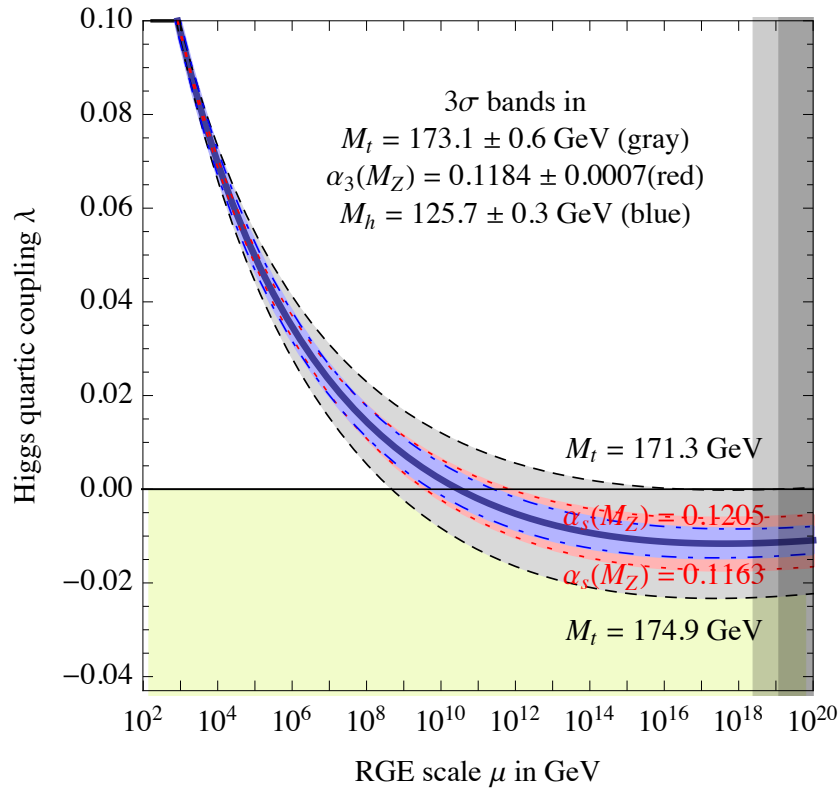
Quartic Coupling Renormalization Group Evolution

- In the SM, the Higgs mass is governed by the value of the quartic coupling at the weak scale. This coupling evolves with energy, affected mostly by top quark loops, weak gauge couplings and self interactions.

$$16\pi^2 \frac{d\lambda}{dt} = 12(\lambda^2 + h_t^2 \lambda - h_t^4) + \mathcal{O}(g^4, g^2 \lambda) \quad t = \log(Q^2)$$






- If the Higgs mass was larger than the weak scale, the quartic coupling would be large and the theory could develop a Landau Pole. However, the observed Higgs mass leads to a value of $\lambda \simeq 0.125$ and therefore the main effects are associated with the top loops.
- The top quark loops tend to push the quartic coupling to negative values, inducing a possible instability of the electroweak symmetry breaking vacuum.
- A careful analysis reveals, solving the coupled RG equations of the quartic and Yukawa couplings up to three loop order show that that the turning point would be at scales of order 10^{10} GeV and therefore the electroweak symmetry breaking minimum is not stable.
- However, careful analyses reveal that possible transitions to these new deep minima are suppressed and the lifetime of the electroweak symmetry breaking vacuum is much larger than the age of the Universe. No new physics is implied !
- On the other hand, this shows that a theory that would predict small values of the quartic coupling at these large energies would lead to the right Higgs mass..

Stability Bounds and the running quartic coupling

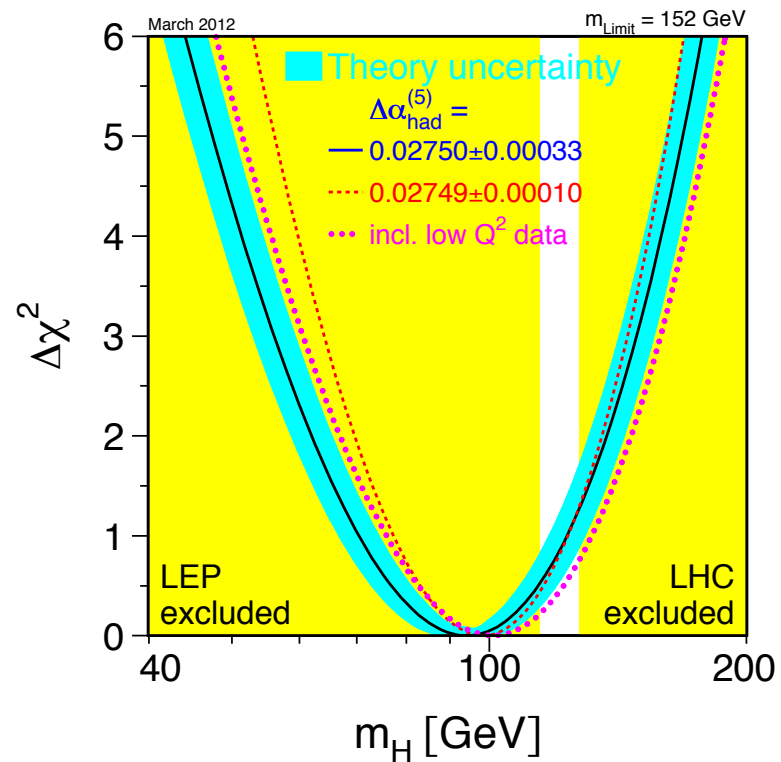
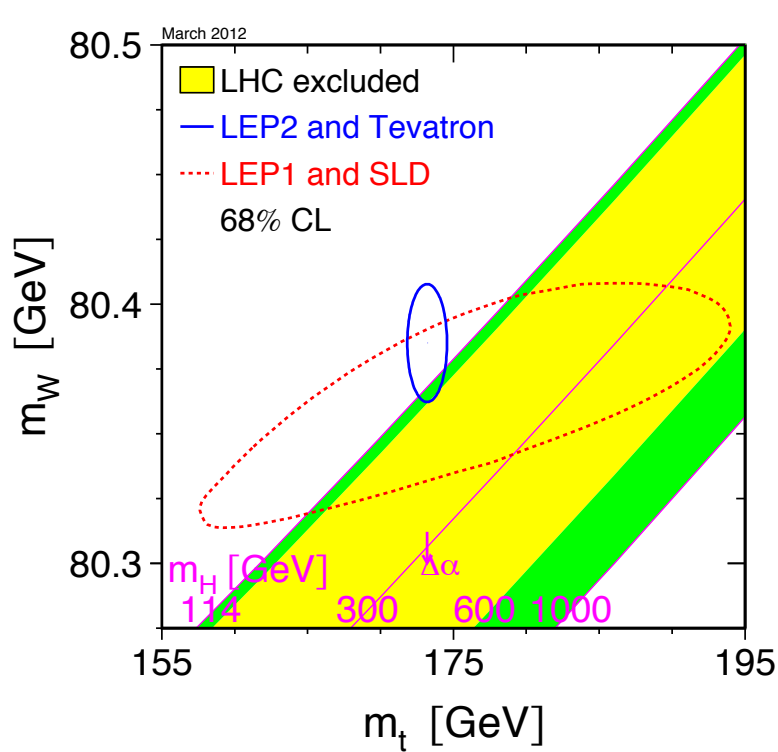


D. Buttazzo et al., arXiv:1307.3536

Constraints on the Higgs Mass Before Discovery

-  The existence of the Higgs bosons at the weak scale tames the rise of the amplitude of the scattering cross section of the longitudinal massive gauge bosons. In order to preserve perturbation theory, a Higgs, with mass below the TeV scale was required (or something with similar properties).
-  Precision electroweak observables, like the ratio of the W and Z masses, the Z partial and total width, and the lepton and quark forward-backward asymmetries, depend via radiative corrections logarithmically on the Higgs mass.
-  Here it is very important the precise value of the mass and the couplings of the Higgs to the gauge bosons, which is governed by the gauge couplings in the SM.
-  Departures of the Higgs couplings from their SM values demand the appearance of new states that tame the logarithmic divergences appearing in the computation of precision observables.
-  Assuming a Higgs like particle, one can obtain information on the Higgs mass from a combination of the precision electroweak observables measured at LEP, SLC and the Tevatron colliders.

Precision Measurements preferred a light Higgs



For a comprehensive discussion, see the LEP Electroweak Working Group page, <http://lepewwg.web.cern.ch>

Comment on Gauge Fixing

- As we said before, the unphysical Goldstone modes may be removed by a choice of gauge. This gauge is called the unitary gauge, in which all gauge and Higgs modes are physical.
- The gauge propagator in this gauge, however, has a hard ultraviolet behavior what leads to complications in the computation of the ultraviolet behavior of quantum processes

$$\frac{-i(g_{\mu\nu} - k_\mu k_\nu / M_V^2)}{k^2 - M_V^2}$$

- It is better to work on what are call general renormalizable gauges, in which the Goldstones are kept in the spectrum and are included as degrees of freedom participating in quantum corrections.
- Due to the fact that the neutral and charged currents involve linear terms in the Higgs field times derivatives of the Goldstone modes, keeping the Goldstone modes lead to terms in the Lagrangian of the form

$$\mathcal{L} \sim g v V^\mu \partial_\mu G \equiv M_V V^\mu \partial_\mu G$$

- This terms may be removed by an appropriate gauge fixing term, which in the abelian case reads

$$\mathcal{L}_{g.f.} = -\frac{1}{2\xi} (\partial_\mu V^\mu + \xi M G)^2$$

- This removes the mixing terms, modifies the gauge boson propagator and provides a mass term for the Goldstone modes,

$$M_G^2 = \xi M_V^2$$

Renormalizable Gauges

- Similar gauge fixing conditions can be used in the non-abelian case to remove the mixing terms between Goldstone modes and gauge bosons,

$$\mathcal{L}_{g.f.} = -\frac{1}{2\xi} \left[\partial_\mu A^{\mu,a} + ig\xi \left(\langle \phi^\dagger \rangle T^a \phi' - (\phi')^\dagger T^a \langle \phi \rangle \right) \right]^2$$

$$\mathcal{L}_{g.f.} = -\frac{1}{2\xi} \left[\partial_\mu B^\mu + ig'\xi \left(\langle \phi^\dagger \rangle \phi' - (\phi')^\dagger \langle \phi \rangle \right) \right]^2$$

$$\phi' = \phi - \langle \phi \rangle$$

- The appearance of a dependence on the Higgs field of the gauge fixing condition leads also to a mass terms for the Ghost fields appearing from the determinant of the variation of the gauge fixing condition in the Path integral quantization

$$\int \mathcal{D}\alpha \delta[F_{g.f.}(A^{\mu,\alpha}, \phi^\alpha)] \det \left[\frac{\delta F_{g.f.}}{\delta \alpha} \right] = 1$$

- Both Goldstone and Ghost modes must be included in quantum computations in order to project into the physical states of the theory.

Propagators

- Gauge Bosons

$$\frac{-i}{k^2 - M_V^2 + i\epsilon} \left[g_{\mu\nu} + (\xi - 1) \frac{k_\mu k_\nu}{k^2 - \xi M_V^2} \right]$$

- Goldstone Modes

$$\frac{i}{k^2 - \xi M_V^2 + i\epsilon}$$

- Ghosts

$$\frac{-i}{k^2 - \xi M_V^2 + i\epsilon}$$

- Observe that when ξ tends to infinity, one recovers the Unitary gauge.
- Physical results must be gauge independent. For many computations the gauge $\xi = 1$ is a very convenient one.

Goldstone Mode interactions with fermions.

- The neutral Goldstone mode is just the imaginary part of the neutral Higgs component and therefore its interactions are similar to the Higgs ones.

$$\bar{d}_L(\phi_1 + v + i\phi_2) \frac{M_d^{\text{diag}}}{v} d_R$$

- For the charged Goldstones, the relevant Lagrangian term may be rewritten by inserting several identity factors

$$G^+ \frac{\sqrt{2}}{v} \bar{u}_L U_L^\dagger U_L D_L^\dagger D_L M_d D_R^\dagger D_R d_R + h.c.$$

- One can see the appearance of the mass eigenstates and the diagonal mass matrix. What is left is the same CKM matrix that appeared in the charged currents. In the mass basis, one gets.

$$G^+ \frac{\sqrt{2}}{v} \bar{u}_L V_{CKM} M_d^{\text{diag}} d_R + h.c.$$

SM Higgs decay rates

Due to the proportionality of fermion Higgs couplings to their mass, and the fact that the measured Higgs mass is below twice the top and gauge boson masses, one would expect the Higgs decays to be dominated by the decay into the heavier fermions, excluding the top. These are bottom-quarks and tau-leptons

This reasoning is affected by the fact that the fermion masses are so small that the fermion decay widths start competing with three body decays mediated by gauge bosons and even loop effects induced by the top-quarks. Fermion decay widths

$$\Gamma(h \rightarrow f \bar{f}) = m_h \frac{N_c m_f^2}{8\pi v^2} \left(1 - \frac{4 m_f^2}{m_h^2} \right)^{3/2}$$

The three body decay width induced by the vector bosons is

$$\Gamma(h \rightarrow V V^*) = \frac{3 M_V^4}{32\pi^3 v^2} M_H \delta_V R(x) \quad x = \frac{M_V^2}{M_H^2}$$

$$\delta_W = 1, \quad \delta_Z = 7/12 - 10/9 \sin^2 \theta_W + 40/9 \sin^4 \theta_W$$

$$R_T(x) = \frac{3(1 - 8x + 20x^2)}{(4x - 1)^{1/2}} \arccos \left(\frac{3x - 1}{2x^{3/2}} \right) - \frac{1 - x}{2x} (2 - 13x + 47x^2) - \frac{3}{2} (1 - 6x + 4x^2) \log x$$

QCD Corrections to Higgs Decays

- The partial decay width depend on the quark masses. The question is what masses should be used.
- It turns out that using the running masses at the Higgs mass scale reduces in great part the size of the QCD corrections, which however remain relevant, but not sizable

$$\Gamma(h \rightarrow b\bar{b}) \simeq \frac{3M_h}{8 v^2 \pi} m_b(m_h)^2 \Delta_{\text{QCD}}$$
$$\Delta_{\text{QCD}} = 1 + 5.7 \frac{\alpha_s(m_h)}{\pi} + 30 \left(\frac{\alpha_s(m_h)}{\pi} \right)^2 + \dots$$

- Similar expressions for other quarks.
- Observe that there is a significant variation of the running masses from low energies to the Higgs mass. The bottom mass at the “pole” bottom mass is about 4.15 GeV, while at the Higgs mass scale is about 2.9 GeV ! When squared, this gives a variation of order 2 between these values, that must be compensated by QCD corrections.

Comments

Although not realistic phenomenologically, it is interesting to consider the decay of a heavy Higgs to gauge bosons.

In such a case, it is easy to prove that the rate is given by

$$\Gamma(h \rightarrow W^+W^-) \simeq \frac{1}{16\pi} \frac{M_W^4}{v^2} \frac{m_h^3}{M_W^2}$$

The first factor comes from the coupling, while the second factor comes from the contribution of the longitudinal components of the W bosons.

The growth with the cube of the Higgs mass may be also understood from the fact that one is producing the Goldstone modes, which are the longitudinal components of the gauge bosons.

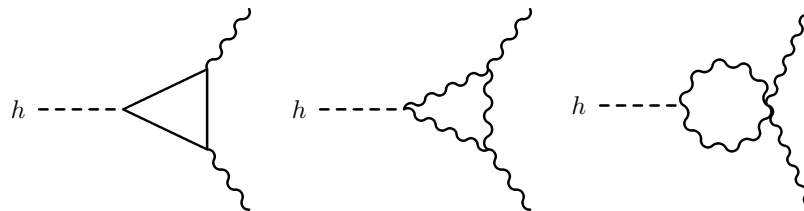
The rate is then proportional to

$$\Gamma(h \rightarrow G^+G^-) \propto \frac{(\lambda v)^2}{m_h}$$

The factor in the numerator comes from the coupling of the Higgs to the Goldstone and the factor in the denominator comes from phase space integration (dimensional analysis). And now, knowing the relation of the quartic coupling with the Higgs mass square, one obtains the behavior shown above.

Higgs Loop-induced Decays

- The most important loop-induced decays are into gluons and photons.
- The decay into gluons in the Standard Model is mostly mediated by loops of top quarks. These also contribute to the decay into photons, but the most important contribution comes from loop of W-bosons



- Both particles cannot be produced on-shell from Higgs decays, and their contributions may be approximated by

$$\Gamma(h \rightarrow \gamma\gamma) = \frac{\alpha^2 m_h^3}{256\pi^3 v^2} \left| \sum_i N_c^i Q_i^2 F_i \right|^2$$

where the factors $F_1 \simeq -7$, $F_{1/2} \simeq 4/3$ are related to the contributions of W bosons and top quarks to the electromagnetic coupling beta function.

- Similarly, for the decay into gluons, one obtains after considering the color structure

$$\Gamma(h \rightarrow gg) \simeq \frac{\alpha_s^2 m_h^3}{128 \pi^3} |F_{1/2}|^2$$

Low Energy Theorems

- The one-loop corrections to the QED Lagrangian induced by heavy particles is given by

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \sum_i \frac{b_i e^2}{16\pi^2} \log \frac{\Lambda^2}{m_i^2(h)} + \dots$$

- The coefficients b_i are the beta function contribution of the corresponding particle

$$b_{1/2} = \frac{4}{3}N_c Q^2 \quad \text{for a Dirac fermion}$$

$$b_1 = -7 \quad \text{for the W bosons}$$

$$b_0 = \frac{1}{3}N_c Q^2 \quad \text{for a complex scalar}$$

- From here it is easy to find the effective Higgs coupling by performing derivatives with respect to the Higgs field

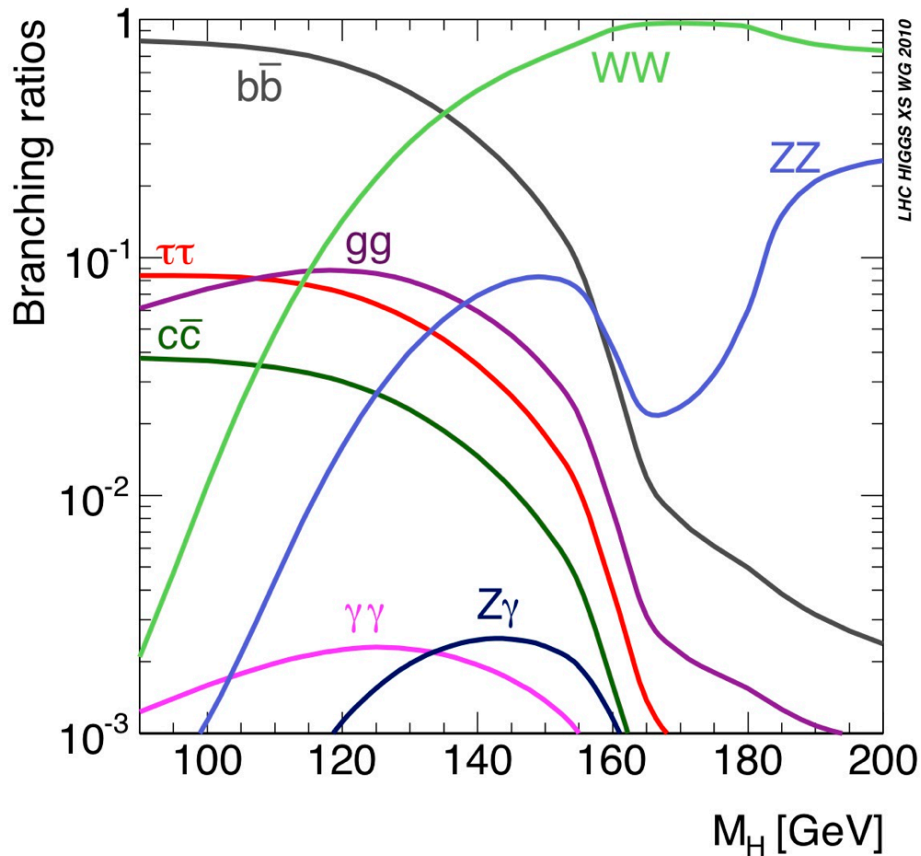
$$\mathcal{L} = \frac{\alpha}{16\pi} \frac{h}{v} \left[\sum_i 2b_i \frac{\partial \log m_i(v)}{\partial \log v} \right] F_{\mu\nu}F^{\mu\nu}$$

- In the presence of several fermion and boson sectors of similar charge, one obtains

$$\mathcal{L} = \frac{\alpha}{16\pi} \frac{h}{v} \left[\sum_i b_i \frac{\partial \log (\det M_{F,i}^\dagger M_{F,i})}{\partial \log v} + \sum_i b_i \frac{\partial \log (\det M_{B,i}^2)}{\partial \log v} \right] F_{\mu\nu}F^{\mu\nu}$$

- This gives an excellent parametrization to consider the contribution of new particles beyond the SM ones. A similar expression exists for the gluon case, by changing the beta functions by the corresponding QCD ones.

Numerical values of the Higgs Decay Branching Ratios



Higher order QCD
corrections included

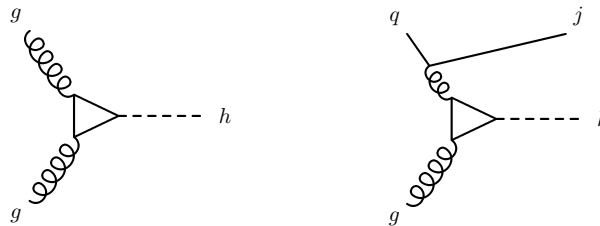
Decay mode	BR
bb	58%
WW^*	22%
gg	8.6%
$\tau\tau$	6.3%
$c\bar{c}$	2.9%
ZZ^*	2.6%
$\gamma\gamma$	0.23%
$Z\gamma$	0.15%
$\mu\mu$	0.022%
Γ_{tot}	4.1 MeV

Branching Ratios
for a mass of 125 GeV

At the observed Higgs mass, several decay Branching ratios of the SM Higgs are larger than a few percent. Decay into photons and into ZZ allow the Higgs mass reconstruction and were discovery modes.

Higgs Production Cross Section at the LHC

- The dominant Higgs Production Mode at the LHC is given by gluon fusion
- At one loop it can be computed from the effective Lagrangian defined above, but the NLO and NNLO corrections are sizable



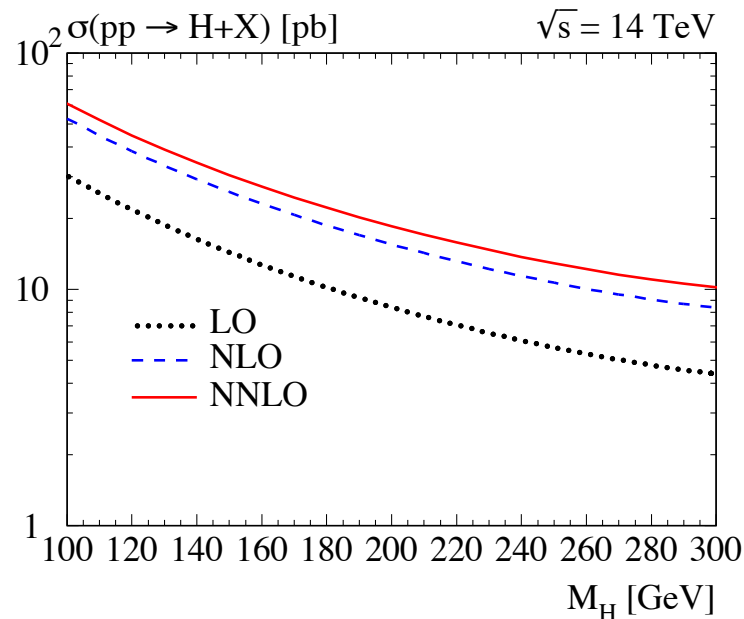
$$\sigma_{LO} = \frac{\alpha_s(\mu)^2}{576 v^2 \pi} |F_{1/2}|^2$$

$$\mathcal{L}_{eff} = \frac{H}{4v} C(\alpha_s) G^{a\mu\nu} G_{\mu\nu}^a$$

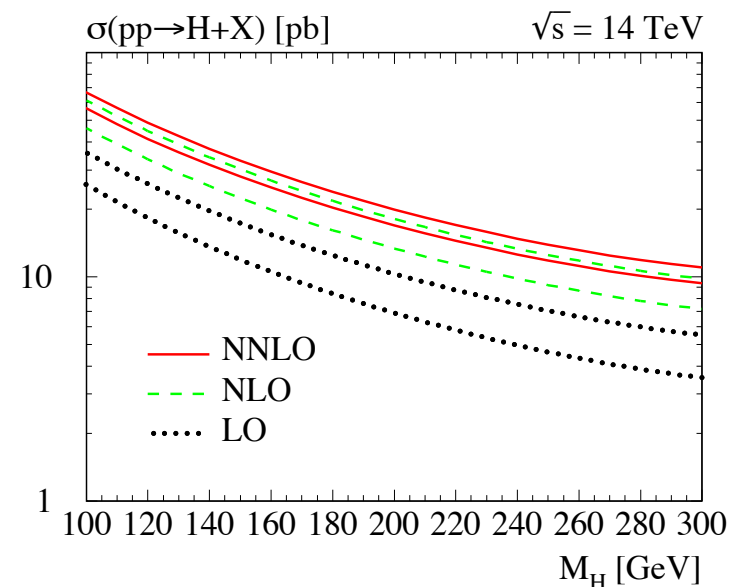
$$C(\alpha_s) = \frac{1}{3} \frac{\alpha_s}{\pi} \left[1 + c_1 \frac{\alpha_s}{\pi} + c_2 \left(\frac{\alpha_s}{\pi} \right)^2 + \dots \right]$$

- Effective Lagrangian ignore top quark mass dependence, and provides a very good approximation. Corrections up to NNLO are today known and show a good degree of convergence, and a small scale dependence.

Higgs Production Cross Section in the Gluon Fusion Channel



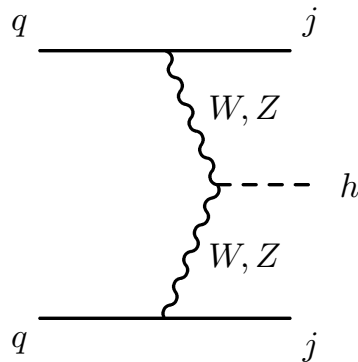
Convergence of the computed Higgs Cross Section at different orders in QCD



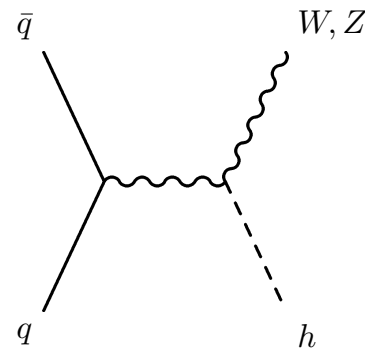
Bands show the renormalization/factorization scale dependence varying up and down by a factor 2 with respect to a reference scale equal to a fourth of the Higgs Mass

Additional Production Modes

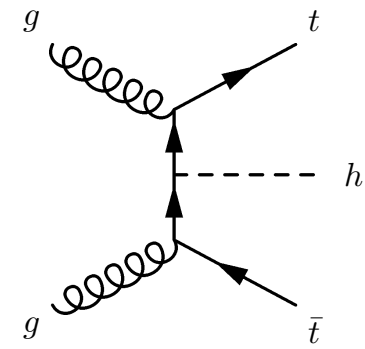
- Apart from gluon fusion, three important production modes are weak boson fusion, associated production with gauge bosons and associated production with heavy quarks.



Weak Boson Fusion
Colinear Jets



Associated Production
with Gauge Bosons

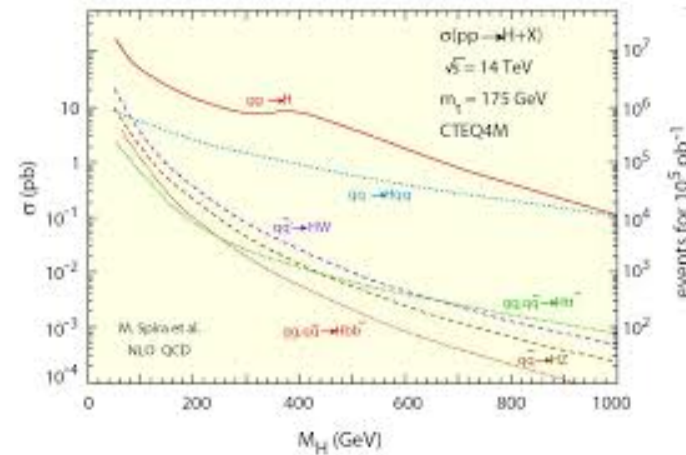
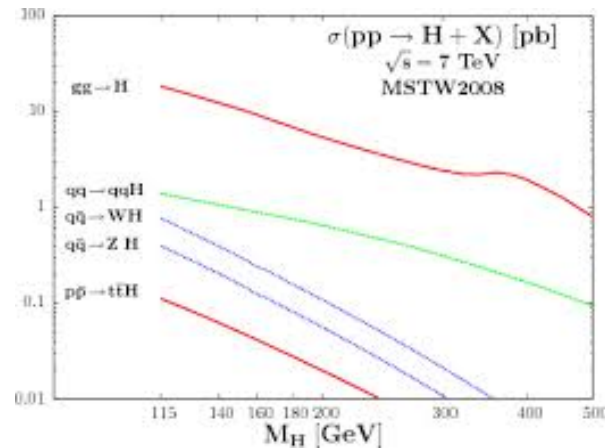


Associated Production
with Top Quarks

- All these processes, together with the decay branching ratios are important to determine the Higgs couplings
- The self couplings may be probed by double Higgs production, which is mediated by Higgs and also by loops of top-quarks. Very challenging measurement at the LHC

Standard Model Higgs Production Cross Section

- Significant hierarchy between dominant production cross section and subdominant ones.
- Subdominant production cross section contain co-linear jets or leptons in the final state and can therefore be interesting production modes that may be measured at the LHC.

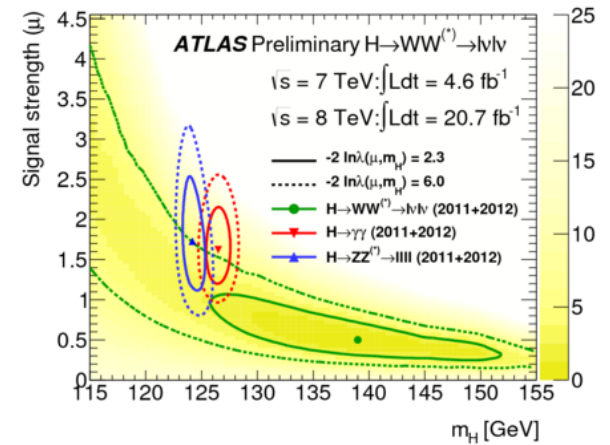
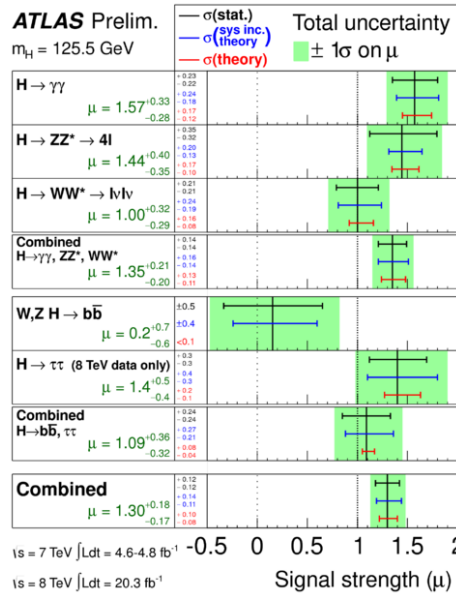
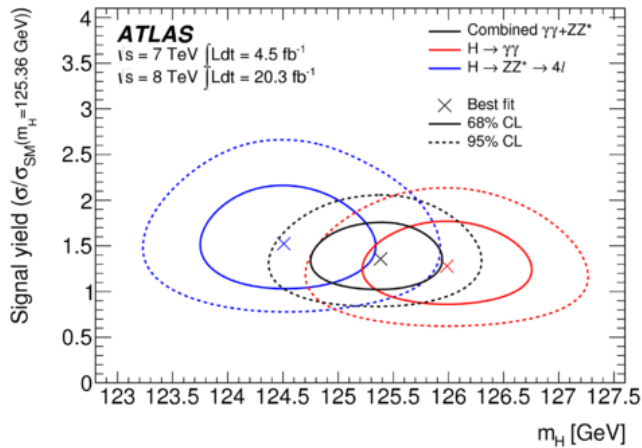
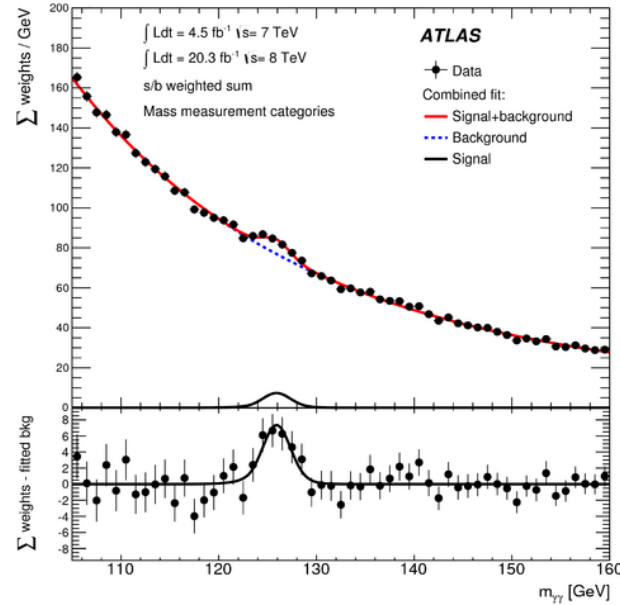
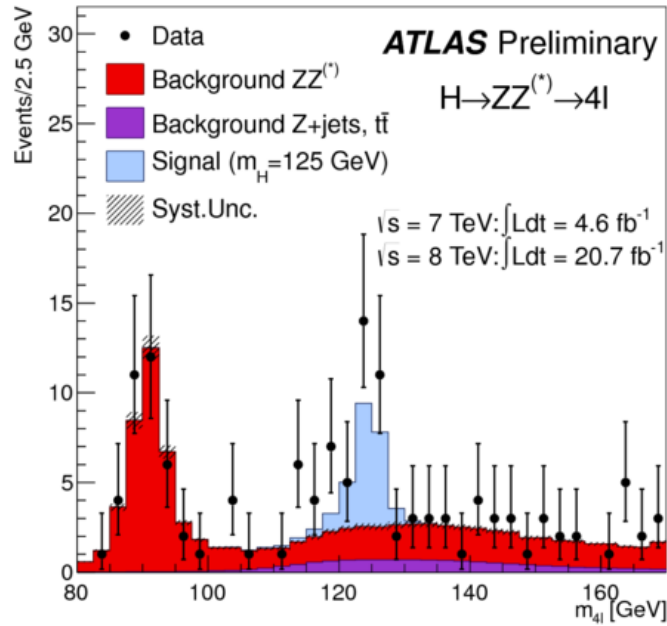


- Discovery modes were mostly in the production of Higgs in gluon fusion, with subsequent decays into ZZ and diphotons.

Higg Discovery at the LHC

- The Discovery of the scalar resonance at the LHC, with properties similar to the ones of a SM-Higgs has been of great importance.
- It has put on solid grounds the mechanism of mass generation based on the spontaneous breakdown of gauge symmetries.
- Nature seems to have chosen the simplest way of generating masses, as described in the Standard Model. No hint of physics beyond the Standard Model exists
- All fundamental particles we know acquire masses via the vacuum expectation value of the Higgs field, including in certain way the Higgs itself. But what sets that scale ? Why aren't there other scalars with gauge invariant masses at that scale ?
- It is too early to say. We need to explore physics at higher energies, and the next run of the LHC will help.
- In the meantime, the Higgs properties are being studied in many different channels, and these studies will continue in the next run of the LHC.

Higgs Production Channels at ATLAS



Higgs Production Channels at CMS

