Programa de Pós Graduação em Física

Renormalization Group Improvement and Dynamical Breaking of Symmetry in a Supersymmetric Abelian Chern-Simons Model

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Motivation

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Renormalization group and conformal symmetry breaking in the Chern-Simons theory coupled to matter

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The three-dimensional Abelian Chern-Simons theory coupled to a scalar and a fermionic field of arbitrary charge is considered in order to study conformal symmetry breakdown and the effective potential stability. We present an improved effective potential computation based on two-loop calculations and the renormalization group equation. The latter allows us to sum up series of terms in the effective potential where the power of the logarithms are one, two, and three units smaller than the total power of coupling constants (i.e., leading, next-to-leading, and next-to-next-to-leading logarithms). For the sake of this computation we determined the beta function of the fermion-fermion-scalar-scalar interaction and the anomalous dimension of the scalar field. We show that the improved effective potential provides a much more precise determination of the properties of the theory in the broken phase, compared to the standard effective potential obtained directly from the loop calculations. This happens because the region of the parameter space where dynamical symmetry breaking occurs is drastically reduced by the improvement discussed here.

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$$\mathcal{L} = \frac{1}{2}\epsilon_{\mu\nu\rho}A^{\mu}\partial^{\nu}A^{\rho} + i\bar{\psi}\gamma^{\mu}D_{\mu}\psi + (D^{\mu}\varphi)^{\dagger}(D_{\mu}\varphi) - \frac{\nu}{6}(\varphi^{\dagger}\varphi)^{3} - \alpha\varphi^{\dagger}\varphi\bar{\psi}\psi.$$

Gap equation and Effective superpotential

The classical action in $\mathcal{N}=1$ superspace of a Chern-Simons superfield coupled to N massless scalars superfields, with a quartic self-interaction,

$$\mathcal{S} = \int d^5z \left\{ -\frac{1}{2}\Gamma^\alpha W_\alpha - \frac{1}{2}\overline{\nabla^\alpha \Phi_a}\nabla_\alpha \Phi_a + \frac{\lambda}{N}\left(\overline{\Phi_a}\Phi_a\right)^2 \right\}, \qquad (1)$$

where $W^\alpha = \frac{1}{2}D^\beta D^\alpha \Gamma_\beta$, $\nabla^\alpha = \left(D^\alpha - i\frac{g}{\sqrt{N}}\Gamma^\alpha\right)$ and $a = 1, \ldots N$.

A.C. Lehum and A.J. da Silva. Phys.Lett., B693:393–398, 2010.
S. J. Gates, Marcus T. Grisaru, M. Rocek, and W. Siegel. Front. Phys., 58:1–548, 1983.

$$\Phi_N = \frac{1}{\sqrt{2}}\left(\Sigma + \sqrt{N}\sigma_{cl} + i\Pi\right),$$

Component form of Eq. (1)

$$\mathcal{S} = \int d^3x \left\{ \frac{1}{2}A_M \epsilon^{MNP}\partial_N A_P - \frac{1}{2}\partial^M \bar{\varphi}_a \partial_M \varphi_a + i\,\overline{\Psi}_a \gamma^M \left(\partial_M - i\,\frac{g}{\sqrt{N}}A_M\right)\Psi_a \right.$$
$$\left. -\frac{g}{\sqrt{N}}\epsilon_{ab}\,A_M \bar{\varphi}_a \partial^M \varphi_b - \frac{1}{2N}g^2 A^2 \bar{\varphi}_a \varphi_a + \frac{1}{N}\left(\frac{1}{2}g^2 + \lambda\right)\overline{\Psi}_a \Psi_a \bar{\varphi}_b \varphi_b - \frac{1}{2}\left(\frac{\lambda}{N}\right)^2 (\bar{\varphi}_a \varphi_a)^3 \right\},$$

Alex G. Dias, M. Gomes, and A. J. da Silva. Phys. Rev., D69:065011, 2004.

A.G. Dias and A.F. Ferrari. Phys.Rev., D82:085006, 2010.

Gap equation and Effective superpotential

Using an appropriate gauge fixing and Faddeev-Popov terms,

$$\mathcal{S}_{GF+FP} = \int d^5z \left[-\frac{1}{2\alpha}\left(\mathcal{F}_G\right)^2 + \bar{c}D^2c + \frac{\alpha}{2}g^2\sigma_{cl}^2\bar{c}c + \frac{\alpha g^2}{2\sqrt{N}}\sigma_{cl}\bar{c}\Sigma c \right], \tag{2}$$

with $\mathcal{F}_G = \left(D^\alpha\Gamma_\alpha + \alpha\frac{g}{2\sqrt{N}}\,\sigma_{cl}\Pi \right)$,

We may write the regularized action for this model as follows,

$$\begin{aligned}\mathcal{S}_{\mathcal{R}} = \int d^5z \Big\{ & -\frac{1}{2}\Gamma^\alpha W_\alpha - M_\Gamma\Gamma^2 - \frac{1}{4\alpha}D^\alpha\Gamma_\alpha D^\beta\Gamma_\beta + \overline{\Phi_j}\left[D^2 + M_{\Phi_j}\right]\Phi_j + \frac{1}{2}\Sigma\left[D^2 + M_\Sigma\right]\Sigma \\ & + \frac{1}{2}\Pi\left[D^2 + M_\Pi\right]\Pi + \bar{c}\left[D^2 + \alpha M_\Gamma\right]c + \frac{ig}{2\sqrt{N}}\left(\left[D^\alpha\overline{\Phi_j}\right]\Gamma_\alpha\Phi_j + \overline{\Phi_j}\Gamma_\alpha D^\alpha\Phi_j\right) \\ & + \frac{g}{2\sqrt{N}}\left(D^\alpha\Pi\,\Gamma_\alpha\Sigma + \Pi\,\Gamma_\alpha D^\alpha\Sigma\right) - \frac{g^2}{2N}\left[\overline{\Phi_j}\Phi_j + \Sigma^2 + \Pi^2\right]\Gamma^2 + \frac{\lambda}{N}\left(\overline{\Phi_j}\Phi_j\right)^2 \\ & + \frac{\lambda}{4N}\left(\Sigma^2 + \Pi^2\right)^2 + \frac{\lambda}{N}\left(\Sigma^2 + \Pi^2\right)\overline{\Phi_j}\Phi_j + \frac{\lambda}{\sqrt{N}}\sigma_{cl}\Sigma\left[\overline{\Phi_j}\Phi_j + \Sigma^2 + \Pi^2 - \frac{g^2}{\lambda}\Gamma^2 + \frac{\alpha}{2\lambda}g^2\sigma_{cl}\bar{c}c\right] \\ & + \sqrt{N}\lambda\sigma_{cl}^3\Sigma + N\frac{\lambda}{4}\sigma_{cl}^4 + \mathcal{L}_{ct} \Big\},\end{aligned}$$

Gap equation and Effective superpotential

Propagators

$$\langle|\mathrm{T}\ \Sigma(k,\theta_1)\,\Sigma(-k,\theta_2)|\rangle = -i\,\frac{D^2 - M_\Sigma}{k^2 + M_\Sigma^2}\delta_{12}^2\,, \qquad \delta_{12}^2 = \delta^2(\theta_1 - \theta_2)$$

$$\langle|\mathrm{T}\ \Phi_i(k,\theta_1)\,\Phi_j(-k,\theta_2)|\rangle = -i\,\delta_{ij}\,\frac{D^2 - M_{\Phi_j}}{k^2 + M_{\Phi_j}^2}\delta_{12}^2\,,$$

$$\langle|\mathrm{T}\ \Pi(k,\theta_1)\,\Pi(-k,\theta_2)|\rangle = -i\,\frac{D^2 - M_\Pi}{k^2 + M_\Pi^2}\delta_{12}^2\,,$$

$$\langle|\mathrm{T}\ \Gamma_\alpha(k,\theta_1)\,\Gamma_\beta(-k,\theta_2)|\rangle = -\frac{i}{2}\left\{\frac{D^2\left(D^2 - M_\Gamma\right)}{k^2\left(k^2 + M_\Gamma^2\right)}D_\alpha D_\beta - \alpha\,\frac{D^2\left(D^2 - M_\Gamma\right)}{k^2\left(k^2 + \alpha^2 M_\Gamma^2\right)}D_\beta D_\alpha\right\}\delta_{12}^2\,,$$

where

$$M_\Gamma = \frac{g^2}{2}\sigma_{cl}^2\,, \qquad M_{\Phi_j} = \lambda\sigma_{cl}^2\,, \qquad M_\Sigma = 3\lambda\sigma_{cl}^2\,,$$

$$M_c = \alpha M_\Gamma\,, \quad M_\Pi = \lambda\sigma_{cl}^2 - M_c\,.$$

Gap equation and Effective superpotential

$$\mathcal{S}^{(1)}_{\Sigma\,1l} = \frac{i\sigma^3_{cl}}{4\pi\sqrt{N}}\left[-\left[9+N\right]\lambda^2 + \frac{g^4}{4}\right]\int \frac{d^3p}{(2\pi)^3}\, d^2\theta\, \Sigma\left(p,\theta\right). \qquad (3)$$

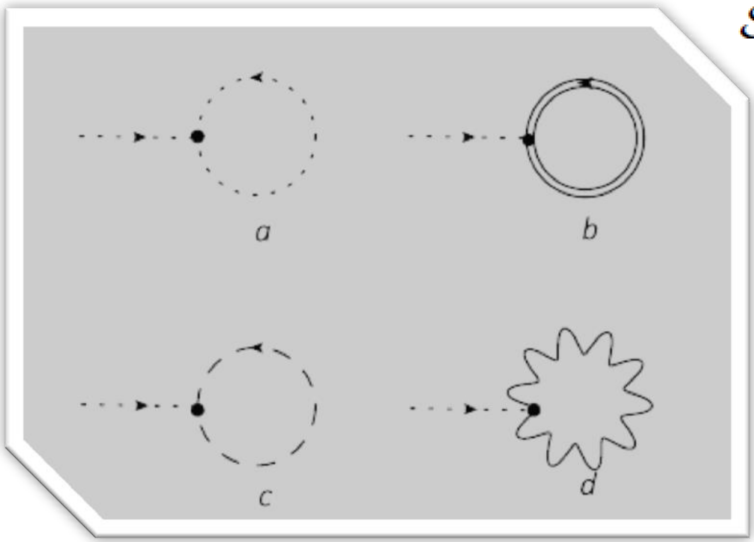

Figure 1: One-loop contribution to the one point vertex function. The $\sum$ superfield is represented by the dotted line, the Φ_j superfield by double line, the Π by dashed line and the Γ_α by the wave line.

with $\sigma_{cl} = \sigma_1 - \theta^2\sigma_2\,,$

Imposing $\mathcal{S}^{(1)}_{\Sigma\,(1)l} = 0$, one verifies that the one-loop correction is not sufficient to ensure a nontrivial solution to the **gap equation**, i.e., the only solution is $\sigma_{cl} = 0$. Therefore, there is not DSB in the first loop correction.

Gap equation and Effective superpotential

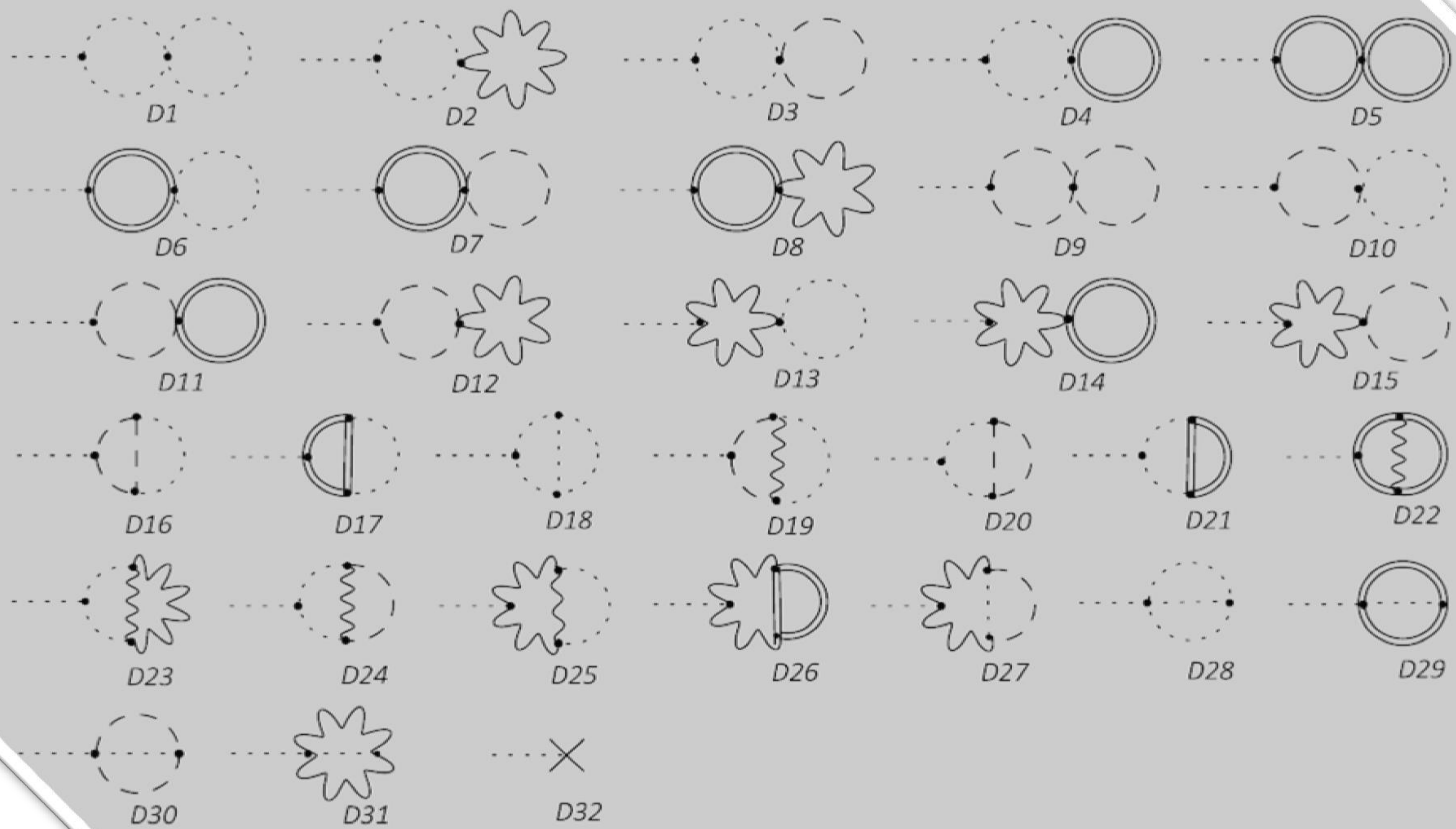

Figure 2: Two-loops contribution to the one-point vertex function. The countertem is represented by a cross line.

Gap equation and Effective superpotential

Table I: The result of the diagrams appearing in Fig. 2, omitting an overall factor of $\frac{i}{N^{3/2}}\,\sigma_{cl}^3\int_p d^2\theta\,\Sigma\,(p,\theta)$.

D1	$-\frac{27}{6}\left(\frac{1}{16\pi^2}\right)\lambda^3$	D9	$-3\left(\frac{1}{16\pi^2}\right)\lambda^3$	D17	$-(N-1)\,\lambda^3\,\zeta$	D25	$-g^6\,\zeta$
D2	$\frac{3}{2}\left(\frac{1}{16\pi^2}\right)\lambda\,g^4$	D10	$-3\left(\frac{1}{16\pi^2}\right)\lambda^3$	D18	$-54\,\lambda^3\,\zeta$	D26	$-\frac{3}{4}\,(N-1)\,\lambda\,g^4\,\zeta$
D3	$-3\left(\frac{1}{16\pi^2}\right)\lambda^3$	D11	$-2\,(N-1)\,\lambda^3$	D19	$\left(-\frac{1}{8}\,\lambda\,g^4-\lambda^2\,g^2\right)\zeta$	D27	$\frac{9}{4}\,\lambda\,g^4\,\zeta$
D4	$-6\left(\frac{1}{16\pi^2}\right)(N-1)\,\lambda^3$	D12	$\frac{1}{2}\left(\frac{1}{16\pi^2}\right)\lambda\,g^4$	D20	$-6\,\lambda^3\,\zeta$	D28	$-54\,\lambda^3\,\zeta$
D5	$-4\left(\frac{1}{16\pi^2}\right)(N-1)^2\,\lambda^3$	D13	$3\left(\frac{1}{16\pi^2}\right)\lambda\,g^4$	D21	$-3\,(N-1)\,\lambda^3\,\zeta$	D29	$-10\,(N-1)\,\lambda^3\,\zeta$
D6	$-3\left(\frac{1}{16\pi^2}\right)(N-1)\,\lambda^3$	D14	$\left(\frac{1}{16\pi^2}\right)(N-1)\,\lambda\,g^4$	D22	$\frac{1}{4}\,(N-1)\,\lambda\,g^4\,\zeta$	D30	$-10\,\lambda^3\,\zeta$
D7	$-\left(\frac{1}{16\pi^2}\right)(N-1)$	D15	$\left(\frac{1}{16\pi^2}\right)\lambda\,g^4$	D23	$\frac{3}{2}\,\lambda\,g^4\,\zeta$	D31	$\left(3\,\lambda\,g^4+\frac{3}{2}\,g^6\right)\zeta$
D8	$\frac{1}{4}\left(\frac{1}{16\pi^2}\right)(N-1)\,\lambda\,g^4$	D16	$-4\,\lambda^3\,\zeta$	D24	$\left(-\frac{3}{2}\,\lambda^2\,g^2-\frac{3}{4}\,\lambda\,g^4\right)\zeta$		

$$\mathcal{S}^{(1)}_{\Sigma\,(1+2)l}=i\sigma_{cl}^3N^{1/2}\left[\frac{1}{4\pi N}\,A_1+\frac{1}{N^2}\left(\zeta\,A_2+\frac{1}{16\pi^2}\,A_3\right)+A_4\right]\int_p d^2\theta\,\Sigma\,(p,\theta)\,, \qquad (4)$$

where

$$A_1=-\left[9+N\right]\lambda^2+\frac{g^4}{4},\quad A_2=-\left[114+14N\right]\lambda^3-\frac{5}{2}\lambda^2g^2+\left[\frac{51}{8}-\frac{1}{2}N\right]\lambda g^4+\frac{1}{2}g^6,$$

$$A_3=\left(\frac{11}{2}-4N-4N^2\right)\lambda^3+\left(\frac{23}{4}+\frac{5}{4}N\right)\lambda\,g^4$$

A. F. Ferrari. Comput. Phys. Commun., 176:334–346, 2007.

Gap equation and Effective superpotential

$$\zeta = \frac{1}{32\pi^2}\left\{\frac{1}{\epsilon} + \ln\left[4\pi e^{1-\gamma}\right]\right\} - \frac{1}{16\pi^2}\ln\left[\frac{\sigma_{cl}^2}{\mu}\right],$$

The unimproved Kählerian effective superpotential

$$K_{eff}^{(0+1+2)l} = K_{eff}^{(0)l} + K_{eff}^{(1+2)l} = \sigma_{cl}^4\,\mathcal{S}_{eff}^{(0+1+2)l} \tag{5}$$

where

$$\mathcal{S}_{eff}^{(0+1+2)l} = \left[-\frac{N}{4}\lambda - \frac{1}{16\pi N^{1/2}}A_1 + \frac{1}{64\pi^2 N^{3/2}}A_2\left(\frac{1}{2} - L\right) + \frac{1}{64\pi^2 N^{3/2}}A_3\right],$$

with $L \equiv \ln\left[\sigma_{cl}^2/\mu\right]$.

β and γ functions

Propagators in symmetric phase

$$\langle | \mathrm{T}\ \Phi_k\,(k,\theta_1)\,\Phi_l\,(-k,\theta_2) | \rangle = -i\,\delta_{kl}\,\frac{D^2}{k^2}\delta_{12}^2\,,$$

$$\langle | \mathrm{T}\ \Gamma_\alpha\,(k,\theta_1)\,\Gamma_\beta\,(-k,\theta_2) | \rangle = \frac{i}{2}\frac{1}{k^2}\,\{\,D_\alpha D_\beta - \alpha\,D_\beta D_\alpha\}\,\delta_{12}^2\,,$$

Table II: Divergent parts of the diagrams appearing in Fig. 3 omitting an overall factor of $i\,\zeta\int_p d^2\theta\,\Phi_a\,(-p,\theta)\,D^2\,\bar{\Phi}_a\,(p,\theta)$.

D1	0	D4	$-\frac{1}{2}\,g^4$	D7	$-\frac{1}{2}N\,g^4$	D10	0
D2	0	D5	0	D8	$-\frac{1}{4}\,g^4$	D11	0
D3	$-\frac{1}{2}\,g^4$	D6	$-\frac{1}{4}\,g^4$	D9	$8N^2\,\lambda^2$		

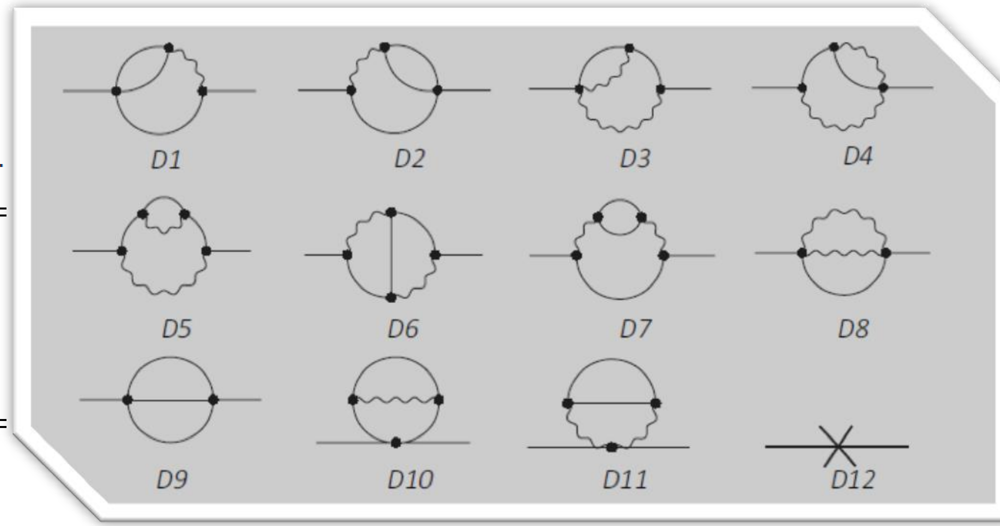

Figure 3: Two-loop contribution to two-point vertex function of the Φ_a superfield.

$$\mathcal{S}^{(2)}_{\Phi\Phi\,2l} = \left[\frac{1}{N^2}\,i\zeta\left\{8\,N^2\,\lambda^2 - \frac{1}{2}\,(3+N)\;g^4\right\} + \frac{1}{2}\,i\,\delta_\Phi\right]\int_p d^2\theta\,\Phi_a\,(-p,\theta)\,D^2\,\bar{\Phi}_a\,(p,\theta)\,, \tag{6}$$

β and γ functions

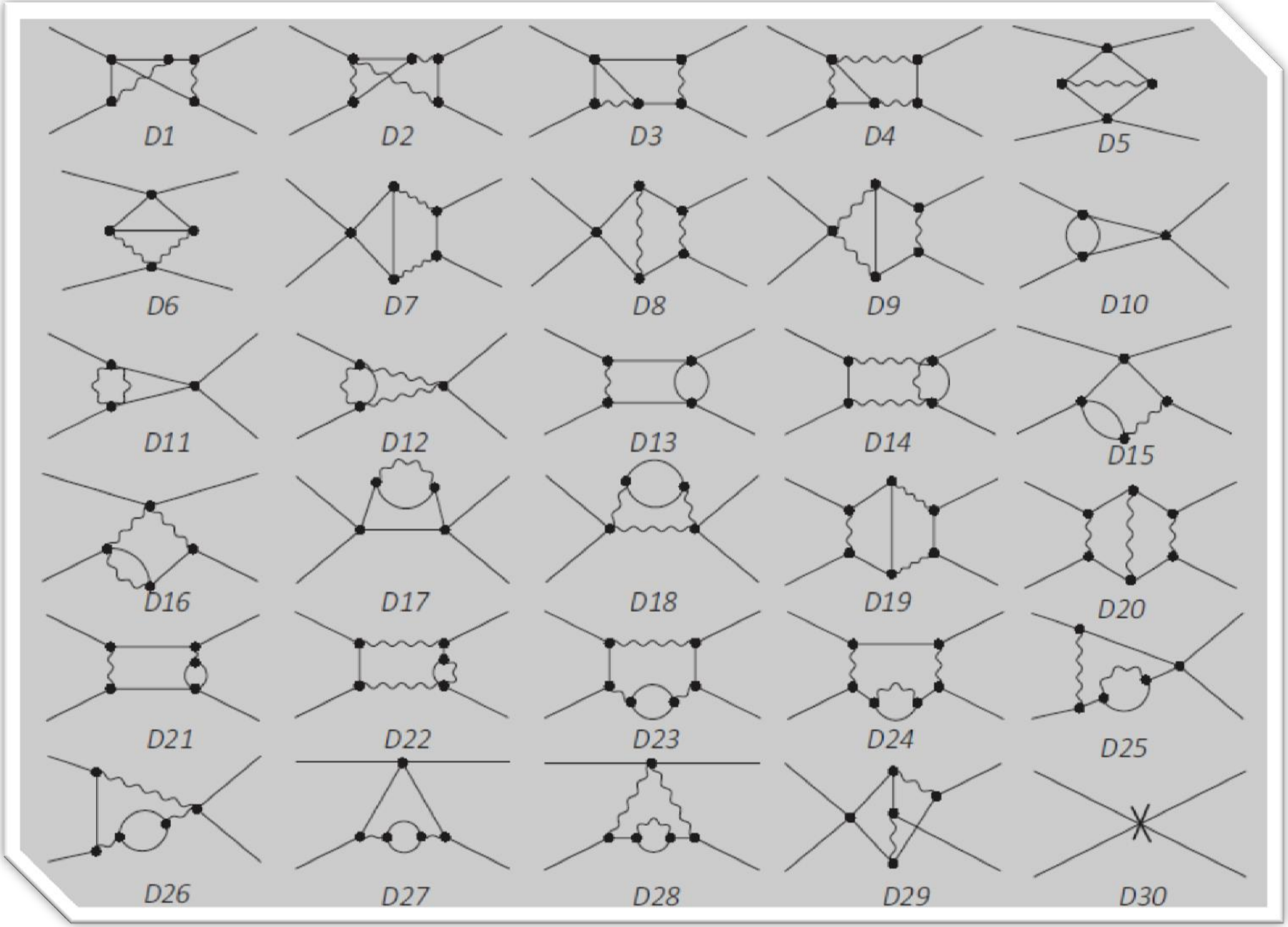

Figure 6: Two-loop contribution to four-point vertex function.

β and γ functions

Table V: Divergent parts of the diagrams appearing in Fig. 6, omitting an overall factor of $i\,\zeta\int_{p,\,s,\,l} d^2\theta\,\Phi_a\,(p,\theta)\,\bar{\Phi}_a\,(s,\theta)\,\Phi_a\,(l,\theta)\,\bar{\Phi}_a\,(-p-s-l,\theta)$.

D1	$2\,\lambda\,g^4$	D7	0	D13	$-8N\,\lambda^2\,g^2$	D19	0	D25	$8\,\lambda\,g^4$
D2	0	D8	$8\,\lambda\,g^4$	D14	$-\frac{1}{2}\,g^6$	D20	$\frac{1}{2}\,g^6$	D26	$N\,g^6$
D3	$-2\,\lambda\,g^4$	D9	$-g^6$	D15	$16N\,\lambda^2\,g^2$	D21	$-2N\,\lambda\,g^4$	D27	$-4N\,\lambda\,g^4$
D4	0	D10	$-64N^2\,\lambda^3$	D16	$\frac{1}{2}\,g^6$	D22	$-\frac{1}{2}g^6$	D28	g^6
D5	$-64N\,\lambda^2\,g^2$	D11	$-\lambda\,g^4$	D17	$64N\,\lambda^2\,g^2$	D23	$\frac{1}{2}N\,g^6$	D29	$2\,\lambda\,g^4$
D6	$12N\,\lambda\,g^4$	D12	$-\frac{1}{4}\,g^6$	D18	$N\,g^6$	D24	$\frac{1}{2}\,g^6$		

$$\mathcal{S}^{(2)}_{\Phi\bar{\Phi}\Phi\bar{\Phi}\,2l} = \left[\frac{1}{N^3} i\zeta\,F_1 + i\,\frac{\delta_\lambda}{N}\right]\int_{p,\,s,\,l} d^2\theta\,\Phi_a\,(p,\theta)\,\bar{\Phi}_a\,(s,\theta)\,\Phi_a\,(l,\theta)\,\bar{\Phi}_a\,(-p-s-l,\theta)\,, \qquad (7)$$

where $F_1 = -64N^2\,\lambda^3 + 8N\,\lambda^2\,g^2 + (17+6N)\lambda\,g^4 + \frac{1}{2}\,(1+5N)\,g^6$.

β and γ functions

The counterterms in the minimal subtraction scheme as

$$\delta_\Phi = -\frac{1}{32\pi^2 N^2}\frac{1}{\epsilon}\left\{16\,N^2\,\lambda^2 - (3+N)\,g^4\right\},$$

$$\delta_\lambda = -\frac{1}{32\pi^2 N^2}\frac{1}{\epsilon}\,F_1.$$

The relation between bare quantities (labeled by subscript zero) and renormalized ones are,

$$\Phi_{a\,0} = Z_\Phi^{\frac{1}{2}}\,\Phi_a\,,$$

$$\lambda_0\left(\Phi_{a\,0}\bar{\Phi}_{a\,0}\right)^2 = \mu^{2\epsilon}\left(\lambda+\delta_\lambda\right)\left(\Phi_a\bar{\Phi}_a\right)^2,$$

where $Z_\Phi = (1+\delta_\Phi)$.

$$\beta_\lambda \equiv \mu\frac{d}{d\mu}\lambda = \beta_\lambda^{(3)}, \qquad (8)$$

$$\beta_\lambda^{(3)} = \frac{12}{\pi^2}\,\lambda^3 - \frac{1}{\pi^2}\,\lambda^2\,y - \frac{1}{8\,\pi^2\,N^2}\,(11+4N)\,\lambda\,y^2 - \frac{1}{16\,\pi^2\,N^2}\,(1+5N)\,y^3,$$

where $y = g^2$.

$$\beta_g \equiv \mu\frac{d}{d\mu}g = 0, \qquad (9)$$

$$\gamma_\Gamma \equiv -\frac{\mu}{\Gamma_\alpha}\frac{d}{d\mu}\Gamma_\alpha = 0. \qquad (10)$$

L. Avdeev, G. Grigorev, and D. Kazakov, Nucl.Phys. B382, 561 (1992).
S. R. Coleman and B. R. Hill, Phys.Lett. B159, 184 (1985).

β and γ functions

$$\gamma_\Phi \equiv -\frac{\mu}{\Phi_a}\frac{d}{d\mu}\Phi_a = \gamma_\Phi^{(2)} = \frac{1}{\pi^2}\lambda^2 - \frac{1}{16\,\pi^2\,N^2}\,(3+N)\;y^2, \quad (11)$$

Now, using the RGE and the β functions

$$\left[\frac{1}{32\pi^2 N^{3/2}}A_2 - \frac{N}{4}\beta_\lambda + N\,\gamma_\sigma\,\lambda'\right]\;\sigma_{cl}^4 = \;0, \quad (12)$$

where

$$\lambda' = \lambda + \frac{1}{4\pi N^{3/2}}\,A_1 - \frac{1}{16\pi^2 N^{5/2}}\,A_2\left(\frac{1}{2} - L\right) + \frac{1}{16\pi^2 N^{5/2}}\,A_3\,,$$

For convenience, we solve (13) for γ_σ assuming $\lambda \neq 0$ as well as $y \lesssim \lambda < 1$, in which case $\lambda' \sim \lambda$. With this approximation, we obtain

$$\gamma_\sigma = \gamma_\sigma^{(2)} = \frac{1}{\pi^2}\left\{\frac{1}{16N^{5/2}}\,[57+7\,N]+3\right\}\lambda^2 + \frac{1}{\pi^2}\left\{-\frac{1}{64\,N^{5/2}}\left[\frac{51}{4} - N\right] - \frac{1}{32N^2}\,(11+4N)\right\}y^2$$
$$-\frac{1}{\pi^2}\left\{-\frac{5}{64N^{5/2}} + \frac{1}{4N}\right\}\lambda\,y - \frac{1}{64\pi^2}\left\{\frac{1}{N^{5/2}} + \frac{1}{N^2}\,(1+5N)\right\}\lambda^{-1}\,y^3,$$

Renormalization group equation

$$\left[-2\left(1+\gamma_\sigma\right)\frac{\partial}{\partial L}+\beta_\lambda\frac{\partial}{\partial\lambda}-\gamma_\sigma\sigma_{cl}\frac{\partial}{\partial\sigma_{cl}}\right]K_{eff} \;=\; 0. \qquad (13)$$

$$K^{I}_{eff}\equiv\sigma_{cl}^{4}S_{eff}\left(\lambda,y,L\left(\sigma\left(\mu\right),\mu\right)\right)\,,$$

The contributions to $\mathcal{S}_{eff}$ according to the relation between the aggregate powers of coupling constants and the power of the logarithm L,

$$\begin{aligned}\mathcal{S}_{eff} =&\mathcal{S}^{LL}_{eff}+\mathcal{S}^{NLL}_{eff}+\mathcal{S}^{2NLL}_{eff}+\ldots\\ &=\sum_{\substack{n,m\\(n+m\geq 1)}} C^{LL}_{n,m}\,\lambda^{n}\,y^{m}\,L^{n+m-1}+\sum_{\substack{n,m\\(n+m\geq 2)}} C^{NLL}_{n,m}\,\lambda^{n}\,y^{m}\,L^{n+m-2}\\ &\quad+\sum_{\substack{n,m\\(n+m\geq 3)}} C^{2NLL}_{n,m}\,\lambda^{n}\,y^{m}\,L^{n+m-3}+\ldots\,.\end{aligned}$$

Hereafter, LL stands for Leading Logarithms, NLL for Next-to-Leading Logarithms, and so on.

Renormalization group equation

The unimproved potential in function of C constants

$$K_{eff}^{(0+1+2)l} = K_{eff}^{(0)l} + K_{eff}^{(1+2)l} = \sigma_{cl}^4 \mathcal{S}_{eff}^{(0+1+2)l}$$

$$\mathcal{S}_{eff}^{(0+1+2)l} = C_{1,0}^{LL}\,\lambda + C_{2,0}^{NLL}\,\lambda^2 + C_{0,2}^{NLL}\,y^2 + C_{3,0}^{2NLL}\,\lambda^3 + C_{2,1}^{2NLL}\,\lambda^2\,y + C_{1,2}^{2NLL}\,\lambda\,y^2 + C_{0,3}^{2NLL}\,y^3 + [C_{3,0}^{NLL}\,\lambda^3 + C_{2,1}^{NLL}\,\lambda^2\,y + C_{1,2}^{NLL}\,\lambda\,y^2 + C_{0,3}^{NLL}\,y^3]\,L, \quad (14)$$

where

$$C_{1,0}^{LL} = -\frac{N}{4}, \quad C_{2,0}^{NLL} = \frac{9+N}{16\pi N^{1/2}},$$

$$C_{3,0}^{NLL} = \frac{114+14N}{64\pi^2 N^{3/2}}, \quad C_{3,0}^{2NLL} = \frac{1}{64\pi^2 N^{3/2}}\left(\frac{103}{2} - 11N - 4N^2\right),$$

$$C_{2,1}^{NLL} = \frac{5}{128\pi^2 N^{3/2}}, \quad C_{2,1}^{2NLL} = -\frac{5}{256\pi^2 N^{3/2}},$$

$$C_{0,2}^{NLL} = -\frac{1}{64\pi N^{1/2}}, \quad C_{1,2}^{NLL} = -\frac{1}{128\pi^2 N^{3/2}}\left[\frac{51}{4} - N\right],$$

$$C_{1,2}^{2NLL} = \frac{1}{64\pi^2 N^{3/2}}\left(\frac{143}{16} + N\right), \quad C_{0,3}^{NLL} = -\frac{1}{128\pi^2 N^{3/2}},$$

$$C_{0,3}^{2NLL} = \frac{1}{256\pi^2 N^{3/2}}.$$

Renormalization group equation

The renormalization group equation is given by

$$-\frac{\partial}{\partial L}\,\mathcal{S}_{eff}^{LL}+\left[\beta_{\lambda}^{(3)}\frac{\partial}{\partial\lambda}-4\,\gamma_{\sigma}^{(2)}\right]\mathcal{S}_{eff}^{LL}-2\frac{\partial}{\partial L}\,\mathcal{S}_{eff}^{NLL}+$$
$$\left\{\left[-2\,\gamma_{\sigma}^{(2)}\frac{\partial}{\partial L}\right]\mathcal{S}_{eff}^{LL}+\left[\beta_{\lambda}^{(3)}\frac{\partial}{\partial\lambda}-4\,\gamma_{\sigma}^{(2)}\right]\mathcal{S}_{eff}^{NLL}-2\frac{\partial}{\partial L}\,\mathcal{S}_{eff}^{2NLL}\right\}=0. \qquad (15)$$

Terms of order $\lambda^{n}y^{m}L^{n+m-2}$

$$-\frac{\partial}{\partial L}\,\mathcal{S}_{eff}^{LL}=0, \qquad (n+m-1)\,C_{n,m}^{LL}=0\,,\;\;(n+m\geq 1)\;.$$

$$\mathcal{S}_{eff}^{LL}=C_{1,0}^{LL}\lambda\,.$$

$$C_{n,m}^{LL}=0 \qquad n+m\geq 2.$$

There are not leading logarithms corrections at higher loop orders.

That complies with the general picture that supersymmetric models have less divergences and have simpler Renormalization properties.

Renormalization group equation

Terms of order $\lambda^n y^m L^{n+m-3}$

$$\left[\beta_\lambda^{(3)} \frac{\partial}{\partial \lambda} - 4\,\gamma_\sigma^{(2)}\right] \mathcal{S}_{eff}^{LL} - 2\frac{\partial}{\partial L}\,\mathcal{S}_{eff}^{NLL} = 0\,. \qquad (16)$$

with

$$C_{n,m}^{NLL} = \frac{[n+m-2]^{-1}}{64\pi^2 N^2}\left\{\left[384N^2\,(n-3) - \frac{4}{N^{1/2}}\,[114+14N]\right] C_{n-2,m}^{LL} - \left[32N\,(n-2) + \frac{1}{N^{1/2}}10\right] C_{n-1,m-1}^{LL} - \left[(44+16N)\,(n-1) - \frac{1}{N^{1/2}}\left[\frac{51}{2} - 2\,N\right]\right] C_{n,m-2}^{LL} - 2\left[(1+5N)\,(n) - \frac{1}{N^{1/2}}\right] C_{n+1,m-3}^{LL}\right\}$$

The solution associate to this order

$$\mathcal{S}_{eff}^{NLL} = C_{2,0}^{NLL}\,\lambda^2 + C_{0,2}^{NLL}\,y^2 + \left[C_{3,0}^{NLL}\,\lambda^3 + C_{2,1}^{NLL}\,\lambda^2\,y + C_{1,2}^{NLL}\,\lambda\,y^2 + C_{0,3}^{NLL}\,y^3\right]\,L,$$

Again there are not new corrections.

Renormalization group equation

Terms of order $\lambda^n y^m L^{n+m-4}$

$$\left\{\left[-2\,\gamma_\sigma^{(2)}\frac{\partial}{\partial L}\right]\mathcal{S}_{eff}^{LL}+\left[\beta_\lambda^{(3)}\frac{\partial}{\partial\lambda}-4\,\gamma_\sigma^{(2)}\right]\mathcal{S}_{eff}^{NLL}-2\frac{\partial}{\partial L}\,\mathcal{S}_{eff}^{2NLL}\right\}=0 \tag{17}$$

with

$$\begin{aligned}
C_{n,m}^{2NLL} &= \frac{1}{32\pi^2N^2}\left\{\left(\frac{1}{N^{1/2}}\left[[114+14N]+96N^{5/2}\right]C_{n-2,m}^{LL}\right.\right.\\
&+\frac{1}{N^{1/2}}\left[\frac{5}{2}-8N^{3/2}\right]C_{n-1,m-1}^{LL}-\frac{1}{N^{1/2}}\left[\frac{51}{8}-\frac{1}{2}N+N^{1/2}\left(11+4N\right)\right]C_{n,m-2}^{LL}\\
&\left.-\frac{1}{N^{1/2}}\left[\frac{1}{2}+\frac{1}{2}N^{1/2}\left(1+5N\right)\right]C_{n+1,m-3}^{LL}\right)+\\
&+\frac{1}{2}\left(n+m-3\right)^{-1}\left(\left[384N^2\left(n-3\right)-\frac{4}{N^{1/2}}\left[114+14N\right]\right]C_{n-2,m}^{NLL}\right.\\
&-\left[32N\left(n-2\right)+\frac{10}{N^{1/2}}\right]C_{n-1,m-1}^{NLL}-\left[\left(44+16N\right)\left(n-1\right)-\frac{4}{N^{1/2}}\left[\frac{51}{8}-\frac{1}{2}N\right]\right]C_{n,m-2}^{NLL}\\
&\left.\left.-2\left[\left(1+5N\right)n-\frac{1}{N^{1/2}}\right]C_{n+1,m-3}^{NLL}\right)\right\},\left(n+m\geq 4\right)
\end{aligned}$$

Renormalization group equation

$$S_{eff}^{2NLL} = C_{3,0}^{2NLL}\,\lambda^3 + C_{2,1}^{2NLL}\,\lambda^2\,y + C_{1,2}^{2NLL}\,\lambda\,y^2 + C_{0,3}^{2NLL}\,y^3 + \left[C_{5,0}^{2NLL}\,\lambda^5 + C_{4,1}^{2NLL}\,\lambda^4\,y + C_{3,2}^{2NLL}\,\lambda^3\,y^2 + C_{2,3}^{2NLL}\,\lambda^2\,y^3 + C_{1,4}^{2NLL}\,\lambda\,y^4 + C_{0,5}^{2NLL}\,y^5\right] L^2.$$

Here, in this order there are new contribution to C values:

$$C_{4,0}^{2NLL} = C_{3,1}^{2NLL} = C_{2,2}^{2NLL} = C_{1,3}^{2NLL} = C_{0,4}^{2NLL} = 0,$$

$$C_{5,0}^{2NLL} = \frac{1}{32\pi^2 N^2}\left[192\,N^3 - \frac{1}{N^{1/2}}\,[114+14N]\right] C_{3,0}^{NLL},$$

$$C_{4,1}^{2NLL} = \frac{1}{32\pi^2 N^2}\left\{\left[96\,N^2 - \frac{1}{N^{1/2}}\,[114+14N]\right] C_{2,1}^{NLL} - \left[16\,N + \frac{5}{2\,N^{1/2}}\right] C_{3,0}^{NLL}\right\},$$

$$C_{3,2}^{2NLL} = -\frac{1}{32\pi^2 N^2}\left\{\frac{1}{N^{1/2}}\,[114+14N]\,C_{1,2}^{NLL} + \left[8\,N + \frac{5}{2\,N^{1/2}}\right] C_{2,1}^{NLL} + \left[\frac{1}{2}\,(44+16N) + \frac{1}{N^{1/2}}\left[\frac{51}{8} - \frac{1}{2}N\right]\right] C_{3,0}^{NLL}\right\},$$

Renormalization group equation

$$C_{2,3}^{2NLL} = -\frac{1}{32\pi^2 N^2}\left\{\left[96\,N^2 + \frac{1}{N^{1/2}}\left[114+14N\right]\right] C_{0,3}^{NLL} + \frac{5}{2\,N^{1/2}}\,C_{1,2}^{NLL} + \left[\frac{1}{4}\left(44+16N\right) - \left[\frac{51}{8} - \frac{1}{2}N\right]\right] C_{2,1}^{NLL} + \left[\left(1+5N\right) - \frac{1}{2\,N^{1/2}}\right] C_{3,0}^{NLL}\right\},$$

$$C_{1,4}^{2NLL} = -\frac{1}{32\pi^2 N^2}\left\{\left[\frac{5}{2\,N^{1/2}} - 8\,N\right] C_{0,3}^{NLL} + \frac{1}{N^{1/2}}\left[\frac{51}{8} - \frac{1}{2}N\right] C_{1,2}^{NLL} + \frac{1}{2}\left[\left(1+5N\right) - \frac{1}{N^{1/2}}\right] C_{2,1}^{NLL}\right\},$$

$$C_{0,5}^{2NLL} = \frac{1}{32\pi^2 N^2}\left\{\left[\frac{1}{4}\left(44+16N\right) + \frac{1}{N^{1/2}}\left[\frac{51}{8} - \frac{1}{2}N\right]\right] C_{0,3}^{NLL} + \frac{1}{2\,N^{1/2}}\,C_{1,2}^{NLL}\right\},$$

while other values $C_{n,m}^{2NLL}$ are zero.

Dinamical breaking of symmetry

The improved Kählerian effective superpotential

$$K^I_{eff}(\sigma_{cl}) = \sigma^4_{cl}\left\{S^{LL}_{eff} + S^{NLL}_{eff} + S^{2NLL}_{eff} + \rho\right\}, \qquad (18)$$

The component version of the improved Kählerian effective superpotential

$$V^I_{eff} = \int d^2\theta\, K^I_{eff}(\sigma_{cl}) = \sigma_2 \frac{\partial}{\partial\sigma_1} K^I_{eff}(\sigma_1) = \left[\frac{\partial}{\partial\sigma_1} K^I_{eff}(\sigma_1)\right]^2, \qquad (19)$$

The renormalization constant ρ is fixed using the renormalization condition.

$$\left.\frac{d^6}{d\sigma_1^6} V^I_{eff}(\sigma_1)\right|_{\sigma_1^2=\mu} = 6!\,\frac{N}{2}\lambda^2, \qquad (20)$$

$$\left.\frac{d}{d\sigma_1} V^I_{eff}(\sigma_1)\right|_{\sigma_1^2=\mu} = 0. \qquad (21)$$

This equation is used to determine the value of λ as a function of the free parameters **y** and **N**.

$$M_\Sigma = \left.\frac{d^2}{d\sigma_1^2} V^I_{eff}(\sigma_1)\right|_{\sigma_1^2=\mu} > 0. \qquad (22)$$

Minimum of the potential.

Dinamical breaking of symmetry

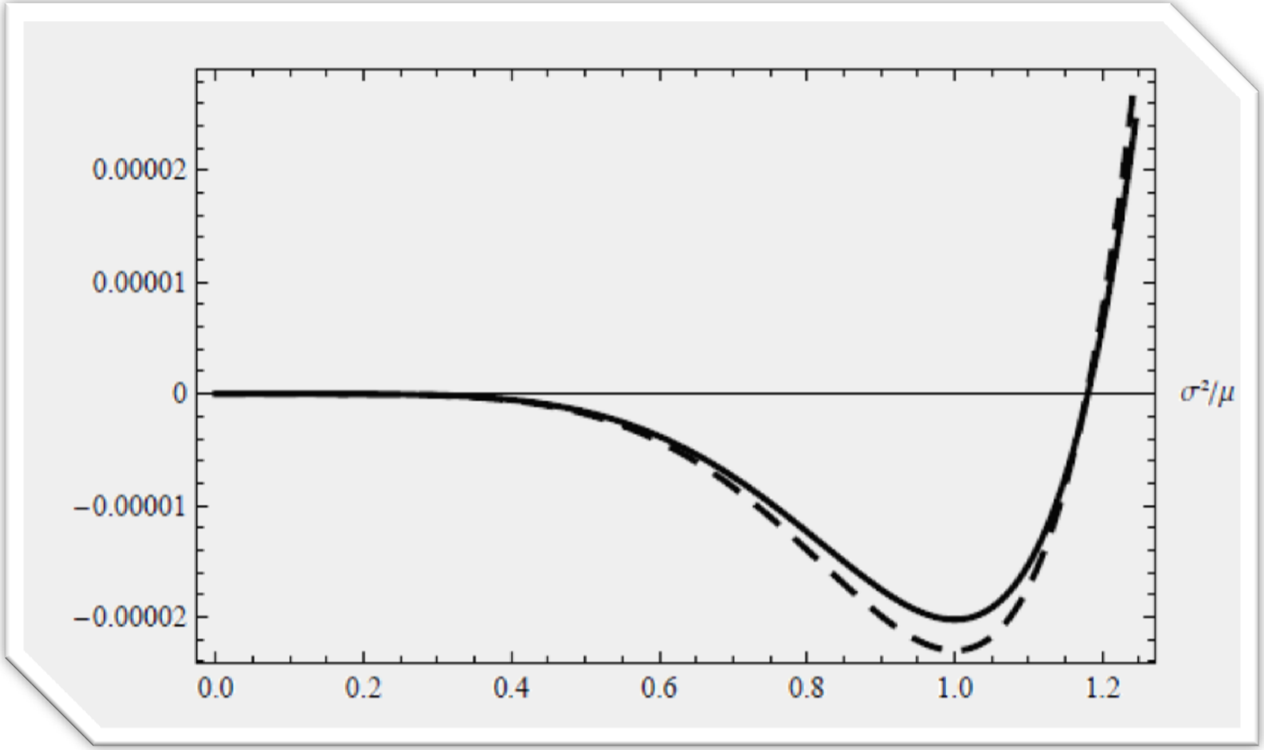

Figure 7: Comparison of the unimproved (continuous line) and improved (dashed line) potentials, with the same numerical values of y, N and as the example in the text.

Example: Choosing **g = 0.6** and **N = 1**, we found that **λ = 0.0242** for the unimproved effective potential and **λ^I = 0.0269** for its improved version. We see that the enforcement of the RGE on the calculation of the effective potential provides only a quantitative improvement on the parameters of the DSB.

Conclusions

- We had to calculate perturbative corrections to the vertex functions up to the two-loop level, finding the renormalization group **β and γ functions**.

- With theses function, together with the RGE, was possible to compute K^{I}_{eff} and, from this, we found **improved component effective potential** V^{I}_{eff} **.**

- This potential was used to study **DSB** in our model, and compared it with V_{eff}.

- The end result was that DSB is operational for all reasonable values of the free parameters, and that the RGE improvement produces only a small quantitative change in the properties of the model. **In this particular model, the effects of the improvement in the phase structure of the model were not so dramatic as in its non supersymmetric counterpart, however the question remains whether the same might happen in different models.**

- A future work we going to study for the calculation of the auxiliary field effective superpotential F $\Gamma[\sigma_{cl}] = \int d^5z\, K(\sigma_{cl}) + \int d^5z\, F(\sigma_{cl}, D_\alpha\sigma_{cl}, D^2\sigma_{cl})$).

THANKS!

Integrals (Apendix A)

$$\mathcal{I}_1(m_1,m_2,m_3)=\int\frac{d^Dkd^Dq}{(2\pi)^{2D}}\frac{1}{\left((q+k)^2+m_1^2\right)\left(q^2+m_2^2\right)\left(k^2+m_3^2\right)}$$

$$d^Dq\equiv\mu^\epsilon d^{3-\epsilon}q.$$

$$=-\frac{1}{32\pi^2}\left\{\frac{1}{\epsilon}-\gamma+\ln 4\pi+1\right\}+\frac{1}{16\pi^2}\ln\left[\frac{m_1+m_2+m_3}{\mu}\right],$$

$$\mathcal{I}_2(m_1,m_2,m_3)=\int\frac{d^Dkd^Dq}{(2\pi)^{2D}}\frac{2(k\cdot q)}{\left((q+k)^2+m_1^2\right)\left(q^2+m_2^2\right)\left(k^2+m_3^2\right)}$$

$$=\left(-m_1^2+m_2^2+m_3^2\right)\mathcal{I}_1(M)+\frac{1}{16\pi^2}\left[m_2m_3-m_1m_2-m_1m_3\right],$$

$$\mathcal{I}_3(m_1,m_2,m_3)=\int\frac{d^Dkd^Dq}{(2\pi)^{2D}}\frac{(k\cdot q)^2}{\left((q+k)^2+m_1^2\right)\left(q^2+m_2^2\right)\left(k^2+m_3^2\right)}$$

$$=-\frac{1}{64\pi^2}\left\{m_1^2\left(m_1m_2+m_1m_3-m_2m_3\right)\right.$$

$$\left.-m_1\left(3m_2^3+3m_3^3+m_2^2m_3+m_2m_3^2\right)+\left(m_2^2+m_3^2\right)m_2m_3\right\}$$

$$-\frac{1}{4}\left(4m_2m_3-\left(-m_1^2+m_2^2+m_3^2\right)\right)^2\mathcal{I}_1(M),$$

Superfileds conventions (Apendix B)

$$\Phi(x,\theta) = \varphi(x) + \theta^\alpha \Psi_\alpha(x) - \theta^2 F(x)\,, \qquad \varphi(x) = \Phi(x,\theta)|\,, \qquad F(x) = D^2\Phi(x,\theta)|\,.$$

$$\Psi_\alpha(x) = D_\alpha \Phi(x,\theta)|\,,$$

$$W_\alpha(x,\theta) = \rho_\alpha(x) + \theta^\beta f_{\beta\alpha}(x) - i\theta^2 \partial_\alpha{}^\beta \rho_\beta(x)\,,$$

$$\rho_\alpha(x) = W_\alpha(x,\theta)|\,, \qquad f_{\beta\alpha}(x) = \frac{1}{2}\left(\partial_{\beta\rho}A^\rho{}_\alpha + \partial_{\alpha\rho}A^\rho{}_\beta\right) = D_\beta W_\alpha(x,\theta)|\,,$$

$$i\partial_\alpha{}^\beta \rho_\beta(x) = D^2 W_\alpha(x,\theta)|\,,$$

$$\Gamma_\alpha(x,\theta) = \chi_\alpha(x) - \theta_\alpha B(x) + i\theta^\beta A_{\alpha\beta}(x) - \theta^2\left[2\rho_\alpha(x) + i\partial_{\beta\alpha}\chi^\beta(x)\right],$$

$$\chi_\alpha(x) = \Gamma_\alpha(x,\theta)|\,, \qquad B(x) = \frac{1}{2} D^\alpha \Gamma_\alpha(x,\theta)|\,,$$

$$A_{\alpha\beta}(x) = -\frac{i}{2}\left[D_\alpha \Gamma_\beta(x,\theta) + D_\beta \Gamma_\alpha(x,\theta)\right]|\,, \qquad 2\rho_\alpha(x) + i\partial_{\beta\alpha}\chi^\beta(x) = D^2\Gamma_\alpha(x,\theta)|\,.$$