Electric Dipole Moments and Supersymmetry

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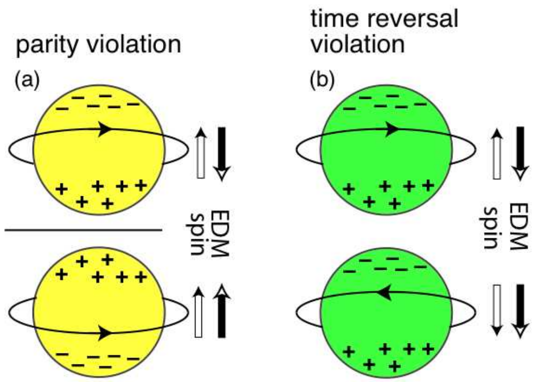
* JHEP 0810 (2008) 049 [arXiv:0808.1819v6 [hep-ph]] with J. Ellis and A. Pilaftsis

♠ *EDMs*

- Electric Dipole Moments (EDMs): T violation ⇒ CP violation (under CPT)



$$\mathcal{H}^{\rm EDM} = -d\ \mathbf{E}\cdot\hat{\mathbf{s}}$$

$$|d_{\rm Tl}| < 9\times 10^{-25}\ {\rm e\,cm}\,, \quad |d_{\rm n}| < 2.9\times 10^{-26}\ {\rm e\,cm}\,,$$

$$|d_{\rm Hg}| < 3.1\times 10^{-29}\ {\rm e\,cm}\,, \quad |d_{\rm ThO}/\mathcal{F}_{\rm ThO}| < 8.7\times 10^{-29}\ {\rm e\,cm}\,,$$

B. C. Regan, E. D. Commins, C. J. Schmidt and D. DeMille, PRL**88** (2002) 071805;

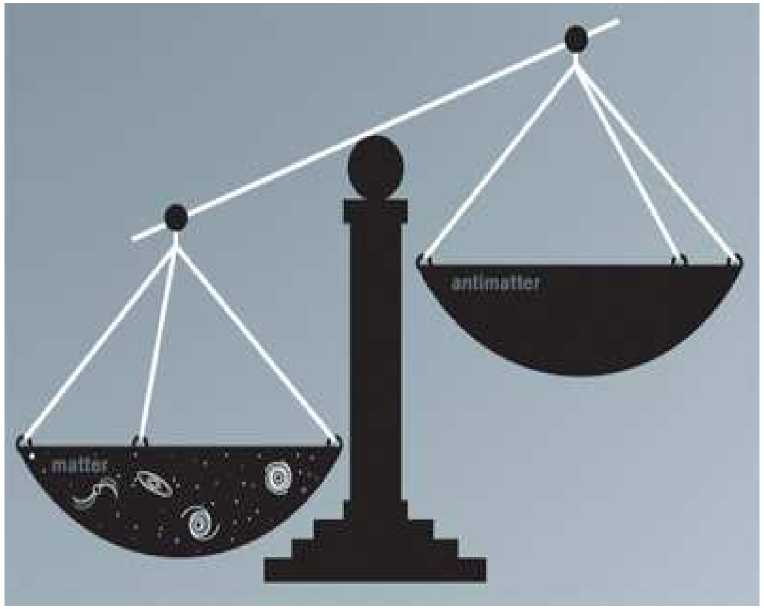
C. A. Baker *et al.*, PRL**97** (2006) 131801;

W. C. Griffith, M. D. Swallows, T. H. Loftus, M. V. Romalis, B. R. Heckel and E. N. Fortson, PRL**102** (2009) 101601

J. Baron et al. [ACME Collaboration], arXiv:1310.7534

♠ *Why more CP violation?*

- Then, who ordered "more" CP violation beyond the SM CKM phase? A. D. Sakharov, JETP Letters **5**(1967)24



CP violation in the SM is too weak to explain the matter dominance of the Universe J. Cline, arXiv:hep-ph/0609145

The matter-dominated Universe did!

♠ *Sources of CPV with Supersymmetry (Breaking)*

- V_{CKM}
- V_{PMNS}
- QCD θ term
- Supersymmetric terms:
  - MSSM: $W \supset \mu \hat{H}_2 \cdot \hat{H}_1$
  - NMSSM: $W \supset \lambda \hat{S}\hat{H}_2 \cdot \hat{H}_1 + \frac{\kappa}{3}\hat{S}^3$
- Soft SUSY breaking terms: ... *next pages*

♠ Sources of CPV with Supersymmetry (Breaking)

- A lot of CPV sources in soft SUSY breaking terms!:

  – Φ_i [3]: $-\mathcal{L}_{\rm soft} \supset \frac{1}{2}(M_3\,\widetilde{g}\widetilde{g} + M_2\,\widetilde{W}\widetilde{W} + M_1\,\widetilde{B}\widetilde{B} + {\rm h.c.})$

  – $\mathrm{Arg}\left(\mathbf{M}^2_{\widetilde{Q},\widetilde{L},\widetilde{u},\widetilde{d},\widetilde{e}}\right)_{i<j}$ $[5\times 3 \to 0_{\rm NFV}]$:

$$-\mathcal{L}_{\rm soft} \supset \widetilde{Q}^\dagger\,\mathbf{M^2_{\widetilde{Q}}}\,\widetilde{Q} + \widetilde{L}^\dagger\,\mathbf{M^2_{\widetilde{L}}}\,\widetilde{L} + \widetilde{u}^*_R\,\mathbf{M^2_{\widetilde{u}}}\,\widetilde{u}_R + \widetilde{d}^*_R\,\mathbf{M^2_{\widetilde{d}}}\,\widetilde{d}_R + \widetilde{e}^*_R\,\mathbf{M^2_{\widetilde{e}}}\,\widetilde{e}_R$$

  – $\mathrm{Arg}\,(\mathbf{A}_{u,d,e})_{i,j}$ $[3\times 9 \to (3\times 3)_{\rm NFV}]$:

$$-\mathcal{L}_{\rm soft} \supset +(\widetilde{u}^*_R\,\mathbf{A_u}\,\widetilde{Q}H_2 - \widetilde{d}^*_R\,\mathbf{A_d}\,\widetilde{Q}H_1 - \widetilde{e}^*_R\,\mathbf{A_e}\,\widetilde{L}H_1 + {\rm h.c.})$$

  – $\mathrm{Arg}\,(m^2_{12})$ [1]: $-\mathcal{L}_{\rm soft} \supset -(m^2_{12}H_1H_2 + {\rm h.c.})$

  – $\mathrm{Arg}(A_{\lambda,\kappa})$ (NMSSM): $-\mathcal{L}_{\rm soft} \supset \left(\lambda A_\lambda S H_2 H_1 - \frac{1}{3}\kappa A_\kappa S^3 + {\rm h.c.}\right)$

♠ *Sources of CPV with Supersymmetry (Breaking)*

- Minimal Flavour Violation?: Ellis, JSL, Pilaftsis, PRD, arXiv:0708.2079

  One may define a scheme of Maximally CP-violating MSSM with Minimal Flavour Violation, MCPMFV framework, with the maximal set of flavour-singlet mass scales:

$$M_{1,2,3}\,, \quad M^2_{H_{u,d}}\,, \quad \widetilde{\mathbf{M}}^2_{Q,L,U,D,E} = \widetilde{M}^2_{Q,L,U,D,E}\,\mathbf{1}_3\,, \quad \mathbf{A}_{u,d,e} = A_{u,d,e}\,\mathbf{1}_3$$

$$3 \oplus 3 \qquad 2 \qquad 5 \qquad 3 \oplus 3$$

$13 \oplus 6 = 19$ Parameters !

The 19 parameters are given at the MFV scale which could be the GUT scale

♠ *Sources of CPV with Supersymmetry (Breaking)*

- Rephasing invariance?: Physical observables depend on : Arg($M_i\,\mu\,(m_{12}^2)^*$) and Arg($\mathbf{A_f}\,\mu\,(m_{12}^2)^*$) M. Dugan, B. Grinstein and L. J. Hall, Nucl. Phys. B **255** (1985) 413; S. Dimopoulos and S. Thomas, Nucl. Phys. B **465** (1996) 23

  Without flavour-mixing terms, in the MSSM, one may have the following 12 physical SUSY CP phases most generally

$$\mathrm{Arg}(M_1\,\mu)\,,\ \mathrm{Arg}(M_2\,\mu)\,,\ \mathrm{Arg}(M_3\,\mu)\,;$$

$$\mathrm{Arg}(A_e\,\mu)\,,\ \mathrm{Arg}(A_\mu\,\mu)\,,\ \mathrm{Arg}(A_\tau\,\mu)\,;$$

$$\mathrm{Arg}(A_d\,\mu)\,,\ \mathrm{Arg}(A_s\,\mu)\,,\ \mathrm{Arg}(A_b\,\mu)\,;$$

$$\mathrm{Arg}(A_u\,\mu)\,,\ \mathrm{Arg}(A_c\,\mu)\,,\ \mathrm{Arg}(A_t\,\mu)$$

$*$ We will take the Arg(m_{12}^2) $= 0$ convention throughout this talk

♠ *Sources of CPV with Supersymmetry (Breaking)*

- Rephasing invariance?: In the NMSSM, Kingman Cheung, Tie-Jiun Hou, JSL, Eibun Senaha, arXiv:1006.1458 [hep-ph] [PRD82(2010)075007]; arXiv:1102.5679 [hep-ph] [PRD84(2011)015002]

$$One\ more\ CP\ Phase\ :\ \phi'_\lambda - \phi'_\kappa$$

where $\phi'_\lambda \equiv \phi_\lambda + \theta + \varphi$; $\phi'_\kappa \equiv \phi_\kappa + 3\varphi$ with θ and φ defined in

$$H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + \phi_d^0 + i a_d) \\ \phi_d^- \end{pmatrix} \ ; \quad H_u = e^{i\theta} \begin{pmatrix} \phi_u^+ \\ \frac{1}{\sqrt{2}}(v_u + \phi_u^0 + i a_u) \end{pmatrix}$$

$$S = \frac{e^{i\varphi}}{\sqrt{2}}(v_S + \phi_S^0 + i a_S)$$

*** Another two rephasing invariant combinations $\phi'_\lambda + \phi_{A_\lambda}$ and $\phi'_\kappa + \phi_{A_\kappa}$ are determined by the two CP-odd tadpole conditions

♠ Contents

♠ *Synopsis of the observable EDMs*

- The CP-violating interactions: *in terms of photons, gluons, quarks and leptons*

$$
\begin{aligned}
\mathcal{L} \;\; = \;\; & \frac{\alpha_s}{8\pi}\,\bar{\theta}\,G^a_{\mu\nu}\tilde{G}^{\mu\nu,a} \\
& -\frac{i}{2}\,d^E_f\,F^{\mu\nu}\,\bar{f}\,\sigma_{\mu\nu}\gamma_5\,f \;-\; \frac{i}{2}\,d^C_q\,G^{a\,\mu\nu}\,\bar{q}\,\sigma_{\mu\nu}\gamma_5 T^a\,q \\
& +\frac{1}{3}\,d^G\,f_{abc}\,G^a_{\rho\mu}\,\widetilde{G}^{b\,\mu\nu}\,G^c{}_\nu{}^\rho \;+\; \sum_{f,f'} C_{f\,f'}(\bar{f}f)(\bar{f}'i\gamma_5 f')
\end{aligned}
$$

where $\bar{\theta} = \theta_{\mathrm{QCD}} + \mathrm{Arg}\,\mathrm{Det}\,M_q$

♠ *Synopsis of the observable EDMs*

- The CP-violating interactions: *in terms of nucleons and pions*

$$\mathcal{L} = C_S\, \bar{e} i\gamma_5 e\, \bar{N}N$$
$$+C_P\, \bar{e}e\, \bar{N}i\gamma_5 N + C'_P\, \bar{e}e\, \bar{N}i\gamma_5\tau_3 N$$
$$+\bar{g}^{(0)}_{\pi NN}\overline{N}\tau^a N\pi^a + \bar{g}^{(1)}_{\pi NN}\overline{N}N\pi^0 + \bar{g}^{(2)}_{\pi NN}\left(\overline{N}\tau^a N\pi^a - 3\overline{N}\tau^3 N\pi^0\right)$$

The last term, for example, takes the explicit form of

$$\bar{g}^{(0)}_{\pi NN}\left[\sqrt{2}\left(\bar{p}n\pi^+ + \bar{n}p\pi^-\right) + \left(\bar{p}p - \bar{n}n\right)\pi^0\right] + \bar{g}^{(1)}_{\pi NN}\left(\bar{p}p + \bar{n}n\right)\pi^0$$
$$+ \bar{g}^{(2)}_{\pi NN}\left[\sqrt{2}\left(\bar{p}n\pi^+ + \bar{n}p\pi^-\right) - 2\left(\bar{p}p - \bar{n}n\right)\pi^0\right]$$

♠ Synopsis of the observable EDMs

- The CP-violating interactions: *connections*

$$C_S \simeq C_{de}\frac{29\,\mathrm{MeV}}{m_d} + C_{se}\frac{\kappa\times 220\,\mathrm{MeV}}{m_s} + C_{be}\frac{66\ \mathrm{MeV}\,(1-0.25\kappa)}{m_b}$$

with $\kappa \equiv \langle N|m_s\bar{s}s|N\rangle/220\ \mathrm{MeV} \simeq 0.50 \pm 0.25$. When the 4-fermion interactions are induced by the Higgs mediations, including the squark contributions, the last term might be replaced with

$$(C_S)^g = (0.1\,\mathrm{GeV})\,\frac{m_e}{v^2}\sum_{i=1}^{3}\frac{g^S_{H_igg}\,g^P_{H_i\bar{e}e}}{M^2_{H_i}}$$

$$\mathcal{L}_{Hff} = -g_f\,H_i\,\bar{f}\,(g^S_{H_i\bar{f}f} + i g^P_{H_i\bar{f}f}\gamma_5)\,f\,;$$

$$g^S_{H_igg} = \sum_{q=t,b}\left\{\frac{2\,x_q}{3}g^S_{H_i\bar{q}q} - \frac{v^2}{12}\sum_{j=1,2}\frac{g_{H_i\tilde{q}^*_j\tilde{q}_j}}{m^2_{\tilde{q}_j}}\right\}\ \text{with } x_t = 1 \text{ and } x_b = (1-0.25\kappa)$$

♠ Synopsis of the observable EDMs

- The CP-violating interactions: *connections ... continued*

$$C_P \simeq -375\ \text{MeV} \sum_{q=c,s,t,b} \frac{C_{eq}}{m_q},$$

$$C'_P \simeq -806\ \text{MeV}\,\frac{C_{ed}}{m_d} - 181\ \text{MeV} \sum_{q=c,s,t,b} \frac{C_{eq}}{m_q}$$

And

$$\bar{g}^{(0)}_{\pi NN} = 0.4 \times 10^{-12}\,\frac{(d_u^C + d_d^C)/g_s}{10^{-26}\text{cm}}\,\frac{|\langle \bar{q}q\rangle|}{(225\,\text{MeV})^3},$$

$$\bar{g}^{(1)}_{\pi NN} = 2^{+4}_{-1} \times 10^{-12}\,\frac{(d_u^C - d_d^C)/g_s}{10^{-26}\text{cm}}\,\frac{|\langle \bar{q}q\rangle|}{(225\,\text{MeV})^3}$$
$$- 8 \times 10^{-3}\text{GeV}^3 \left[\frac{0.5C_{dd}}{m_d} + 3.3\kappa\frac{C_{sd}}{m_s} + (1-0.25\kappa)\frac{C_{bd}}{m_b}\right]$$

The coupling $\bar{g}^{(2)}_{\pi NN}$ is irrelevant in most cases.

♠ *Synopsis of the observable EDMs*

- Thallium EDM; I.B. Khriplovich and S.K. Lamoreaux, *CP Violation Without Strangeness* (Springer, New York, 1997); M. Pospelov and A. Ritz, Annals Phys. **318** (2005) 119,[arXiv:hep-ph/0504231]

$$\begin{aligned} d_{\rm Tl}\,[e\,{\rm cm}] &= -585\cdot d_e^E\,[e\,{\rm cm}] - 8.5\times 10^{-19}\,[e\,{\rm cm}]\cdot(C_S\,{\rm TeV}^2) + \cdots \\ &= -585\,\left\{d_e^E\,[e\,{\rm cm}] + 1.45\times 10^{-21}\,[e\,{\rm cm}]\cdot(C_S\,{\rm TeV}^2)\right\} + \cdots \end{aligned}$$

- Thorium monoxide EDM; V.A. Dzuba, V.V. Flambaum, C. Harabati, Phys. Rev. A 84, 052108 (2011)

$$d_{\rm ThO}\,[e\,{\rm cm}] = \mathcal{F}_{\rm ThO}\,\left\{d_e^E\,[e\,{\rm cm}] + 1.6\times 10^{-21}\,[e\,{\rm cm}]\cdot(C_S\,{\rm TeV}^2)\right\} + \cdots$$

*** Neglecting C_S and $\cdots$,

$$|d_{\rm Tl}| < 9\times 10^{-25}\ {\rm e\,cm} \quad\Longrightarrow\quad \left|d_e^E\right| < 1.5\times 10^{-27}\ {\rm e\,cm}$$

$$|d_{\rm ThO}/\mathcal{F}_{\rm ThO}| < 8.7\times 10^{-29}\ {\rm e\,cm} \quad\Longrightarrow\quad \left|d_e^E\right| < 8.7\times 10^{-29}\ {\rm e\,cm}$$

♠ *Synopsis of the observable EDMs*

- Neutron EDM [Chiral Quark Model (CQM)];A. Manohar and H. Georgi, Nucl. Phys. B **234** (1984) 189; R. Arnowitt, J. L. Lopez and D. V. Nanopoulos, Phys. Rev. D **42** (1990) 2423; R. Arnowitt, M. J. Duff and K. S. Stelle, Phys. Rev. D **43** (1991) 3085; T. Ibrahim and P. Nath, Phys. Rev. D **57** (1998) 478 [Erratum-ibid. D **58** (1998 ERRAT,D60,079903.1999 ERRAT,D60,119901.1999) 019901] [arXiv:hep-ph/9708456]

$$d_n = \frac{4}{3}\, d_d^{\rm NDA} - \frac{1}{3}\, d_u^{\rm NDA},$$

$$d_{q=u,d}^{\rm NDA} = \eta^E\, d_q^E + \eta^C\, \frac{e}{4\pi}\, d_q^C + \eta^G\, \frac{e\Lambda}{4\pi}\, d^G,$$

where $\eta^E \simeq 1.53$, $\eta^C \simeq 3.4$, $\eta^G \simeq 0.45 - 3.4$ depending on models. Dekens, deVries, JHEP 1305 (2013) 149 arXiv:1303.3156 And the chiral symmetry breaking scale $\Lambda \simeq 1.19$ GeV.

♠ *Synopsis of the observable EDMs*

- Neutron EDM [Parton Quark Model (PQM)]; J. R. Ellis and R. A. Flores, Phys. Lett. B **377** (1996) 83, [arXiv:hep-ph/9602211]

$$d_n = \eta^E \left(\Delta_d^{\rm PQM}\, d_d^E + \Delta_u^{\rm PQM}\, d_u^E + \Delta_s^{\rm PQM}\, d_s^E\right) ,$$

with

$$\Delta_d^{\rm PQM} = 0.746\,, \qquad \Delta_u^{\rm PQM} = -0.508\,, \qquad \Delta_s^{\rm PQM} = -0.226$$

The isospin symmetry between the neutron n and the proton p implies that $\Delta_d = (\Delta_u)_p = 4/3$, $\Delta_u = (\Delta_d)_p = -1/3$. Furthermore, in the relativistic Naive Quark Model (NQM), one has $\Delta_s = (\Delta_s)_p = 0$.

♠ *Synopsis of the observable EDMs*

- Neutron EDM [QCD sum rule techniques (QCD)];M. Pospelov and A. Ritz, Phys. Rev. Lett. **83** (1999) 2526, [arXiv:hep-ph/9904483]; M. Pospelov and A. Ritz, Nucl. Phys. B **573** (2000) 177, [arXiv:hep-ph/9908508]; M. Pospelov and A. Ritz, Phys. Rev. D **63** (2001) 073015, [arXiv:hep-ph/0010037]; D. A. Demir, M. Pospelov and A. Ritz, Phys. Rev. D **67** (2003) 015007,[arXiv:hep-ph/0208257]; D. A. Demir, O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Nucl. Phys. B **680** (2004) 339, [arXiv:hep-ph/0311314]; K. A. Olive, M. Pospelov, A. Ritz and Y. Santoso, Phys. Rev. D **72** (2005) 075001, [arXiv:hep-ph/0506106]

$$d_n = d_n(d_q^E\,, d_q^C) + d_n(d^G) + d_n(C_{bd}) + \cdots ,$$

$$\begin{aligned} d_n(d_q^E\,, d_q^C) &= (1.4 \pm 0.6)\,(d_d^E - 0.25\,d_u^E) + (1.1 \pm 0.5)\,e\,(d_d^C + 0.5\,d_u^C)/g_s\,, \\ d_n(d^G) &\sim \pm\,e\,(20 \pm 10)\ \mathrm{MeV}\,d^G\,, \\ d_n(C_{bd}) &\sim \pm\,e\,2.6 \times 10^{-3}\ \mathrm{GeV}^2 \left[\frac{C_{bd}}{m_b} + 0.75\frac{C_{db}}{m_b}\right] \end{aligned}$$

where $d^G = d^G(1\ \mathrm{GeV}) \simeq (\eta^G/0.4)\,d^G(\mathrm{EW})$

♠ *Synopsis of the observable EDMs*

- Mercury EDM; D. A. Demir, O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Nucl. Phys. B **680** (2004) 339, [arXiv:hep-ph/0311314]; K. A. Olive, M. Pospelov, A. Ritz and Y. Santoso, Phys. Rev. D **72** (2005) 075001, [arXiv:hep-ph/0506106]

$$
\begin{aligned}
d_{\rm Hg}^{\rm I\,,II\,,III\,,IV} &= d_{\rm Hg}^{\rm I\,,II\,,III\,,IV}[S] + 10^{-2} d_e^E + (3.5\times 10^{-3}{\rm GeV})\, e\, C_S \\
&+ (4\times 10^{-4}\ {\rm GeV})\, e\, \left[C_P + \left(\frac{Z-N}{A}\right)_{\rm Hg} C_P' \right]
\end{aligned}
$$

where $d_{\rm Hg}^{\rm I\,,II\,,III\,,IV}[S]$ denotes the Mercury EDM induced by the Schiff moment taking account of the theoretical uncertainties.

♠ Synopsis of the observable EDMs

- Mercury EDM ... *continued*: We take account of the uncertainties in the calculation of the Schiff-moment induced Mercury EDM: [13] V. F. Dmitriev and R. A. Senkov, Phys. Atom. Nucl. 66, 1940 (2003) [Yad. Fiz. 66, 1988 (2003)]; [16] J. H. de Jesus and J. Engel, Phys. Rev. C 72 (2005) 045503; [12] S. Ban, J. Dobaczewski, J. Engel and A. Shukla, arXiv:1003.2598 [nucl-th]

Atom	Ref.	Interaction	$-a_0$	$-a_1$	a_2	$-b$
^{199}Hg	[13]	—	0.0004	0.055	0.009	—
	[16]	SkO′	0.010	0.074	0.018	—
		(average)	0.007	0.071	0.018	—
	[12]	SLy4 (HF)	0.013	−0.006	0.022	0.003
		SIII (HF)	0.012	0.005	0.016	0.004
		SV (HF)	0.009	−0.0001	0.016	0.002
		SLy4 (HFB)	0.013	−0.006	0.024	0.007
		SkM* (HFB)	0.041	−0.027	0.069	0.013

$$S = (a_0 + b)\, g_{\pi NN} \bar{g}^{(0)}_{\pi NN} + a_1\, g_{\pi NN} \bar{g}^{(1)}_{\pi NN} + (a_2 - b)\, g_{\pi NN} \bar{g}^{(2)}_{\pi NN}$$

♠ *Synopsis of the observable EDMs*

- Mercury EDM ... *continued*: Taking values from I Pospelov and Ritz, Annals Phys. 318 (2005) 119, II Ref.[13], III average of Ref.[16], and IV SIII(HF) of Ref.[12] J. Ellis, JSL, and A. Pilaftsis, JHEP 1102 (2011) 045, arXiv:1101.3529 [hep-ph]

$$
\begin{aligned}
d_{\rm Hg}^{\rm I}[S] &\simeq 1.8\times 10^{-3}\, e\, \bar{g}^{(1)}_{\pi NN}\,/{\rm GeV}\,,\\
d_{\rm Hg}^{\rm II}[S] &\simeq 7.6\times 10^{-6}\, e\, \bar{g}^{(0)}_{\pi NN}\,/{\rm GeV} + 1.0\times 10^{-3}\, e\, \bar{g}^{(1)}_{\pi NN}\,/{\rm GeV}\,,\\
d_{\rm Hg}^{\rm III}[S] &\simeq 1.3\times 10^{-4}\, e\, \bar{g}^{(0)}_{\pi NN}\,/{\rm GeV} + 1.4\times 10^{-3}\, e\, \bar{g}^{(1)}_{\pi NN}\,/{\rm GeV}\,,\\
d_{\rm Hg}^{\rm IV}[S] &\simeq 3.1\times 10^{-4}\, e\, \bar{g}^{(0)}_{\pi NN}\,/{\rm GeV} + 9.5\times 10^{-5}\, e\, \bar{g}^{(1)}_{\pi NN}\,/{\rm GeV}
\end{aligned}
$$

♠ *Synopsis of the observable EDMs*

- Deuteron EDM: O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Phys. Rev. D **70** (2004) 016003,[arXiv:hep-ph/0402023]

$$d_D = d_D(d_n + d_p) - (1.3 \pm 0.3)\, e\, \frac{g^{(1)}_{\pi NN}}{\text{GeV}} \pm\, e\,(20 \pm 10)\ \text{MeV}\, d^G$$

$$\begin{aligned} d_D(d_n + d_p) &\simeq (0.5 \pm 0.3)(d_u^E + d_d^E) \\ &\quad -(0.6 \pm 0.3)\, e\, \left[(d_u^C - d_d^C)/g_s + 0.3(d_d^C + d_d^C)/g_s)\right] \end{aligned}$$

The projective sensitivity: Y. K. Semertzidis *et al.* [EDM Collaboration], AIP Conf. Proc. **698** (2004) 200, [arXiv:hep-ex/0308063]; Y. F. Orlov, W. M. Morse and Y. K. Semertzidis, Phys. Rev. Lett. **96** (2006) 214802, [arXiv:hep-ex/0605022]; and also see `http://www.bnl.gov/edm/deuteron_proposal_080423_final.pdf`

$$|d_D| < 3 \times 10^{-27} - 10^{-29}\ e\,\text{cm}$$

For our numerical study, we take $3 \times 10^{-27}\ e\,\text{cm}$ as a representative expected value

♠ *Synopsis of the observable EDMs*

- Radium EDM: J. Ellis, JSL, and A. Pilaftsis, JHEP 1102 (2011) 045, arXiv:1101.3529 [hep-ph]

$$d_{\rm Ra} \simeq d_{\rm Ra}[S] \simeq -8.7 \times 10^{-2}\, e\, \bar{g}^{(0)}_{\pi NN}\, /{\rm GeV} + 3.5 \times 10^{-1}\, e\, \bar{g}^{(1)}_{\pi NN}\, /{\rm GeV}$$

The projective one-day sensitivity: L. Willmann, K. Jungmann, H. W. Wilschut, "Searches for permanent electric dipole moments in Radium Isotopes", Letter of Intent to the ISOLDE and Neutron Time-of-Flight Experiments Committee for experiments with HIE-ISOLDE, CERN-INTC-2010-049 / INTC-I-115

$$|d_{\rm Ra}| \sim 1 \times 10^{-27}\, e\, {\rm cm}$$

♠ *Synopsis of the observable EDMs*

- CP-violating QCD θ-term: O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Phys. Rev. D **70** (2004) 016003, [arXiv:hep-ph/0402023]; M. Pospelov and A. Ritz, Annals Phys. **318** (2005) 119, [arXiv:hep-ph/0504231]; J. Ellis, JSL, and A. Pilaftsis, arXiv:1006.3087 [hep-ph] The QCD θ-term contributes to the neutron, Deuteron, Mercury and Radium EDMs:

$$\begin{aligned} d_{\rm n}(\bar\theta) &\simeq 2.5\times 10^{-16}\,\bar\theta\; e\cdot{\rm cm}\,,\\ d_D(\bar\theta) &\simeq -9.7\times 10^{-17}\,\bar\theta\; e\cdot{\rm cm}\,,\\ d_{\rm Hg}^{\rm I\,,II\,,III\,,IV}(\bar\theta) &\simeq (C_{\rm Hg}^{\rm I\,,II\,,III\,,IV}\times 10^{-3}\,{\rm GeV}^{-1})\,e\,\bar g^{(1)}_{\pi NN}(\bar\theta)\,,\\ d_{\rm Ra}(\bar\theta) &\simeq (3.5\times 10^{-1}\,{\rm GeV}^{-1})\,e\,\bar g^{(1)}_{\pi NN}(\bar\theta) \end{aligned}$$

where $C_{\rm Hg}^{\rm I}=1.8$, $C_{\rm Hg}^{\rm II}=1.0$, $C_{\rm Hg}^{\rm III}=1.4$ and $C_{\rm Hg}^{\rm IV}=9.5\times 10^{-2}$, and

$$\bar g^{(1)}_{\pi NN}(\bar\theta)\;\simeq\;1.1\times 10^{-3}\,\bar\theta$$

♠ *So, it seems ...*

$$\textit{The task is to find the expressions of:}$$

$$d_f^E,\ d_q^C,\ d^G,\ \textit{and}\ C_{ff'}$$

$$\textit{in terms of the model parameters}$$

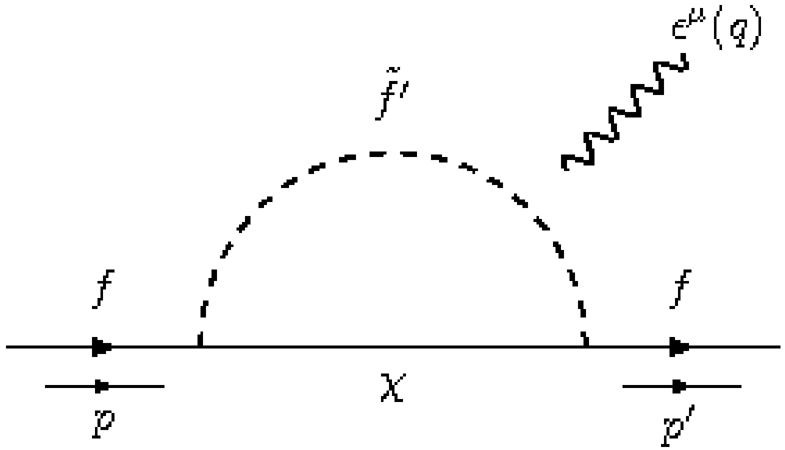
♠ *But ...*

- SUSY parameters $\longrightarrow \bar{\theta},\ d_f^E,\ d_q^C,\ d^G,\ C_{ff'}$ at SUSY scale
- $\bar{\theta},\ d_f^E,\ d_q^C,\ d^G,\ C_{ff'}$ at 1 GeV: QCD, anomalous dimension matrix, operator mixing
- $C_S,\ C_P,\ C'_P,\ g_{\pi NN}^{(0)},\ g_{\pi NN}^{(1)},\ g_{\pi NN}^{(2)}$: Nuclear physics
- $d_{\rm Tl},\ d_{\rm ThO},\ d_n,\ d_{\rm Hg},\ d_D,\ d_{\rm Ra}$: More Nuclear physics, Atomic physics, ...

Very complicated ...

♠ EDMs in the MSSM : One-loop EDMs $(1/5)$

- Generically, the χ-mediated one-loop f EDM is given by See, for example, T. Ibrahim and P. Nath,Rev. Mod. Phys. **80** (2008) 577,[arXiv:0705.2008 [hep-ph]]; S. Abel, S. Khalil and O. Lebedev,Nucl. Phys. B **606** (2001) 151, [arXiv:hep-ph/0103320] $(\chi = \tilde{\chi}^0, \tilde{\chi}^\pm, \tilde{g})$



$$\mathcal{L}_{\chi\chi A} = -e\, Q_\chi\, (\bar{\chi}\gamma_\mu\chi) A^\mu$$

$$\mathcal{L}_{\tilde{f}'\tilde{f}'A} = -ie\, Q_{\tilde{f}'}\, \tilde{f}'^* \overleftrightarrow{\partial}_\mu\, \tilde{f}' A^\mu$$

$$\mathcal{L}_{\chi f \tilde{f}'} = g_{L\,ij}^{\chi f \tilde{f}'}\, (\bar{\chi}_i P_L f)\, \tilde{f}_j'^* + g_{R\,ij}^{\chi f \tilde{f}'}\, (\bar{\chi}_i P_R f)\, \tilde{f}_j'^* + \text{h.c.}$$

$$\left(\frac{d_f^E}{e}\right)^\chi = \frac{m_{\chi_i}}{16\pi^2 m_{\tilde{f}_j'}^2} \Im\text{m}[(g_{R\,ij}^{\chi f \tilde{f}'})^* g_{L\,ij}^{\chi f \tilde{f}'}] \left[Q_\chi\, A(m_{\chi_i}^2/m_{\tilde{f}_j'}^2) + Q_{\tilde{f}'}\, B(m_{\chi_i}^2/m_{\tilde{f}_j'}^2) \right]$$

$$A(r) = \frac{1}{2(1-r)^2}\left(3 - r + \frac{2\ln r}{1-r}\right), \quad B(r) = \frac{1}{2(1-r)^2}\left(1 + r + \frac{2r\ln r}{1-r}\right)$$

♠ EDMs in the MSSM : One-loop EDMs (2/5)

- Explicitly, the chargino-mediated one-loop EDMs of charged leptons and quarks:

$$\left(\frac{d_l^E}{e}\right)^{\tilde{\chi}^\pm} = \frac{1}{16\pi^2}\sum_i \frac{m_{\tilde{\chi}_i^\pm}}{m_{\tilde{\nu}_l}^2}\,\Im\mathrm{m}[(g_{R\,i}^{\tilde{\chi}^\pm l\tilde{\nu}})^*\, g_{L\,i}^{\tilde{\chi}^\pm l\tilde{\nu}}]\; Q_{\tilde{\chi}^-}\, A(m_{\tilde{\chi}_i^\pm}^2/m_{\tilde{\nu}_l}^2)$$

$$\left(\frac{d_u^E}{e}\right)^{\tilde{\chi}^\pm} = \frac{1}{16\pi^2}\sum_{i,j} \frac{m_{\tilde{\chi}_i^\pm}}{m_{\tilde{d}_j}^2}\,\Im\mathrm{m}[(g_{R\,ij}^{\tilde{\chi}^\pm u\tilde{d}})^*\, g_{L\,ij}^{\tilde{\chi}^\pm u\tilde{d}}]\;[Q_{\tilde{\chi}^+}\, A(m_{\tilde{\chi}_i^\pm}^2/m_{\tilde{d}_j}^2) + Q_{\tilde{d}} B(m_{\tilde{\chi}_i^\pm}^2/m_{\tilde{d}_j}^2)]$$

$$\left(\frac{d_d^E}{e}\right)^{\tilde{\chi}^\pm} = \frac{1}{16\pi^2}\sum_{i,j} \frac{m_{\tilde{\chi}_i^\pm}}{m_{\tilde{u}_j}^2}\,\Im\mathrm{m}[(g_{R\,ij}^{\tilde{\chi}^\pm d\tilde{u}})^*\, g_{L\,ij}^{\tilde{\chi}^\pm d\tilde{u}}]\;[Q_{\tilde{\chi}^-}\, A(m_{\tilde{\chi}_i^\pm}^2/m_{\tilde{u}_j}^2) + Q_{\tilde{u}} B(m_{\tilde{\chi}_i^\pm}^2/m_{\tilde{u}_j}^2)]$$

where $Q_{\tilde{\chi}^\pm} = \pm 1$, $Q_{\tilde{u}} = 2/3$, $Q_{\tilde{d}} = -1/3$, and

$$g_{L\,i}^{\tilde{\chi}^\pm l\tilde{\nu}} = -g(C_R)_{i1}\,, \qquad g_{R\,i}^{\tilde{\chi}^\pm l\tilde{\nu}} = h_l^*(C_L)_{i2}\,,$$

$$g_{L\,ij}^{\tilde{\chi}^\pm u\tilde{d}} = -g(C_L)_{i1}^*(U^{\tilde{d}})_{1j}^* + h_d(C_L)_{i2}^*(U^{\tilde{d}})_{2j}^*\,, \qquad g_{R\,ij}^{\tilde{\chi}^\pm u\tilde{d}} = h_u^*(C_R)_{i2}^*(U^{\tilde{d}})_{1j}^*\,,$$

$$g_{L\,ij}^{\tilde{\chi}^\pm d\tilde{u}} = -g(C_R)_{i1}(U^{\tilde{u}})_{1j}^* + h_u(C_R)_{i2}(U^{\tilde{u}})_{2j}^*\,, \qquad g_{R\,ij}^{\tilde{\chi}^\pm d\tilde{u}} = h_d^*(C_L)_{i2}(U^{\tilde{u}})_{1j}^*$$

♠ EDMs in the MSSM : One-loop EDMs (3/5)

- The neutralino-mediated one-loop EDMs of charged leptons and quarks:

$$\left(\frac{d_f^E}{e}\right)^{\tilde\chi^0} = \frac{1}{16\pi^2}\sum_{i,j}\frac{m_{\tilde\chi_i^0}}{m_{\tilde f_j}^2}\Im\mathrm{m}[(g_{R\,ij}^{\tilde\chi^0 f\tilde f})^* g_{L\,ij}^{\tilde\chi^0 f\tilde f}]\,Q_{\tilde f}\,B(m_{\tilde\chi_i^0}^2/m_{\tilde f_j}^2)$$

with $f=l,u,d$. The neutralino-fermion-sfermion couplings are

$$g_{L\,ij}^{\tilde\chi^0 f\tilde f} = -\sqrt{2}\,g\,T_3^f\,N_{i2}^*(U^{\tilde f})_{1j}^* - \sqrt{2}\,g\,t_W\,(Q_f - T_3^f)N_{i1}^*(U^{\tilde f})_{1j}^* - h_f N_{i\alpha}^*(U^{\tilde f})_{2j}^*\,,$$

$$g_{R\,ij}^{\tilde\chi^0 f\tilde f} = \sqrt{2}\,g\,t_W\,Q_f\,N_{i1}(U^{\tilde f})_{2j}^* - h_f^* N_{i\alpha}(U^{\tilde f})_{1j}^*$$

where the Higgsino index $\alpha = 3\,(f=l,d)$ or $4\,(f=u)$

- The gluino-mediated one-loop EDMs of quarks:

$$\left(\frac{d_q^E}{e}\right)^{\tilde g} = \frac{1}{3\pi^2}\sum_j\frac{|M_3|}{m_{\tilde q_j}^2}\Im\mathrm{m}[(g_{R\,j}^{\tilde g q\tilde q})^* g_{L\,j}^{\tilde g q\tilde q}]\,Q_{\tilde q}B(|M_3|^2/m_{\tilde q_j}^2)$$

$$g_{L\,j}^{\tilde g q\tilde q} = -\frac{g_s}{\sqrt{2}}\,\mathrm{e}^{-i\Phi_3/2}(U^{\tilde q})_{1j}^*\,,\qquad g_{R\,j}^{\tilde g q\tilde q} = +\frac{g_s}{\sqrt{2}}\,\mathrm{e}^{+i\Phi_3/2}(U^{\tilde q})_{2j}^*$$

- The chargino-, neutralino-, and gluino-mediated one-loop CEDMs of quarks:

$$\left(d_u^C\right)^{\tilde{\chi}^\pm} = \frac{g_s}{16\pi^2}\sum_{i,j}\frac{m_{\tilde{\chi}_i^\pm}}{m_{\tilde{d}_j}^2}\Im\mathrm{m}[(g_{R\,ij}^{\tilde{\chi}^\pm u\tilde{d}})^*\,g_{L\,ij}^{\tilde{\chi}^\pm u\tilde{d}}]\,B(m_{\tilde{\chi}_i^\pm}^2/m_{\tilde{d}_j}^2)\,,$$

$$\left(d_d^C\right)^{\tilde{\chi}^\pm} = \frac{g_s}{16\pi^2}\sum_{i,j}\frac{m_{\tilde{\chi}_i^\pm}}{m_{\tilde{u}_j}^2}\Im\mathrm{m}[(g_{R\,ij}^{\tilde{\chi}^\pm d\tilde{u}})^*\,g_{L\,ij}^{\tilde{\chi}^\pm d\tilde{u}}]\,B(m_{\tilde{\chi}_i^\pm}^2/m_{\tilde{u}_j}^2)\,,$$

$$\left(d_{q=u,d}^C\right)^{\tilde{\chi}^0} = \frac{g_s}{16\pi^2}\sum_{i,j}\frac{m_{\tilde{\chi}_i^0}}{m_{\tilde{q}_j}^2}\Im\mathrm{m}[(g_{R\,ij}^{\tilde{\chi}^0 q\tilde{q}})^*\,g_{L\,ij}^{\tilde{\chi}^0 q\tilde{q}}]\,B(m_{\tilde{\chi}_i^0}^2/m_{\tilde{q}_j}^2)\,,$$

$$(d_{q=u,d}^C)^{\tilde{g}} = -\frac{g_s}{8\pi^2}\sum_{j}\frac{|M_3|}{m_{\tilde{q}_j}^2}\Im\mathrm{m}[(g_{R\,j}^{\tilde{g}q\tilde{q}})^*g_{L\,j}^{\tilde{g}q\tilde{q}}]\,C(|M_3|^2/m_{\tilde{q}_j}^2)$$

where $C(r) \equiv \frac{1}{6(1-r)^2}\left(10r - 26 + \frac{2r\ln r}{1-r} - \frac{18\ln r}{1-r}\right)$, with $C(1) = 19/18$

♠ EDMs in the MSSM : One-loop EDMs (5/5)

- Complex Yukawa couplings; effects of Φ_3 via resummed non-holomorphic threshold corrections:

$$
\begin{aligned}
h_u &= \frac{\sqrt{2}m_u}{v s_\beta}\frac{1}{1+\Delta_u/t_\beta}, \quad & h_c &= \frac{\sqrt{2}m_c}{v s_\beta}\frac{1}{1+\Delta_c/t_\beta}, \\
h_d &= \frac{\sqrt{2}m_d}{v c_\beta}\frac{1}{1+\Delta_d t_\beta}, \quad & h_s &= \frac{\sqrt{2}m_s}{v c_\beta}\frac{1}{1+\Delta_s t_\beta}
\end{aligned}
$$

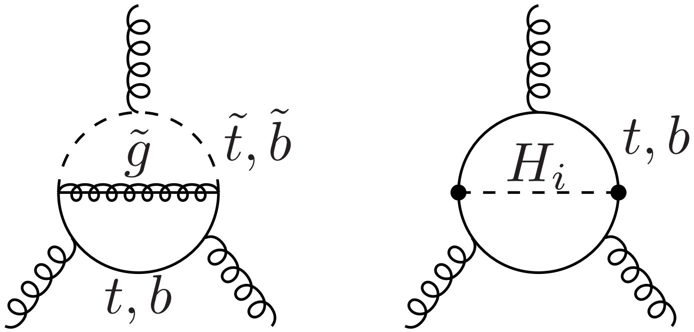
where

$$
\begin{aligned}
\Delta_u &= \frac{2\alpha_s}{3\pi}\,\mu^* M_3^*\, I(M^2_{\tilde{U}_1}, M^2_{\tilde{Q}_1}, |M_3|^2)\,, \quad & \Delta_c &= \frac{2\alpha_s}{3\pi}\,\mu^* M_3^*\, I(M^2_{\tilde{U}_2}, M^2_{\tilde{Q}_2}, |M_3|^2)\,, \\
\Delta_d &= \frac{2\alpha_s}{3\pi}\,\mu^* M_3^*\, I(M^2_{\tilde{D}_1}, M^2_{\tilde{Q}_1}, |M_3|^2)\,, \quad & \Delta_s &= \frac{2\alpha_s}{3\pi}\,\mu^* M_3^*\, I(M^2_{\tilde{D}_2}, M^2_{\tilde{Q}_2}, |M_3|^2)
\end{aligned}
$$

where $I(x,y,z) \equiv \frac{xy\,\ln(x/y) + yz\,\ln(y/z) + xz\,\ln(z/x)}{(x-y)\,(y-z)\,(x-z)}$

♠ *EDMs in the MSSM : Higher-order Contributions* (1/4)

- Weinberg operator;S. Weinberg,Phys. Rev. Lett. **63** (1989) 2333; J. Dai, H. Dykstra, R. G. Leigh, S. Paban and D. Dicus,Phys. Lett. B **237** (1990) 216 [Erratum-ibid. B **242** (1990) 547]; D. A. Dicus,Phys. Rev. D **41** (1990) 999



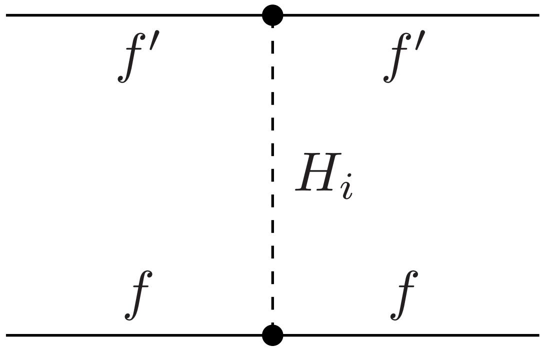
$$\mathcal{L}_{\rm Weinberg} = -\frac{1}{6} d^G f_{abc} G^a_{\mu\rho} G^{b\,\rho}_{\ \ \nu} G^c_{\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma} = \frac{1}{3} d^G f_{abc} G^a_{\rho\mu} \tilde{G}^{b\,\mu\nu} G^{c}_{\ \ \nu}{}^{\rho}$$

where

$$d^G = (d^G)^{\tilde{g}} + (d^G)^H$$

♠ *EDMs in the MSSM : Higher-order Contributions* $(2/4)$

- Higgs-mediated Four-fermion interactions; which are one-loop contributions formally but independent of the first two generations.



$$\mathcal{L}_{\rm 4f} \;=\; \sum_{f,f'} C_{ff'}(\bar{f}f)(\bar{f}'i\gamma_5 f')$$

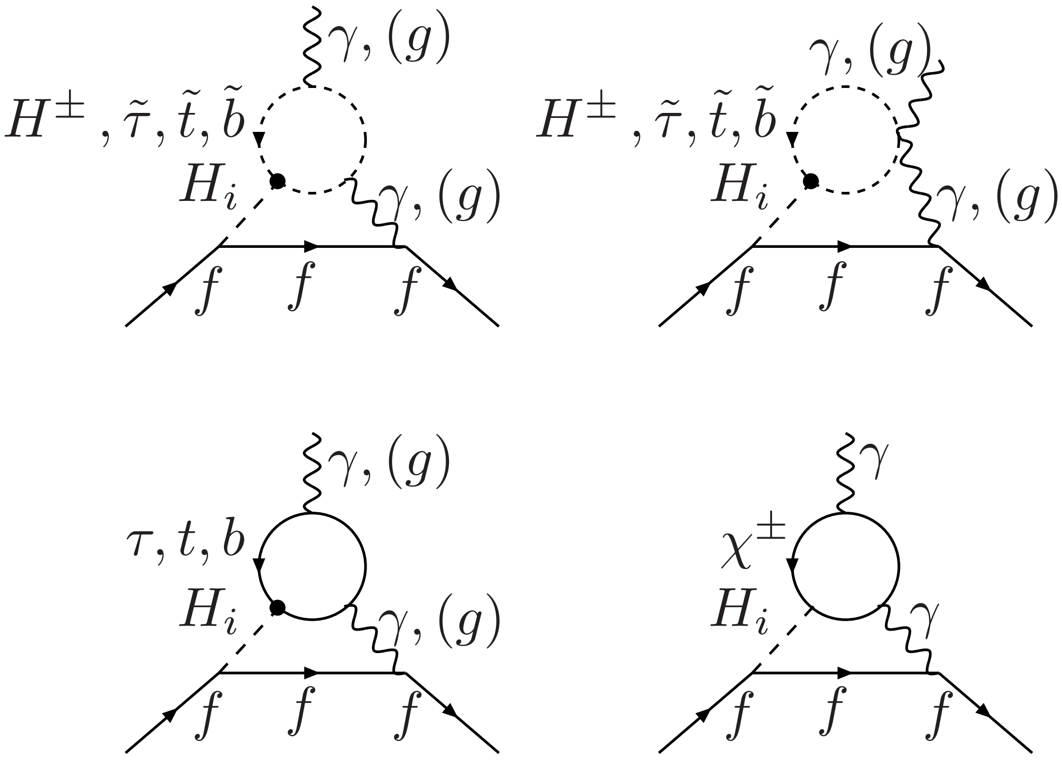
where

$$(C_{ff'})^H \;=\; g_f\,g_{f'}\sum_i \frac{g^S_{H_i\bar{f}f}\,g^P_{H_i\bar{f}'f'}}{M^2_{H_i}}$$

$$\mathcal{L}_{Hff} = -g_f\,H_i\,\bar{f}\,(g^S_{H_i\bar{f}f} + ig^P_{H_i\bar{f}f}\gamma_5)\,f$$

♠ *EDMs in the MSSM : Higher-order Contributions* $(3/4)$

- Two-loop Barr-Zee EDMs; D. Chang, W. Y. Keung and A. Pilaftsis,Phys. Rev. Lett. **82** (1999) 900 [Erratum-ibid. **83** (1999) 3972], [arXiv:hep-ph/9811202]; A. Pilaftsis, Nucl. Phys. B **644** (2002) 263, [arXiv:hep-ph/0207277]; J. R. Ellis, JSL and A. Pilaftsis, Phys. Rev. D **72** (2005) 095006 [arXiv:hep-ph/0507046]



♠ *EDMs in the MSSM : Higher-order Contributions* (3/4)

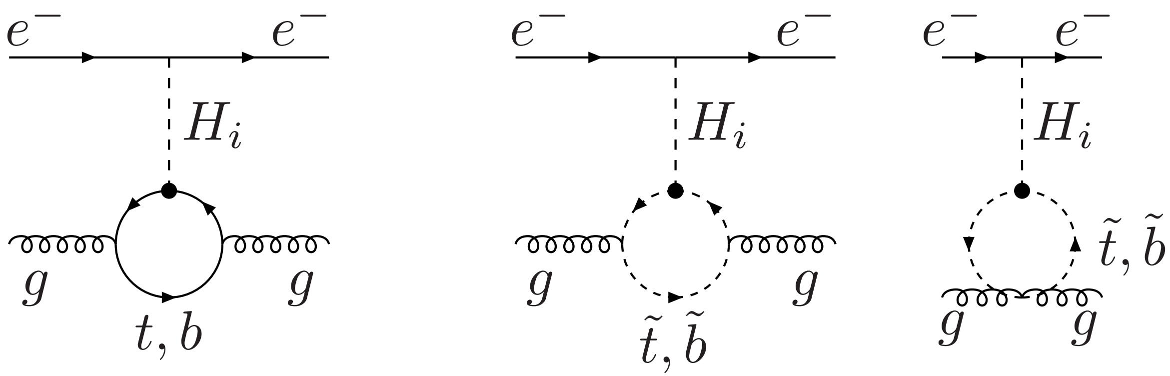
- Two-loop Barr-Zee EDMs; *continued ...*

Also included are the diagrams mediated by the the γ-$H^{\pm}$-$W^{\mp}$, γ-$W^{\pm}$-$W^{\mp}$, and γ-H^0-Z couplings: Kingman Cheung, Otto C.W. Kong, JSL, JHEP 0906 (2009) 020 [arXiv:0904.4352 [hep-ph]]; J. R. Ellis, JSL and A. Pilaftsis, JHEP 1010 (2010) 049 [arXiv:1006.3087 [hep-ph]]; Kingman Cheung, Tie-Jiun Hou, JSL, Eibun Senaha, arXiv:1102.5679 [hep-ph] [PRD84(2011)015002]

$$\left(d_f^E\right)^{\rm BZ} = \left(d_f^E\right)^{\gamma H^0} + \left(d_f^E\right)^{W^{\mp} H^{\pm}} + \left(d_f^E\right)^{W^{\mp} W^{\pm}} + \left(d_f^E\right)^{Z H^0}$$

♠ EDMs in the MSSM : Higher-order Contributions (4/4)

- The gluon-gluon-Higgs contribution to C_S, $\mathcal{L}_{C_S} = C_S\, \bar{e} i\gamma_5 e\, \bar{N}N$;

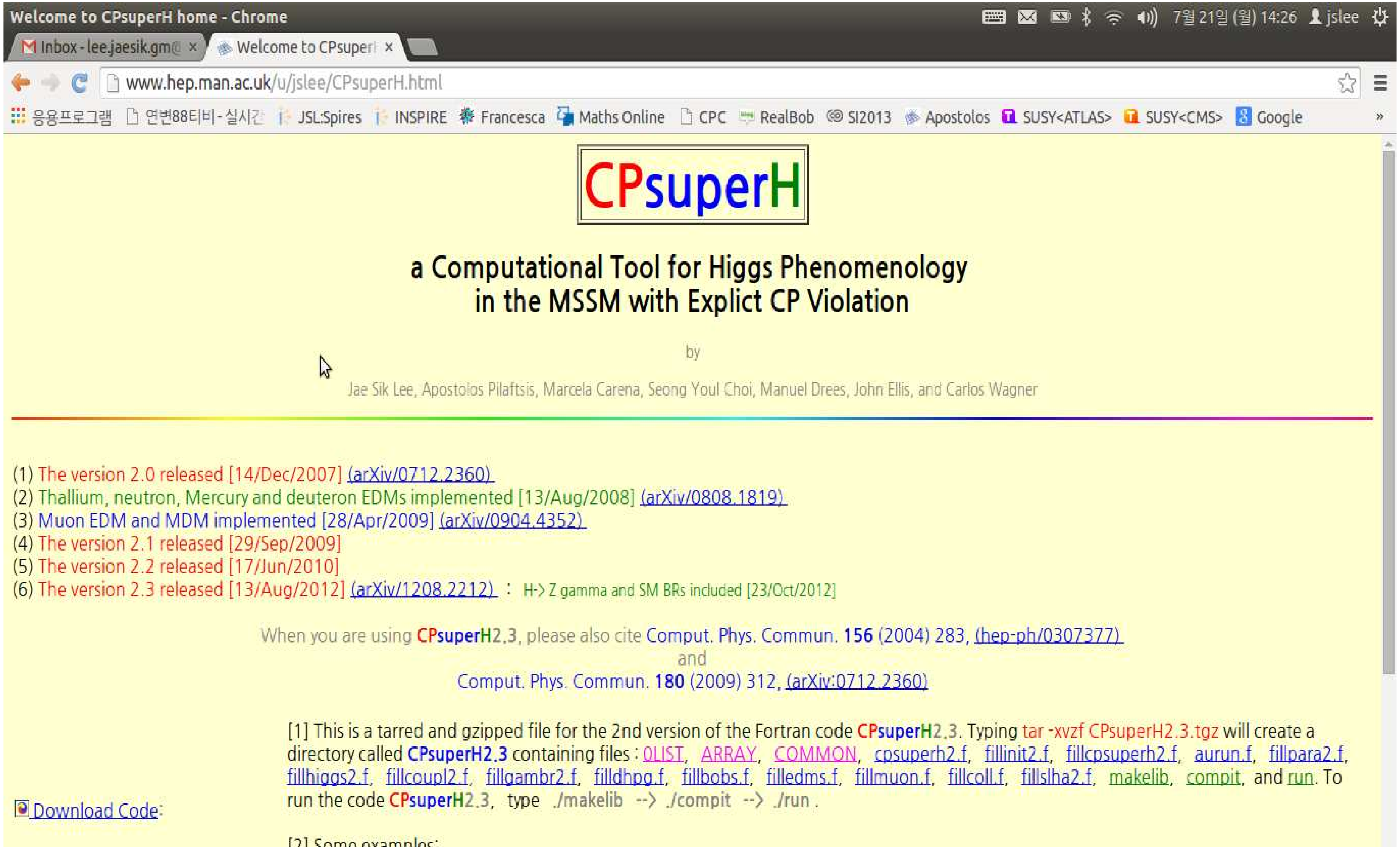


$$(C_S)^g = (0.1\,\mathrm{GeV})\,\frac{m_e}{v^2}\sum_{i=1}^{3}\frac{g^S_{H_i gg}\, g^P_{H_i \bar{e}e}}{M^2_{H_i}}$$

$$g^S_{H_i gg} = \sum_{q=t,b}\left\{\frac{2\,x_q}{3}\, g^S_{H_i \bar{q}q} - \frac{v^2}{12}\sum_{j=1,2}\frac{g_{H_i \tilde{q}^*_j \tilde{q}_j}}{m^2_{\tilde{q}_j}}\right\}$$

♠ *Ready?*

- Ready for the numerical estimations?: The Thallium, neutron, Mercury and deuteron EDMs are implemented in `CPsuperH2.3` ; Thorium monoxide not includeded, yet



QUESTION

Non-observation of EDMs necessarily implies small CP phases ?

♠ *EDM Constraints on CP phases (Illustration of Cancellation)*

- CPX scenario:

$$\text{Fixed}:$$

$$M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3} = M_{\tilde{L}_3} = M_{\tilde{E}_3} = M_{\rm SUSY}\,,$$
$$|\mu| = 4\,M_{\rm SUSY}\,,\quad |A_{t,b,\tau}| = 2\,M_{\rm SUSY}\,,\quad |M_3| = 1\ \ \text{TeV}$$

$$|M_2| = 2|M_1| = 100\ \ \text{GeV}\,, M_{H^\pm} = 300\ \ \text{GeV}\,, M_{\rm SUSY} = 0.5\ \ \text{TeV}$$
$$|A_e| = |A_\tau|, |A_{u,c}| = |A_t|, |A_{d,s}| = |A_b|$$
$$\Phi_\mu = \Phi_{A_\tau} = \Phi_{A_e} = \Phi_{A_u} = \Phi_{A_c} = \Phi_{A_d} = \Phi_{A_s} = 0^\circ$$

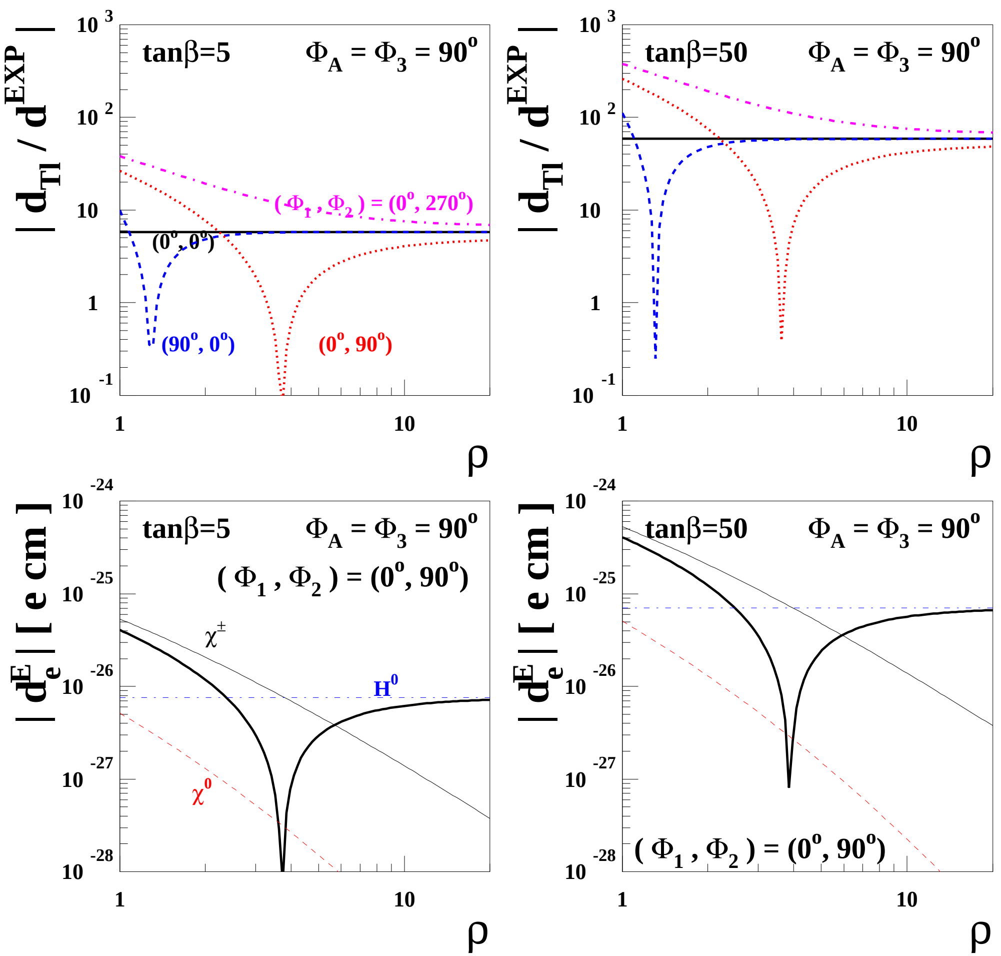
$$\text{Varying}:$$

$$\tan\beta\,;\quad \Phi_{A_{t,b}}\,,\ \Phi_3\,;\quad \Phi_1\,,\ \Phi_2\,,\ \rho$$

where the ρ parameter is defined as: $M_{\tilde{X}_{1,2}} = \rho\, M_{\tilde{X}_3}$ with $X = Q, U, D, L, E$

♠ EDM Constraints on CP phases (Illustration of Cancellation)

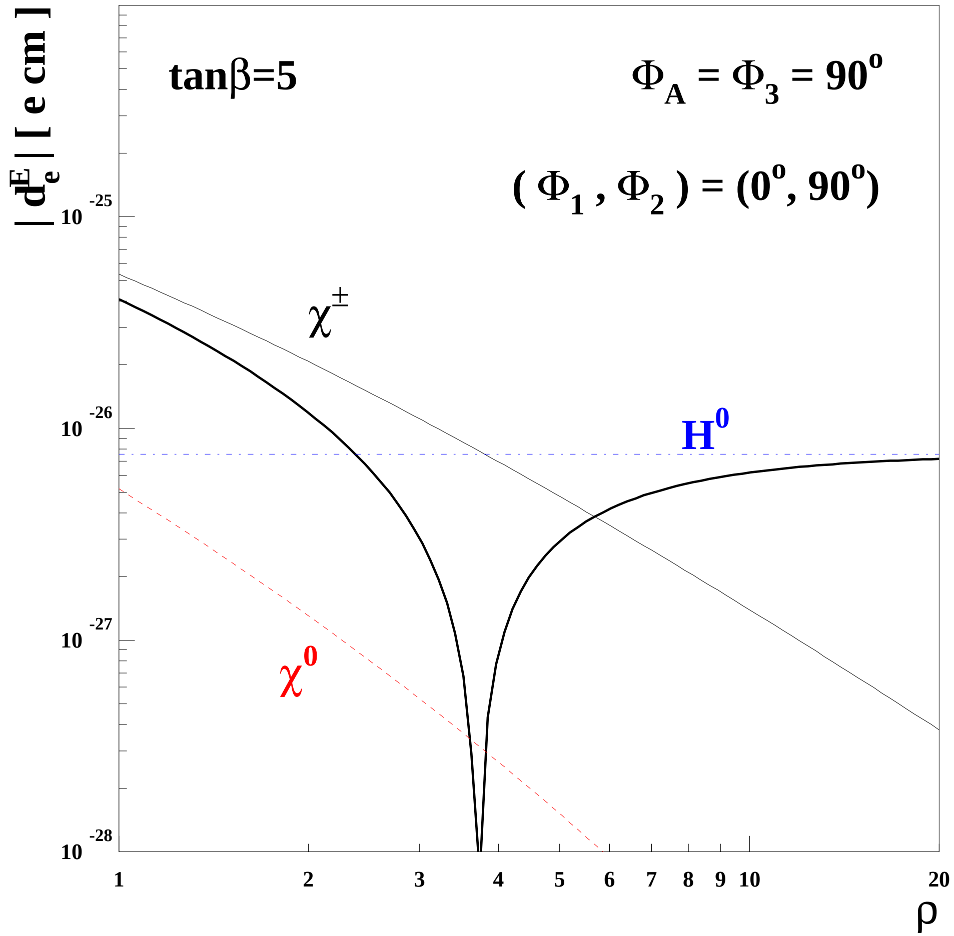
- Thallium EDM: CPX with $\Phi_A = \Phi_3 = 90^\circ$ as functions of ρ;



  - When $(\Phi_1, \Phi_2) = (0^\circ, 0^\circ)$, the one-loop contribtions to $d_{\rm Tl}$ vanish and it becomes independent of ρ
  - As $\rho \uparrow$; 'decrease' $\rightarrow$ 'dip' $\rightarrow$ 'flat'
  - 'decrease': suppressed one-loop contribution
  - 'dip': cancellation between one- and two-loop contributions
  - 'flat': two-loop (higher-order) contribution dominates

♠ *EDM Constraints on CP phases (Illustration of Cancellation)*

- Thallium EDM: CPX with $\Phi_A = \Phi_3 = 90^\circ$ as functions of ρ;



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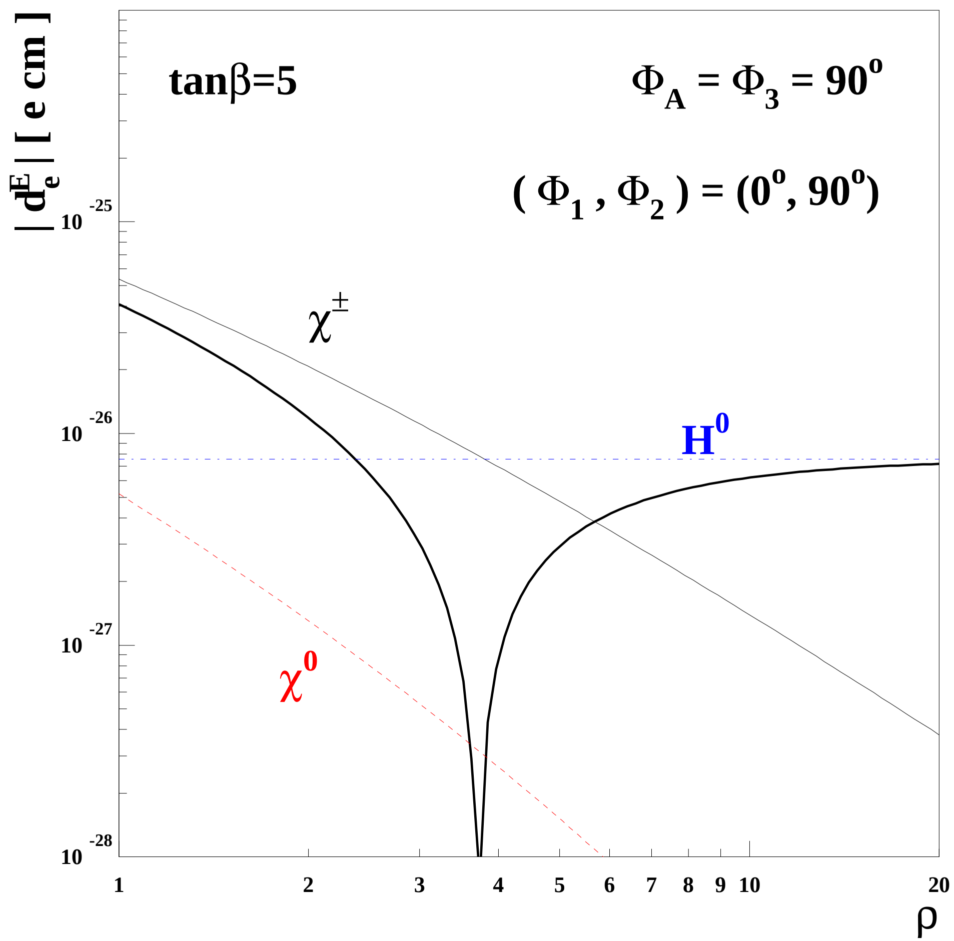
  - As $\rho \uparrow$; 'decrease' $\rightarrow$ 'dip' $\rightarrow$ 'flat'
  - 'decrease': suppressed one-loop contribution
  - 'dip': cancellation between one- and two-loop contributions
  - 'flat': two-loop (higher-order) contribution dominates

♠ *EDM Constraints on CP phases (Illustration of Cancellation)*

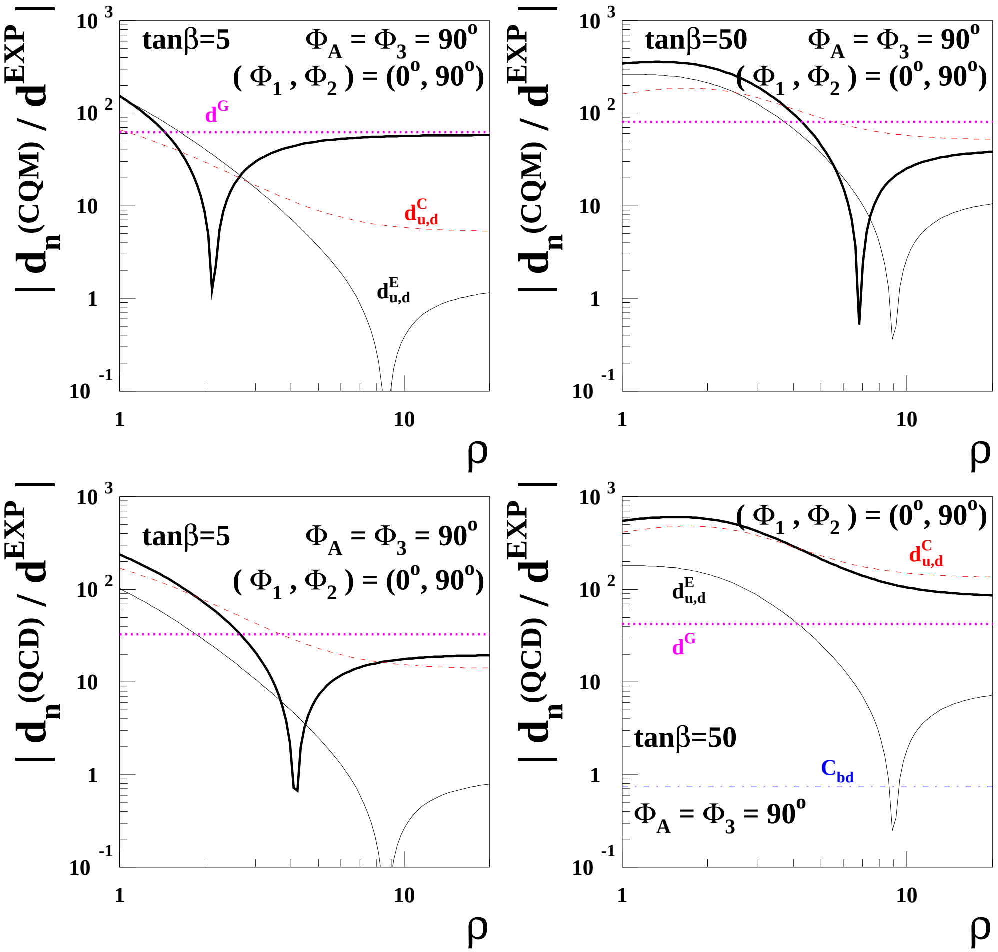
- Thorium monoxide EDM: $\left|d_e^E\right| \lesssim 8.7 \times 10^{-29}\ \mathrm{e\,cm}$ functions of ρ;



- Sensitive to $\rho > 20$ or $m_{\tilde{\nu}_e} \gtrsim 10$ TeV
- EDM can probe SUSY effects (far) beyond 10 TeV !!!

♠ *EDM Constraints on CP phases (Illustration of Cancellation)*

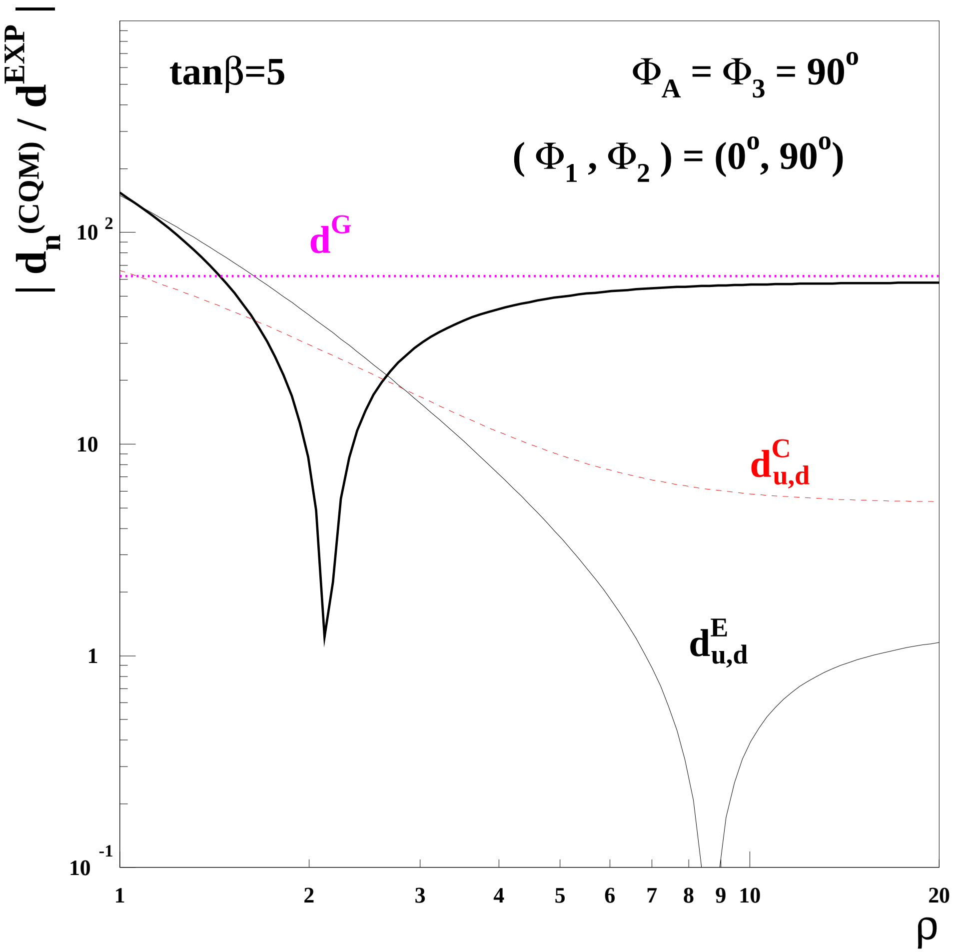
- Neutron EDM (CQM and QCD): CPX with $\Phi_A = \Phi_3 = 90^\circ$ as functions of ρ;



- cancellation between $d_n(d_d^E)$[CQM] / $d_n(d_d^C)$[QCD] and $d_n(d^G)$
- 'flat': d^G and $(d_d^{E,C})^{\rm BZ}$
- $\tan\beta = 5$: CQM dip around $\rho = 2$ and QCD dip around $\rho = 4$
- $\pm 50\%$ $d_n(d^G)$ uncertainty: no cancellation and/or dip position $\delta\rho \sim \pm 1$
- more significant $d_n(d_d^E)$ in CQM $\rightarrow$ more sensitive to Φ_2

♠ *EDM Constraints on CP phases (Illustration of Cancellation)*

- Neutron EDM (CQM and QCD): CPX with $\Phi_A = \Phi_3 = 90^\circ$ as functions of ρ;



  - cancellation between $d_n(d_d^E)$[CQM] / $d_n(d_d^C)$[QCD] and $d_n(d^G)$
  - 'flat': d^G and $(d_d^{E,C})^{\rm BZ}$
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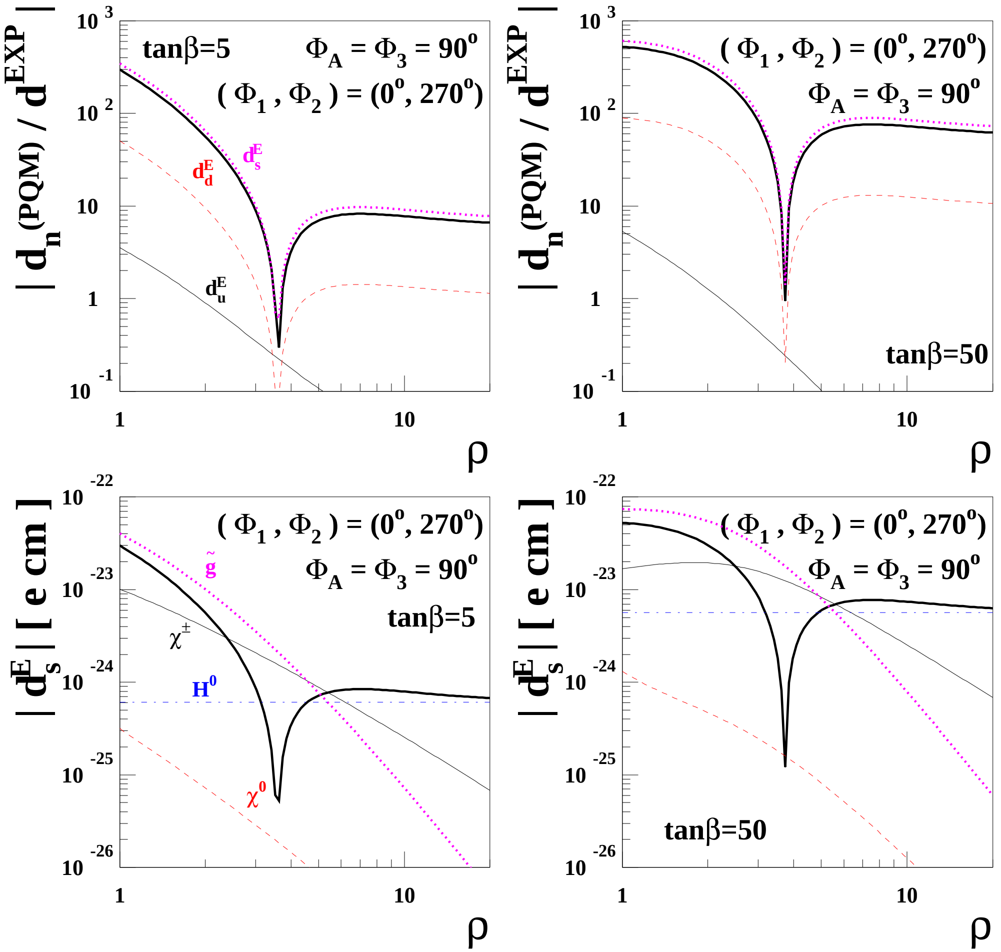
  - more significant $d_n(d_d^E)$ in CQM $\rightarrow$ more sensitive to Φ_2

♠ EDM Constraints on CP phases (Illustration of Cancellation)

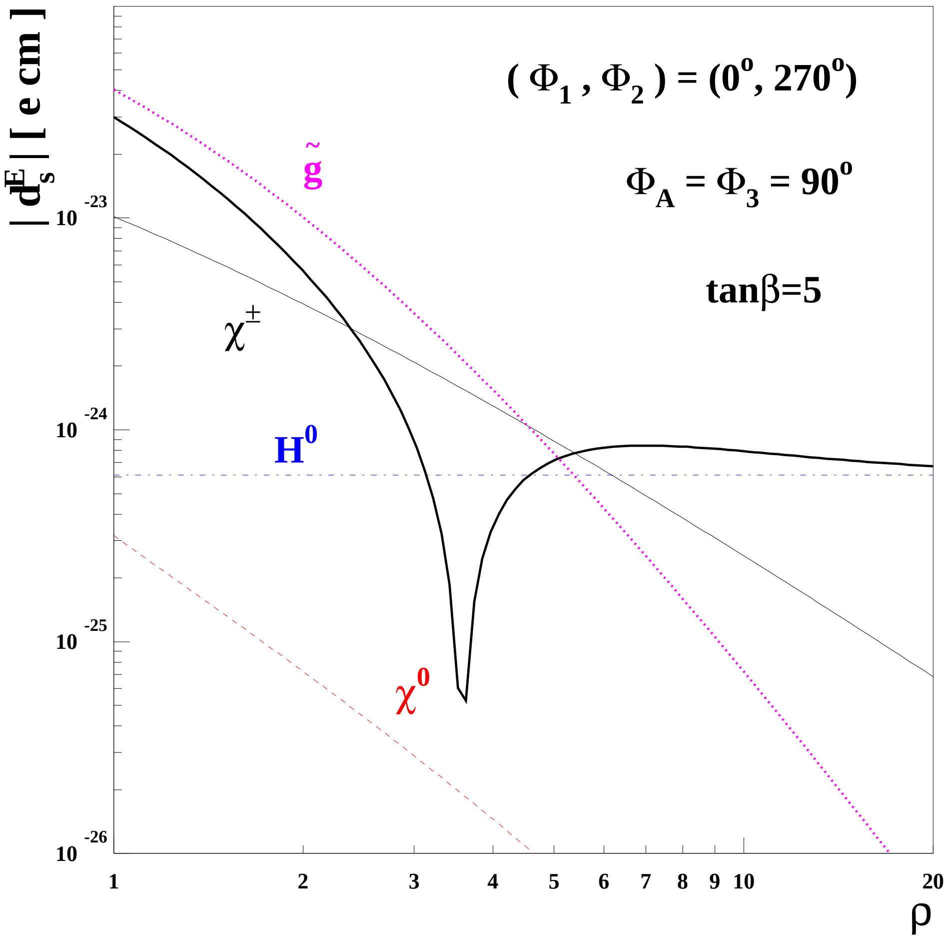
- Neutron EDM (PQM): CPX with $\Phi_A = \Phi_3 = 90^\circ$ as functions of ρ;



- $d_n \sim d_n(d_s^E)$
- $d_s^E \sim (d_s^E)^{\tilde{\chi}^\pm} + (d_s^E)^{\tilde{g}}$
- cancellation between the two dominant one-loop EDMs
- large $(d_s^E)^{\tilde{\chi}^\pm} \rightarrow$ sensitive to Φ_2

♠ *EDM Constraints on CP phases (Illustration of Cancellation)*

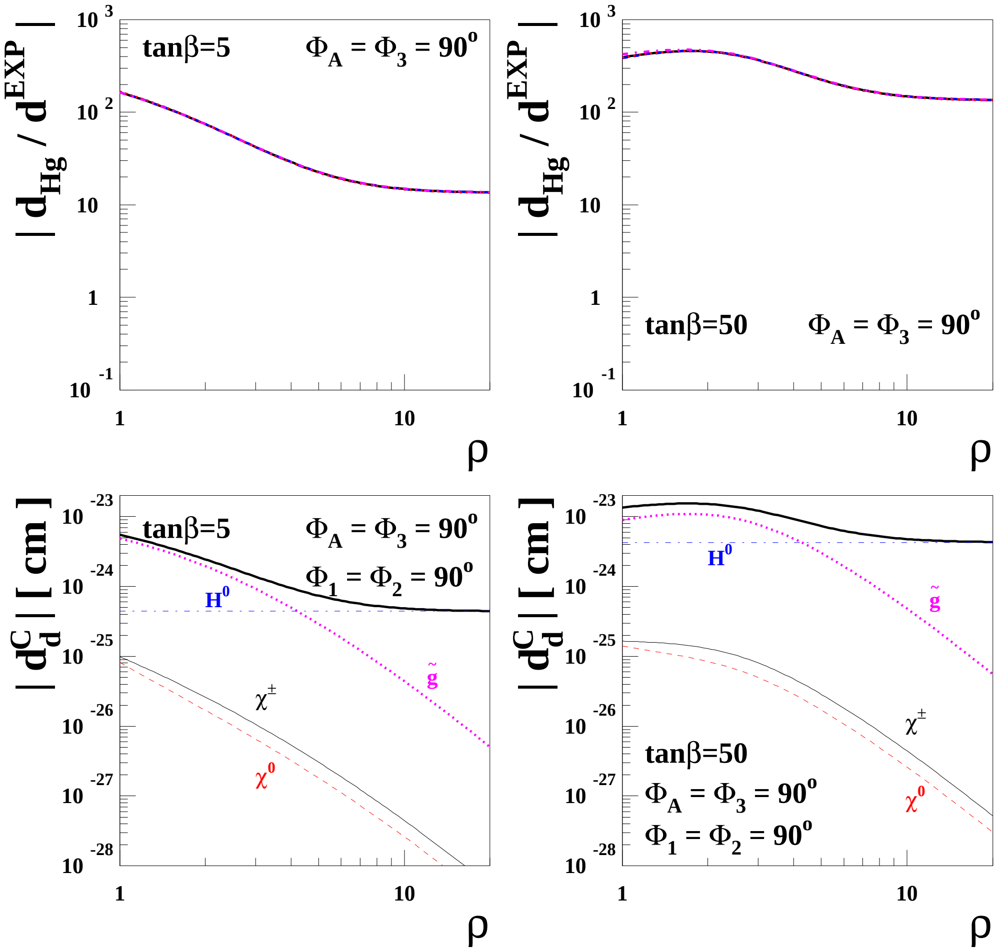
- Neutron EDM (PQM): CPX with $\Phi_A = \Phi_3 = 90^\circ$ as functions of ρ;



- $d_n \sim d_n(d_s^E)$
- $d_s^E \sim (d_s^E)^{\tilde{\chi}^\pm} + (d_s^E)^{\tilde{g}}$
- cancellation between the two dominant one-loop EDMs
- large $(d_s^E)^{\tilde{\chi}^\pm} \rightarrow$ sensitive to Φ_2

♠ EDM Constraints on CP phases (Illustration of Cancellation)

- Mercury EDM: CPX with $\Phi_A = \Phi_3 = 90^\circ$ as functions of ρ;

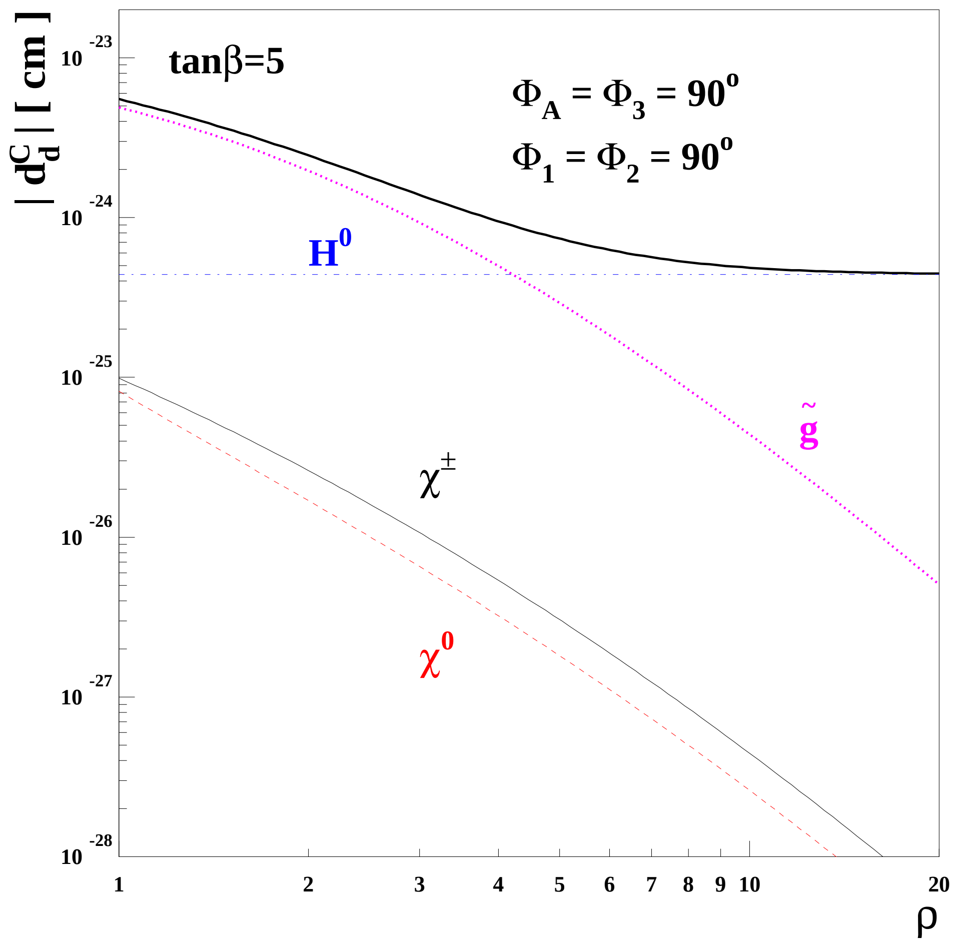


- dominance of $d_{\rm Hg}(d_d^C)$ $\rightarrow$ no sensitive to $\Phi_{1,2}$
- no cancellation between $(d_d^C)^{\tilde{g}}$ and $(d_d^C)^{\rm BZ}$
- 'flat': $(d_d^C)^{\rm BZ}$

$$*(d_{\rm Hg}) = d_{\rm Hg}^{\rm I}$$

♠ EDM Constraints on CP phases (Illustration of Cancellation)

- Mercury EDM: CPX with $\Phi_A = \Phi_3 = 90^\circ$ as functions of ρ;



– dominance of $d_{\rm Hg}(d_d^C)$ $\to$ no sensitive to $\Phi_{1,2}$

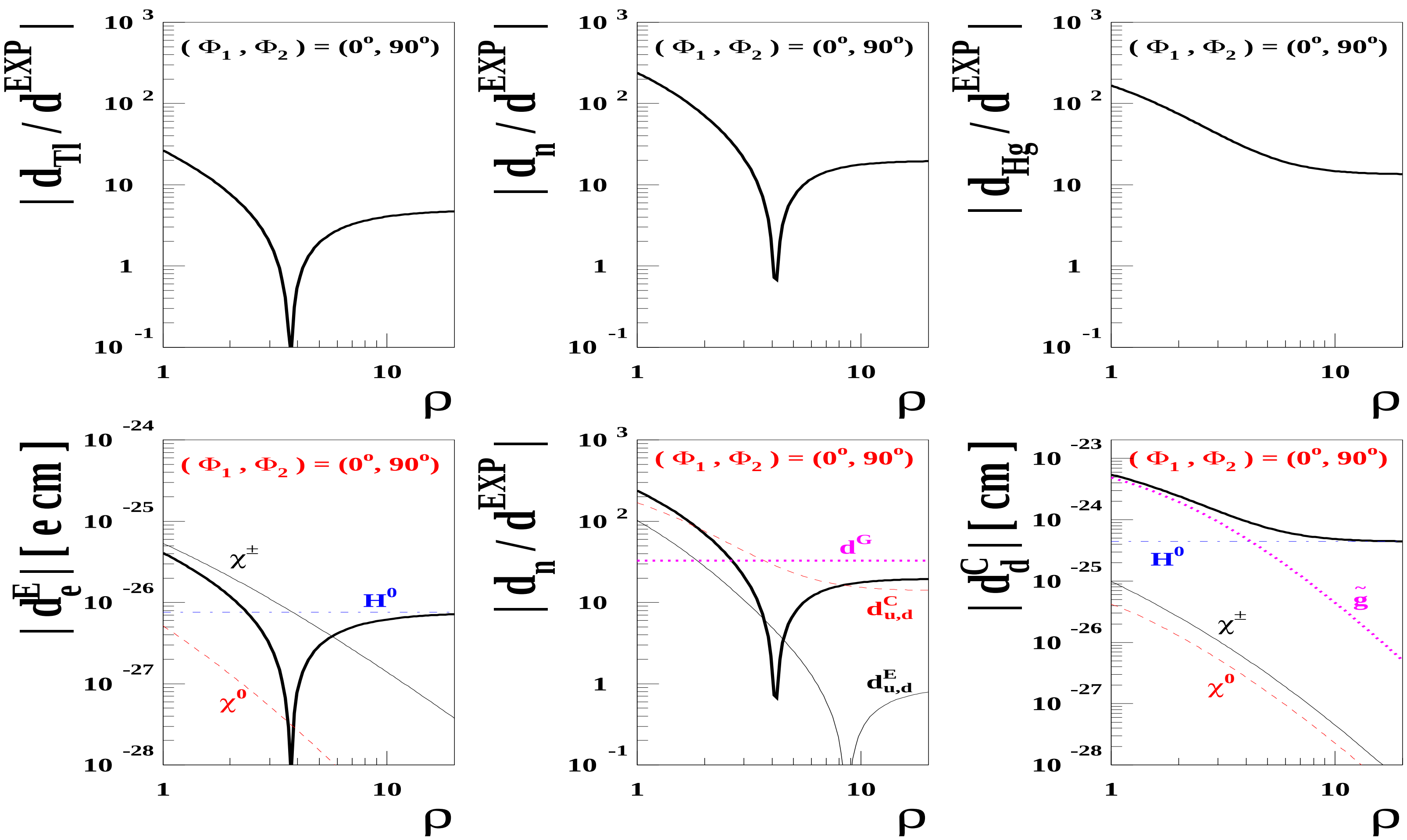
– no cancellation between $(d_d^C)^{\tilde{g}}$ and $(d_d^C)^{\rm BZ}$, *but with* $\Phi_3 = -90^\circ$ ...

– 'flat': $(d_d^C)^{\rm BZ}$

$*(d_{\rm Hg}) = d_{\rm Hg}^{\rm I}$

♠ *EDM Constraints on CP phases (Illustration of ancellation)*

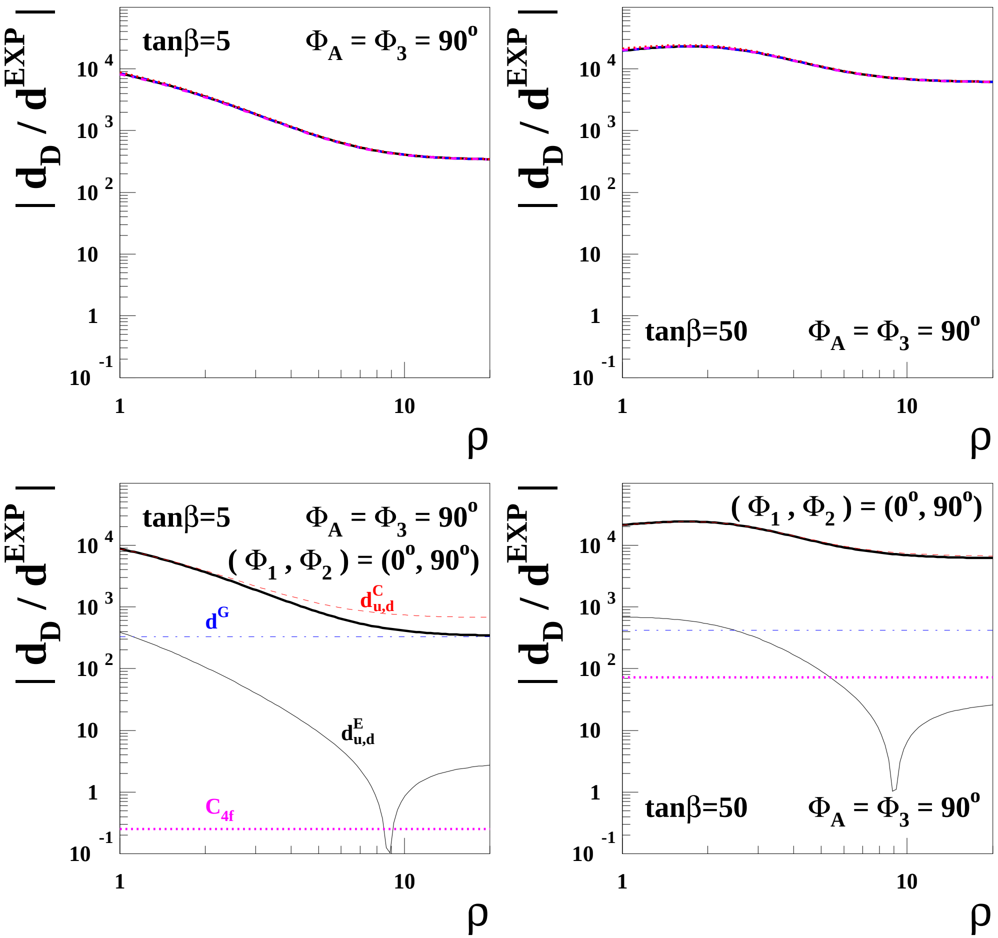
- Thallium, neutron(QCD), and Mercury EDMs: CPX with $\Phi_1 = 0°, \Phi_2 = 90°, \Phi_A = \Phi_3 = 90°$ as functions of ρ when $\tan\beta = 5$;



  - $\rho \sim 4$ and $\Phi_3 = -90°$?

♠ *EDM Constraints on CP phases (Illustration of Cancellation)*

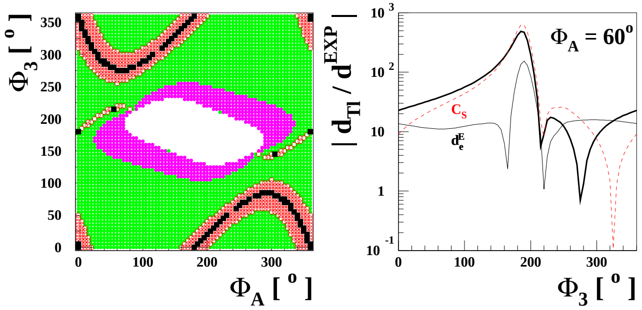
- Deuteron EDM: CPX with $\Phi_A = \Phi_3 = 90^\circ$ as functions of ρ;



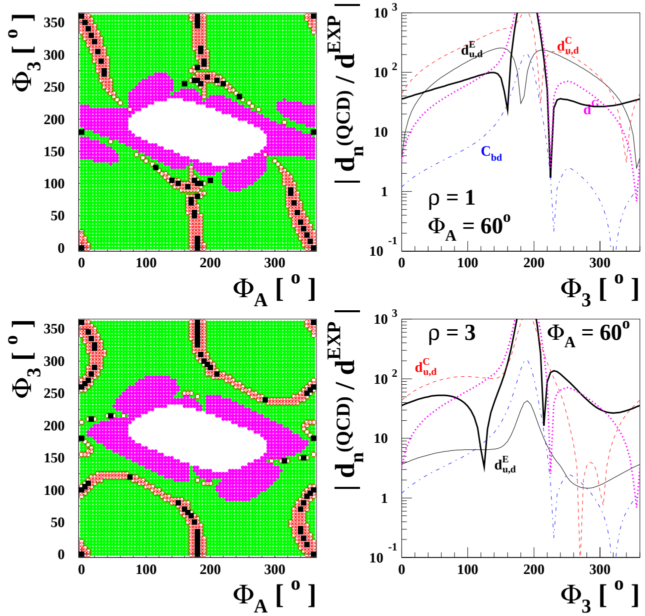
  - no sensitive to $\Phi_{1,2}$
  - Very sensitive even to the higher-order corrections
  - 300 times more sensitive if the projective $10^{-29} e\,cm$ achieved

♠ *EDM Constraints on CP phases (Geometric approach)*

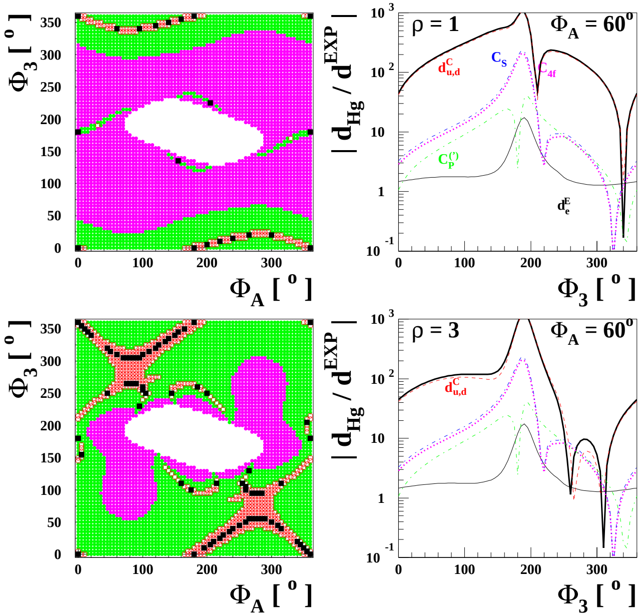
- A scan method: J. R. Ellis, JSL and A. Pilaftsis, JHEP **0810** (2008) 049 [arXiv:0808.1819 [hep-ph]]



d_{Tl}



$d_{\mathrm{n}}^{\mathrm{QCD}}$



d_{Hg}

♠ *EDM Constraints on CP phases (Geometric approach)*

- A scan method is like *"shooting in the dark"* ...

blind,

time consuming,

no guiding principle, etc

Any analytic, exact, and more effective method?

$\Rightarrow$ **A Geometric Approach** to CP violation

♠ EDM Constraints on CP phases (Geometric approach)

- A LINEAR APPROXIMATION: We consider the case with N CP-violating phases

In the N-dimensional CP-phase space, we define

$$N\text{−D phase vector}\quad \mathbf{\Phi} = (\Phi_1, \Phi_2, \cdots, \Phi_N)$$

and then any CP-odd observable O and any EDM E can be expanded as

$$O = \mathbf{\Phi} \cdot \mathbf{O} + \cdots\,; \quad E = \mathbf{\Phi} \cdot \mathbf{E} + \cdots$$

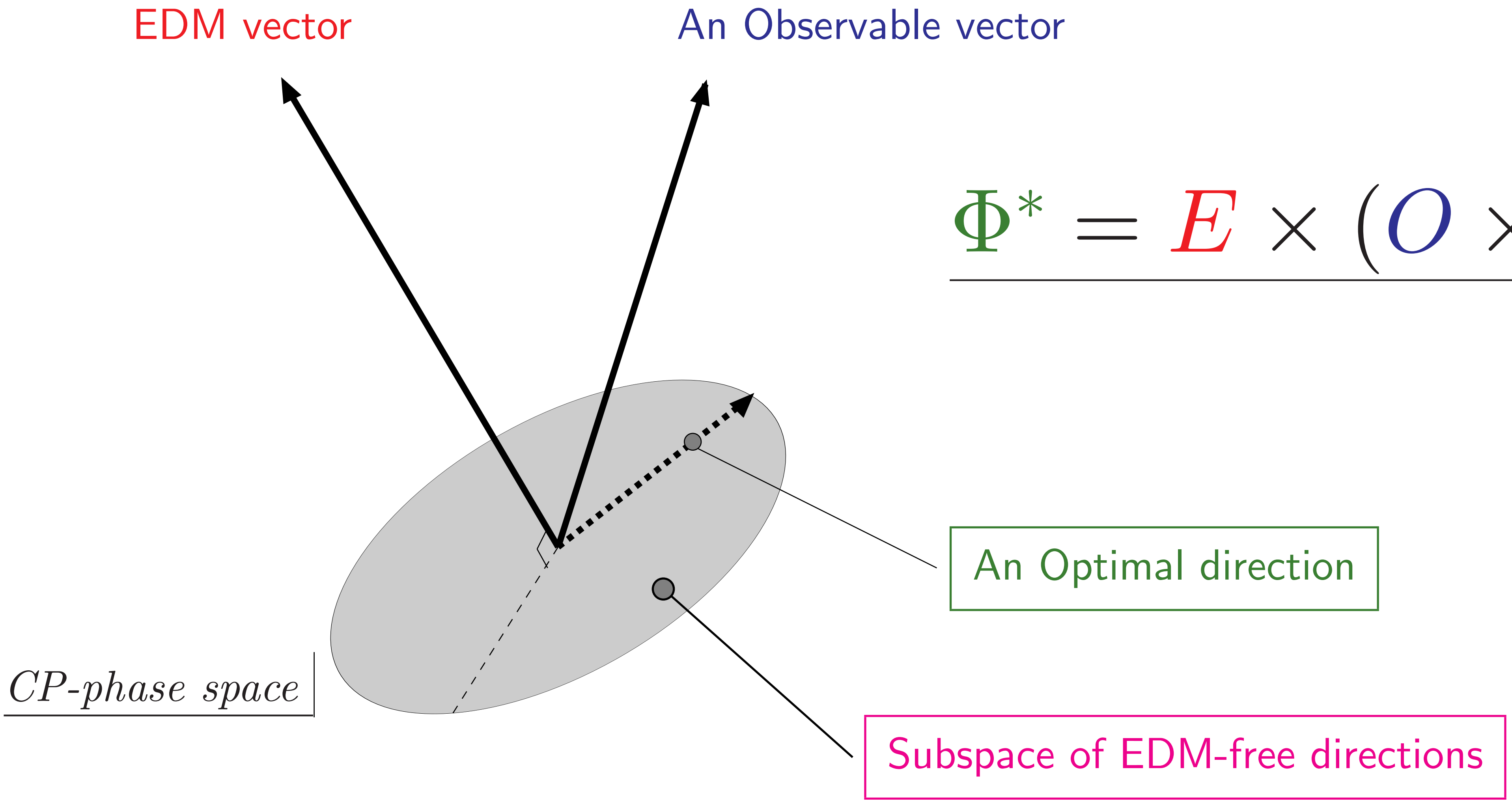
Formally, we define

$$\mathbf{O} \equiv \nabla O\,; \qquad \mathbf{E} \equiv \nabla E$$

with $\nabla \equiv (\partial/\partial\phi_1, \partial/\partial\phi_2, \cdots, \partial/\partial\phi_N)$

♠ *EDM Constraints on CP phases (Geometric approach)*

- [Simple 3D example] with 3 CP phases and 1 EDM constraint: EDM-free subspace and Optimal direction in the linear approximation



$$\Phi^* = E \times (O \times E)$$

♠ EDM Constraints on CP phases (Geometric approach)

- THE HIGHER-DIMENSIONAL GENERALIZATION with

N CP phases and n EDM constraints

The N-dimensional vector of the optimal direction

$$\Phi^*{}_\alpha = \epsilon_{\alpha\beta_1\cdots\beta_n\gamma_1\cdots\gamma_{N-n-1}} E^{(1)}_{\beta_1} \cdots E^{(n)}_{\beta_n} B_{\gamma_1\cdots\gamma_{N-n-1}}$$

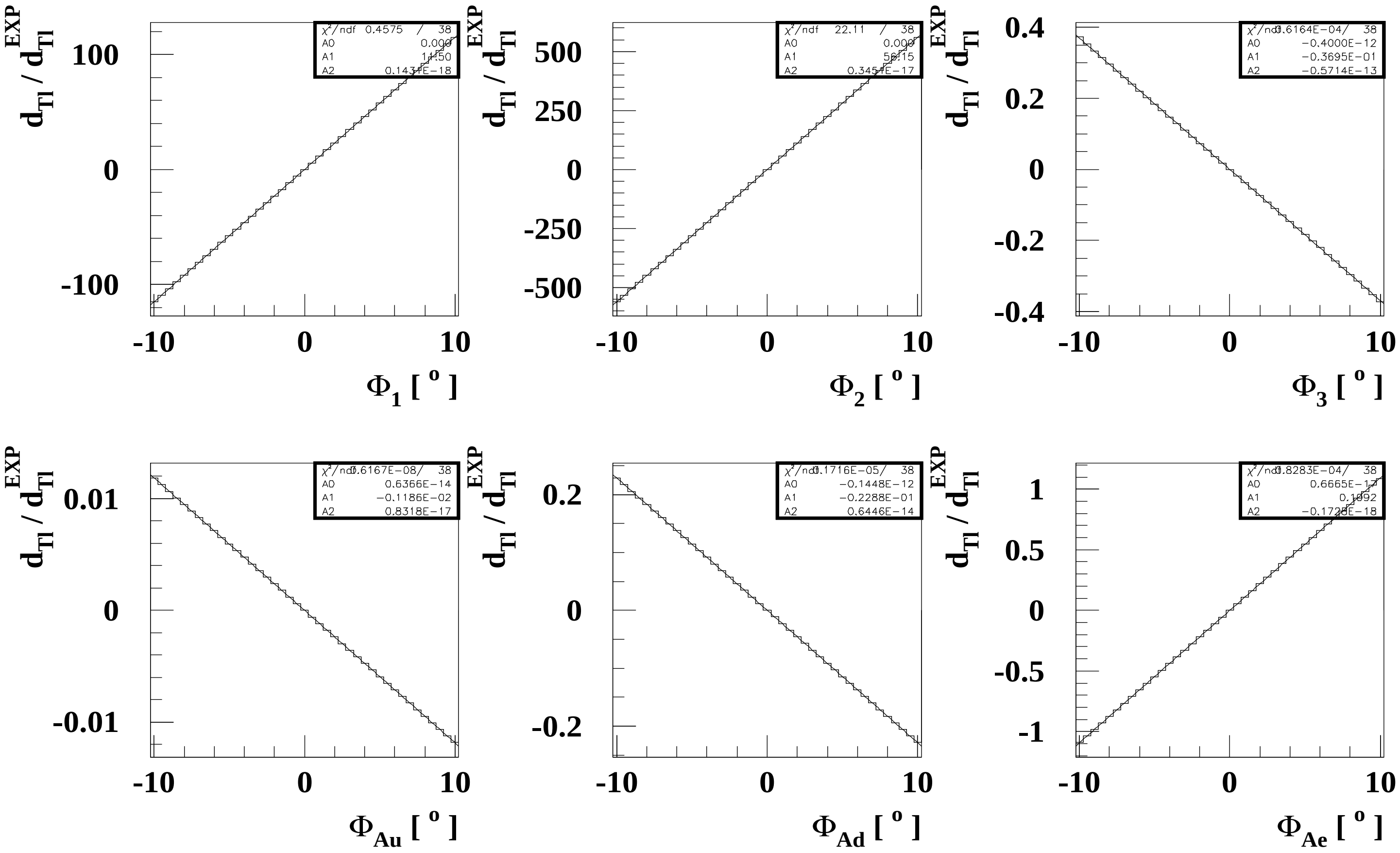
where $(N$-n-$1)$-dimensional B form is

$$B_{\gamma_1\cdots\gamma_{N-n-1}} = \epsilon_{\gamma_1\cdots\gamma_{N-n-1}\sigma\beta_1\cdots\beta_n} O_\sigma E^{(1)}_{\beta_1} \cdots E^{(n)}_{\beta_n}$$

The maximum allowed value of O is: $O^{\max} = (\phi^*)^{\max}\, \widehat{\mathbf{\Phi}}^* \cdot \mathbf{O}$ with the normalized optimal-direction vector $\widehat{\mathbf{\Phi}}^*$ and ϕ^* which may be practically determined by the validity of the small-phase approximation of the EDMs.

♠ EDM Constraints on CP phases (Geometric approach)

- *How good is the linear approximation?* The quadratic fit to the Thallium EDM that is used to obtain the 6D vector $\mathbf{E}^{d_{\rm Tl}} \equiv \nabla(d_{\rm Tl}/d_{\rm Tl}^{\rm EXP})$ in an expansion around $\widetilde{\varphi}_\alpha = 0^\circ$ for the MCPMFV scenario: $|M_{1,2,3}| = 250\ \ {\rm GeV}\,, M^2_{H_u} = M^2_{H_d} = \widetilde{M}^2_Q = \widetilde{M}^2_U = \widetilde{M}^2_D = \widetilde{M}^2_L = \widetilde{M}^2_E = (100\ \ {\rm GeV})^2\,, |A_u| = |A_d| = |A_e| = 100\ \ {\rm GeV}\,$, and $\tan\beta = 40$.



♠ *EDM Constraints on CP phases (Geometric approach)*

- *A demonstration of the geometric approach*

$$\textbf{Observable} = d_{\rm Ra} \quad \text{and} \quad \textbf{EDMs} = d_{\rm Tl}\,,\ d_{\rm n}\,,\ d_{\rm Hg}$$

taking the scenario

$$|M_{1,2,3}| = 350 \ \ \text{GeV}\,,$$
$$M^2_{H_u} = M^2_{H_d} = \widetilde{M}^2_Q = \widetilde{M}^2_U = \widetilde{M}^2_D = \widetilde{M}^2_L = \widetilde{M}^2_E = (100 \ \ \text{GeV})^2\,,$$
$$|A_u| = |A_d| = |A_e| = 100 \ \ \text{GeV}\,,$$

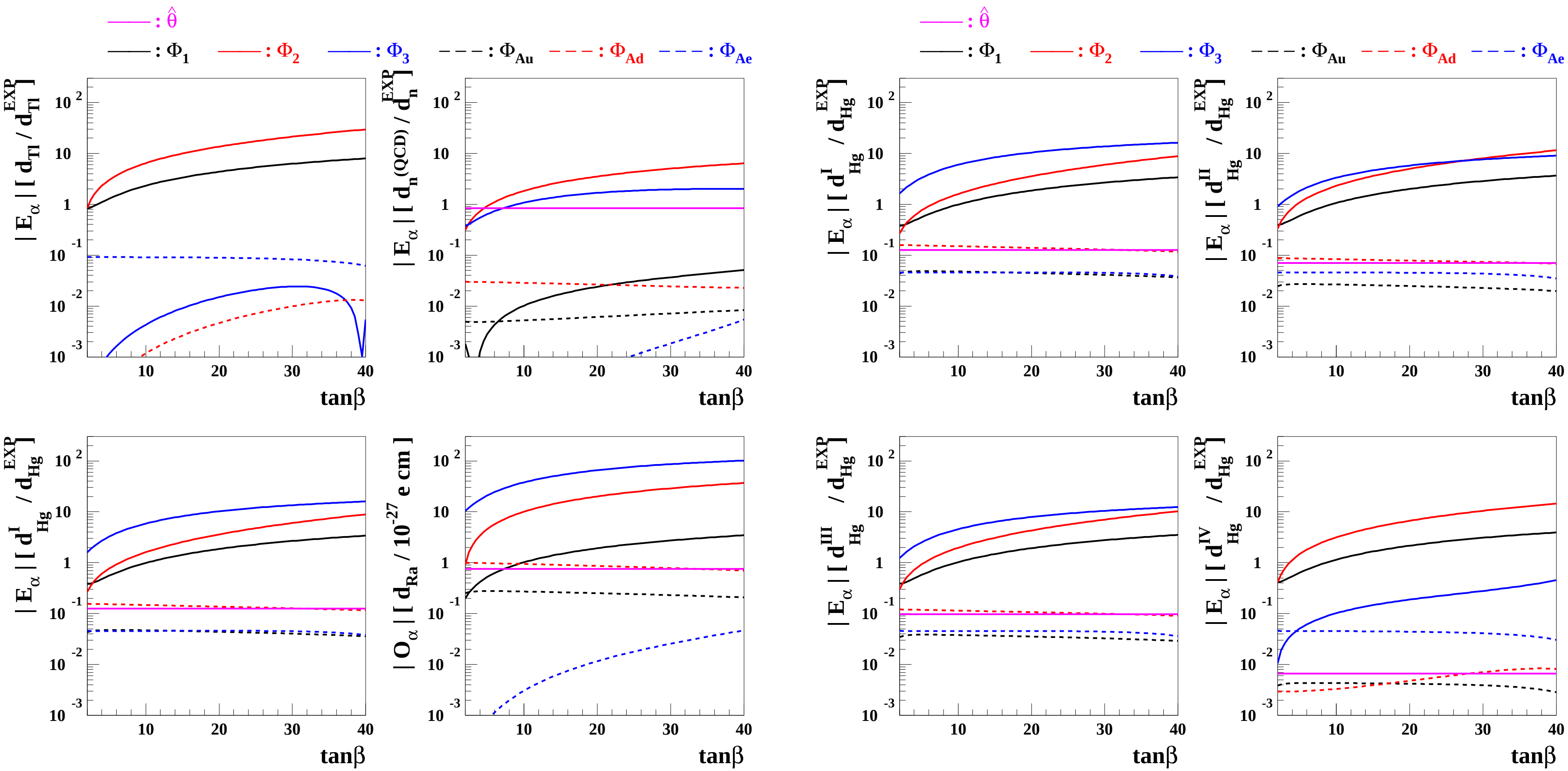
at the GUT scale, varying $\tan\beta\,(M_{\rm SUSY})$ and the following six MCPMFV CP phases at the GUT scale:

$$\Phi_1[^\circ]\,,\quad \Phi_2[^\circ]\,,\quad \Phi_3[^\circ]\,,\quad \Phi_{A_u}[^\circ]\,,\quad \Phi_{A_d}[^\circ]\,,\quad \Phi_{A_e}[^\circ]$$

with and without the QCD term $\hat{\theta} \equiv \bar{\theta} \times 10^{10}$

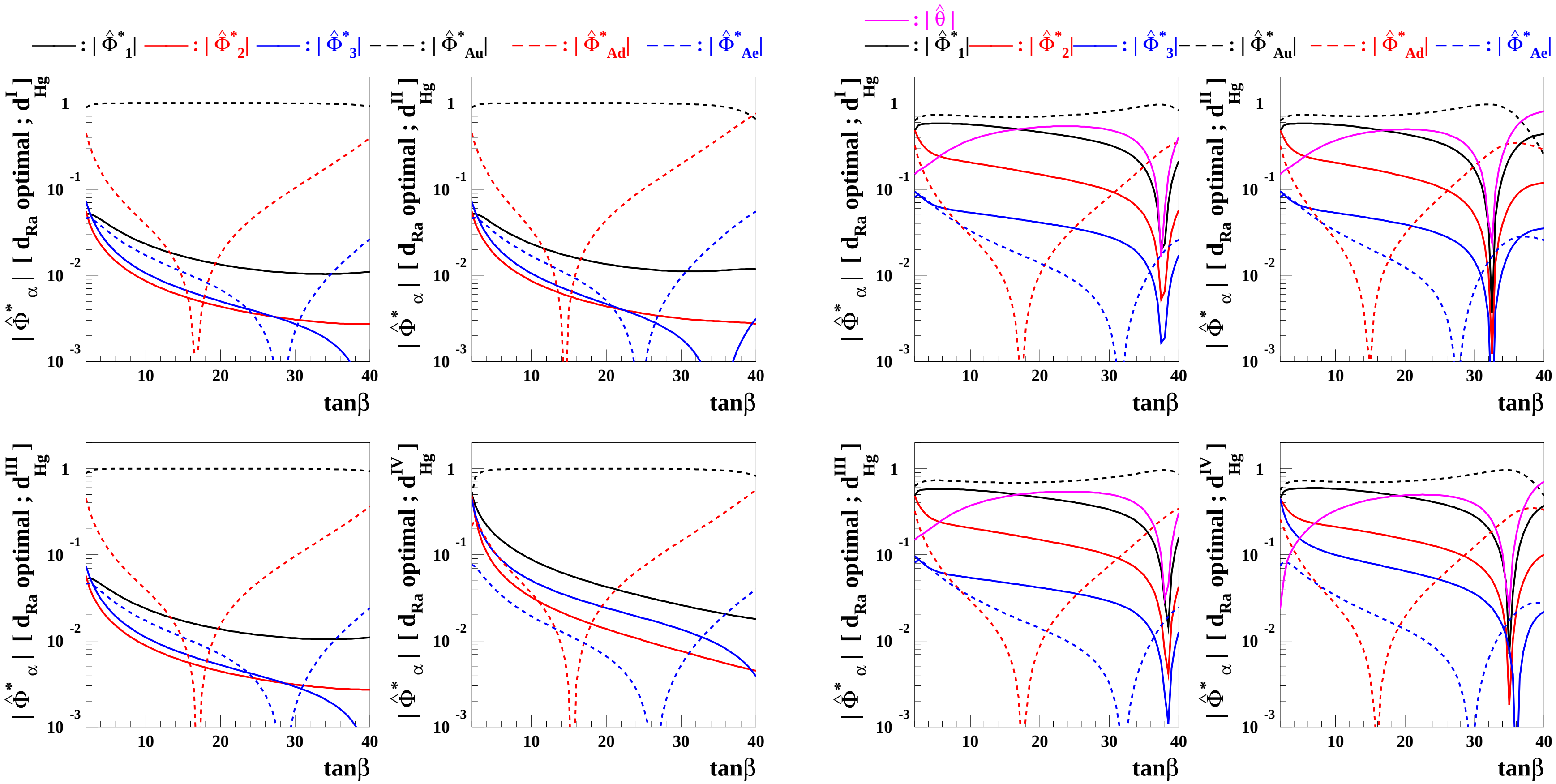
EDM Constraints on CP phases (Geometric approach)

- $\mathbf{O}[d_{\rm Ra}/10^{-27}e{\rm cm}]$ and $\{\mathbf{E}[d_{\rm Tl}/d_{\rm Tl}^{\rm EXP}],\ \mathbf{E}[d_{\rm n}/d_{\rm n}^{\rm EXP}],\ \mathbf{E}[d_{\rm Hg}^{\rm I,II,III,IV}/d_{\rm Hg}^{\rm EXP}]\}$



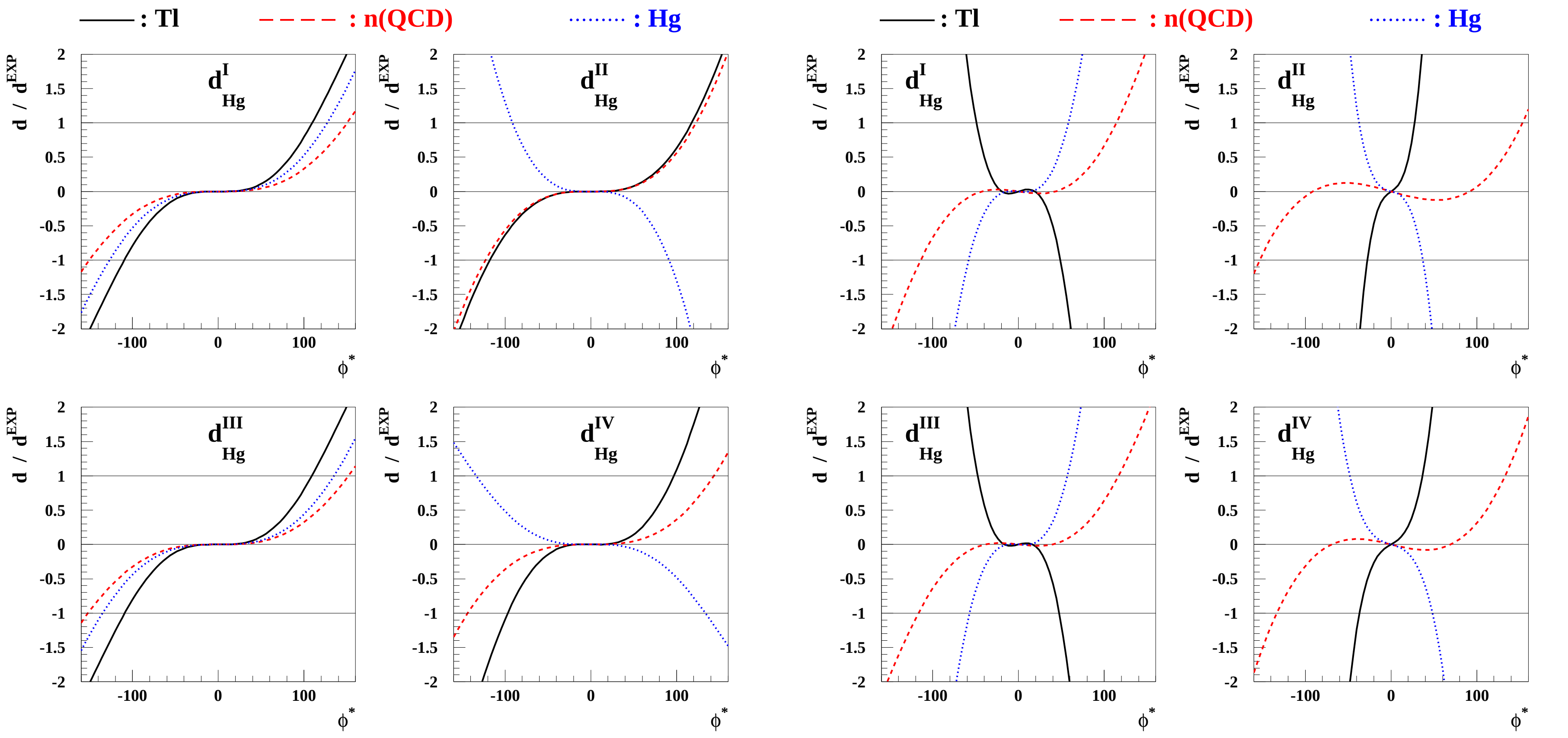
♠ *EDM Constraints on CP phases (Geometric approach)*

- $\hat{\Phi}^*$ with (left) and without (right) $\hat{\theta}$:



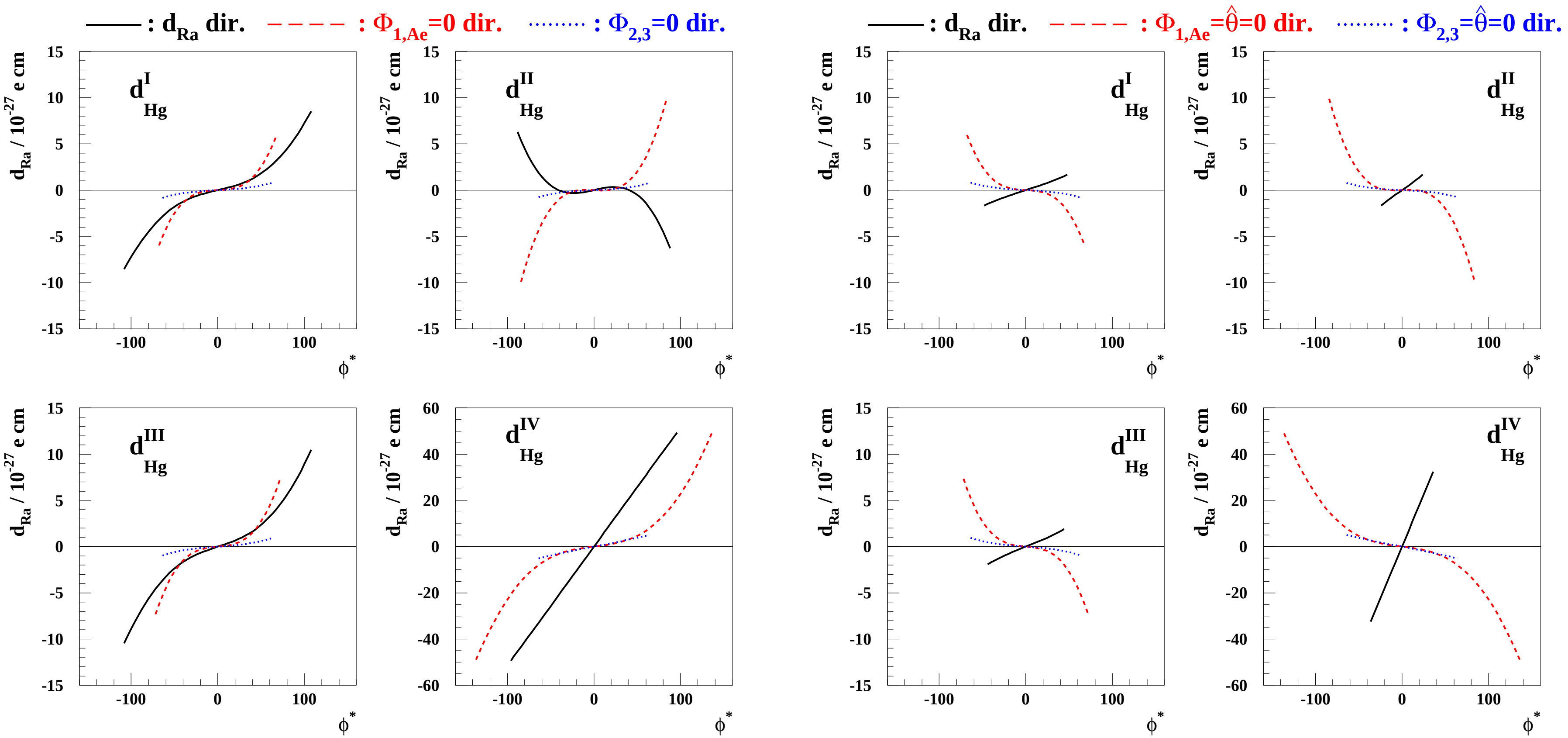
♠ EDM Constraints on CP phases (Geometric approach)

- The Thallium, neutron, Mercury EDMs along the Radium-EDM optimal direction with (left) and without (right) $\hat{\theta}$: $(\phi^*)^{\max}$



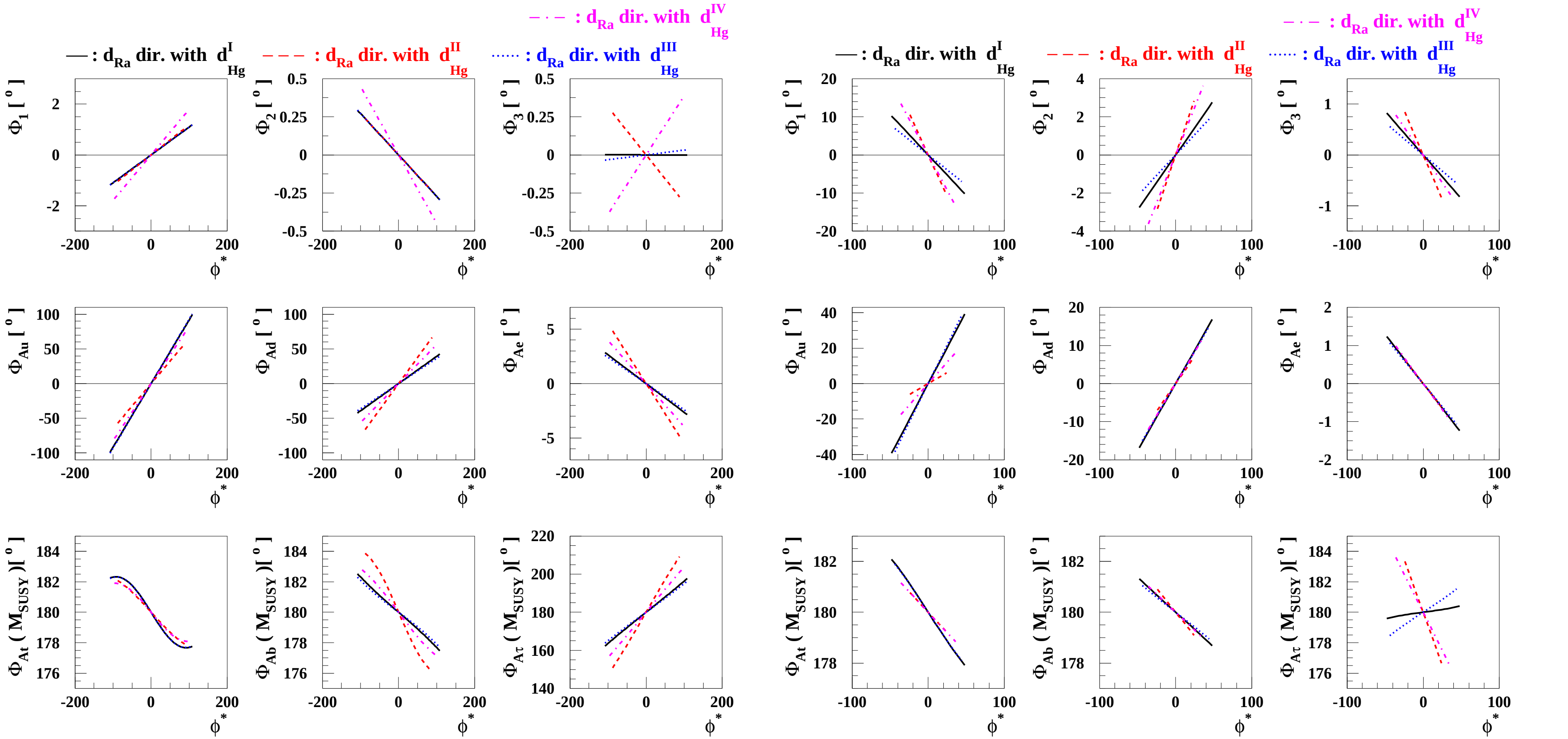
♠ *EDM Constraints on CP phases (Geometric approach)*

- The predictions for the Radium EDM along its optimal direction with (left) and without (right) $\hat{\theta}$: comparisons to those along the two arbitrary directions



♠ EDM Constraints on CP phases (Geometric approach)

- The CP phases with (left) and without (right) $\hat{\theta}$:



♠ *Summary and Future Prospects*

- Theoretical calculation of EDMs requires expertise in various fields of Physics and suffers from large uncertainties
- A geometric method has been developed to predict the maximal size of any CP-violating observable while satisfying all the EDM constraints
- The large/medium CP phases might still be allowed while satisfying the current EDM constraints
- EDMs can probe SUSY beyond 10 TeV
- We eagerly anticipate signals of CP violation in (near) future EDM experiments

Backup slides

♠ NMSSM Higgs Sector I

- Superpotential:

$$W_{\rm NMSSM} \; = \; \widehat{U}^C \mathbf{h}_u \widehat{Q} \widehat{H}_u \; + \; \widehat{D}^C \mathbf{h}_d \widehat{H}_d \widehat{Q} \; + \; \widehat{E}^C \mathbf{h}_e \widehat{H}_d \widehat{L} \; + \; \lambda \widehat{S} \widehat{H}_u \widehat{H}_d \;\; + \; \tfrac{\kappa}{3} \; \widehat{S}^3$$

- Higgs potential at the tree level:

$$V_0 = V_F + V_D + V_{\rm soft}$$

$$\begin{aligned}
V_F &= |\lambda|^2 |S|^2 (H_d^\dagger H_d + H_u^\dagger H_u) + |\lambda H_u H_d + \kappa S^2|^2, \\
V_D &= \frac{g_2^2 + g_1^2}{8} (H_d^\dagger H_d - H_u^\dagger H_u)^2 + \frac{g_2^2}{2} (H_d^\dagger H_u)(H_u^\dagger H_d), \\
V_{\rm soft} &= m_{H_d}^2 H_d^\dagger H_d + m_{H_u}^2 H_u^\dagger H_u + m_S^2 |S|^2 + \left(\lambda A_\lambda S H_u H_d - \frac{1}{3} \kappa A_\kappa S^3 + \text{h.c.} \right)
\end{aligned}$$

Kingman Cheung, Tie-Jiun Hou, JSL, Eibun Senaha, arXiv:1006.1458 [hep-ph] [PRD82(2010)075007]; arXiv:1102.5679 [hep-ph] [PRD84(2011)015002]

♠ *NMSSM Higgs Sector I*

- Components fields and VEVs:

$$H_d = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_d + \phi_d^0 + i a_d) \\ \phi_d^- \end{pmatrix} \;;\; H_u = e^{i\theta} \begin{pmatrix} \phi_u^+ \\ \frac{1}{\sqrt{2}}(v_u + \phi_u^0 + i a_u) \end{pmatrix}$$

$$S = \frac{e^{i\varphi}}{\sqrt{2}}(v_S + \phi_S^0 + i a_S)$$

The next step is to minimize the Higgs potential V_0 *by inserting the above component fields and VEVs which may give* FIVE *tadpole conditions; three CP-even and two CP-odd ones*

♠ NMSSM Higgs Sector I

- Tadpole conditions: $\mathcal{R} = |\lambda||\kappa|\cos(\phi'_\lambda - \phi'_\kappa)$ and $\mathcal{I} = |\lambda||\kappa|\sin(\phi'_\lambda - \phi'_\kappa)$ with $\phi'_\lambda \equiv \phi_\lambda + \theta + \varphi$; $\phi'_\kappa \equiv \phi_\kappa + 3\varphi$

$$\begin{aligned}
\frac{1}{v_d}\left\langle\frac{\partial V_0}{\partial \phi_d^0}\right\rangle &= m_{H_d}^2 + \frac{g_2^2+g_1^2}{8}(v_d^2 - v_u^2) - R_\lambda \frac{v_u v_S}{v_d} + \frac{|\lambda|^2}{2}(v_u^2 + v_S^2) - \frac{1}{2}\mathcal{R}\frac{v_u v_S^2}{v_d} = 0 \\
\frac{1}{v_u}\left\langle\frac{\partial V_0}{\partial \phi_u^0}\right\rangle &= m_{H_u}^2 - \frac{g_2^2+g_1^2}{8}(v_d^2 - v_u^2) - R_\lambda \frac{v_d v_S}{v_u} + \frac{|\lambda|^2}{2}(v_d^2 + v_S^2) - \frac{1}{2}\mathcal{R}\frac{v_d v_S^2}{v_u} = 0 \\
\frac{1}{v_S}\left\langle\frac{\partial V_0}{\partial \phi_S^0}\right\rangle &= m_S^2 - R_\lambda \frac{v_d v_u}{v_S} + \frac{|\lambda|^2}{2}(v_d^2 + v_u^2) + |\kappa|^2 v_S^2 - \mathcal{R} v_d v_u - R_\kappa v_S = 0 \\
\frac{1}{v_u}\left\langle\frac{\partial V_0}{\partial a_d}\right\rangle &= \frac{1}{v_d}\left\langle\frac{\partial V_0}{\partial a_u}\right\rangle = I_\lambda v_S + \frac{1}{2}\mathcal{I} v_S^2 = 0 \\
\frac{1}{v_S}\left\langle\frac{\partial V_0}{\partial a_S}\right\rangle &= I_\lambda \frac{v_d v_u}{v_S} - \mathcal{I} v_d v_u + I_\kappa v_S = 0
\end{aligned}$$

$$\begin{aligned}
R_\lambda &= \frac{|\lambda||A_\lambda|}{\sqrt{2}}\cos(\phi'_\lambda + \phi_{A_\lambda}), \quad R_\kappa = \frac{|\kappa||A_\kappa|}{\sqrt{2}}\cos(\phi'_\kappa + \phi_{A_\kappa}), \\
I_\lambda &= \frac{|\lambda||A_\lambda|}{\sqrt{2}}\sin(\phi'_\lambda + \phi_{A_\lambda}), \quad I_\kappa = \frac{|\kappa||A_\kappa|}{\sqrt{2}}\sin(\phi'_\kappa + \phi_{A_\kappa}),
\end{aligned}$$

Three CP Phases (still...) : $\phi'_\lambda - \phi'_\kappa$; $\phi'_\lambda + \phi_{A_\lambda}$; $\phi'_\kappa + \phi_{A_\kappa}$

♠ *NMSSM Higgs Sector I*

- Tadpole conditions ... *continued*:
  - The first three CP-even conditions may allows us to reexpress the three soft masses $m^2_{H_d}$, $m^2_{H_u}$, and m^2_S in terms of other parameters
  - The remaining two CP-odd conditions determine the two CP phases $\phi'_\lambda + \phi_{A_\lambda}$ and $\phi'_\kappa + \phi_{A_\kappa}$ up to a two-fold ambiguity

$$
\begin{aligned}
I_\lambda &= \frac{|\lambda||A_\lambda|}{\sqrt{2}} \sin(\phi'_\lambda + \phi_{A_\lambda}) = -\frac{1}{2}\mathcal{I}\, v_S \,, \\
I_\kappa &= \frac{|\kappa||A_\kappa|}{\sqrt{2}} \sin(\phi'_\kappa + \phi_{A_\kappa}) = \frac{3}{2}\mathcal{I}\,\frac{v_d v_u}{v_S}
\end{aligned}
$$

  - Therefore, the only rephasing invariant physical CP phase at the tree level is

$$
\phi'_\lambda - \phi'_\kappa
$$

  - N.B. No tree-level CP violation in the MSSM Higgs sector

♠ *Synopsis of EDMs in the NMSSM*

- <u>What's different from the MSSM?</u>
  - Neutral Higgs sector: 3 states (MSSM) $\to$ 5 states (NMSSM)
  - Neutralino sector: 4 states (MSSM) $\to$ 5 states (NMSSM):

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & -M_Z c_\beta s_W & M_Z s_\beta s_W & 0 \\ & M_2 & M_Z c_\beta c_W & -M_Z s_\beta c_W & 0 \\ & & 0 & -\frac{|\lambda| v_S}{\sqrt{2}}\, e^{i\phi'_\lambda} & -\frac{|\lambda| v s_\beta}{\sqrt{2}}\, e^{i\phi'_\lambda} \\ & & & 0 & -\frac{|\lambda| v c_\beta}{\sqrt{2}}\, e^{i\phi'_\lambda} \\ & & & & \sqrt{2}|\kappa| v_S\, e^{i\phi'_\kappa} \end{pmatrix}$$

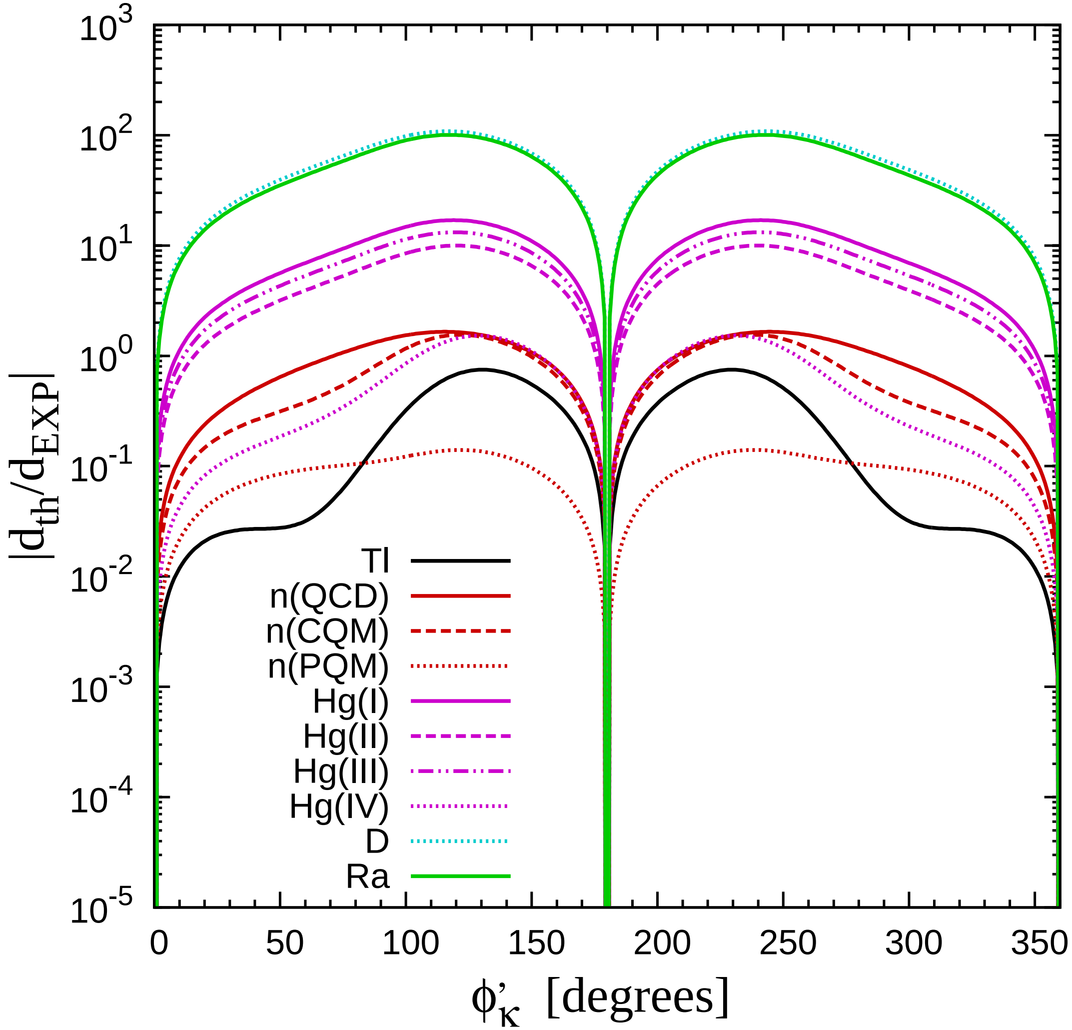
Diagonalization: $N^* \mathcal{M}_N N^\dagger = \mathrm{diag}\,(m_{\widetilde{\chi}^0_1}, m_{\widetilde{\chi}^0_2}, m_{\widetilde{\chi}^0_3}, m_{\widetilde{\chi}^0_4}, m_{\widetilde{\chi}^0_5})$ with the mixing matrix $N_{i\alpha}$ $(\widetilde{B}, \widetilde{W}^0, \widetilde{H}^0_d, \widetilde{H}^0_u, \widetilde{S})^T_\alpha = N^*_{i\alpha} (\widetilde{\chi}^0_1, \widetilde{\chi}^0_2, \widetilde{\chi}^0_3, \widetilde{\chi}^0_4, \widetilde{\chi}^0_5)^T_i$

  - Basically, one additional physical CP phase $(\phi'_\lambda - \phi'_\kappa)$

QUESTION: *What's the EDM constraint on* $(\phi'_\lambda - \phi'_\kappa)$*?*

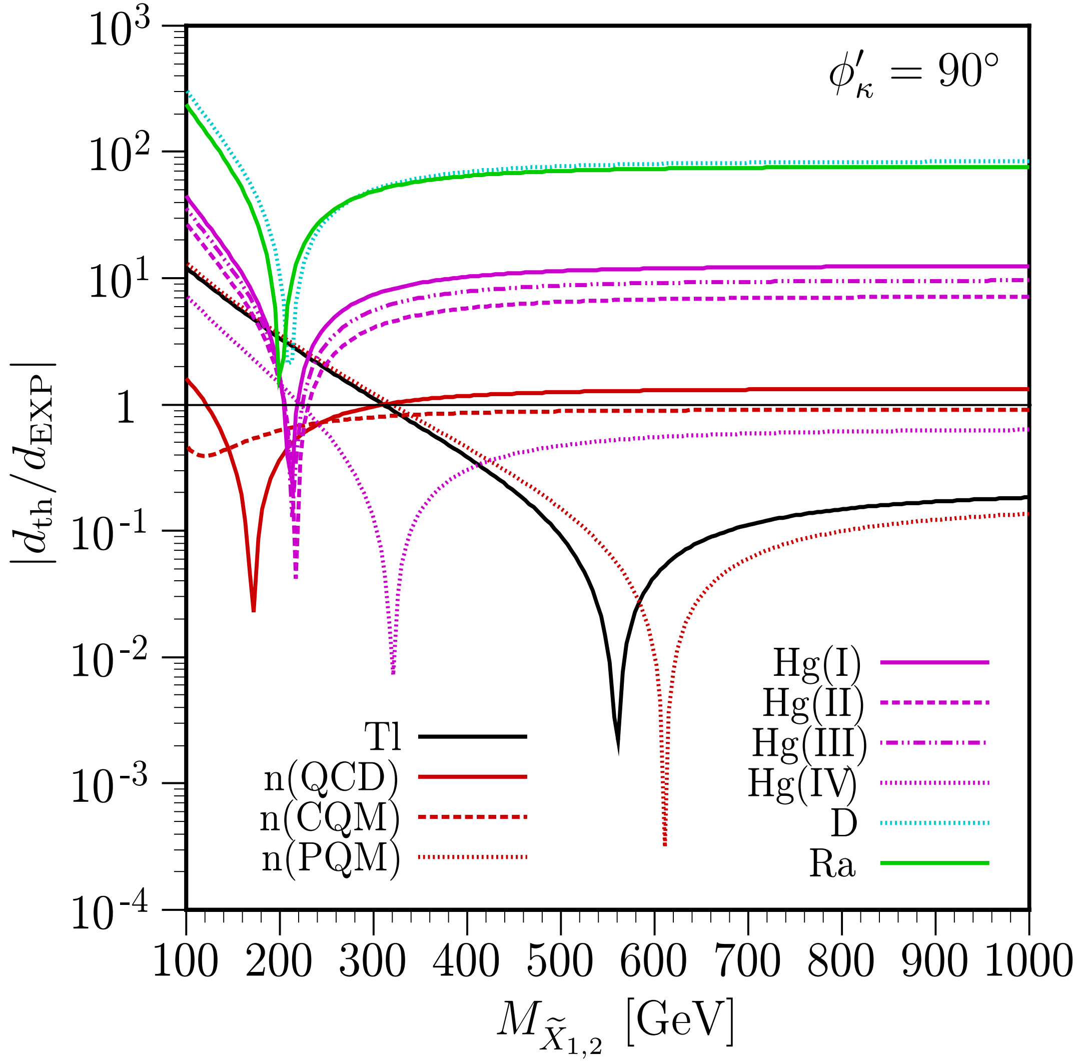
♠ *EDM constraints on* ϕ'_κ

- Taking the EWBG-motivated scenario with $|\lambda| = 0.81$, $|\kappa| = 0.08$, $|A_\lambda| = 575$ GeV, and $|A_\kappa| = 110$ GeV (our convention: $\phi'_\lambda = 0$ and $M_{\rm SUSY}$=1 TeV by default):



♠ *EDM constraints on ϕ'_κ*

- As functions of $M_{\widetilde{X}_{1,2}} \equiv M_{\widetilde{Q}_{1,2}} = M_{\widetilde{U}_{1,2}} = M_{\widetilde{D}_{1,2}} = M_{\widetilde{L}_{1,2}} = M_{\widetilde{E}_{1,2}}$ taking the maximal CP phase $\phi'_\kappa = 90°$:



Kngman Cheung, Tie-Jiun Hou, JSL, Eibun Senaha, arXiv:1102.5679 [hep-ph] [PRD84(2011)015002]

– $d_{\rm Hg}^{\rm IV}$: all the EDM constraints could be fulfilled when $M_{\widetilde{X}_{1,2}} \gtrsim 300$ GeV

– $d_{\rm Hg}^{\rm I,II,III}$: all the EDM constraints could be fulfilled with 90 % cancellation

– Singlino-driven EWBG Kingman Cheung, Tie-Jiun Hou, JSL, Eibun Senaha, arXiv:1201.3781 [hep-ph]