Testing Neutrino Mass Seesaw at the LHC

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Where does neutrino mass come from ?

- Charged fermion masses come from the Higgs vev:

$$m_f = h_f v_{wk} \qquad v_{wk} = < h^0 >$$

★ Discovery of the 125 GeV Higgs h^0 confirms this.

- For neutrinos, this formula gives too large a mass unless $h_\nu \leq 10^{-12}$!!

- This is an indication of new physics as source of neutrino mass !

Weinberg Effective operator as a clue to the new physics

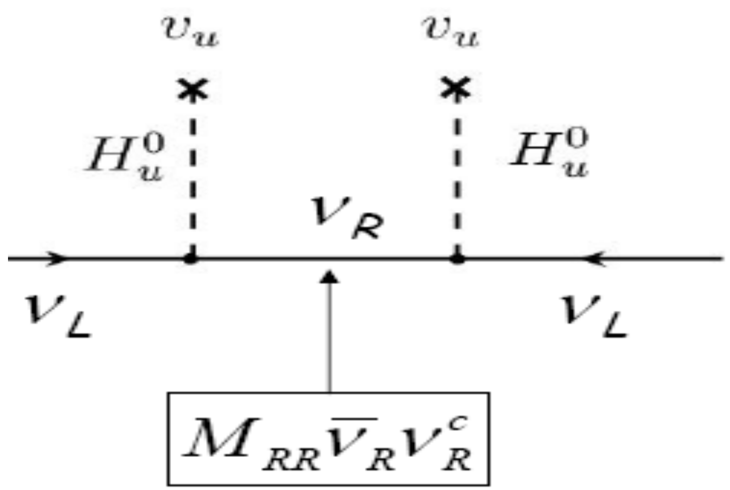
- Add effective operator to SM: $\lambda \frac{LHLH}{M}$

→ $$\boxed{m_\nu = \lambda \frac{v_{wk}^2}{M}}$$

- $\lambda \sim 1; M$ big → $m_\nu \ll m_f$ naturally !
- What is the Physics of M?
- To explore this, seek UV completion of Weinberg operator→ seesaw→**M-physics**

Seesaw paradigm and UV completion of Weinberg Op.

- **Simplest Seesaw** SM+ RH neutrinos ν_R with heavy Majorana mass (Breaks B-L)



$$\rightarrow \quad m_\nu \simeq -\frac{m_D^2}{M_{\nu_R}}$$

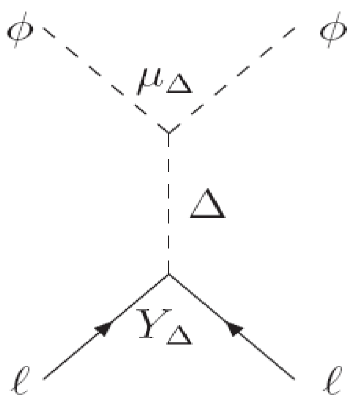
Minkowski; Mohapatra,Senjanovic;Gell-Mann,Ramond, Slansky; Yanagida; Glashow

- **Type I seesaw** (Main focus of Talk)

Other types of seesaws

- SM+ Dirac singlet or Maj. triplet fermions or Higgs $\vec{\Delta}$

Type II

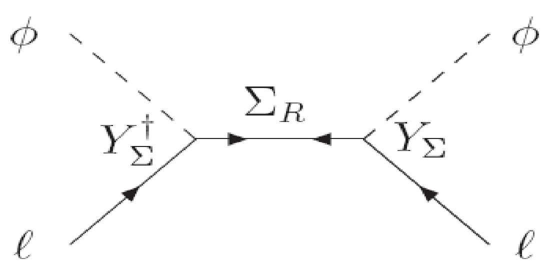


$$m_\nu = -2Y_\Delta v^2 \frac{\mu_\Delta}{M_\Delta^2}\,.$$

Triplet Higgs $\vec{\Delta}$

Lazaridis, Shafi,Wetterich
Schecter,Valle ,RNM,Senjanovic

Type III



$$m_\nu = -\frac{v^2}{2} Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$$

Triplet N Maj.

Foot, He,Joshi,Lew

Inverse

$$\begin{pmatrix} 0 & hv_{wk} & 0 \\ hv_{wk} & 0 & M \\ 0 & M & \mu \end{pmatrix}$$

$$m_\nu \cong -m_D^T M^{-1} \mu M^{-1} m_D$$

N Pseudo Dirac

Mohapatra; Mohapatra,Valle

Weinberg operator, simplest but not the only way ?

- It could be other higher dim operators e.g.

$$\mathcal{O}_2 = L^i L^j L^k e^c H^l \epsilon_{ij} \epsilon_{kl}$$

$$\mathcal{O}_3 = \{L^i L^j Q^k d^c H^l \epsilon_{ij} \epsilon_{kl},\ \ L^i L^j Q^k d^c H^l \epsilon_{ik} \epsilon_{jl}\}$$

$$\mathcal{O}_4 = \{L^i L^j \bar{Q}_i \bar{u^c} H^k \epsilon_{jk},\ \ L^i L^j \bar{Q}_k \bar{u^c} H^k \epsilon_{ij}\}$$

- .... (Babu, Leung'01; de Gouvea, Jenkins'07)

- Different UV completions and different tests !!

(see e.g. Angel, Rodd, Volkas'12)

- This talk deals only with seesaw case:

Testing Seesaw physics in colliders

$$\boxed{m_\nu = \lambda \frac{v_{wk}^2}{M}}$$

- $\lambda \ll 1$, M can be in the TeV range and accessible to colliders:
- Two classes of models discussed here:

  (i) SM gauge group

  (ii) Left-right gauge group (LR)

New seesaw particles for the LHC

- SM seesaw: Singlet neutrinos ν_R (also called N) (Majorana (type I) or pseudo-Dirac (inverse));

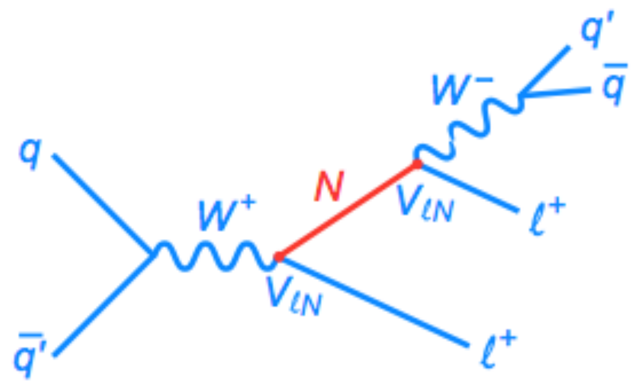
- Scalar SM triplet $(\Delta^{++}, \Delta^{+}, \Delta^{0})$ (type II)

- Fermion SM triplet : $\boxed{\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}.}$ (type III)

- Left-right seesaw: W_R + above

Type I SM seesaw : experimental signals

- **Type I: SM+RH neutrino**: $\boxed{N}$ $\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}$
- Two aspects: m_D and M_N



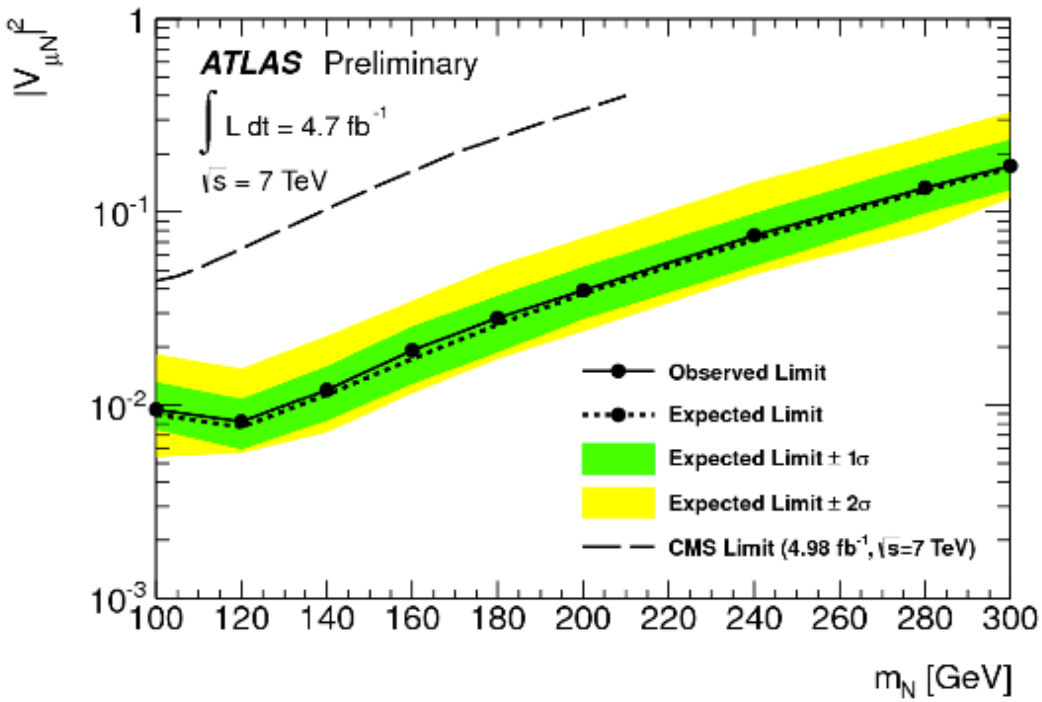
- Majorana N: $N \to l^\pm W^\mp \to l^\pm jj$ *(like sign dileptons+2j)*

(Han, Zhang'06; del Aguila, Aguila-Saavedra, Pittau, 06; Bray, Lee, Pilaftsis, 07)

- SM, only production mode is via νN mixing $V_{\ell N} \simeq \dfrac{m_D}{M_N}$
- Observation as step I to establish seesaw !!

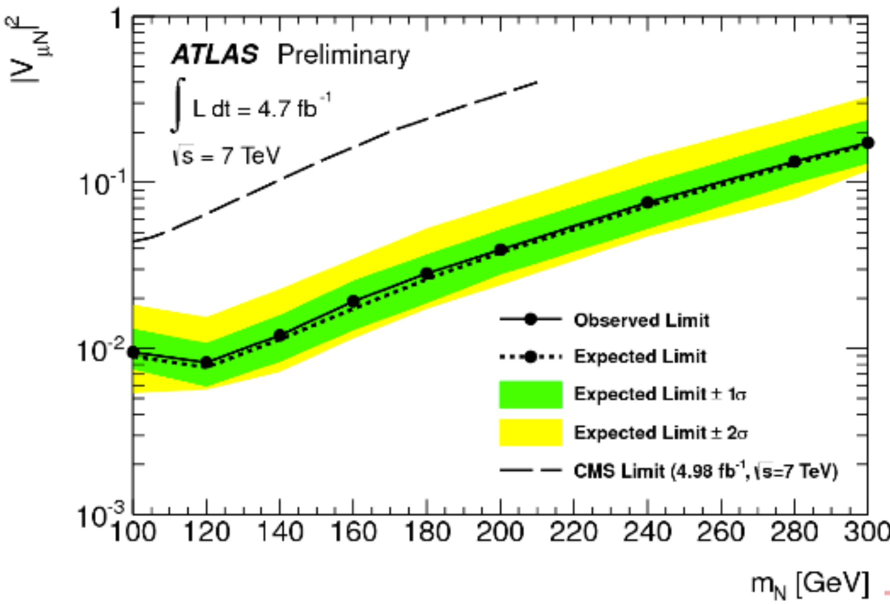
Current LHC results: Type I SM seesaw

- CMS, ATLAS search in $e^{\pm}e^{\pm}, \mu^{\pm}\mu^{\pm}$



Current LHC results: Type I SM seesaw with new graph

- CMS, ATLAS search in $e^{\pm}e^{\pm}, \mu^{\pm}\mu^{\pm}$



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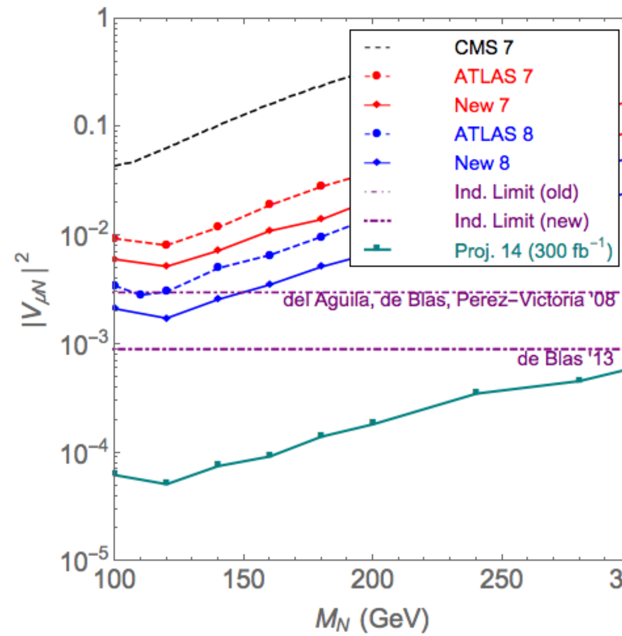
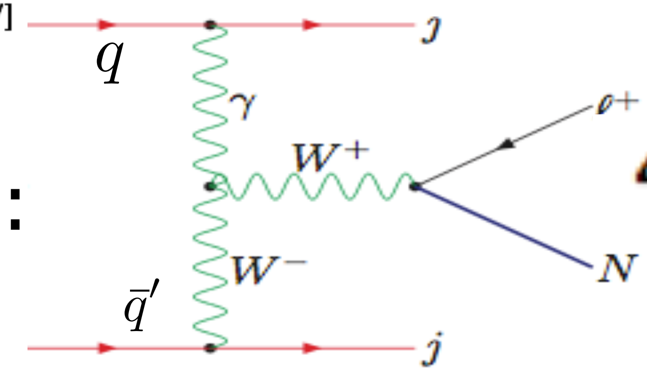


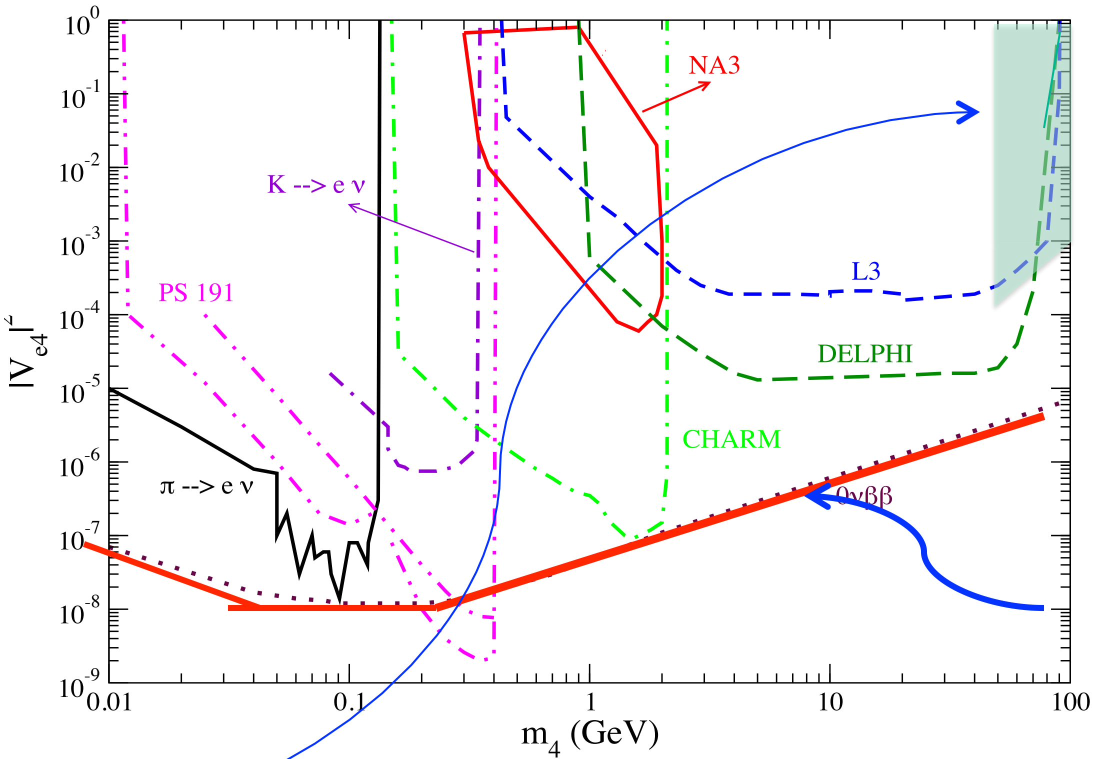
- Improved limit with photon exchange graph:



$4j + \ell^{\pm}\ell^{\pm}$

(Dev, Pilaftsis, Yang'13)

Type I seesaw: lower M_N
Other constraints on $V_{\ell N}$



most relevant for seesaw

(Atre, Han, Pascoli, Zhang)

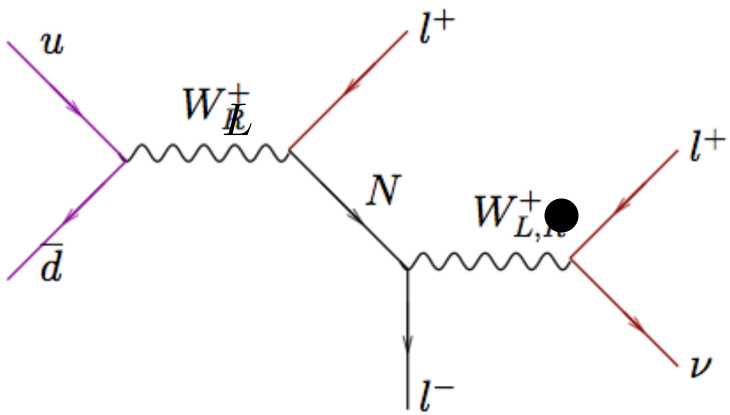
Bounds from LHC Higgs decay to $e^+e^-\not{E}_T$ from $pp \to h \to \ell N$

$N \to W^+\ell^-, Z+\nu$

(Dev, Francischini, RNM'12)

Inverse seesaw In SM

- SM+pseudo-Dirac RH neutrino $\boxed{N}$



$$pp \to \ell^{\pm}\ell^{\mp}\ell^{\pm} + \cancel{E} + X$$

- Key parameter for signal strength in the SM $V_{\ell N} \simeq \frac{m_D}{M_N}$
- $V_{\ell N} \leq 10^{-2}$ Observable: del Aguila.Hirsch et al; Mondal et al;Chen, Dev, Das, Okada
- As in type I, observation first step to establish seesaw!

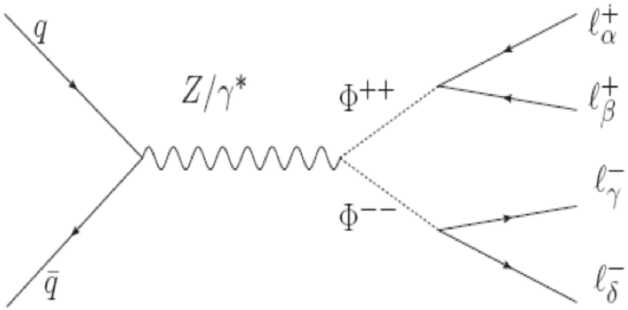
Type II seesaw

- **Scalar triplet**

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}.$$

$$\delta^{++} \to l^+ l^+, W^+ W^+$$

$$\delta^+ \to l^+ \overline{\nu} \quad , W^+ Z$$



Huitu, et al., Han, Perez et al, Mukhopadhyay, Sii; Aoki, Kanemura, ..

$$M_{\Delta^{++}} \geq 450 GeV$$

- Direct production at LHC- without connection to nu's:

- Need detailed coupling profile to connect to seesaw

Type III seesaw

Fermion triplet:

$$\Sigma = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix}.$$

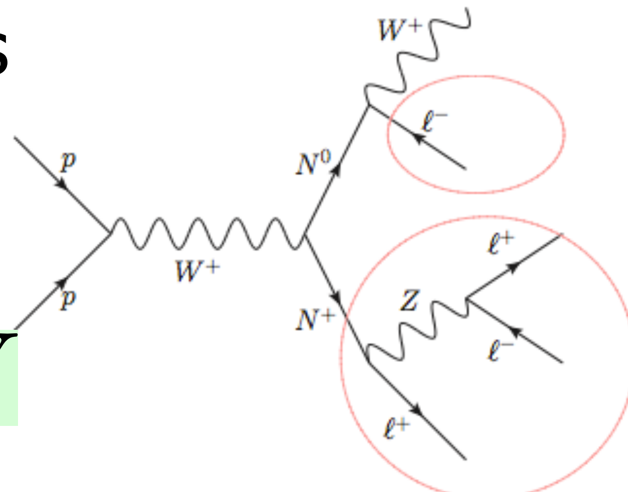
$$\Sigma^0 \to l^+ W^-$$
$$\Sigma^+ \to l^+ Z, ..$$

- Direct production does

not require connection to neutrinos

Bajc, Senjanovic, Nemesvec;....

4-lepton final states.



- LHC limit: $M_\Sigma \geq 245\ GeV$

(S. Vanini, Ph. D. thesis)

- Discovering 4 leptons needs $\Sigma^- \ell^-$ mixing which is a sign of type III seesaw !

Back to Type I Seesaw: Theoretical expectations

- Heavy-light mixing parameter in *generic SM type I* case

$$V_{\ell N} \simeq \frac{m_D}{M_N} = \sqrt{\frac{m_\nu}{M_N}} \leq 10^{-6}$$

- Much too small to be observable at LHC.
- Two ways around: *Heavy fine tuning or*

  (i) *Special textures or*

  (ii) *Beyond SM seesaw*

(i) m_D , M_N special texture

- Enhancing $V_{\ell N}$ while keeping small m_ν

- Idea: $m_\nu^{(0)} = m_D^T M_N^{-1} m_D = 0$

- Nonzero nu mass comes in higher orders of seesaw; may be due to extra symmetries !

- Allows leading order m_D to be large making seesaw effect potentially observable:

(Pilaftsis, Underwood; Kersten, Smirnov; Mitra, Senjanovic, Vissani; Haba, Mimura,Horita; He et al)

Special texture examples

- .

$$m_D = \begin{pmatrix} m_1 & \delta_1 & \epsilon_1 \\ m_2 & \delta_2 & \epsilon_2 \\ m_3 & \delta_3 & \epsilon_3 \end{pmatrix} \quad M_N = \begin{pmatrix} 0 & M_1 & 0 \\ M_1 & 0 & 0 \\ 0 & 0 & M_2 \end{pmatrix}$$

- $\epsilon_i, \delta_i \rightarrow 0, m_\nu^{(0)} = m_D^T M_N^{-1} m_D = 0$

- $\epsilon_i, \delta_i \ll m_i ; m_\nu \sim \dfrac{m_i \delta_j}{M_1} + \dfrac{\epsilon_i \epsilon_j}{M_2}$ small;

→ $A_{SM}^{LHC}(\ell^+\ell^+ jj) \propto m_D^T M_N^{-1} m_D M_N^{-1}$

Special texture examples

- .
$$m_D = \begin{pmatrix} m_1 & \delta_1 & \epsilon_1 \\ m_2 & \delta_2 & \epsilon_2 \\ m_3 & \delta_3 & \epsilon_3 \end{pmatrix} \quad M_N = \begin{pmatrix} 0 & M_1 & 0 \\ M_1 & 0 & 0 \\ 0 & 0 & M_2 \end{pmatrix}$$

- $\epsilon_i, \delta_i \to 0,\ m_\nu^{(0)} = m_D^T M_N^{-1} m_D = 0$
$\epsilon_i, \delta_i \ll m_i$;

$$A_{SM}^{LHC}(\ell^+\ell^+ jj) \propto m_D^T M_N^{-1} m_D M_N^{-1}$$

→ *like sign dilepton LHC signal suppressed* ~ $O(\frac{\delta, \epsilon}{M_N})$

(ii) Going Beyond SM for type I seesaw

- Why beyond SM ? Questions raised by seesaw:
- Where did N come from ?
- Where did the seesaw scale come from and what is its value ?
- Two simple theories that provide answers:

(i) Left-right model where N is the parity partner of ν and seesaw scale is $SU(2)_R$ scale !!

(ii) SO(10) GUT where N+15 SM fermions =16 spinor and seesaw scale = GUT scale.

Theoretical suggestions for type I Seesaw scale

(ii) GUT embedding e.g. SO(10)→ very natural since GUT→ $h_\nu \sim h_q$ → $M_R \sim 10^{14}$ GeV: (Generally not possible to test in colliders !)

(ii) Left-right can have M_R TeV scale and hence collider accessible !

- Rest of the talk: LR Models with observable signals of TeV M_{WR} with type I seesaw !

Left-Right Model Realization of Seesaw

- LR basics: Gauge group: $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$
- Fermions

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \overset{P}{\Leftrightarrow} \begin{pmatrix} u_R \\ d_R \end{pmatrix} \quad \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \overset{P}{\Leftrightarrow} \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$$

$$L = \frac{g}{2}[\vec{J}_L^{\ \mu} \cdot \vec{W}_{\mu L} + \vec{J}_R^{\ \mu} \cdot \vec{W}_{\mu R}]$$

- Parity a spontaneously broken symmetry: $M_{W_R} \gg M_{W_L}$

Seesaw scale is Parity breaking Scale

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$\Big\downarrow v_R$$

$$SU(2)_L \times U(1)_Y$$

$$\Big\downarrow \kappa$$

$$U(1)_{em}$$

$$M_N = f v_R$$

Seesaw

$$M_{\nu,N} = \begin{pmatrix} 0 & h\kappa \\ h\kappa & f v_R \end{pmatrix}$$

- Case (i): Generic type I: $h \sim 10^{-5.5}$ tiny $V_{\ell N}$ as before yet visible signals for TeV W_R!
- Case (ii): Special textures with enhanced $V_{\ell N}$!

A Tale of two symmetries

- Two symmetries: P and $SU(2)_R$

  Two Scales: M_P M_{WR}

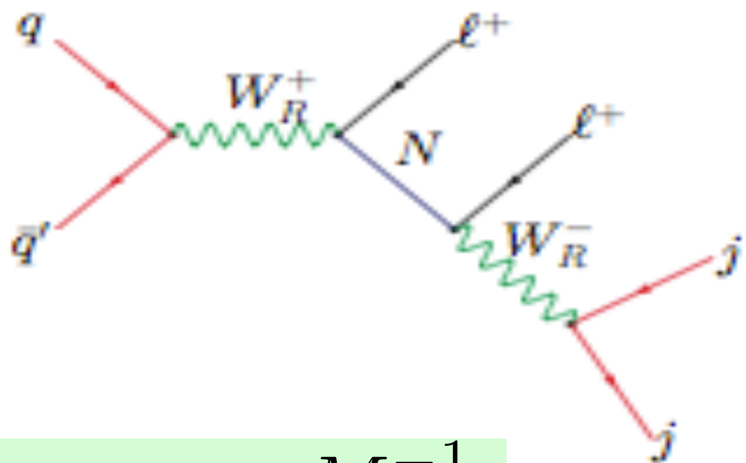
- $M_P = M_{WR} \rightarrow g_L = g_R$

- $M_P >> M_{WR} \rightarrow g_L > g_R$ *(favored by coupling unification:* (Chang, Parida, RNM'84))

Case (i): Majorana N production via W_R

- Live WR production: $u\bar{d} \to W_R \to l^+ N$
- Subsequent N-decay via (a) νN mixing (b) W_R exchange
- Generic type I : tiny $V_{\ell N}$ (a) negligible;

(b)→



$N \to l^\pm jj$

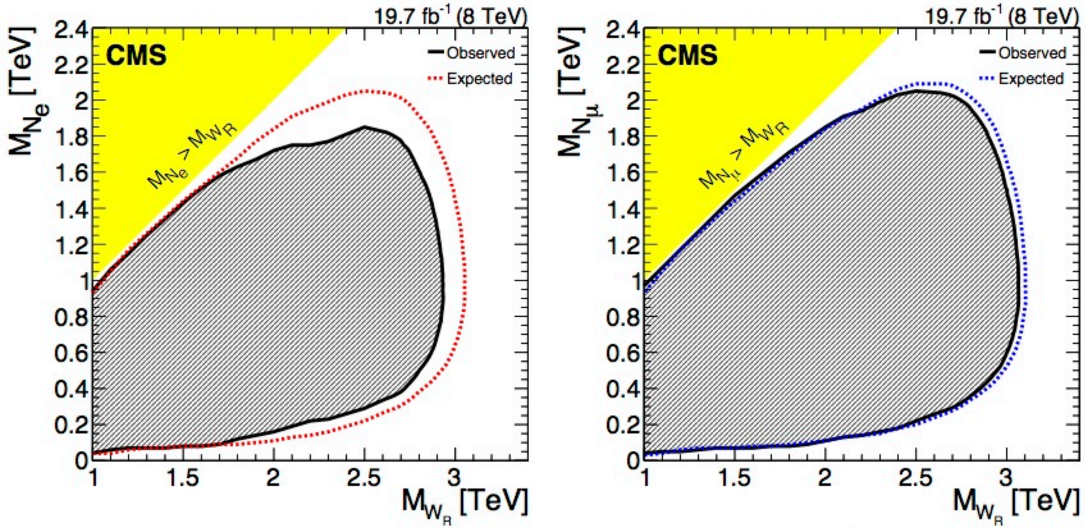
(Keung, Senjanovic'82)

$$A_{\ell^+\ell^+jj} \propto M_{N,ik}^{-1}$$

- Golden channel: $\ell_i \ell_k jj$; probes M_N flavor pattern

Current LHC analysis: only W_R graph

- Current W_R limits from LHC 2.9 TeV: ($g_L = g_R$)



CMS arXiv:1407.3683

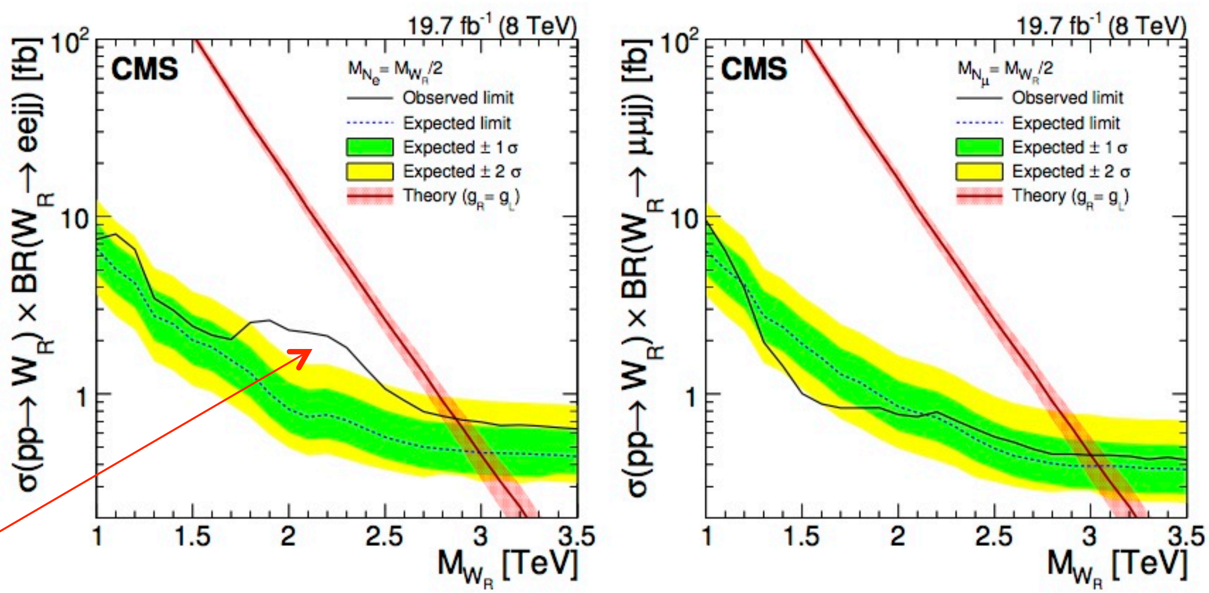
- 14-TeV LHC reach for M_{WR} upto 6 TeV with 300 fb^{-1}

Datta et al; del Aguila, Aguilar Saavedra; Ferrari et al., Gninenko et al, Maiezza, Nemevsek,.Nestii, Senjanovic, Zhang;Tello, Vissani; Chakrabortty, Gluza, Sevillano and Szafron; .Das, Deppisch, .Kittel, Valle;

- Helicity of W_R :tb mode;angular distribution (Han, Lewis, Ruiz, Si)

Any Hints from expts ?

- 2.8 **σ** excess in ee channel seen in CMS:



- (details in U. K. Yang talk)

- Possible interpretation: $g_R/g_L=0.6$, $M_{WR} = 2.1$ TeV; $V_{eN} = 0.9$; (Deppisch, Gonzalo, Patra, Sahu, Sarkar: arXiv:1407.5384)
- *Caution: no evidence for N (ljj) peak in CMS data*

Case (ii) TeV LR seesaw with enhanced $V_{\ell N}$

- LR embedding of previous texture (Dev, Lee, RNM'13)

e.g. $m_D = \begin{pmatrix} m_1 & \delta_1 & \epsilon_1 \\ m_2 & \delta_2 & \epsilon_2 \\ m_3 & \delta_3 & \epsilon_3 \end{pmatrix}$ $M_N = \begin{pmatrix} 0 & M_1 & 0 \\ M_1 & 0 & 0 \\ 0 & 0 & M_2 \end{pmatrix}$

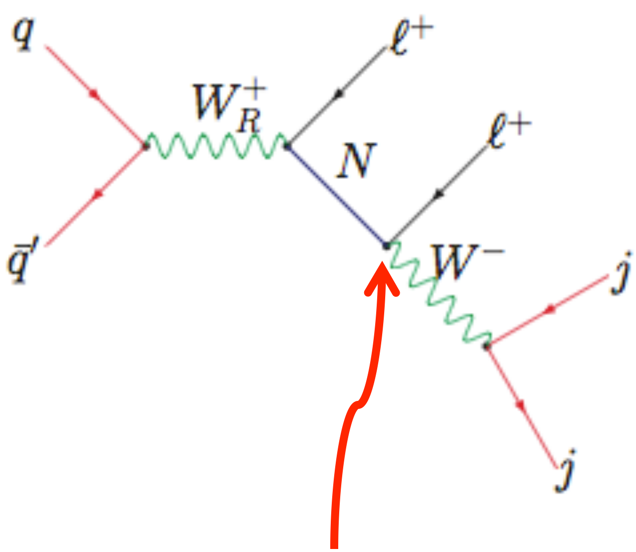
- $V_{\ell N} \sim \frac{m_D}{M_N}$ "large" $A_{LR}^{LHC}(\ell^+\ell^+ jj) \propto m_D M_N^{-1}$

- *Observable Collider signals reappear !!*

New RL contribution to like sign dilepton signal

$V_{\ell N} \sim 0.01 - 0.001$ can be probed:

- New graphs can dominate WR signal (Chen, Dev, RNM' arXiv: 1306.2342- PRD)



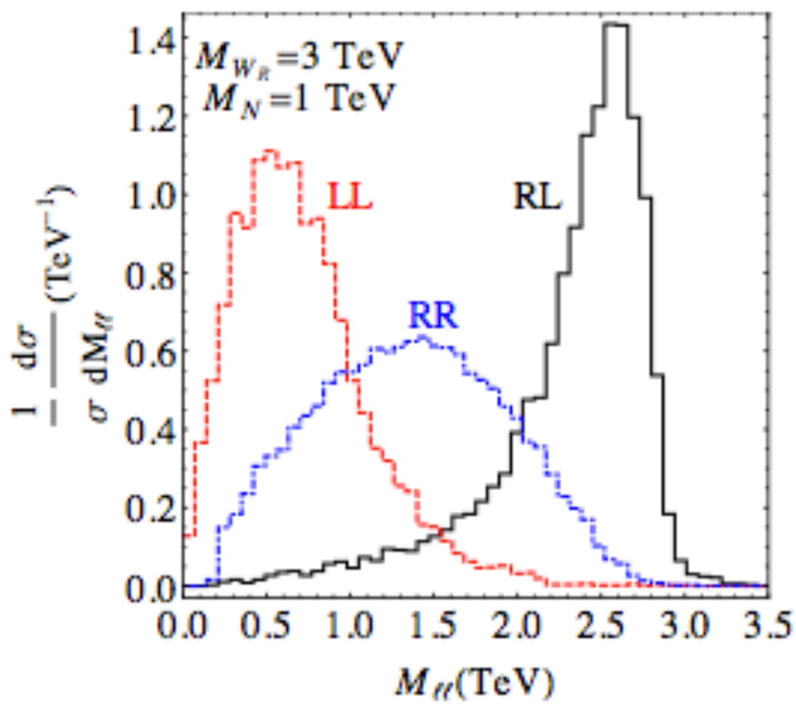
$$q\bar{q} \to W_R \to \ell + N;$$
$$N \to \ell W_L$$

(RL diagram)

- Flavor dependence will probe Dirac mass M_D profile:

Distinguishing RR from RL

- Post-observation of W_R Dilepton invariant mass plots can distinguish RL from RR



(Chen et al using Han, Lewis, Ruiz, Si)

LR seesaw at colliders as a probe of origin of matter

- Leptogenesis attractive feature of seesaw:
- Can we learn anything about leptogenesis from W_R searches at LHC ?
- Analysis of this started by Frere, Hambye, Vertongen ($M_{WR} > M_N$)

for generic models assuming maximal CP asym. $\varepsilon \sim 1$

- *Requires M_{WR} > 18 TeV due to strong washout;*
- Reinvestigated in models, with larger Yukawas and lepton mass fit (Dev, Lee, RNM'14)
- Larger Yukawa, flavor effects→ M_{WR} > 3 TeV

Parameteric dependence on RH Majorana mass

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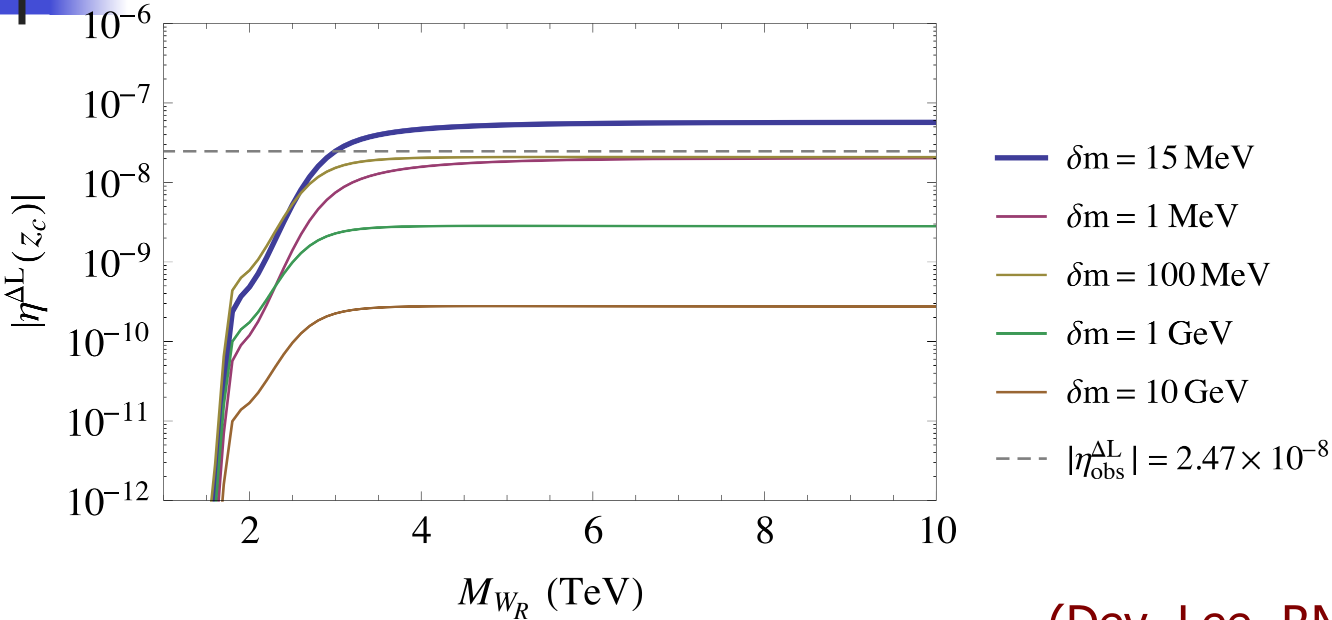


(Dev, Lee, RNM'14, 1407.xxxx)

- $\delta m = M_{N,11}$ → M_{WR} >3 TeV (in LHC reach)
- $M_N > M_{WR}$, leptogenesis not viable (Deppisch, Harz, Hirsch'14)

Summary:

- LHC can be an effective probe of TeV scale SM and left-right seesaw for small m_ν
- Premium channel for probing WR at LHC: *like sign dileptons (type I) or trileptons (Inverse)*

  -can probe details of seesaw flavor structure;
- TeV WR → observable LFV $(\mu \to e + \gamma)$ (Talks by Ilakovac, Weiland, Morrisi)i and $\beta\beta_{0\nu}$ can provide supplementary info!
- 100 TeV machine – a powerful tool and can extend the W_R mass reach to 30 TeV (Rizzo'14)!!

Possible Origin of $\delta\kappa$ from quark sector

- Finite $\delta\kappa$ generated at one loop e.g.with quark seesaw :

$$\rightarrow \delta\mu^2 Tr(\phi^\dagger \tilde{\phi})$$

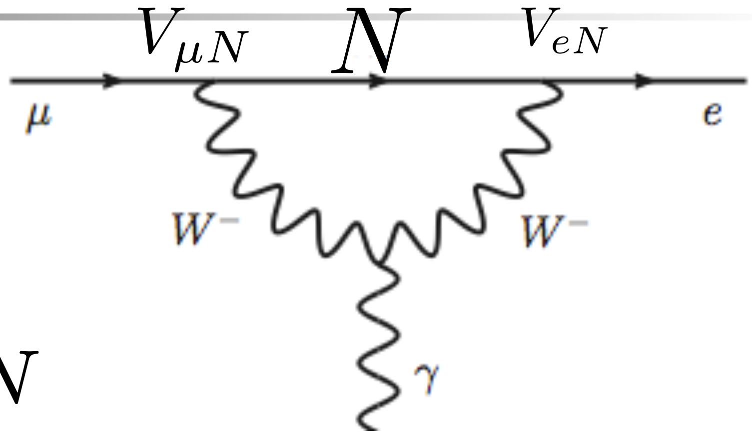
$$\delta\mu^2 \sim \frac{\delta m^2_{W_L W_R} m^2_{\psi_T}}{v_R^2}$$

- $\delta\kappa \sim \frac{\delta\mu^2}{\kappa}$ → v_R < 10-100 TeV

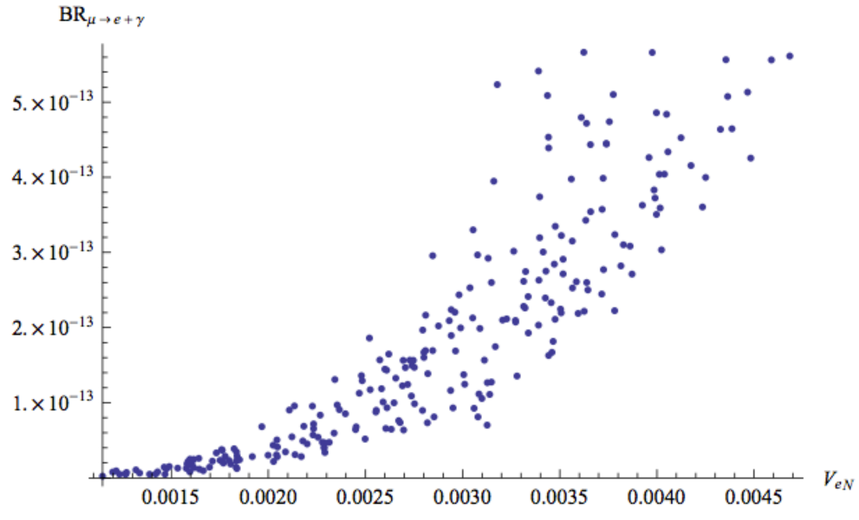
"Large" $V_{\ell N}$ and $\mu \to e + \gamma$

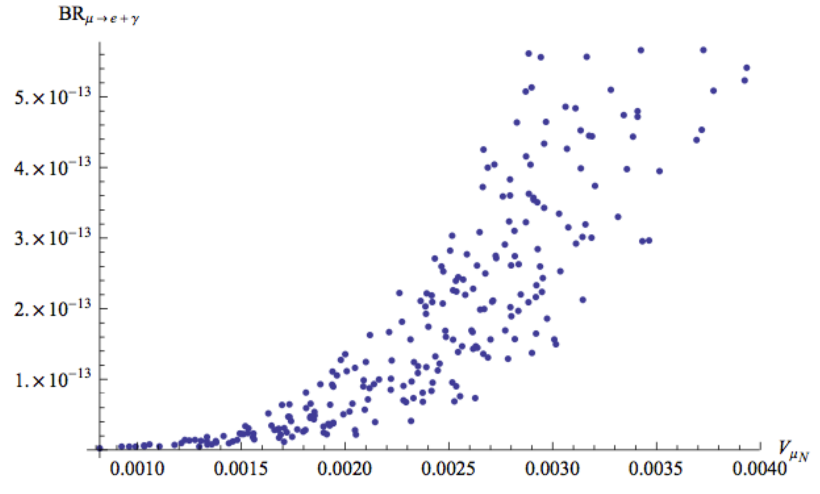
- New graphs:

(Pilaftsis; de Gouvea; Alonso, Gavela, Dhen, Hambye;…)



- Predictions of the model vs $V_{\ell N}$





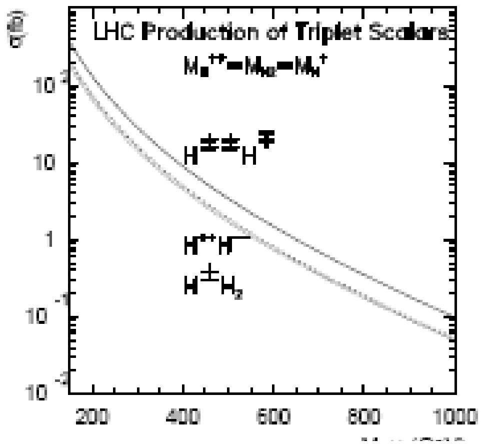
Testing Type II:

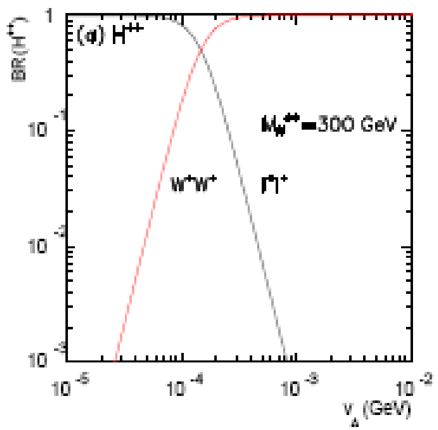
- Doubly charged member →Striking signal

$$u\bar{u} \to \delta^{++}\delta^{--}; u\bar{d} \to \delta^{++}\delta^{-} \qquad \delta^{++} \to l^{+}l^{+}, W^{+}W^{+}$$

Production

Decay



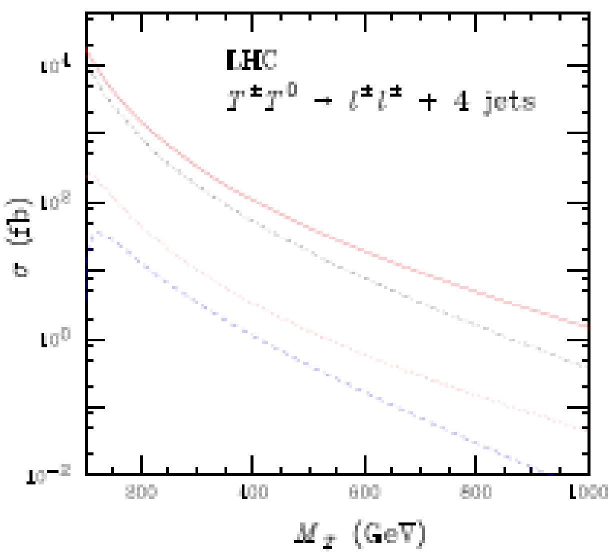


- Final state: $\ell^{+}\ell^{+}\ell^{-}\ell^{-}$ inv mass can be used to reduce bg.
- LHC reach ~TeV; leptonic couplings give nu mass matrix (roughly) (Han, Perez, Huang, Li, Wang; Akyroid, Aoki; Azuelos,…)

Signals of Type III

- Y=0, fermion triplet: (Bajc, Senjanovic, Nemesvec,..)

$$q\bar{q} \rightarrow Z^*/\gamma^* \rightarrow \Sigma^+\Sigma^-$$
$$q\bar{q}' \rightarrow W^* \rightarrow \Sigma^+\Sigma^0 \qquad \Sigma^0 \rightarrow l^+W^-$$
$$W^- \rightarrow jj$$



Like sign dileptons+jets

LHC Reach <TeV

Bound on LR scale

- Most stringent bounds come from CP viol. Observables e.g. $\varepsilon, \varepsilon', d_n^e$ depends on how CP is introduced:

<u>**Two minimal scenarios**</u>

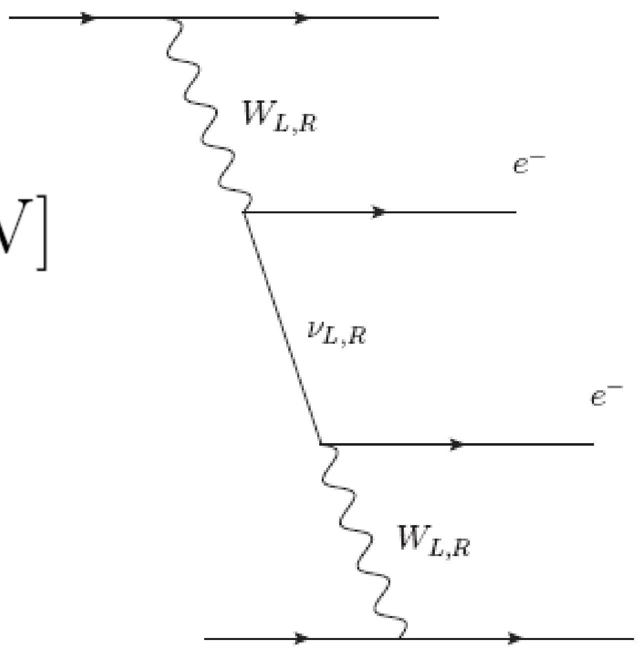
- **Parity defined as usual:($\psi_L \leftrightarrow \psi_R$) minimal model:**
- $\theta_L^{CKM} = \theta_R$;2 CP phases $M_{W_R} \geq 4TeV$ **(An,Ji,Zhang,RNM ' 07)**
- **Parity as C (as in SUSY i.e.** $\psi \leftrightarrow \psi^c$ **)** $\theta_L^{CKM} = \theta_R$ **more CP phases**
(Maezza, Nesti Nemevsek,Senjanovic' 10)

$$M_{W_R} \geq 2.5TeV$$

- **With SUSY: bounds weaker: > 1-2 TeV** **(An, Ji, Zhang' 08)**
- **Collider (CDF,D0) 640-750 GeV;** $M_{Z'} > 1.3 - 1.7 M_{W_R}$

Bounds from Nu-less double beta decay

- New contributions from WR-N exchange **(only for Case I)** **(RNM, 86; Hirsch, Klapdor, Panella 96)**
- **Diagram:**



$$\rightarrow \quad m_{W_R} \geq 1.1 \left(\frac{\langle m_N^{(V)} \rangle}{1\mathrm{TeV}} \right)^{(-1/4)} [\mathrm{TeV}]$$

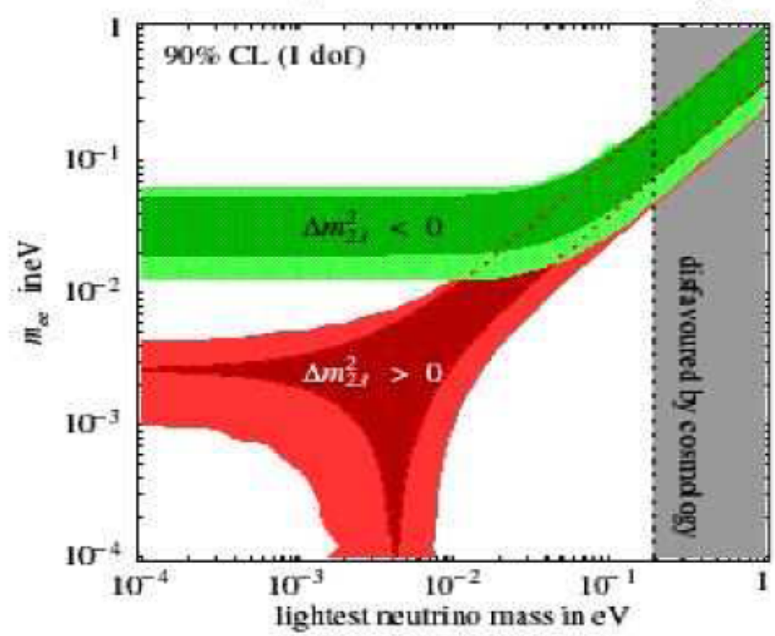
From Ge76:

TeV Seesaw signal from $\beta\beta_{0\nu}$

- **Nu contribution**:

Inverse hierarchy

Normal hierarchy



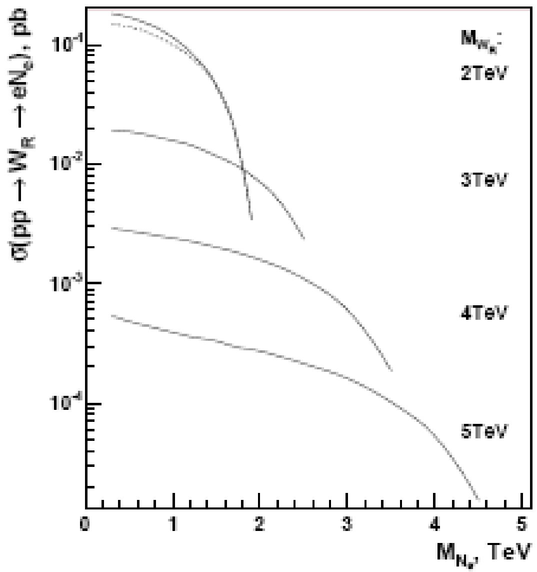
Punch line:

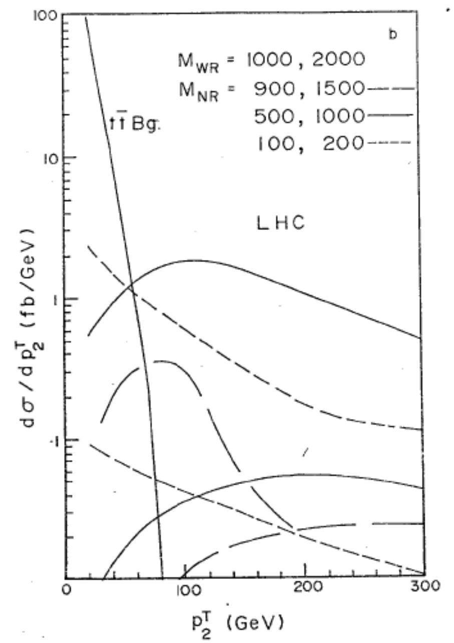
- **Suppose long baseline** → $\Delta m^2_{31} > 0$
- **and nonzero signal for** $\beta\beta_{0\nu}$ (+ RP if susy)

→could be a signal of TeV WR and type I

LHC Reach for WR

- (Ferrari et al' 00 ; Gninenko et al, 07) Datta, Guchait, Roy' 92





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m_{W_R}[TeV]	m_{ν_R} [TeV]	$\int L$	energy
4 (2)	2 (1)	30 /fb	14(7) TeV
2.1 (1.5)	2.1	100/pb	14(10) TeV

(Large PT cut to reduce tt-bar bg)

Generating neutrino masses

- Break Discrete sym.

$$< \phi_i >= \begin{pmatrix} \delta\kappa_i & 0 \\ 0 & \kappa_i \end{pmatrix}$$

with $\delta\kappa_i \ll \kappa_i$ by sym.(loops)

- Leads naturally to

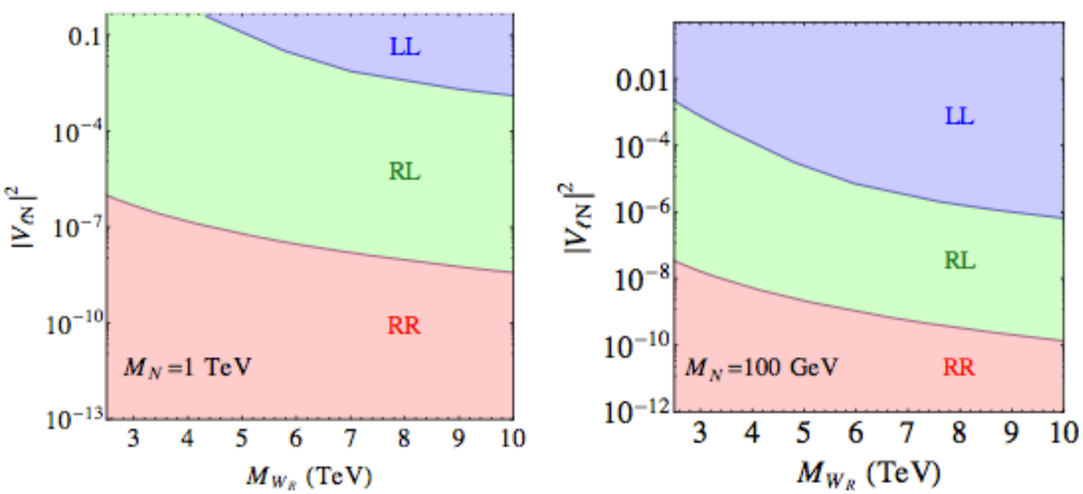
$$m_D = \begin{pmatrix} m_1 & \delta_1 & \epsilon_1 \\ m_2 & \delta_2 & \epsilon_2 \\ m_3 & \delta_3 & \epsilon_3 \end{pmatrix} M_N = \begin{pmatrix} 0 & M_1 & 0 \\ M_1 & 0 & 0 \\ 0 & 0 & M_2 \end{pmatrix} \delta_i, \epsilon_i \ll m_i$$

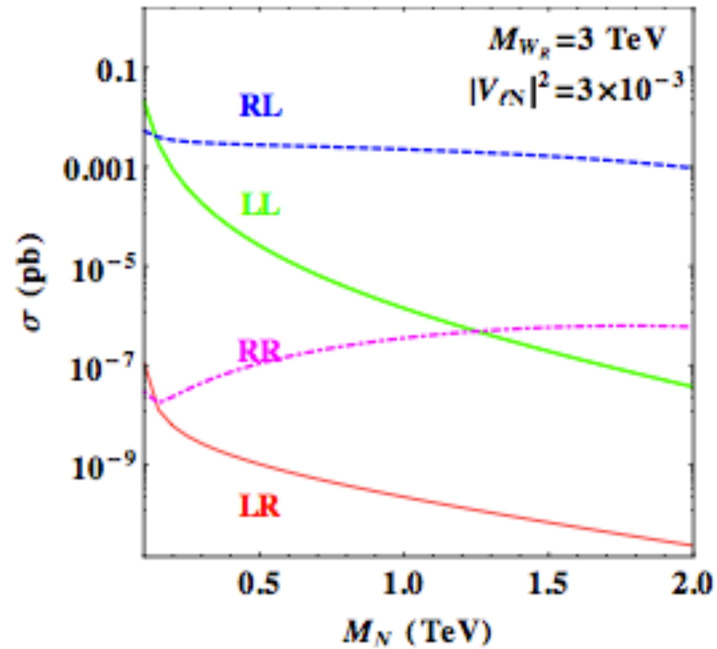
- $\delta\kappa_2 \sim 10^{-5}; \delta\kappa_{1,3} \sim 10^{-3}$ required for fit !!

- Induced by loops with right magnitude if $v_{R,i}$ < 10 TeV.

Domains where RL dominates over RR

- Phase diagram: (Chen, Dev, RNM)





- Relative signal strength: RR vs RL: (mu channel)

LR embedding nontrivial

- SM doublet gets replaced by a bi-doublet→

  same Yukawas responsible for both neutrino

  Dirac mass and charged leptn mass:

- A working example: (Dev, Lee and R. N. M'2013, PRD)

- $$M_\ell = \begin{pmatrix} 0 & h_{12}\kappa_3 & h_{13}\kappa_2 \\ 0 & h_{22}\kappa_3 & h_{23}\kappa_2 \\ 0 & h_{32}\kappa_3 & h_{33}\kappa_2 \end{pmatrix}$$ + small terms $$M_D = \begin{pmatrix} h_{11}\kappa_1 & 0 & 0 \\ h_{21}\kappa_1 & 0 & 0 \\ h_{31}\kappa_1 & 0 & 0 \end{pmatrix}$$ +small

- $$M_R = \begin{pmatrix} 0 & M_1 & 0 \\ M_1 & 0 & 0 \\ 0 & 0 & M_2 \end{pmatrix}$$ → neutrino fit with $V_{\ell N} \sim 10^{-2}$

NEUTRINO FITS WITH ENHANCED $V_{\ell N}$

$$M_D = \begin{pmatrix} 14.0638 & -7.51379\times 10^{-10} & -0.000179257 \\ 0 & 1.41139\times 10^{-9} & -0.0000407079 \\ 0 & 0 & -0.0000718846 \end{pmatrix} \quad M_e = \begin{pmatrix} 0.00153973 & -0.0511895 & -1.61367 \\ 0 & 0.0961545 & -0.366453 \\ 0 & 0 & -0.647105 \end{pmatrix}$$

$$M_R = \begin{bmatrix} 0 & 814.118 & 0 \\ 814.118 & 0 & 0 \\ 0 & 0 & -2549.95 \end{bmatrix}$$

$$V_{\mathrm{PMNS\ (fit)}} = V_e^T V_\nu = \begin{pmatrix} 0.821407 & 0.550361 & -0.149532 \\ -0.35362 & 0.697233 & 0.623538 \\ 0.447484 & -0.459255 & 0.7672 \end{pmatrix}$$

New feature of model: $V_{\ell N} \simeq \frac{m_D}{M_N}$ is "large" $V_{\ell N} \sim 10^{-2}$

(Lee, Dev, RNM'13)