Beyond Randall-Sundrum: The Flavour of Warped Backgrounds

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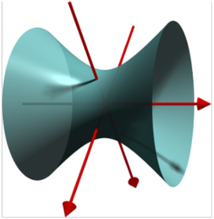
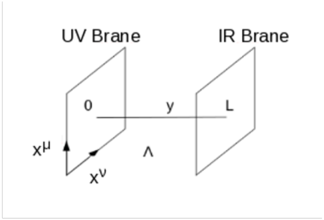
Warped Extra Dimensions

The "hard wall" RS model looks like:





The spacetime is bounded by 3-branes at $y = 0$ and $y = L$ with an AdS background $ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 = e^{-2A(z)}\eta_{MN}dx^M dx^N$

The warping of the fifth dimension results in a suppression of the Higgs v.e.v. towards the IR $\boxed{v_{eff} = ve^{-k|y|}}$

Explanation of the hierarchy $M_{Pl} >> M_{EW}$ and fermion masses.

Warped Extra Dimension With a Soft Wall

Can remove the IR brane and let the fifth dimension go to infinity.

Avoid KK continuum with decaying exponential of dilaton field that goes to zero as $z \to \infty$

$$S = \int d^5x \sqrt{g} e^{-\phi(z)} \mathcal{L} \tag{1}$$

This is known as the Soft Wall model. The action for a fermion in the 5-dimensional bulk can be expressed with z-dependant mass term

$$S_{matter} = \int d^5x \sqrt{g} e^{-\phi} (\bar{\Psi}(i\Gamma^M \nabla_M - M(z))\Psi) \tag{2}$$

where $\nabla_M = D_M + \omega_M$ and D_M is a component of the gauge covariant derivative and ω_M is a component of the spin connection. The Γ matrix carries a spacetime index since $\Gamma^M = E^M_A \gamma^A$ where E^M_A is a Fünfbein component.

For a gauge field:

$$\mathcal{L}_{gauge} = -\frac{1}{4} F_{MN} F^{MN} + \frac{1}{2} M_A^2 A_M A^M \tag{3}$$

KK reduction

Decompose each field into a part depending on the first four coordinates and another depending only on the fifth.

Treat the situation like a particle in a box (the compact extra dimension), coupled to a usual Poincaré invariant QFT in 4D.

Fermions

$$\Psi_{L,R} = \sum_{n=0}^{\infty} e^{2A(z)} \psi_{L,R}^{(n)}(x) f_{L,R}^{(n)}(z)$$

$$\int_R^\infty dz\, f_{L,R}^{(n)} f_{L,R}^{(m)} = \delta_{nm}$$

Bosons

$$A_\mu(x,z) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x) f^{(n)}(z)$$

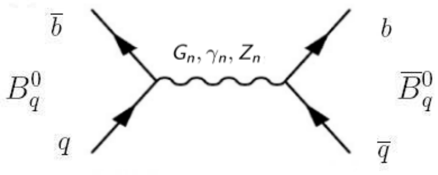
$$\int_R^\infty dz\, e^{-A(z)} f_{L,R}^{(n)} f_{L,R}^{(m)} = \delta_{nm}$$

The label n runs to infinity and is analogous to the harmonic number in a standing wave in a box scenario.

Two-two processes

Can we detect the extra dimension via interactions of KK bosons and SM fermions?

For example, we can look for:



with heavy intermediate virtual bosons - difficult to produce on shell but can probe with flavour physics.

In particular, flavour changing neutral current processes provide tight constraints on NP theories, due to low SM background (the diagram above doesn't exist in the SM).

Non-Universal Gauge Couplings

Consider term coupling fermion to guage field in the 5D bulk

$$-g\bar{\Psi}iE_A^M\gamma^A A_M\Psi$$

After KK expansion, find 4D coupling

$$\boxed{g_{ijn} = g\int_R^\infty dz\, f_i f_j f_n^A} \tag{4}$$

where f^A is the bosonic profile and the f's the fermionic profiles.

For massive modes (all but zero modes of photon/gluon) the gauge profile will not be flat.

Non-universal couplings $\Rightarrow$ flavour violating vertices between KK modes of the neutral bosons and zero-mode (SM) fermions.

Extent of flavour violation depends on the steepness of the gauge profile.

In a flat region, the generation of FCNC's are suppressed, since degree of non-universality is small. Note $M(z) = \frac{c_0}{R} + \frac{c_1}{R}\frac{z^{\alpha}}{R'^{\alpha}}$

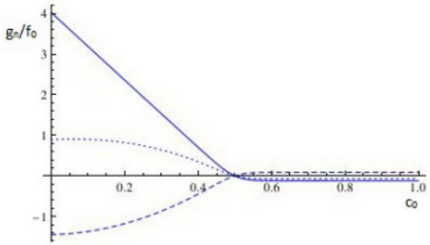
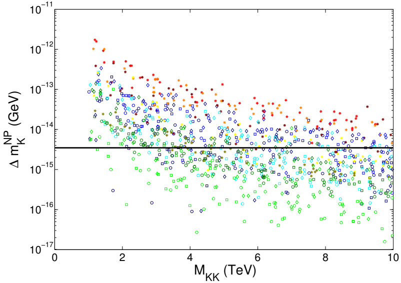
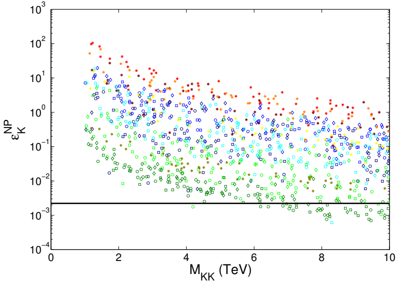


Figure: Relative coupling of the first three (solid, dashed, dotted) KK modes of a massless gauge field to the fermion zero modes localised at c0

Kaon physics in the Soft-Wall with quadratic dilaton ($\phi = z^2/R'^2$) has been studied $\Rightarrow$ amending the geometry can help satisfy constraints.

Conversely, flavour physics can probe the structure of the extra dimension.





P. R. Archer, S. J. Huber, and S. Jager JHEP 1112 (2011) 101, [arXiv:1108.1433]

Boson Propagators

Need to evaluate boson propagators.

$$\boxed{A_\mu(x,z) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x) f^{(n)}(z)} \tag{5}$$

$$\langle 0 | T A^\mu(x,z) A^\nu(x',z') | 0 \rangle = \sum_{n=0}^{\infty} \int \frac{d^4p}{(2\pi)^4} i \frac{f^n(z) f^n(z')}{p^2 - m_n^2} e^{-ip.(x-x')} \Pi^{\mu\nu} \tag{6}$$

Can identify the position space fifth dimensional part as

$$G_p(z,z') = \sum_{n=0}^{\infty} \frac{f^n(z) f^n(z')}{p^2 - m_n^2}$$

Boson Propagators

The equations of motion for the profiles imply

$$e^{\phi}\frac{z}{R}\left(\partial z\left(e^{-\phi}\frac{R}{z}\partial z\right)-p^2\right)G_p^{(\gamma/G)}=e^{\phi}\frac{z}{R}\delta(z-z') \tag{7}$$

we may expand (since $m_0=0$):

$$G_p^{(\gamma/G)}(z,z')=\frac{f_A^0(z)f_A^0(z')}{p^2}+\sum_{n=1}^{\infty}\frac{f_A^n(z)f_A^n(z')}{p^2-m_n^2} \tag{8}$$

$$=\frac{A_0}{p^2}+G_0^{(\gamma/G)}(z,z')+p^2G_1^{(\gamma/G)}(z,z')+\mathcal{O}(p^4) \tag{9}$$

with $A_0=$ const. Then to zeroth order, we have

$$\partial_z\left(e^{-\phi(z)}\frac{R}{z}\partial_z\right)G_0^{(\gamma/G)}(z,z')+A_0e^{-\phi(z)}\frac{R}{z}=\delta(z-z') \tag{10}$$

Similar equation with additional mass term exists for W/Z

The Zeroth Order Green's Function

Solving, taking in to account boundary conditions, we find

$$\boxed{G_0(u,v) = \int_R^v e^{\phi(t)} \frac{t}{R} dt + BD + g(u) + g(v)} \qquad (11)$$

where $g(z) = -A_0 \int_R^z dz_1 \left(e^{\phi(z_1)} \int_R^{z_1} e^{-\phi(z_0)} \frac{R}{z}_0 dz_0 \right)$

There is again a similar solution for the W/Z (some further complications due to massive lowest mode).

$\Rightarrow$ Use this to compute two-to-two diagrams and calculate Wilson Coefficients

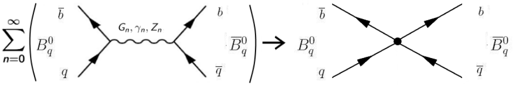
The Effective Approach

Can find "basis" of local operators for all interactions of interest (in this case $\Delta F = 2$) and write an effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \sum_{i=1}^{5} C_i Q_i^{sd} + \sum_{i=1}^{3} \tilde{C}_i \tilde{Q}_i^{sd}. \tag{12}$$

$$\begin{aligned}
Q_1^{sd} &= (\bar{d}_L \gamma^\mu s_L)(\bar{d}_L \gamma_\mu s_L) & \tilde{Q}_1^{sd} &= (\bar{d}_R \gamma^\mu s_R)(\bar{d}_R \gamma_\mu s_R) \\
Q_2^{sd} &= (\bar{d}_R s_L)(\bar{d}_R s_L) & \tilde{Q}_2^{sd} &= (\bar{d}_L s_R)(\bar{d}_L s_R) \\
Q_3^{sd} &= (\bar{d}_R^\alpha s_L^\beta)(\bar{d}_R^\beta s_L^\alpha) & \tilde{Q}_3^{sd} &= (\bar{d}_L^\alpha s_R^\beta)(\bar{d}_L^\beta s_R^\alpha) \\
Q_4^{sd} &= (\bar{d}_R s_L)(\bar{d}_L s_R) \\
Q_5^{sd} &= (\bar{d}_R^\alpha s_L^\beta)(\bar{d}_L^\beta s_R^\alpha)
\end{aligned}$$

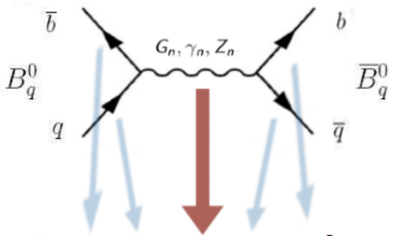
Match full theory onto effective theory $\Rightarrow$



Wilson Coefficients

For $\Delta F = 2$ we find:

$$\boxed{\begin{aligned}
C_1 &= \frac{g_s^2}{6}\mathcal{I}^{(G)}_{L_dL_bL_dL_b} + \frac{e^2}{18}\mathcal{I}^{(\gamma)}_{L_dL_bL_dL_b} + \frac{g^2}{2c_w^2}\Big(\frac{1}{2} - \frac{s_w^2}{3}\Big)\mathcal{I}^{(Z)}_{L_dL_bL_dL_b}\\
\tilde{C}_1 &= \frac{g_s^2}{6}\mathcal{I}^{(G)}_{R_dR_bR_dR_b} + \frac{e^2}{18}\mathcal{I}^{(\gamma)}_{R_dR_bR_dR_b} + \frac{g^2 s_w^4}{18c_w^2}\mathcal{I}^{(Z)}_{R_dR_bR_dR_b}\\
C_4 &= -g_s^2\mathcal{I}^{(G)}_{L_dL_bR_dR_b}\\
C_5 &= -\frac{g_s^2}{8N_c}\mathcal{I}^{(G)}_{L_dL_bR_dR_b}
\end{aligned}}$$



$$\mathcal{I}^{(A)}_{\psi_k\chi_l\xi_m\sigma_n} = \sum_{i,j=1}^{3}(U_\psi^\dagger)^{ki}(U_\chi)^{il}\left[\int_R^\infty dz \int_R^\infty dz'\, f_\psi^i(z) f_\chi^i(z) G_p^{(A)}(u,v) f_\xi^j(z') f_\sigma^j(z')\right](U_\xi^\dagger)^{mj}(U_\sigma)^{jn},$$

Hadronic matrix elements

Hadronic matrix elements for the relevant operators are calculated via

$$\boxed{\begin{aligned}
\langle K^0|Q_1^{sd}(\mu)|\bar{K}^0\rangle &= \tfrac{m_K f_K^2}{3} B_1(\mu)\ , \\
\langle K^0|Q_4^{sd}(\mu)|\bar{K}^0\rangle &= \left(\tfrac{m_K}{m_d(2M_{\mathrm{KK}})+m_s(2M_{\mathrm{KK}})}\right)^2 \tfrac{m_K f_K^2}{4} B_4(\mu)\ , \\
\langle K^0|Q_5^{sd}(\mu)|\bar{K}^0\rangle &= \left(\tfrac{m_K}{m_d(2M_{\mathrm{KK}})+m_s(2M_{\mathrm{KK}})}\right)^2 \tfrac{m_K f_K^2}{12} B_5(\mu)
\end{aligned}}$$

Similar expressions exist for the B mesons. We work at leading log with $\mu = 2M_{KK}$, with the Bag factors[†] and masses run up to this scale.

(The same is done for the strong couplings appearing in the gluonic parts of the Wilson coefficients).

[†] A.J.Buras, S.Jager, and J.Urban Nucl. Phys. B605f (2001) 600-624), [hep-ph 0102316]

RS Phenomenology

- Studies of flavour phenomenology† have shown that in RS with the Higgs localised on the IR brane, quarks must be localised near the IR to have correct zero-mode masses
- FCNC's not suppressed in this region
- Constraints from $K_0 - \bar{K}_0$ mixing forces the mass of the first KK gauge mode to be $20 - 30$ TeV.
- Can modify geometry and study effect on flavour observables

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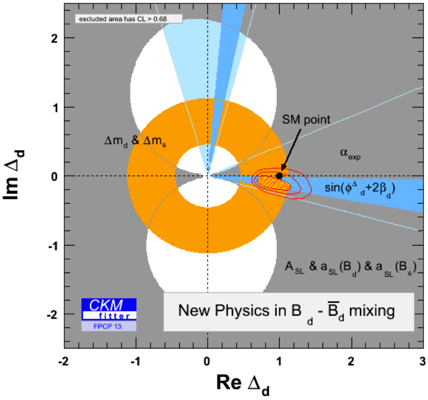
†C Csaki, A. Falkowski, and A. Weiler, JHEP 09 (2008) 008, [arXiv:0804.1954].

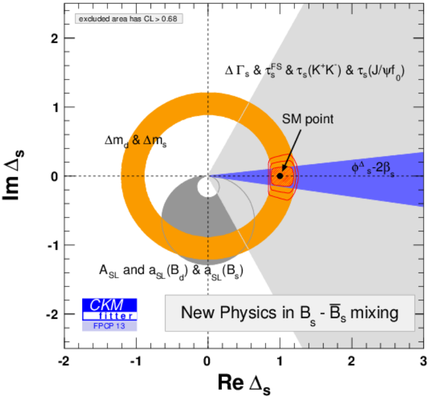
M Blanke, A. J. Buras, B. Duling, S. Gori, and A. Weiler, JHEP 03 (2009) 001, [arXiv:0809.1073].

M Bauer, S. Casagrande, U. Haisch, and M. Neubert, JHEP 09 (2010) 017, [arXiv:0912.1625].

Soft Wall B Mixing

$$M_{12}^{q} \equiv M_{12}^{SM,q}.\Delta_q$$
$$\Delta_q \equiv |\Delta_q| e^{i\phi_q^{\Delta}}$$





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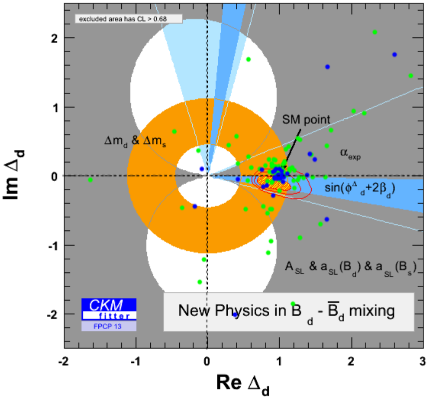

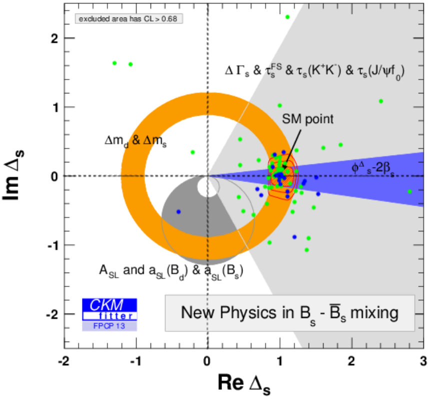

Conclusions

- The new approach to calculating boson 2pt correlators gives a close match to traditional methods.
- A good sample of the points are consistent with experimental bounds from $B - \bar{B}$ mixing for quadratic dilaton model.
- Currently working on $\Delta F = 1$ hadronic decays $b \to s\bar{q}q$ and $b \to d\bar{q}q$.
- Flavour phenomenology is sensitive to slight modification of the background geometry via the dilaton.
- Plans to study these modified backgrounds and their effect on B-physics processes.

SM recap: Diagonalising the Mass Matrix

Masses come from post SSB Yukawa couplings.

$$\mathcal{L}_{Yuk} = \left(\frac{\eta(x)+v}{\sqrt{2}}\right)\left(f^p_{ij}\bar{p}'_i p'_j + f^n_{ij}\bar{n}'_i n'_j + h.c.\right) \tag{13}$$

Matrices $M^{p,n}_{ij} = -\frac{v}{\sqrt{2}} f^{p,n}_{ij}$ might not be diagonal.

But one can always find a biunitary transformation

$$U^\dagger M V = M_d \tag{14}$$

s.t. M_d is diagonal. So we can relate the gauge and mass eigenbases via

$$\bar{\psi}'_L M \psi'_R = (\bar{\psi}_L U)(U^\dagger M V)(V^\dagger \psi'_R) = \bar{\psi}_L M_d \psi_R \tag{15}$$

where $\psi'_L = U\psi_L$ and $\psi'_R = V\psi_R$.

Specifically $p'_L = U_{(p)} p_L$, $n'_L = U_{(n)} n_L$ and $p'_R = V_{(p)} p_R$, $n'_R = V_{(n)} n_R$.

NB: mass states are admixtures of gauge states, defined by CKM matrix.

SM recap: Flavour Changing Currents

Consider now a charged left-handed quark current:

$$J^{\mu}_{+} = \bar{p}'_{iL}\gamma^{\mu} n'_{iL} = \bar{p}_{aL}\gamma^{\mu} U^{\dagger}_{(p)ai} U_{(n)ib} n_{bL}, \tag{16}$$

and in contrast a neutral left-handed quark current:

$$J^{\mu}_{0} = \bar{p}'_{iL}\gamma^{\mu} p'_{iL} = \bar{p}_{aL}\gamma^{\mu} \underbrace{U^{\dagger}_{(p)ai} U_{(p)ib}}_{\mathbb{1}} p_{bL}. \tag{17}$$

Since in the SM, these couple to the gauge bosons via terms of the form $-igA_{\mu}J^{\mu}$, then generically we have

$$-iA_{\mu}\bar{q}_{a}\gamma^{\mu} \underbrace{U^{\star}_{ai} g_{ij} U_{jb}}_{g_{ab}} q_{b} \tag{18}$$

Now since all flavours couple universally in the standard model, $g_{ij} = g\delta_{ij}$. So for neutral currents, the couplings are unaffected by the basis change.

But for charged currents, get non-zero off-diagnonal entries in g_{ab} in the mass basis $\Rightarrow$ vertices between two different fermion flavours and boson.