A naturally light & bent dilaton

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Cornell University, Founded A.D. 1865

MANCHESTER 1824
The University of Manchester

with B.Bellazzini, C.Csaki, J.Hubisz, J.Terning

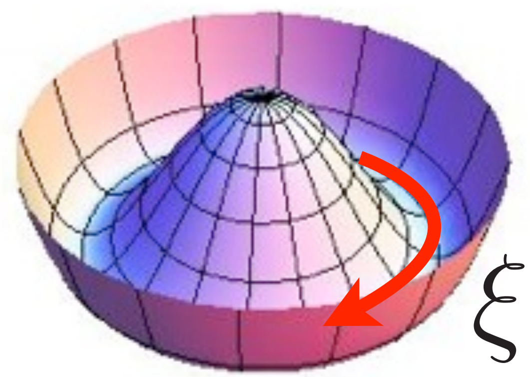
arXiv:1305.3919
arXiv:14xx.xxxx

SUSY 2014
Manchester
July 22, 2014

DILATON =Goldstone Boson of Spontaneous Breaking of Scale Invariance

$$\mathcal{L} = (\partial\Phi^\dagger)(\partial\Phi) + m^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$$

Broken phase



Spontaneous internal Symmetry Breaking: G -> H

(approximately) Massless **Goldstone mode** appear = phase

$$\Phi = e^{i\xi}(\langle\Phi\rangle + \sigma)$$

$$V(\xi) = 0$$

DILATON = Goldstone Boson of Spontaneous Breaking of Scale Invariance

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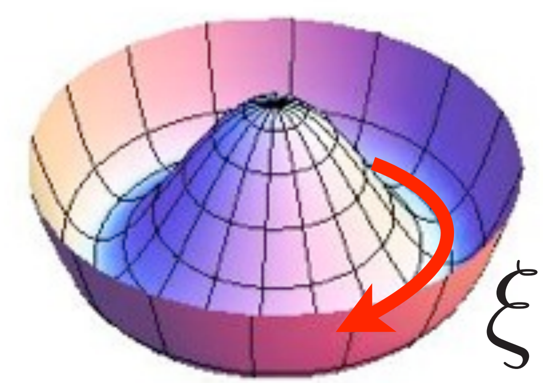


Spontaneous space-time Symmetry Breaking: G -> H

The dilaton behaves more like the **Amplitude mode**

$$\Phi = e^{i\xi}(\langle\Phi\rangle + \sigma)$$

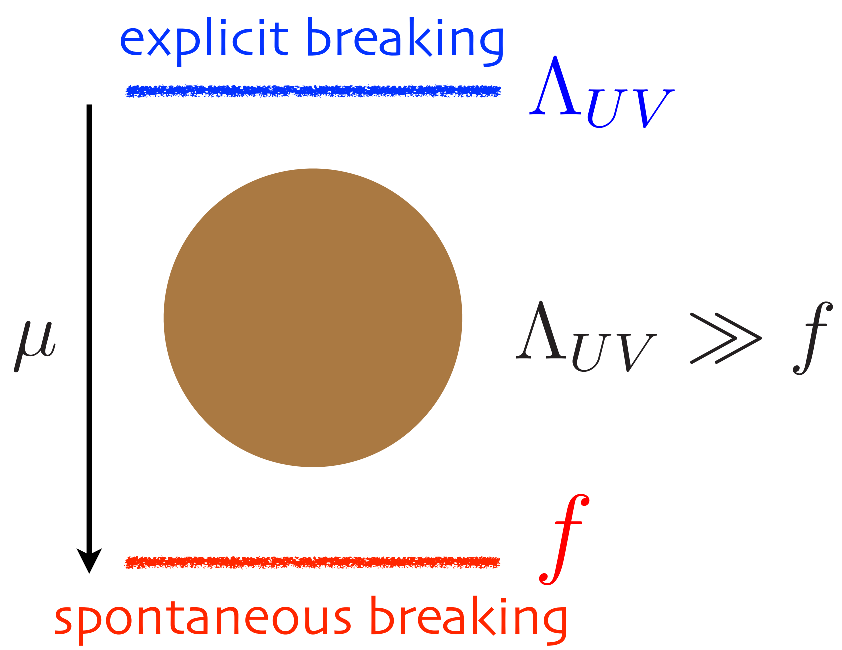
$$V(\sigma) \neq 0$$

What is the Dilaton?

Scale (conformal) invariant sector

$$x \to e^{\alpha} x\,,\ \Phi(x) \to e^{d_\Phi \alpha}\Phi(e^{\alpha}x)$$

$$S_{CFT} = \int d^4x \sum_i \mathcal{O}_i\ , \quad d_i = 4$$



- d>4: Irrelevant invariant terms are unimportant at low energies.
- d < 4: No large relevant invariant terms can be present.

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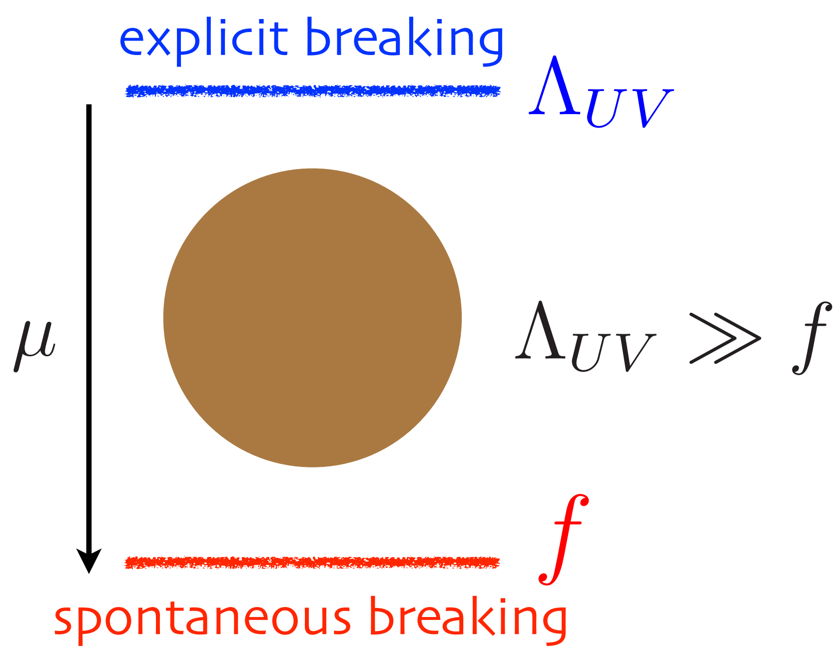

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Spontaneous breaking of scale invariance

$$\langle \Phi(x) \rangle = f^{d_\Phi} \qquad \mathrm{SO}(4,2)/\mathrm{SO}(3,1)$$

$$\sigma(x) \to \sigma(e^{\alpha} x) + \alpha f$$

$$\chi \equiv f e^{\sigma/f} \to e^{\alpha} \chi$$

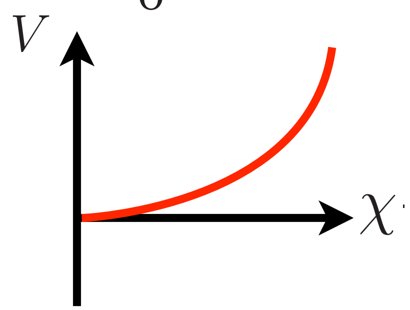
Effective theory of CFT spontaneous breaking

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial\chi)^2 - a_0\chi^4 + \cdots$$

Quartic potential allowed

Is there really a light scalar when scale symmetry spontaneously breaks?

$a_0 > 0$

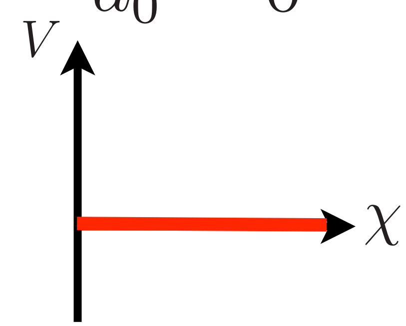


$\langle\chi\rangle \to 0$

CFT/AdS4

runaway

$a_0 = 0$

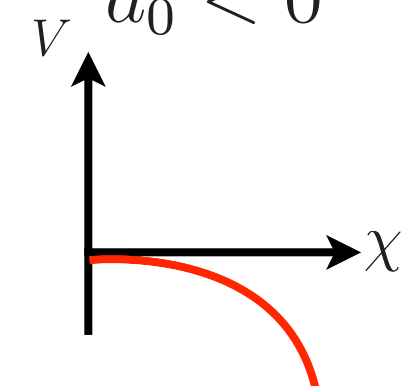


$\langle\chi\rangle = f = \, ?$

CFT/Poincare4

flat direction

$a_0 < 0$



$\langle\chi\rangle \to \infty$

CFT/dS4

runaway

Fubini '76

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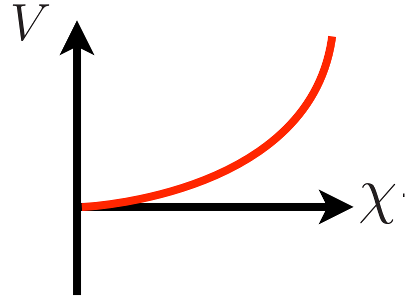

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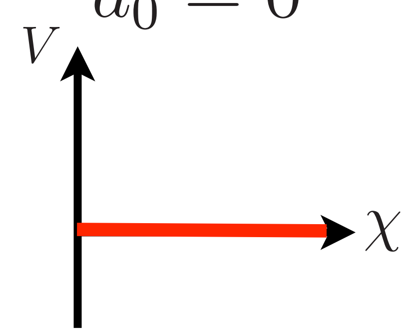

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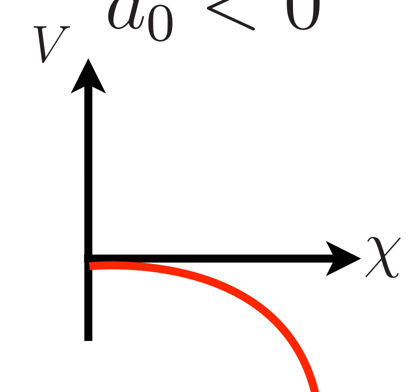

$\langle\chi\rangle \to \infty$

CFT/dS4

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Fubini '76

TUNING

$\langle a_0\chi^4\rangle = a_0 f^4 \sim (4\pi f)^2 f^2$ vacuum energy

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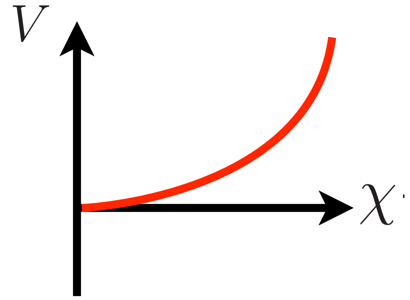

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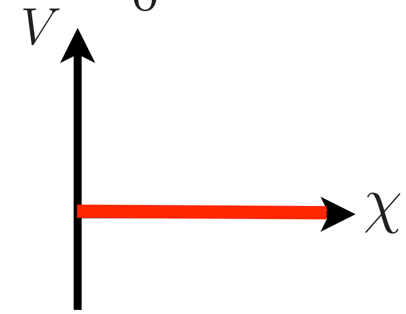

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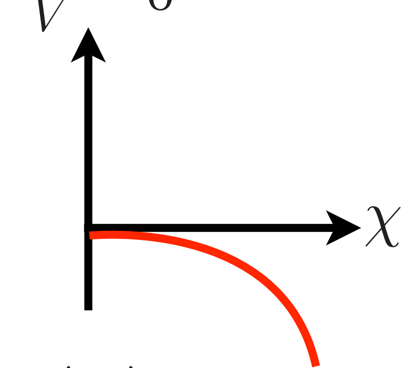

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CFT/dS4

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Fubini '76

$$\langle a_0\chi^4\rangle = a_0 f^4 \sim (4\pi f)^2 f^2$$ vacuum energy

Effective theory of CFT spontaneous breaking

We need a perturbation, aka explicit breaking

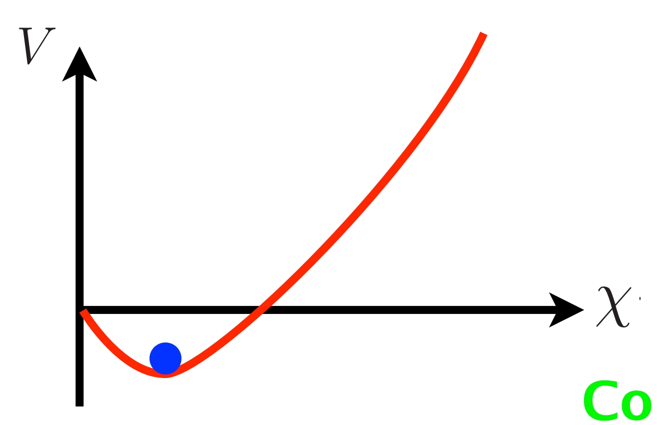
$$\mathcal{L} = \mathcal{L}_{CFT} + \lambda \mathcal{O} \qquad [\mathcal{O}] = 4 - \beta/\lambda \qquad \frac{d\lambda(\mu)}{d\log\mu} = \frac{\beta(\lambda)}{\lambda} \neq 0$$

spurion: $\mu \to \chi$

$$V(\chi) = \chi^4 F(\lambda(\chi))$$

$$F(\lambda(\chi)) = a_0 + \sum_n a_n \lambda^n(\chi)$$

Quartic gets dependence on running coupling.



Coleman, Weinberg '73

The dilaton effectively scans the landscape of quartics.

Minimum and dilaton mass

$$\langle \chi \rangle = f$$

$$V' = f^3[4F(\lambda(f)) + \beta F'(\lambda(f))] = 0$$

$$m_d^2 \simeq 4f^2 \beta F'(\lambda(f)) = -16 f^2 F(\lambda(f)) = -16 V(f)/f^2$$

i) Dilaton mass prop. to explicit breaking at condensate scale, $\beta = \beta(\lambda(f))$

ii) Potential at minimum, aka **vacuum energy**, also prop. to explicit breaking

iii) Hierarchy between UV and IR scales fixed by dimensional transmutation, dependent on the explicit breaking along the whole running

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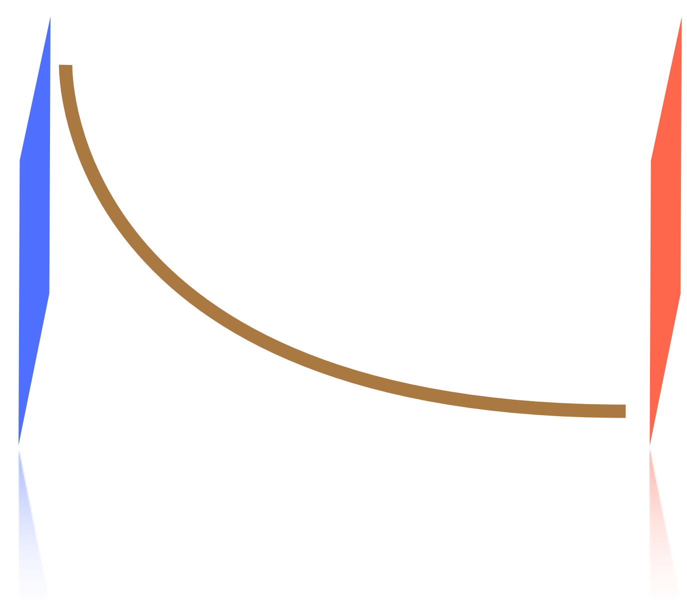
iii) **Hierarchy** between UV and IR scales fixed by dimensional transmutation, dependent on the explicit breaking along the whole running

Dynamical (theory space) requirement

$$\beta(\lambda) = \epsilon\, b(\lambda) \;, \quad \epsilon \ll 1 \;, \quad b(\lambda) = O(1)$$

Large hierarchy, light dilaton, small cosmological constant

An Extra-D Computable Example



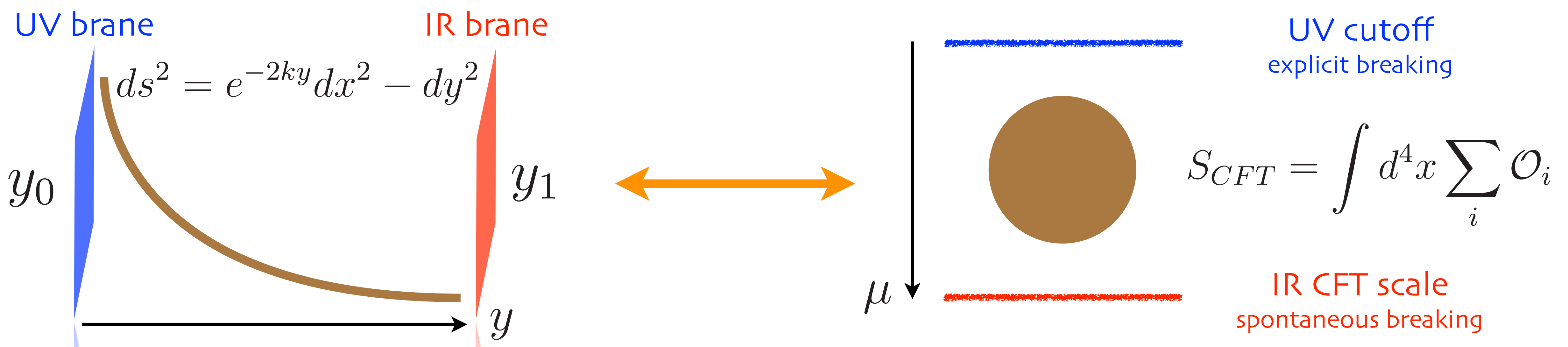
Contino, Pomarol, Rattazzi, ‘10

Bellazzini, Csaki, Hubisz, Terning, JS, ’13

Coradeschi, Lodone, Pappadopulo, Rattazzi, Vitale, ’13

Megias, Pujolas ’14

Randall & Sundrum setup



Randall, Sundrum '99

$$\mathbf{AdS}_5 \longleftrightarrow \mathbf{CFT}_4$$

$$e^{-ky} \longleftrightarrow \mu$$

$$e^{-ky_0} \longleftrightarrow \Lambda_{UV}$$

$$e^{-ky_1} \longleftrightarrow \chi$$

radion $\longleftrightarrow$ **dilaton**

The brane separation – hierarchy of scales, is fixed by $\langle \chi \rangle = f$.

AdS/CFT phenomenological correspondence

Adding explicit breaking perturbation in AdS/CFT

$$\mathbf{AdS}_5 \longleftrightarrow \mathbf{CFT}_4$$

$$\phi \longleftrightarrow \mathcal{O}$$

$$V'(\phi) = dV/d\phi \longleftrightarrow \beta(\lambda) = d\lambda/d\log\mu$$

$V(\phi) = \Lambda_{(5)}$ exactly marginal

$$(\partial\phi)|_{y=y_0} = 0 \qquad \phi|_{y=y_0} \longleftrightarrow \lambda_{UV}$$

4D gravity is readily included

$g_{MN} \longleftrightarrow$ graviton + dilaton

4D effective Lagrangian

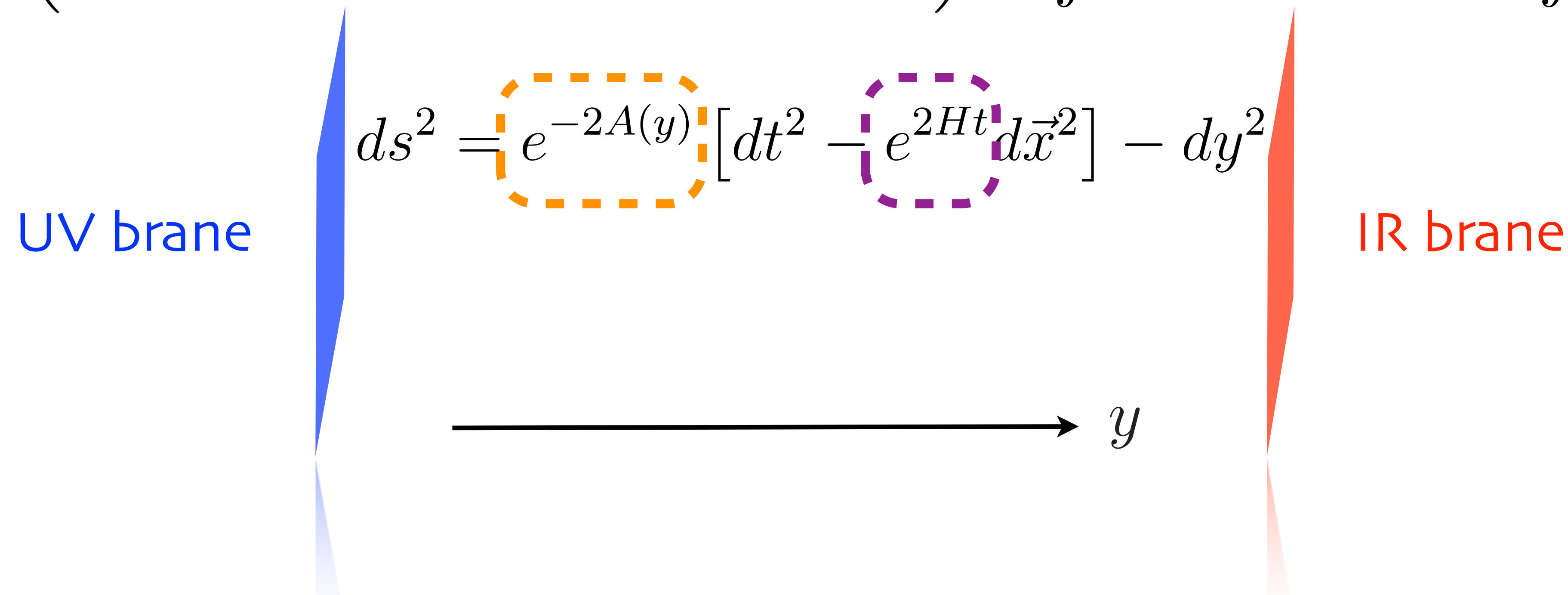
$$\mathcal{L} = \sqrt{g}\left[\mathcal{L}_{CFT} + \lambda\mathcal{O} + M_P^2 R\right]$$

The general & stabilized & bent RS

Bellazzini, Csaki, Hubisz, Terning, JS, '13

$$S=\int d^5x\sqrt{g}\left(-\frac{1}{2\kappa^2}\mathcal{R}+\frac{1}{2}g^{MN}\partial_M\phi\partial_N\phi-V(\phi)\right)-\int d^4x\sqrt{g_0}V_0(\phi)-\int d^4x\sqrt{g_1}V_1(\phi)$$

$$ds^2=e^{-2A(y)}\left[dt^2-e^{2Ht}d\vec{x}^2\right]-dy^2$$



We solve for the scalar & the **warp factor** profiles & **Hubble**

bulk E.O.M.'s

$$A'^2+H^2e^{2A}=\frac{\kappa^2\phi'^2}{12}-\frac{\kappa^2}{6}V(\phi)$$

$$\phi''=4A'\phi'+\frac{\partial V}{\partial\phi}$$

boundary conditions

$$2A'|_{y=y_0,y_1}=\pm\frac{\kappa^2}{3}V_{0,1}(\phi)|_{y=y_0,y_1}$$

$$2\phi'|_{y=y_0,y_1}=\pm\frac{\partial V_{0,1}}{\partial\phi}|_{y=y_0,y_1},$$

General 4D effective potential

$$V_{eff} = V_{IR} + V_R$$

$$V_{IR} = e^{-4A(y_1)}\left[V_1\left(\phi(y_1)\right) \mp \frac{6}{\kappa^2}A'(y_1)\right]$$

$$V_R = -3H^2(y_0, y_1)M_{Pl}^2(y_0, y_1) \qquad M_{Pl}^2 = \frac{2}{\kappa^2}\int_{y_0}^{y_1} dy\, e^{-2A(y)}$$

↕ We obtain just what we expected

Modified quartic dilaton potential from **two** sources of explicit breaking

running UV perturbation

$$V_\lambda = \chi^4 F_\lambda(\lambda(\chi))$$

UV Hubble constant

$$V_H = \chi^4 F_H(H(\chi))$$

$$V(\phi) = \Lambda_{(5)} + m^2\phi^2$$

$$m^2 = -2\epsilon k^2 \qquad \epsilon \ll 1 \qquad d_{\mathcal{O}} \approx 4 - \epsilon$$

↓ expansion in back-reaction

$$F_\lambda(\chi) \simeq \left(\Lambda_1 - \frac{\Lambda_{(5)}}{k}\right) + \left[\lambda_{IR} - \lambda_{UV}\left(\frac{\Lambda_{UV}}{\chi}\right)^\epsilon\right]^2$$

$$F_H(\chi) \simeq \left(\Lambda_0 + \frac{\Lambda_{(5)}}{k}\right)\frac{\Lambda_{UV}^2}{\chi^2}\left(1 + \frac{\Lambda_{UV}^2}{\chi^2}\right)$$

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ϵ → β/λ

$\left(\Lambda_0 + \frac{\Lambda_{(5)}}{k}\right)$ → H_{UV}^2/Λ_{UV}^2

Generalized Randall-Sundrum

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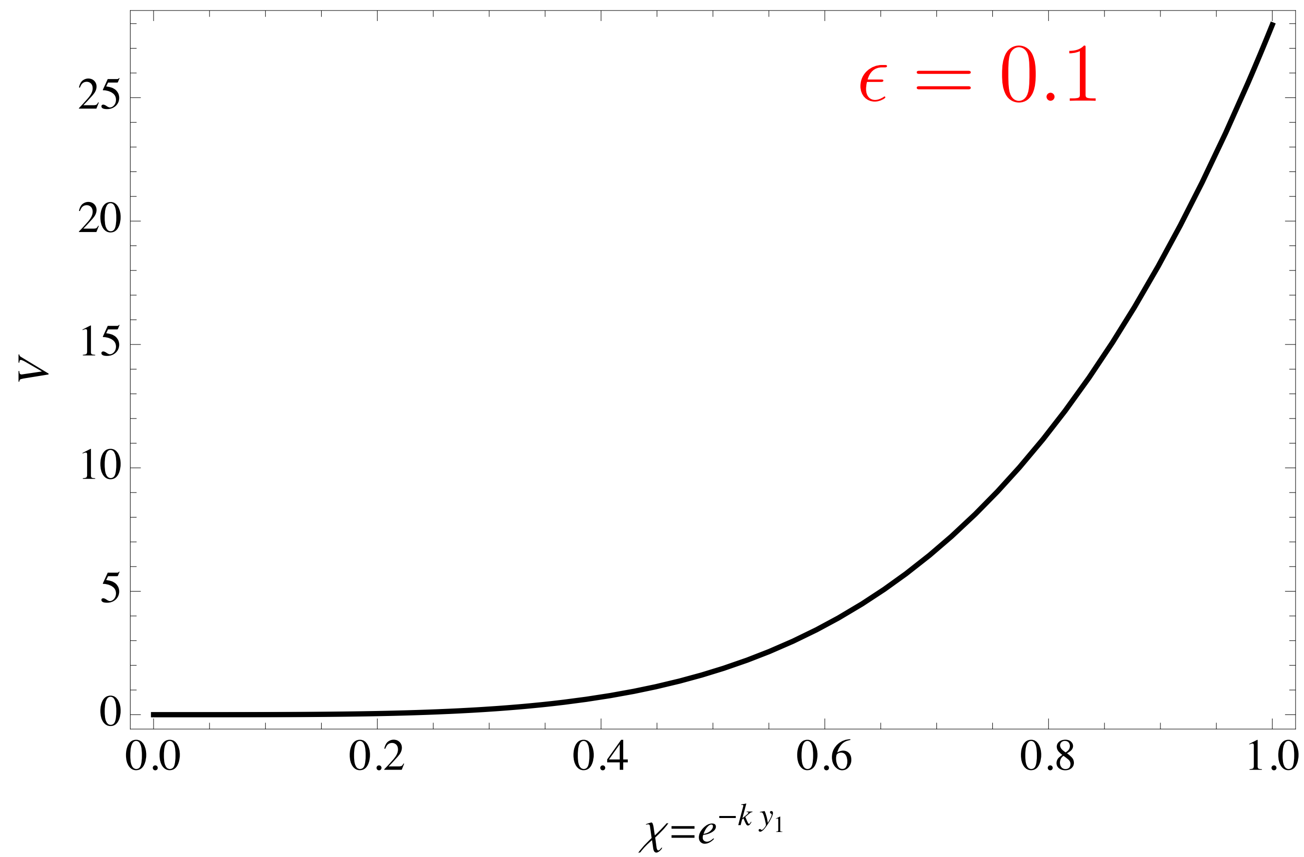
ϵ ↔ β/λ

$\left(\Lambda_0 + \frac{\Lambda_{(5)}}{k}\right)$ ↔ H_{UV}^2/Λ_{UV}^2

And **eliminating cut-off effects** (UV cosmological constant tuning) $H_{UV} = 0$,

$$V_\lambda \simeq \chi^4 \left\{\Lambda_1 - \frac{\Lambda_{(5)}}{k}\cosh\left[\lambda_{IR} - \lambda_{UV}(\Lambda_{UV}/\chi)^{\epsilon}\right]\right\}$$

The large hierarchy

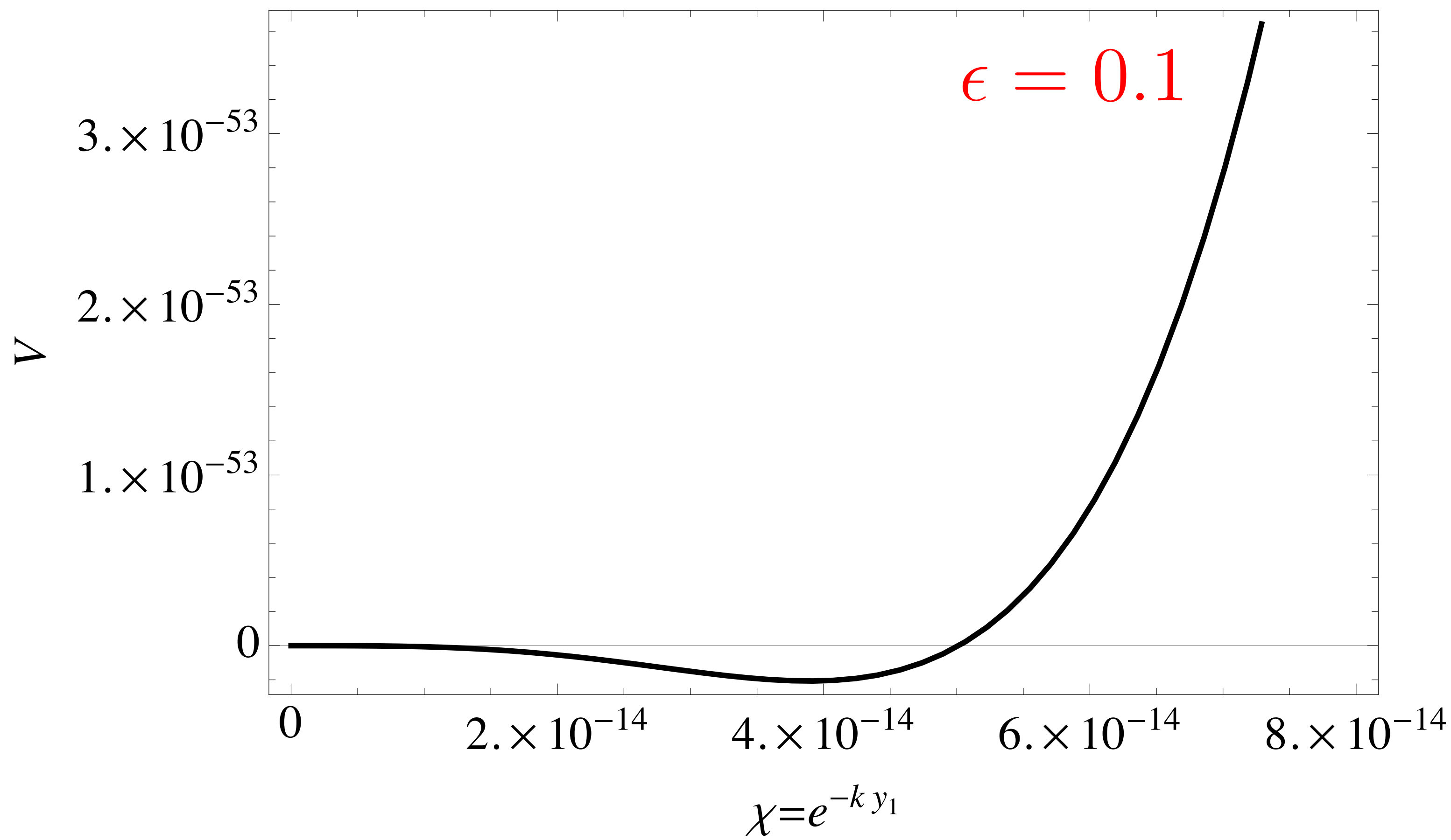


$$\langle \chi \rangle = f$$

$$\frac{f}{\Lambda_{UV}} = \left(\frac{\lambda_{UV}}{\lambda_{IR} - \,\mathrm{sign}(\epsilon)\,\mathrm{arcsech}(\Lambda_{(5)}/k\Lambda_1)} \right)^{1/\epsilon}$$

Because of slow running for a long time.

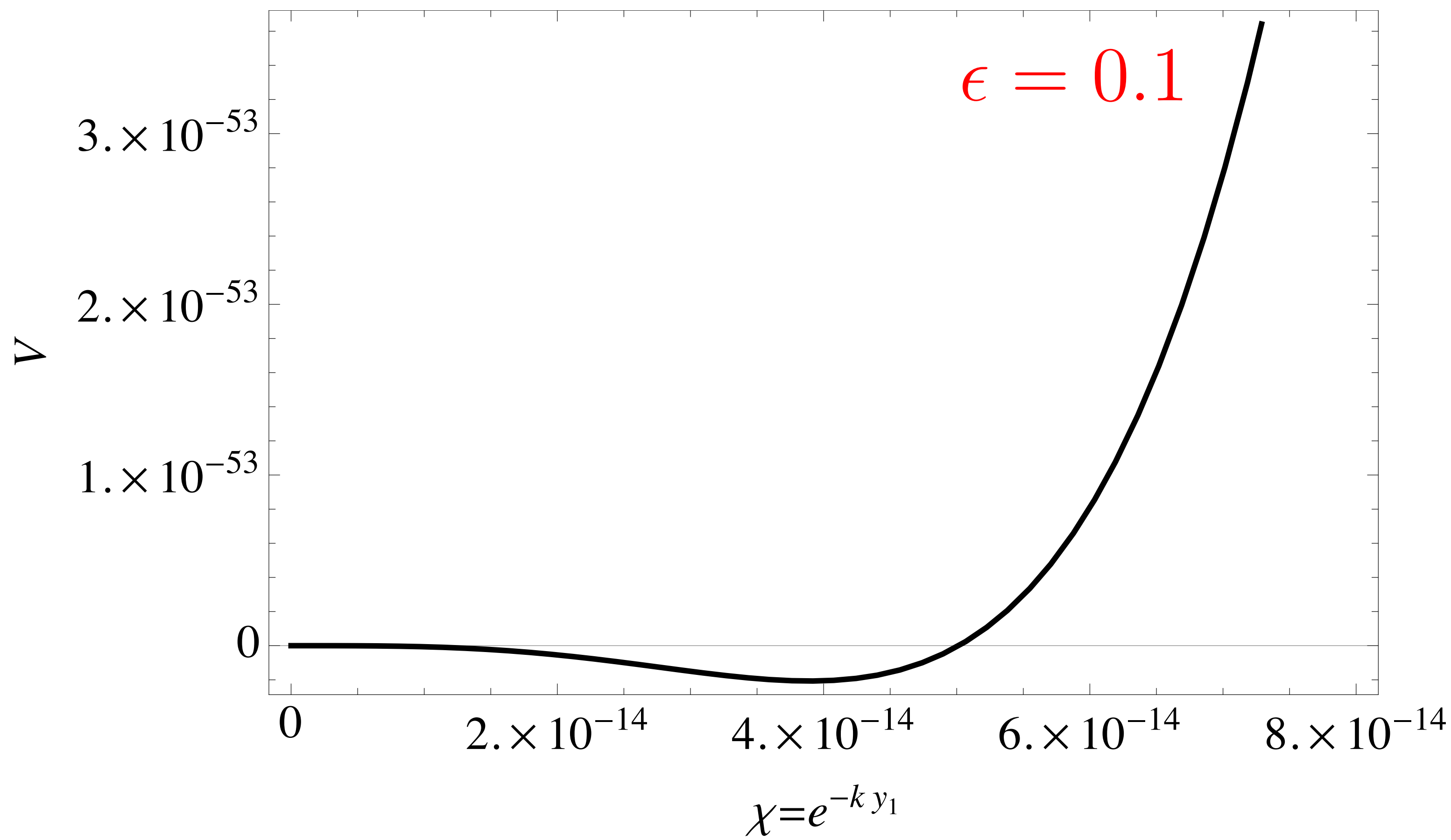
The light dilaton



$$m_\chi^2 \sim \epsilon\, 48 \lambda_{UV} \tanh\left[\frac{1}{2}(\lambda_{IR} - \lambda_{UV}(\Lambda_{UV}/f)^\epsilon)\right](\Lambda_{UV}/f)^\epsilon f^2$$

Because of slow running at the minimum.

The small cosmological constant



$$V_{IR}^{min} = -\epsilon\, 3\lambda_{UV} \tanh\left[\frac{1}{2}(\lambda_{IR} - \lambda_{UV}(\Lambda_{UV}/f)^{\epsilon})\right](\Lambda_{UV}/f)^{\epsilon} f^4$$

Because of slow running at the minimum.

Conclusions

Approximate Spontaneous Breaking of Scale Invariance offers a natural way to obtain a light scalar, the **Dilaton**,

$$\beta(\lambda) = \epsilon\, b(\lambda)\ , \quad \epsilon \ll 1\ , \quad b(\lambda) = O(1)$$

and to suppress the spontaneously generated **Vacuum energy**.

Is this possibility realized in Nature?

Inflaton as Dilaton arXiv:1406.5192

Higgs as Dilaton arXiv:1209.3299

Dilaton in Phase Transitions arXiv:14xx.xxxx

...

We just have to wait and see

Thank you for your attention

Comments on the Cosmological Constant

1) Small cosmological constant & light dilaton signal the approximate scale invariance at the IR scale:

$$V'(\phi) = dV/d\phi \longleftrightarrow \beta(\lambda) = d\lambda/d\log\mu$$

Change the bulk potential, change the running.

Chacko, Mishra, Stolarski '13

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2) The suppression is parametrically better than in SUSY:

SUSY

$$\Lambda_{CC}^{IR} = c(m_b^4 - m_f^4) \simeq c(m_b^2 + m_f^2) g_s^2 F_s^2$$

CFT

$$\Lambda_{CC}^{IR} = \tilde{c}\,\epsilon(4\pi)^2 f^4 \simeq \tilde{c}\,\epsilon\Lambda_{IR}^2 f^2$$

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3) Our result is consistent with <u>Weinberg's no-go theorem</u>:

$\epsilon = 0$ can remove the CC, but $\epsilon \neq 0$ is required for a unique vacuum

A very light state must be in the spectrum.

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A very light state must be in the spectrum.

4) UV contribution to the cosmological constant must be tuned away.

(perturbative) Example: **bulk mass**

$$V(\phi) = \Lambda_{(5)} + m^2\phi^2$$

Scaling dimension of operator

$$d_{\mathcal{O}} = 2 + \sqrt{4 + m^2/k^2}$$

Background scalar solution of E.O.M.

$$\phi(y) = \underbrace{\phi_0 e^{-ky(4-d_{\mathcal{O}})}}_{\text{running}} + \underbrace{\phi_1 e^{-kyd_{\mathcal{O}}}}_{\text{condensate}}$$

$$\phi_0 = \lim_{\Lambda_{UV}\to\infty} \Lambda_{UV}^{4-d_{\mathcal{O}}} \lambda_{UV}$$

$$\phi_1 = \frac{\langle \mathcal{O} \rangle}{2d_{\mathcal{O}} - 4}$$

$$\frac{d\lambda}{d\log\mu} \equiv \beta(\lambda) = (4 - d_{\mathcal{O}})\lambda$$