

# Electroweak Effective Operators and Higgs Physics

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SUSY 2014

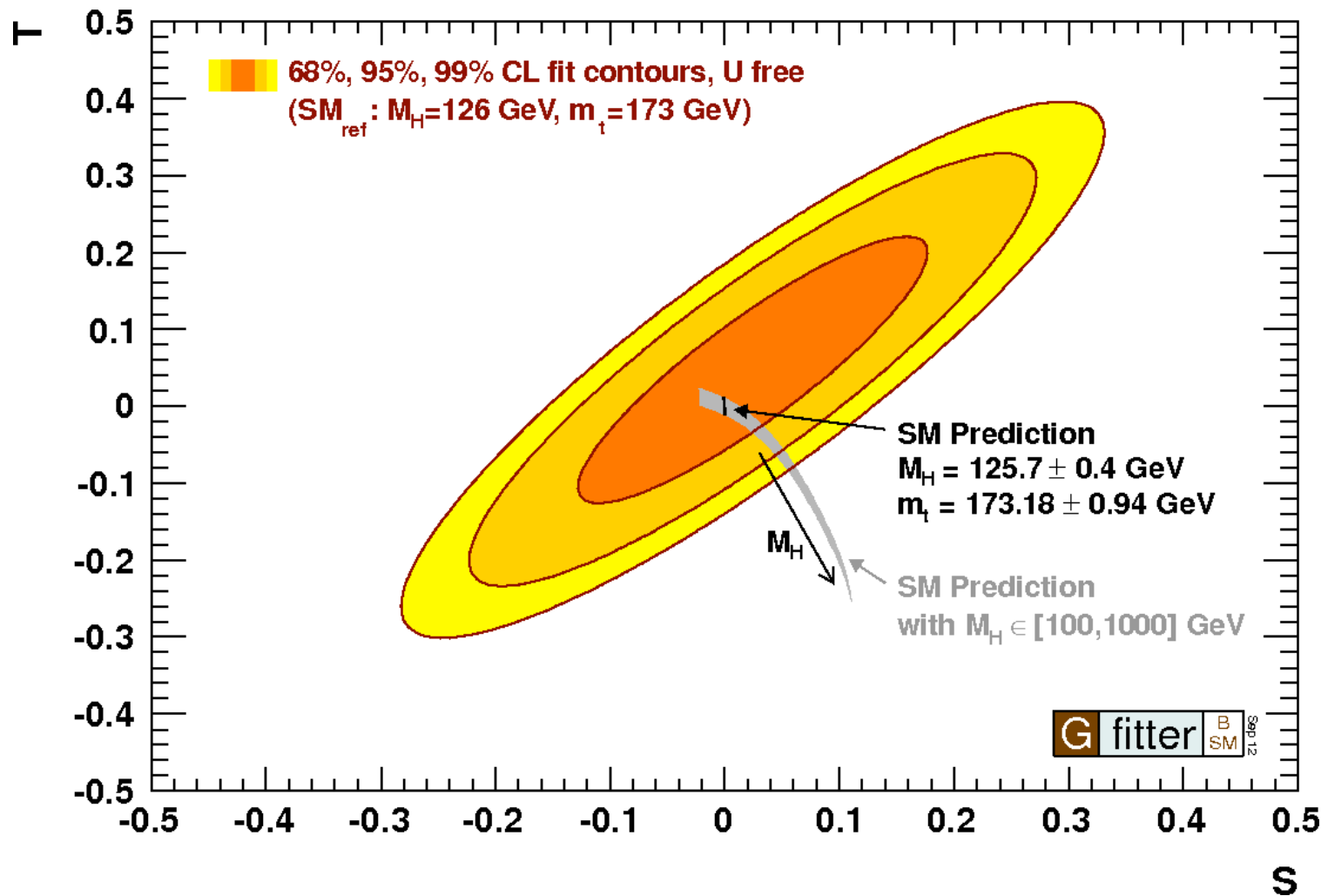
University of Manchester, July 25, 2014

# Motivations

- We've found the Higgs boson! What can we say about its couplings to other particles?
- Can electroweak measurements tell us anything about the Higgs couplings?
- Derive bounds from the oblique parameters and the recent Higgs data on the dimension-6 operators.
- Are constraints on coefficients of the effective operators from **precision test** complementary to those from **direct Higgs production measurements**?

# Oblique parameters

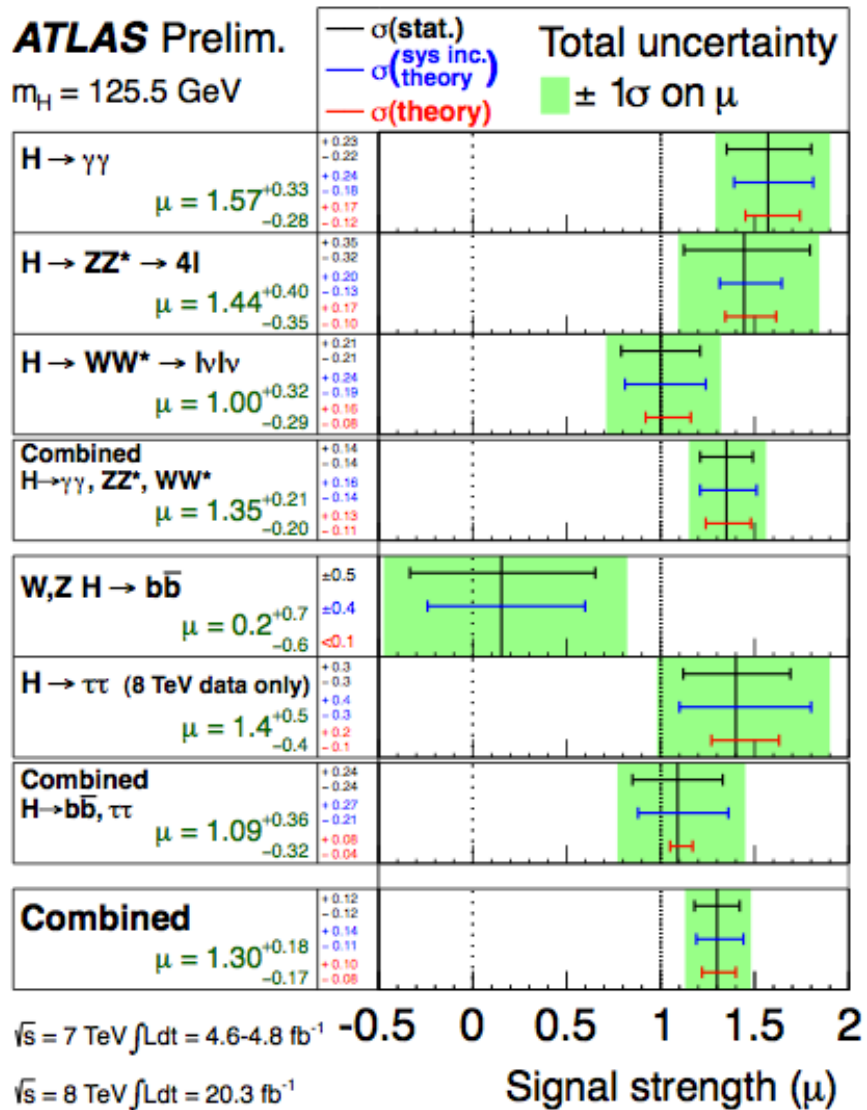
- Most of the effects on electroweak precision observables can be parametrized by  $S$ ,  $T$  and  $U$ . [Phys.Rev.D46, 38, Peskin and Takeuchi]



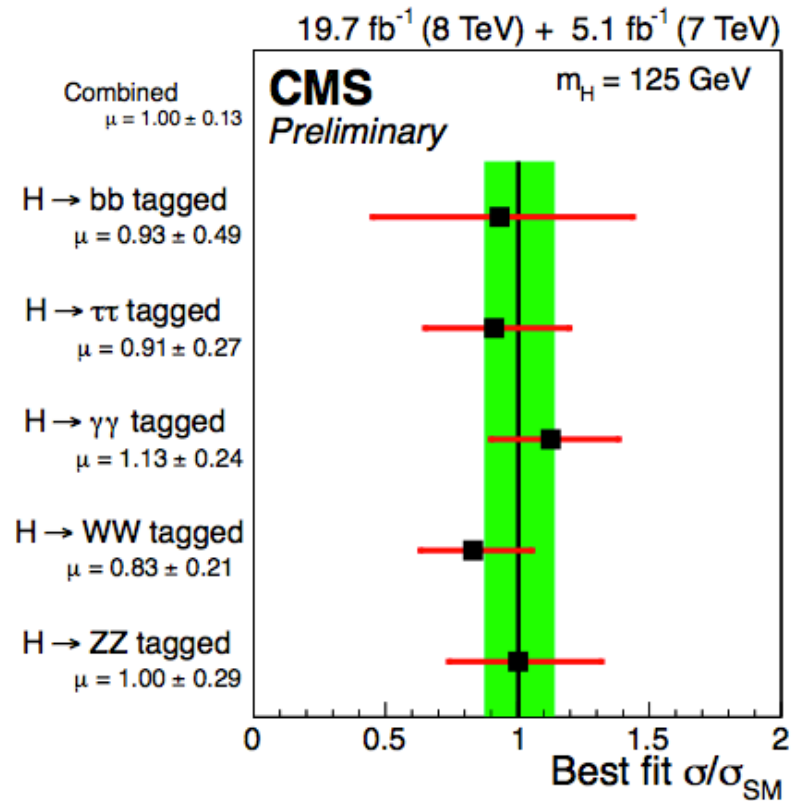
Best fit:

$$S = 0.03 \pm 0.10$$
$$T = 0.05 \pm 0.12$$
$$U = 0.03 \pm 0.10$$

# Higgs Data: signal strength



[ATLAS-CONF-2014-009]



[HIG-14-009-PAS]

$$\mu \equiv \frac{\sum_j \sigma(pp \rightarrow j \rightarrow h) \times B(h \rightarrow \text{decay})|_{\text{observed}}}{\sum_j \sigma(pp \rightarrow j \rightarrow h) \times B(h \rightarrow \text{decay})|_{SM}}$$

- $\mu = 1$  : Standard Model Higgs
- Measuring deviations of the couplings from the SM

# Approach

- Assume new physics is at a scale ( $\Lambda$ ) much higher than that we can probe experimentally.

$$\mathcal{L}_{eff} = \sum_{n=5}^{\infty} \frac{f_n}{\Lambda^{n-4}} \mathcal{O}_n + \dots$$

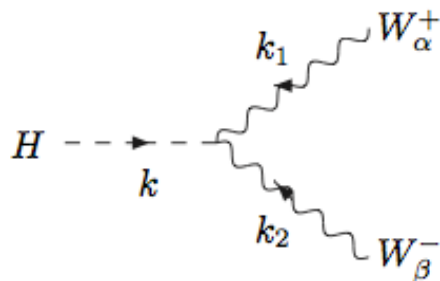
- Focus on electroweak sector of the SM
- The lowest-dimension operators,  $\mathcal{O}_i$ , which contribute to processes involving the **SM gauge bosons** and **Higgs doublets** are dimension six.
- Assume flavor and CP conservation

# Approach

- Lagrangian

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{SM}} + \frac{f_{DW}}{\Lambda^2} \mathcal{O}_{DW} + \frac{f_{DB}}{\Lambda^2} \mathcal{O}_{DB} + \frac{f_{BW}}{\Lambda^2} \mathcal{O}_{BW} + \frac{f_{\Phi,1}}{\Lambda^2} \mathcal{O}_{\Phi,1} \\ & + \frac{f_{\Phi,2}}{\Lambda^2} \mathcal{O}_{\Phi,2} + \frac{f_{WWW}}{\Lambda^2} \mathcal{O}_{WWW} + \frac{f_W}{\Lambda^2} \mathcal{O}_W + \frac{f_B}{\Lambda^2} \mathcal{O}_B \\ & + \frac{f_{WW}}{\Lambda^2} \mathcal{O}_{WW} + \frac{f_{BB}}{\Lambda^2} \mathcal{O}_{BB}. \end{aligned}$$

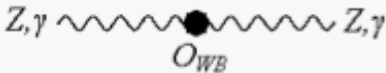
- Derive Feynman rules: e.g. HWW



$$i \frac{gm_W}{\Lambda^2} \left\{ \frac{f_W}{2} [(k_1^\alpha k_1^\beta + k_2^\alpha k_2^\beta) - g^{\alpha\beta} (k_1^2 + k_2^2)] + (f_W - 2f_{WW}) [k_2^\alpha k_1^\beta - g^{\alpha\beta} (k_1 \cdot k_2)] \right\}$$

# Approach

- Four operators that affect the gauge boson two point functions **at tree level**:

tree level: 

$$\mathcal{O}_{DW} = -\frac{g^2}{4} \text{Tr}([D_\mu, \sigma^a \cdot W_{\nu\rho}^a][D^\mu, \sigma^b \cdot W^{b,\nu\rho}])$$

$$\mathcal{O}_{DB} = -\frac{g^2}{2} (\partial_\mu B_{\nu\rho})(\partial^\mu B^{\nu\rho})$$

$$\mathcal{O}_{BW} = -\frac{gg'}{4} \Phi^\dagger B_{\mu\nu} \sigma^a \cdot W^{a,\mu\nu} \Phi$$

$$\mathcal{O}_{\Phi,1} = (D_\mu \Phi)^\dagger (\Phi \Phi^\dagger) (D^\mu \Phi),$$

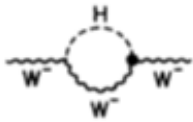
$$D_\mu = \partial_\mu - i\frac{g}{2} B_\mu - i\frac{g'\sigma^a}{2} W_\mu^a \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm \mp ig(W_\mu^3 W_\nu^\pm - W_\nu^3 W_\mu^\pm)$$

$$W_{\mu\nu}^3 = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3 - ig(W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-).$$

# Approach

- There are six bosonic operators which contribute to the oblique parameters **at one loop**



$$\mathcal{O}_{WWW} = -i \frac{g^3}{8} \text{Tr}(\sigma^a \cdot W_\nu^{a,\mu} \sigma^b \cdot W_\rho^{b,\nu} \sigma^c \cdot W_\mu^{c,\rho})$$

$$\mathcal{O}_W = i \frac{g}{2} (D_\mu \Phi)^\dagger \sigma^a \cdot W^{a,\mu\nu} (D_\nu \Phi)$$

$$\mathcal{O}_B = i \frac{g'}{2} (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi)$$

$$\mathcal{O}_{WW} = -\frac{g^2}{4} \Phi^\dagger \sigma^a \cdot W^{a,\mu\nu} \sigma^b \cdot W_{\mu\nu}^b \Phi$$

$$\mathcal{O}_{BB} = -\frac{g'^2}{4} \Phi^\dagger B^{\mu\nu} B_{\mu\nu} \Phi$$

$$\mathcal{O}_{\Phi,2} = \frac{1}{2} \partial^\mu (\Phi^\dagger \Phi) \partial_\mu (\Phi^\dagger \Phi).$$

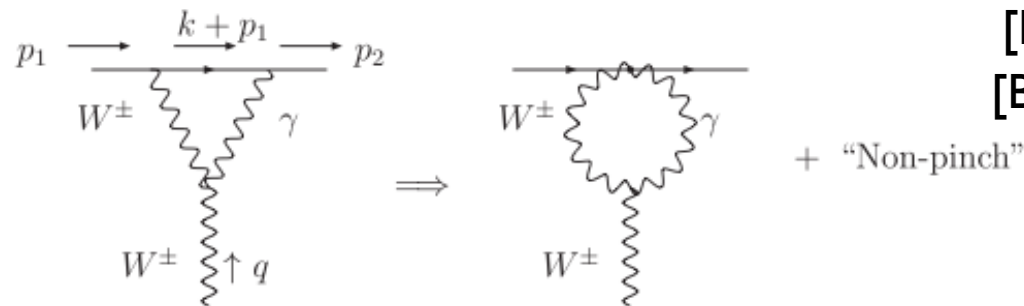
- Neglect

$$\mathcal{O}_{\Phi,3} = \frac{1}{3} (\Phi^\dagger \Phi)^3 \quad \mathcal{O}_{\Phi,4} = (D_\mu \Phi)^\dagger (D^\mu \Phi) (\Phi^\dagger \Phi)$$



# Two point functions

- S,T and U parameters defined in terms of self-energies are gauge dependent
- Need pinch part of the vertex corrections



[Degrassi and Sirlin, 1992]  
[Binosi, Papavassiliou, 2009]

- The gauge dependence of the gauge boson self-energies is exactly canceled by the pinch part of the vertex corrections.

# Two point functions

$$\bar{\Pi}_{WW}(q^2) = \Pi_{WW}(q^2) + 2(q^2 - m_W^2)\Delta\Gamma_L^W(q^2)$$

$$\bar{\Pi}_{ZZ}(q^2) = \Pi_{ZZ}(q^2) + 2c(q^2 - m_Z^2)\Delta\Gamma_L^Z(q^2)$$

$$\bar{\Pi}_{\gamma Z}(q^2) = \Pi_{\gamma Z}(q^2) + sq^2\Delta\Gamma_L^Z(q^2) + c(q^2 - m_Z^2)\Delta\Gamma_L^\gamma(q^2)$$

$$\bar{\Pi}_{\gamma\gamma}(p^2) = \Pi_{\gamma\gamma}(p^2) + 2sq^2\Delta\Gamma_L^\gamma(q^2),$$

where  $c \equiv \cos \theta_W$  and  $s \equiv \sin \theta_W$ .

$$\alpha\Delta S = \left( \frac{4s^2c^2}{m_Z^2} \right) \left\{ \bar{\Pi}_{ZZ}(m_Z^2) - \bar{\Pi}_{ZZ}(0) - \bar{\Pi}_{\gamma\gamma}(m_Z^2) - \frac{c^2 - s^2}{cs} \left( \bar{\Pi}_{\gamma Z}(m_Z^2) \right) \right\}$$

$$\alpha\Delta T = \left( \frac{\bar{\Pi}_{WW}(0)}{m_W^2} - \frac{\bar{\Pi}_{ZZ}(0)}{m_Z^2} \right)$$

$$\alpha\Delta U = 4s^2 \left\{ \frac{\bar{\Pi}_{WW}(m_W^2) - \bar{\Pi}_{WW}(0)}{m_W^2} - c^2 \left( \frac{\bar{\Pi}_{ZZ}(m_Z^2) - \bar{\Pi}_{ZZ}(0)}{m_Z^2} \right) - 2sc \left( \frac{\bar{\Pi}_{\gamma Z}(m_Z^2)}{m_Z^2} \right) - s^2 \frac{\bar{\Pi}_{\gamma\gamma}(m_Z^2)}{m_Z^2} \right\}.$$

# Results

$$\Delta S = C_S \frac{1}{\epsilon} \left( \frac{4\pi\mu^2}{m_Z^2} \right)^\epsilon \Gamma(1 + \epsilon) + R_S$$

$$\Delta T = C_T \frac{1}{\epsilon} \left( \frac{4\pi\mu^2}{m_Z^2} \right)^\epsilon \Gamma(1 + \epsilon) + R_T$$

$$\Delta U = C_U \frac{1}{\epsilon} \left( \frac{4\pi\mu^2}{m_Z^2} \right)^\epsilon \Gamma(1 + \epsilon) + R_U,$$

3 order of magnitude smaller



$$C_S = \frac{m_H^2}{8\pi} \left\{ \frac{f_B + f_W}{\Lambda^2} \right\} + \frac{m_Z^2}{24\pi\Lambda^2} \left\{ f_B(20c^2 + 7) - 3f_W \right. \\ \left. + 24(s^2 f_{BB} + c^2 f_{WW}) + 36c^2 g^2 f_{WWW} + \frac{8c^2}{g^2} f_{\Phi,2} \right\}$$

$$C_T = \frac{1}{16\pi c^2} \left\{ 9m_W^2 \left( \frac{f_B + f_W}{\Lambda^2} \right) + 3m_H^2 \frac{f_B}{\Lambda^2} - 12 \frac{m_W^2}{g^2} \frac{f_{\Phi,2}}{\Lambda^2} \right\}$$

$$C_U = \frac{m_Z^2}{6\pi\Lambda^2} s^2 f_W.$$

$$R_S = \{-0.76f_{BW} + 10^{-3}(1.48f_B - 1.4f_W - 0.2f_{BB} \\ - 0.71f_{WW} + 0.66f_{WWW} + 1.96f_{\Phi,2})\} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2$$

$$R_T = \{-4.0f_{\Phi,1} - 10^{-3}(0.13f_B + 0.12f_W - 3.97f_{\Phi,2})\} \\ \times \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2$$

$$R_U = \{0.20f_{DW} + 10^{-3}(-0.02f_B + 2.06f_W + 0.14f_{WW} \\ + 2.1f_{WWW} - 0.25f_{\Phi,2})\} \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2. \quad (19)$$

# Renormalization

- $\overline{MS}$  scheme
- In  $R_\xi$  Gauge
- The divergences have been eliminated by the renormalization of the tree level couplings.

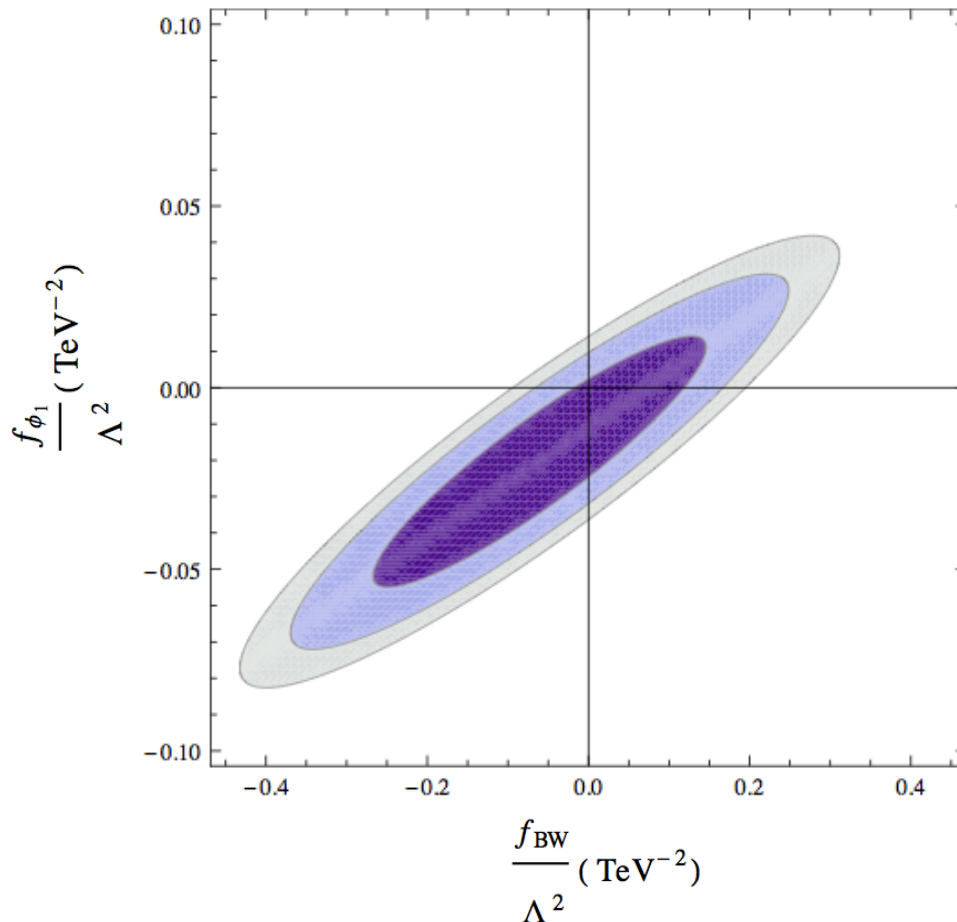
$$f_{BW}(\mu) = f_{BW} - \frac{1}{\epsilon} (4\pi)^\epsilon \Gamma(1 + \epsilon) C_S$$
$$f_{DW}(\mu) = f_{DW} - \frac{1}{\epsilon} (4\pi)^\epsilon \Gamma(1 + \epsilon) C_U$$
$$f_{\Phi,1}(\mu) = f_{\Phi,1} - \frac{1}{\epsilon} (4\pi)^\epsilon \Gamma(1 + \epsilon) C_T.$$

- The only remaining contributions to the oblique corrections are the finite contributions at  $\mu = m_Z$ .

$$\Delta S = R_S \quad \Delta T = R_T \quad \Delta U = R_U.$$

# Results

- Limits from the oblique parameters.
- Coefficients of the operators that contribute at **tree level** are significantly restricted (all other coefficients are set to zero.)

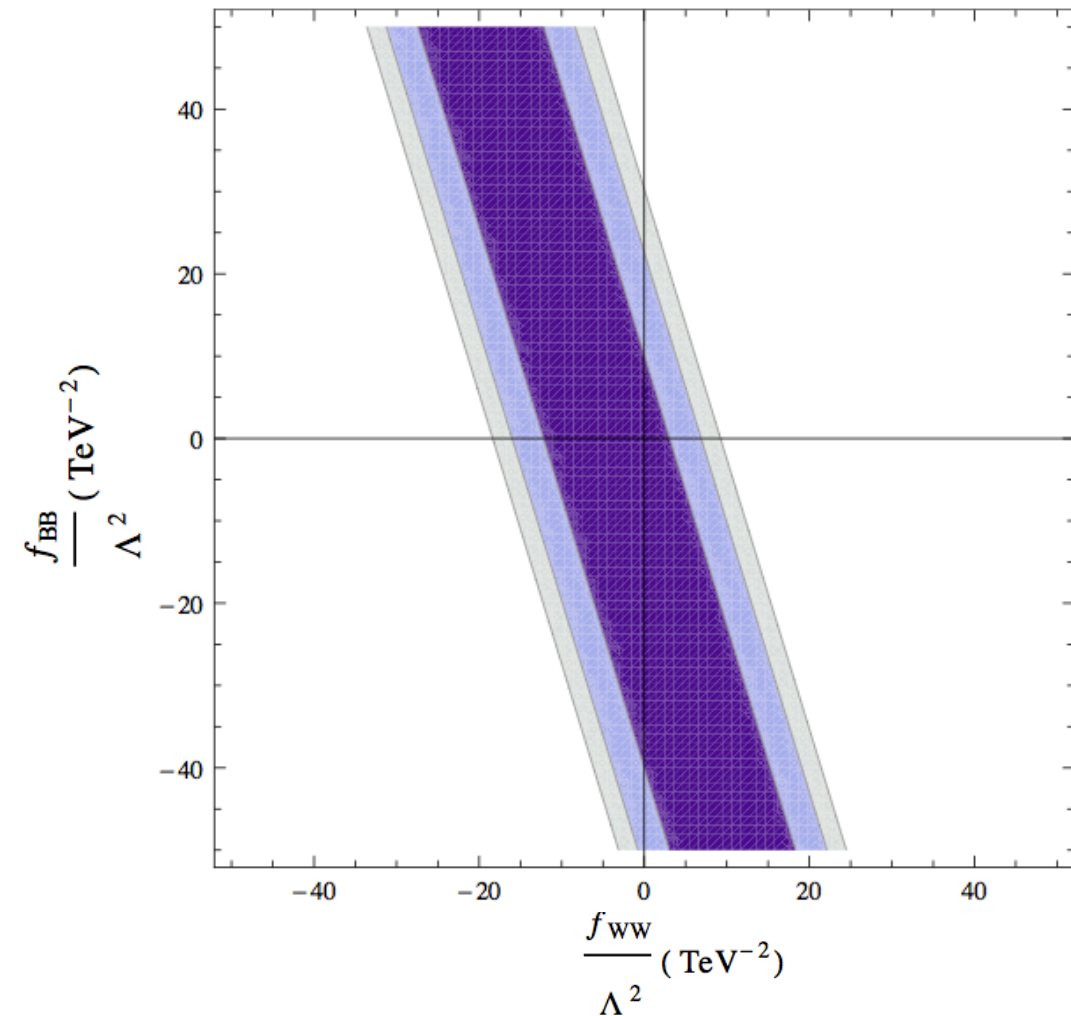


From outer to inner are  
99%, 95% and 68% CL

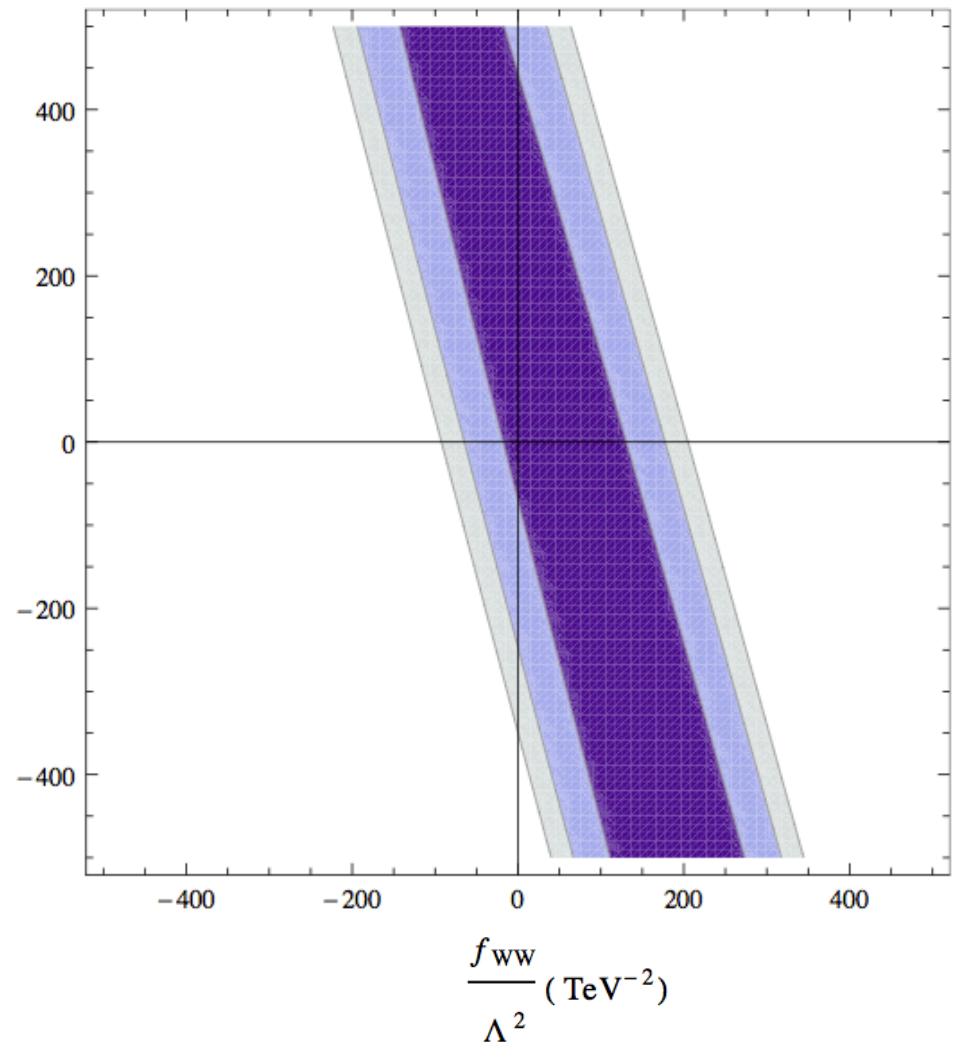
$$\mu = m_Z$$

# Results

- Limits from the oblique parameters on  $f_{WW}$  and  $f_{BB}$  (other coefficients are set to zero.)



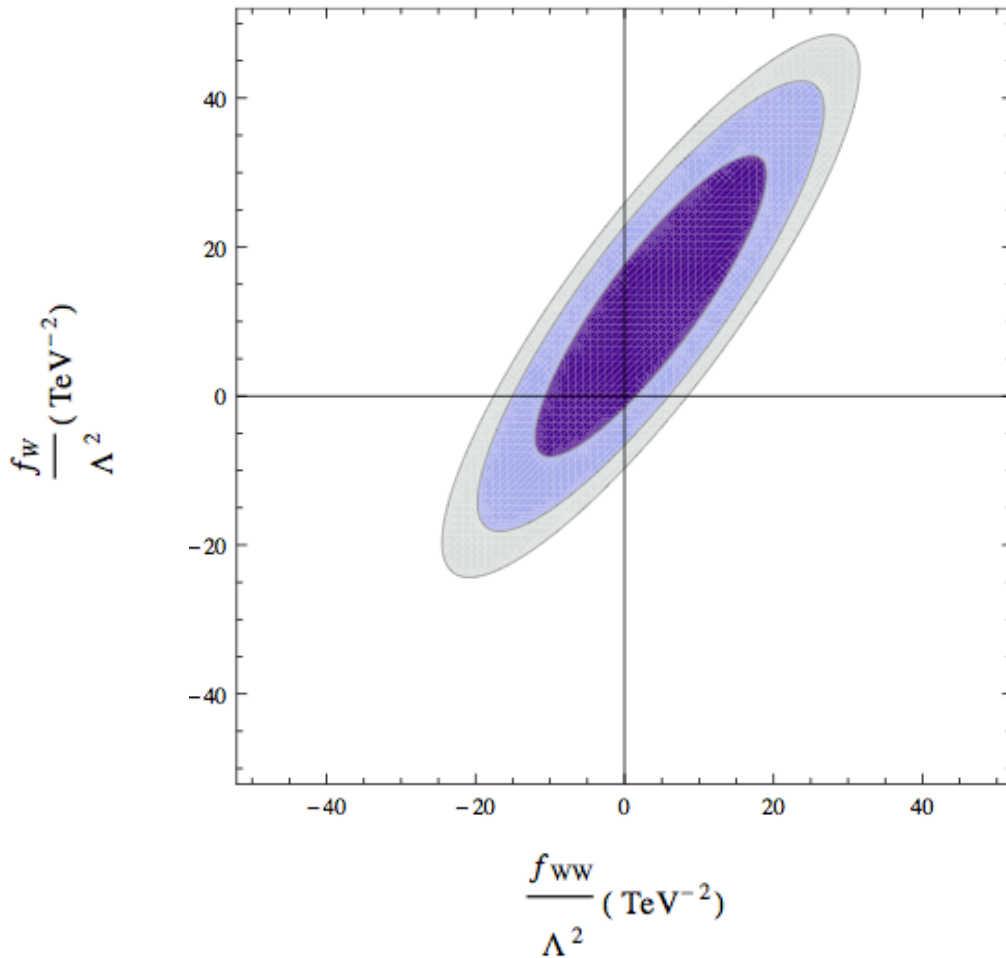
Leading logarithmic result  $\mu = \Lambda$



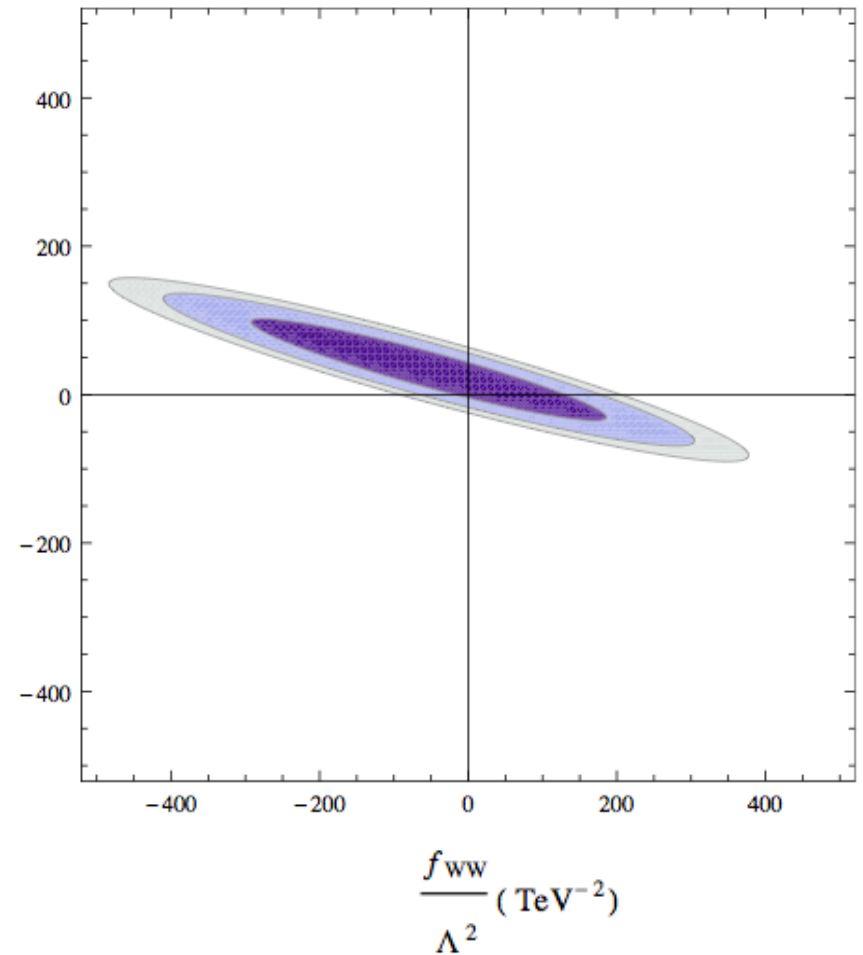
Renormalized result at  $\mu = m_Z$

# Results

- Limits from the oblique parameters on  $f_{WW}$  and  $f_W$  (other coefficients are set to zero.)



Leading logarithmic result  $\mu = \Lambda$

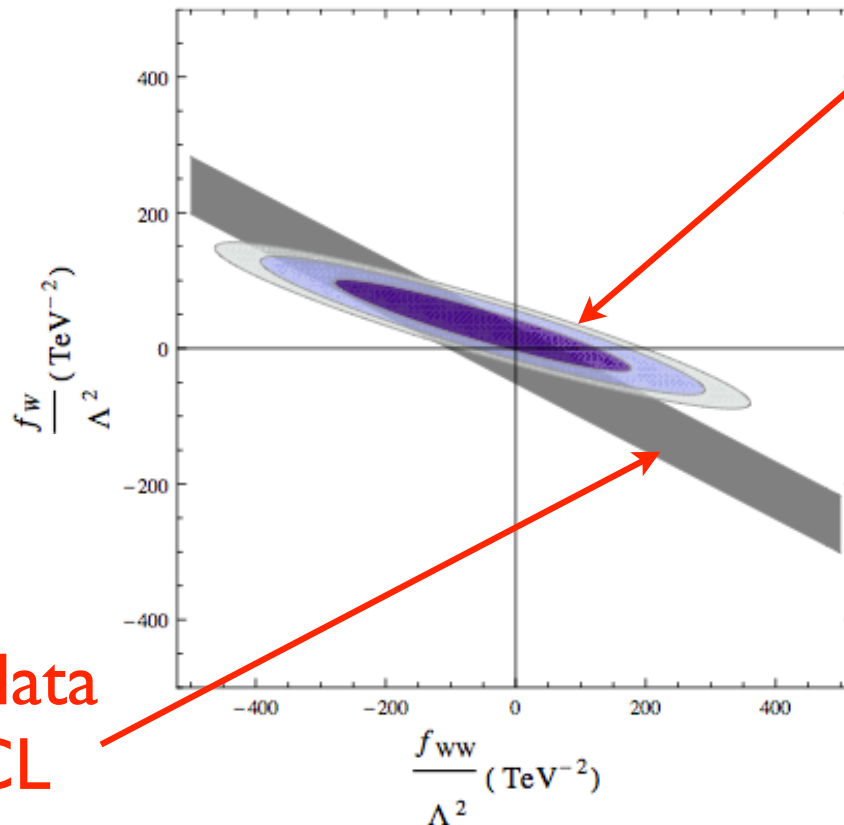


Renormalized result at  $\mu = m_Z$

# Oblique parameters v.s. H-> WW

- Complementary bounds from oblique parameters and Higgs data.

$$\mu_{WW} \sim 1 + [.0086f_{WW}(m_Z) + .017f_W(m_Z) - .03f_{\Phi,1}(m_Z) - .06f_{\Phi,2}(m_Z)] \left(\frac{1 \text{ TeV}}{\Lambda}\right)^2 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$



oblique parameter

$$\mu_{WW} = .68 \pm .20 \quad (CMS)$$

$$\mu_{WW} = .99 \pm .30 \quad (ATLAS)$$

$$\mu = m_Z$$

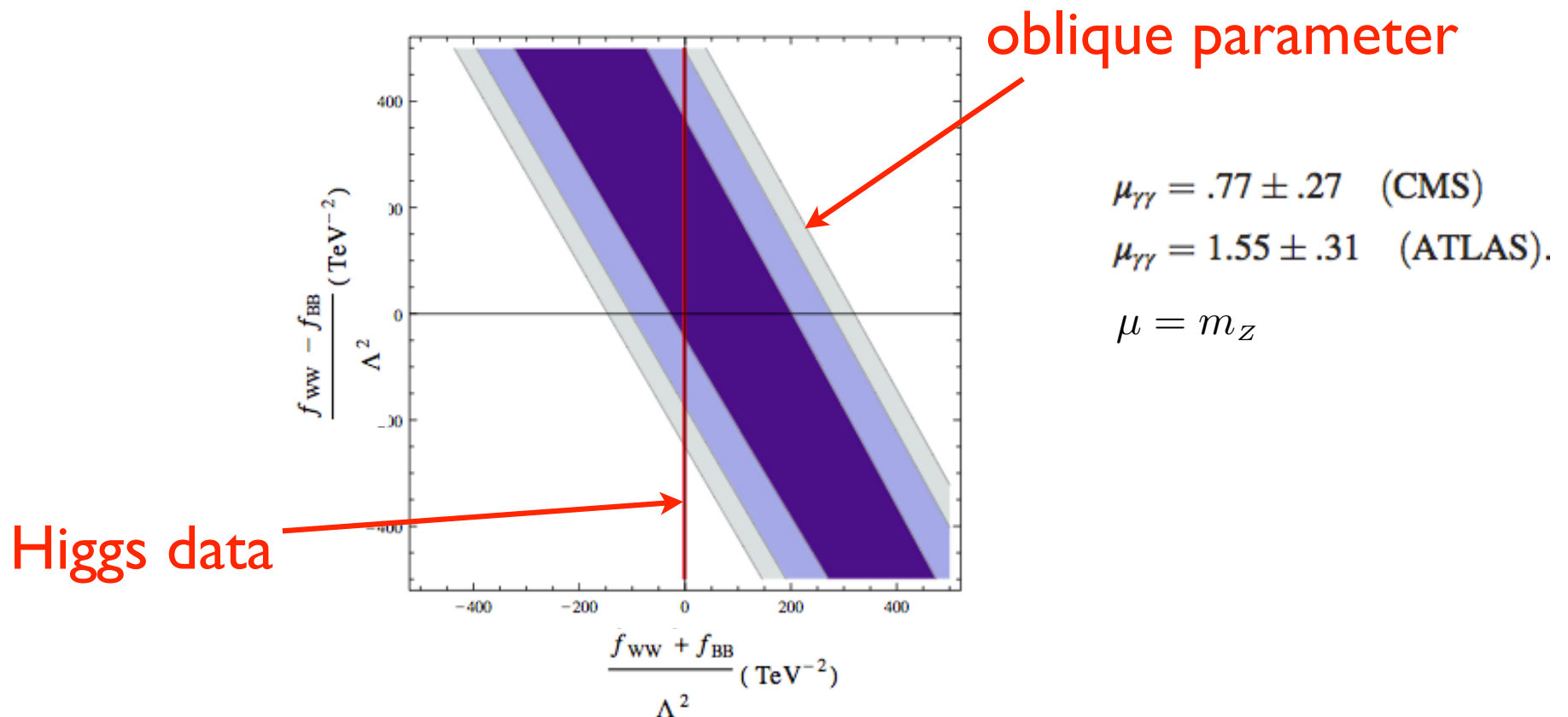
Higgs data  
95%CL



# Oblique parameters v.s. H-> 2 photons

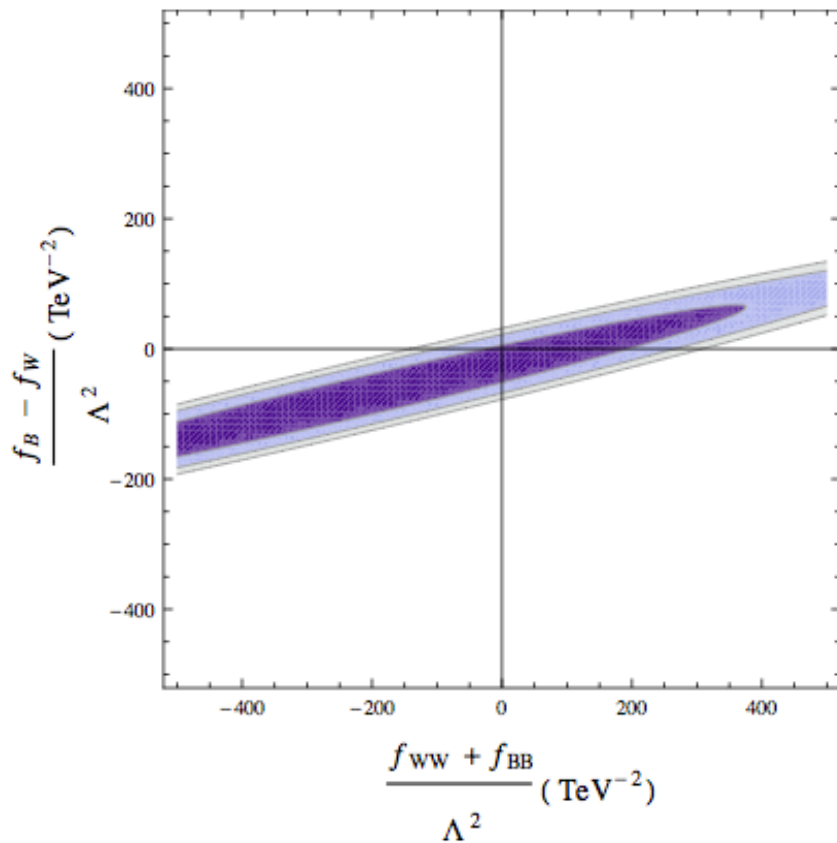
- Comparison of limits from the oblique parameters and H-> 2 photons on  $f_{WW} + f_{BB}$  and  $f_{WW} - f_{BB}$  (other coefficients are set to zero.)

$$\mu_{\gamma\gamma} \equiv \frac{\Gamma(H \rightarrow \gamma\gamma)}{\Gamma(H \rightarrow \gamma\gamma)|_{\text{SM}}} \sim 1 + 1.47 \left( \frac{1 \text{ TeV}}{\Lambda} \right)^2 [f_{BB}(m_Z) + f_{WW}(m_Z) - f_{BW}(m_Z)] + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$



# Results

- Limits from oblique parameters, which influence the decay  $H \rightarrow Z \gamma$ .



$$\mu = m_Z$$

$$\begin{aligned} \mu_{Z\gamma} &\equiv \frac{\Gamma(H \rightarrow Z\gamma)}{\Gamma(H \rightarrow Z\gamma)|_{SM}} \\ &= 1 + \frac{2A_{real}}{A_{real}^2 + A_{imag}^2} \frac{2\pi s c m_Z^2}{\alpha} \frac{1}{\Lambda^2} g_1 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right) \end{aligned}$$

$$\begin{aligned} g_1 &= f_B(m_Z) - f_W(m_Z) + 4s^2 f_{BB}(m_Z) \\ &\quad - 4c^2 f_{WW}(m_Z) + 2(c^2 - s^2) f_{BW}(m_Z) \end{aligned}$$

- Neglect all coefficients except  $f_W(m_Z)$  and  $f_B(m_Z)$ .  

$$-80 < [f_B(m_Z) - f_W(m_Z)] \left(\frac{1 \text{ TeV}}{\Lambda}\right)^2 < 35,$$

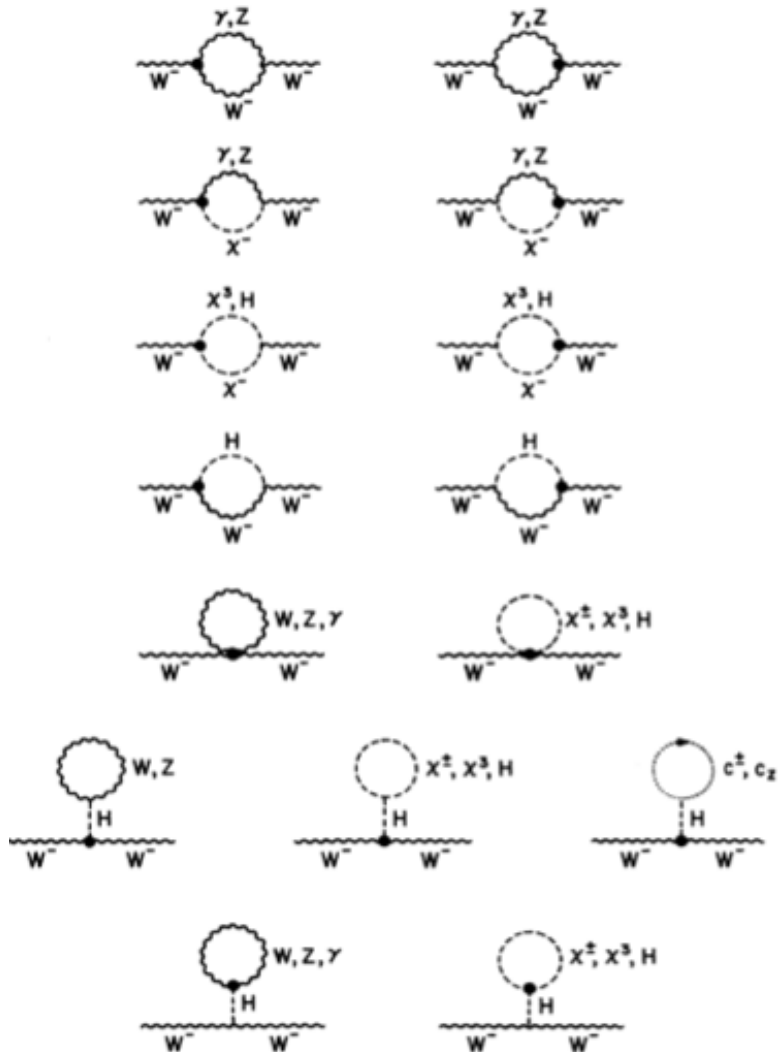
# Conclusions

- Only weak limits on the couplings that contribute at one loop can be obtained from the oblique parameters.
- In contrast, the couplings that contribute at tree level, are tightly constrained.
- Loop contributions to oblique parameters yield complementary information to direct  $H \rightarrow WW$  measurement.

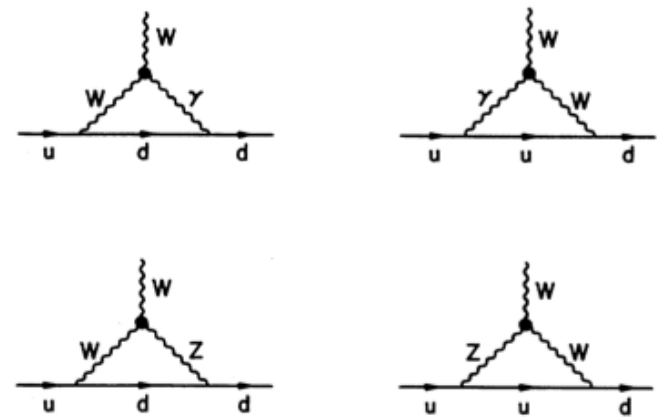
**Backup slides**

# Example

- Anomalous contributions to  $\Pi_{WW}$



- Anomalous contributions to the  $W$   $u$   $d$  vertex  $\Delta\Gamma_L^{Wud}$



$$\Delta\Gamma_L^{Vff}(q^2) = gT_3^f \Delta\Gamma_L^V(q^2) \quad V = Z, \gamma$$

$$\Delta\Gamma_L^{Wff'}(q^2) = \frac{g}{\sqrt{2}} \Delta\Gamma_L^W(q^2).$$

$\chi^\pm$  and  $\chi^3$ : Goldstone bosons  
 $c^\pm$  and  $c_Z$ : Faddeev-Popov ghosts.

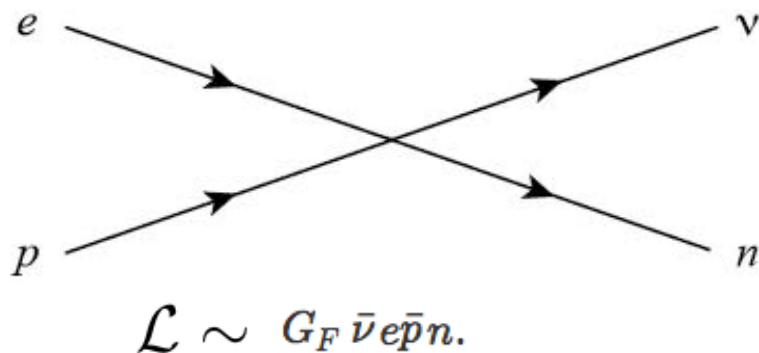
# Effective field theory

- Assume new physics is at a scale ( $\Lambda$ ) much higher than that we can probe experimentally.

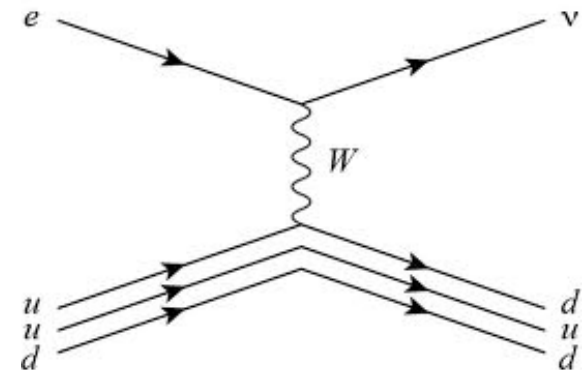
$$\mathcal{L}_{eff} = \sum_{n=5}^{\infty} \frac{f_n}{\Lambda^{n-4}} \mathcal{O}_n + \dots$$

- e.g. Fermi's four fermion interaction

Below  $\Lambda$



Above  $\Lambda$



$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$