Chaotic Inflation and Fractional Powers

The Dynamical Origin of the Inflaton Potential in Chaotic Inflation

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In collaboration with Keisuke Harigaya, Masahiro Ibe and T. T. Yanagida.

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Outline

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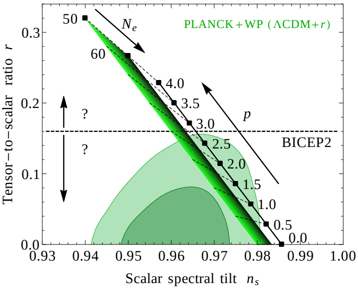
The Case for Dynamical Chaotic Inflation (DCI)



Assume BICEP2 claim survives further scrutiny:

- Large field excursion. $\rightarrow$ Chaotic inflation based on a simple monomial potential.

$$V(\phi) \sim M^4 \left(\frac{|\phi|}{M}\right)^p , \quad p \in \mathbb{Q}^+ .$$

- Perturbative QFT: Only $p \in \mathbb{N}^+$ feasible.

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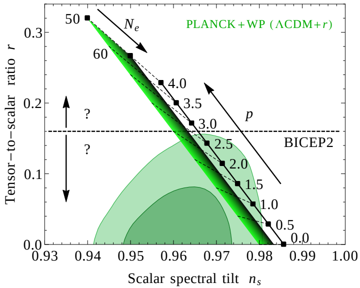

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Why such a potential? What determines the power p and the mass scale M?

- Axion monodromy in *string theory*:
  [Silverstein & Westphal '08] [McAllister, Silverstein & Westphal '08]

$$p = \frac{2}{5}, \frac{2}{3}, 1, 2.$$

- Fractional powers from first principles!

- Embedding into *SUGRA* for $p \in \mathbb{N}^+$:
  [Kawasaki, Yamaguchi & Yanagida '00] [Kallosh & Linde '10]

$$W = X f(\Phi), \quad \Phi \to \Phi + i\alpha.$$

- Only educated guesses of W and K.

Generate fractional power-law potential for chaotic inflation *dynamically* within field theory!

DCI in Strongly Interacting SUSY Gauge Theories (I)

Theoretical ingredients: [Seiberg '94] [Intriligator & Pouliot '95] [Csaki, Schmaltz & Skiba '97]

1. Supersymmetry: Otherwise no control over IR dynamics, quadratic divergences, ...
2. Dynamical SUSY breaking (DSB): Vacuum energy $V(\phi)$ acts as inflaton potential.
3. S-confinement: Smooth effective field theory in terms of composites at low energies.

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Dynamical generation of the inflaton potential (in words):

- In a given DSB model, provide some quark flavors with inflaton-dependent mass:

$$W \supset (\lambda \Phi + M_a) P^a \bar{P}^a , \quad M_a \geq 0 .$$

- For $\lambda \Phi \gtrsim \Lambda$, these quark flavors decouple perturbatively, so that at low energies:

$$\Lambda_{\rm eff} = \Lambda \left(\frac{\lambda \Phi}{\Lambda}\right)^{p/4} , \quad \frac{p}{4} = \frac{b_{\rm eff} - b}{b_{\rm eff}} \quad \Rightarrow \quad W_{\rm eff} \simeq \Lambda_{\rm eff}^2 X , \quad V_{\rm eff} \simeq \Lambda^4 \left(\frac{\lambda \Phi}{\Lambda}\right)^{p} .$$

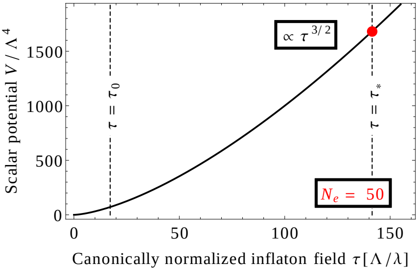
- For $\lambda \Phi \lesssim \Lambda$, s-confined phase with all fields being stabilized around the origin:

$$V \simeq \Lambda^2 \left|\tilde{\phi}\right|^2 , \quad \tilde{\phi} = f(\phi) .$$

Fractional power p. ✔ M identified as Λ. No input scale. ✔ Smooth around $\Phi = 0$. ✔

DCI in Strongly Interacting SUSY Gauge Theories (II)

Example: $p = 3/2$ [based on $SP(3)$ theory]



RGE matching at quark mass threshold:

$$\alpha_{\rm HE}^{-1} = \frac{b}{2\pi} \ln\left(\frac{\mu}{\Lambda}\right)\,, \quad \alpha_{\rm LE}^{-1} = \frac{b_{\rm eff}}{2\pi} \ln\left(\frac{\mu}{\Lambda_{\rm eff}}\right)\,,$$

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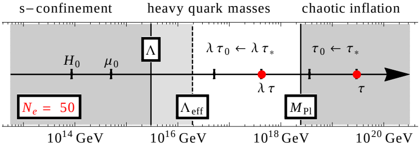
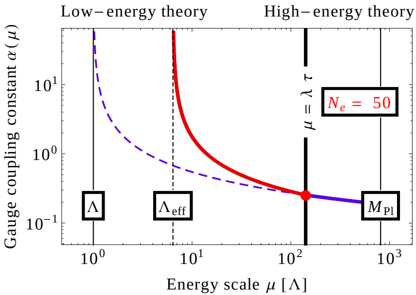
Energy scales involved in DCI:



Running of the gauge coupling $\alpha = \frac{g^2}{4\pi}$:



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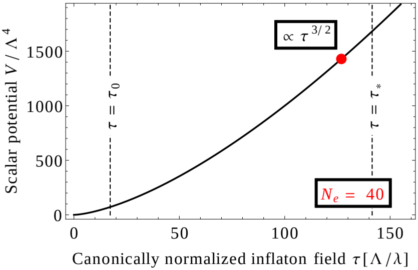

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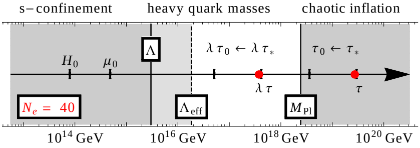

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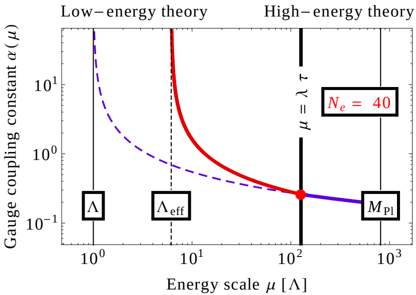

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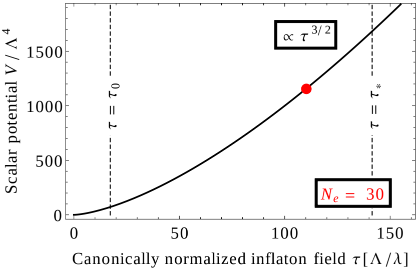

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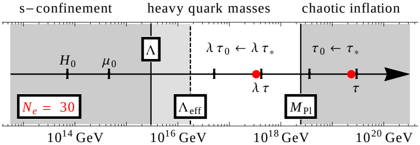

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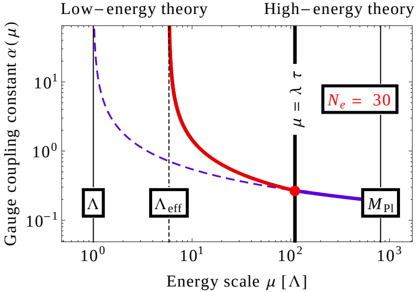

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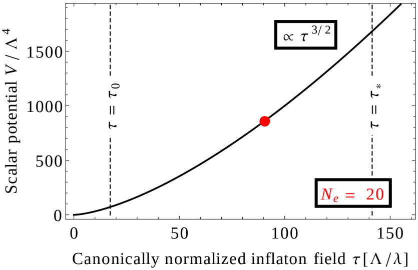

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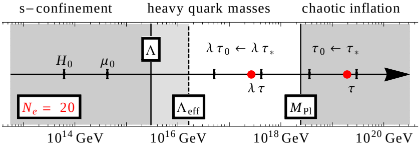

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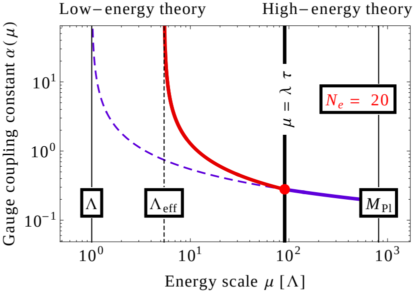

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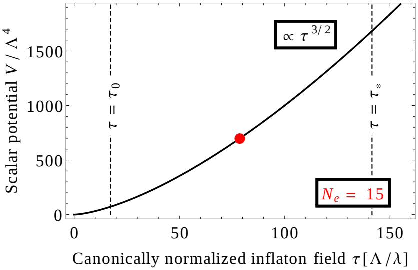

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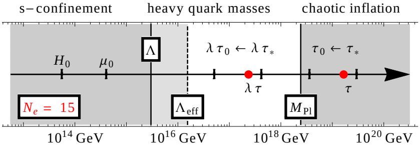

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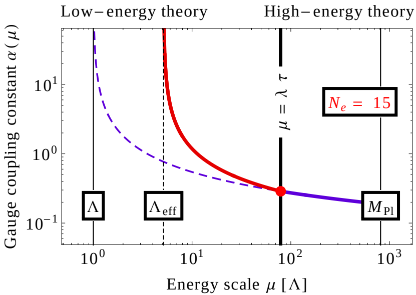

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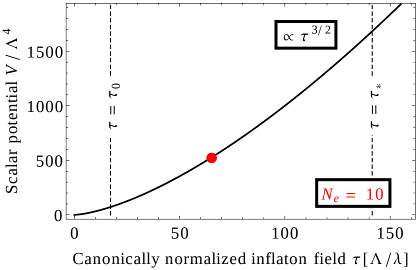

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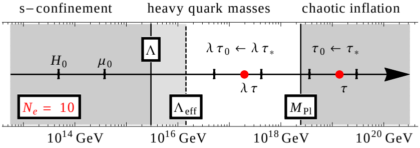

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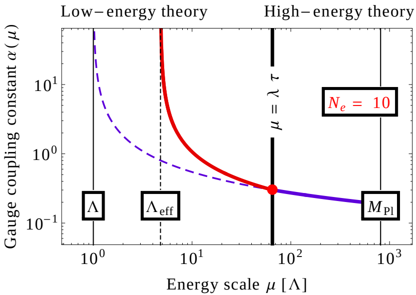

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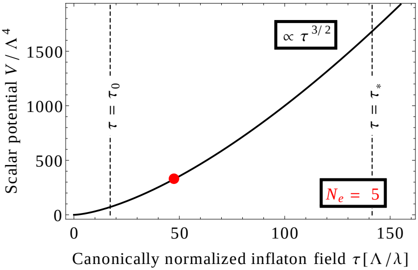

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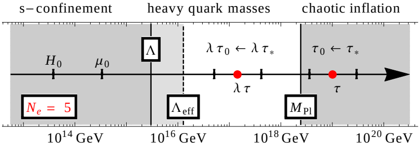

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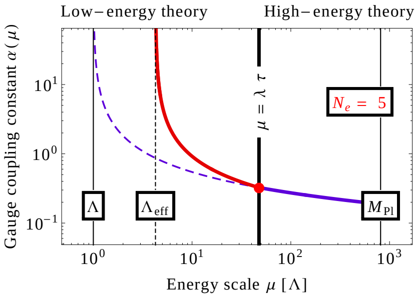

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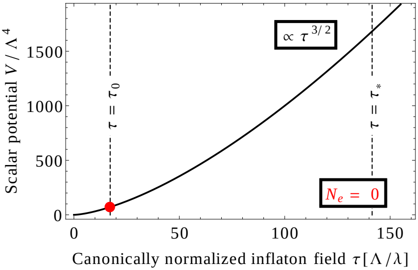

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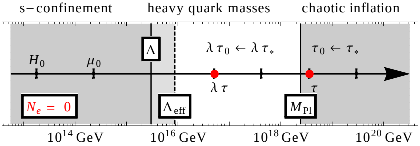

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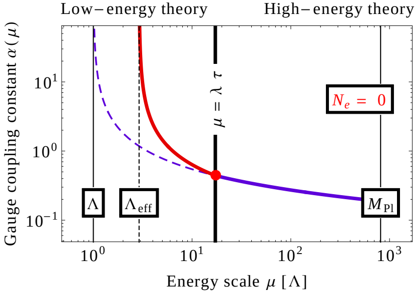

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Minimal Scenario: $SP(N_c)$ Dynamics

Recall ingredient ❷: DSB responsible for $V(\phi) \simeq \Lambda_{\rm eff}^4$ during inflation.

- Need DSB model that flows to s-confining theory when $\phi \to 0$.
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- Or alternatively: s-confining theory flowing to DSB model when $\phi \gg \Lambda$.

Simplest example: $SP(N_c)$ gauge theory with $2N_f$ quarks and $N_f = N_c + 2$.

- For every flat direction in moduli space, introduce one singlet field Z_{IJ}:

$$W = \lambda_{IJ}\, Z_{IJ}\, Q^I Q^J \quad \rightarrow \quad W = \lambda_{ij}\, Z_{ij}\, Q^i Q^j + \lambda\, \Phi\, P\bar{P} + \ldots$$

- Identify one of the singlets as inflaton field Φ, e.g. because $[\Phi]_R = 0$.
- For $\lambda\Phi \lesssim \Lambda$, s-confined phase, dynamical superpotential, SUSY vacuum at origin.
- For $\lambda\Phi \gtrsim \Lambda$, $(P, \bar{P})$ flavor decouples, deformed moduli constraint, SUSY broken

[Izawa & Yanagida '96] [Intriligator & Thomas '96]

$$W \simeq \lambda_{ij}\, \Lambda_{\rm eff}\, Z_{ij}\, M^{ij}\,, \quad \mathrm{Pf}(M) = \Lambda_{\rm eff}^{N_c+1}\,, \quad M^{ij} \simeq Q^i Q^j / \Lambda_{\rm eff}\,, \quad \Lambda_{\rm eff} \simeq \Lambda \left(\frac{\lambda\Phi}{\Lambda}\right)^{p/4}\,.$$

Minimize potential with respect to meson fields M^{ij} $\rightarrow$ low-energy effective theory:

$$W_{\rm eff} \simeq \Lambda_{\rm eff}\, X\,, \quad X \propto \sum Z_{ij} \quad V_{\rm eff}(\phi) \simeq (N_c + 1)\, \Lambda_{\rm eff}^4(\phi)\,, \quad p = \frac{4(b_{\rm eff} - b)}{b_{\rm eff}} = \frac{1}{2N_c + 1}$$

Generalization along Two Different Directions

1. Additional massive matter fields that decouple at energies $\mu \gtrsim \Lambda$.
2. Vacuum energy provided by alternative models of dynamical SUSY breaking.

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❶ For example, $SP(N_c)$ gauge theory with $2N_f$ quarks and $N_f = N_c + 2 + N_m$.

- To retain s-confinement, all extra flavors must decouple above the dynamical scale,

$$W \supset (\lambda \Phi + M_a)\, P^a \bar{P}^a\,, \quad a = 1,..,N_m\,, \quad \mathcal{O}(\Lambda) \lesssim M_a \lesssim \mathcal{O}(M_{\rm Pl})\,.$$

- Extra contribution to the high-energy beta-function coeffcient changes the power p,

$$b = 3(N_c+1) - (N_c + 2 + N_m)\,, \quad b_{\rm eff} = 3(N_c+1) - (N_c+1)\,, \quad p = \frac{2(1+N_m)}{N_c+1}\,.$$

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❷ Seek alternative s-confining theories that can be transformed into DSB models:

Gauge group	S-confining phase	SUSY-breaking phase	Power p
$SP(N_c)$	$Q^i\,,\ i = 1,..,2(N_c+2)$	$Q^i\,,\ i = 1,..,2(N_c+1)$	$2/(N_c+1)$
$SO(10)$	$\mathbf{16}_{0,1}$, $\overline{\mathbf{16}}_1$, $\mathbf{10}_{1,2,3}$	$\mathbf{16}_0$	$14/11$
$SU(5)$	$\mathbf{5}_{1,..,4}$, $\mathbf{5}^*_{0,..,4}$, $\mathbf{10}$	$\mathbf{5}^*_0$, $\mathbf{10}$	$16/13$
$SU(3)\times SU(2)$	q, $\bar{u}$, $\bar{d}$, ℓ, U, $\bar{U}$, D, $\bar{D}$, L, $\bar{L}$	q, $\bar{u}$, $\bar{d}$, ℓ	$8/7$
$SU(N_c)$	Q^i, $\bar{Q}^i\,,\ i = 1,..,N_c+1$	Q^i, $\bar{Q}^i\,,\ i = 1,..,N_c$	1

[Izawa & Yanagida '96] [Intriligator & Thomas '96] [Affleck, Dine & Seiberg '84] [Affleck, Dine & Seiberg '85] [Seiberg '85]

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Constraints on Parameter Space

$V \simeq \Lambda^4 \left(\frac{\lambda \Phi}{\Lambda}\right)^p$ $\rightarrow$ two parameters: dynamical scale Λ and inflaton Yukawa coupling λ.

Eta problem in supergravity:

- Too large η because of e^K corrections.
- Shift symmetry in the direction of Φ:

$$\Phi \rightarrow \Phi + i\, c\, M_{\rm Pl}\,, \quad c \in \mathbb{R}.$$

- $\tau \equiv \sqrt{2}\,\mathrm{Im}\,\{\Phi\}$ is the actual inflaton.
- Shift symmetry explicitly broken in the Kähler potential at one-loop level:

$$K_{\rm eff} \supset \frac{\lambda^2}{16\pi^2}\,|\Phi|^2 \ln\left(\frac{\mu^2}{M_{\rm Pl}^2}\right).$$

- Coupling λ must be small, $\lambda \lesssim 0.1$.

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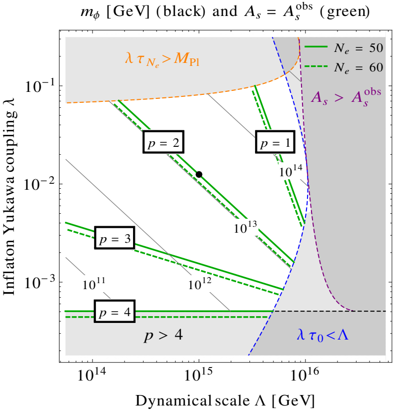
$\Lambda \sim \Lambda_{\rm GUT}\,,$ ✔ $\lambda \sim 10^{-3}..10^{-1}\,.$ ✔

Normalization of the power spectrum and bounds imposed for consistency:

$$A_s \equiv A_s^{\rm obs}\,, \quad \Lambda \lesssim \lambda\tau \lesssim M_{\rm Pl}\,.$$



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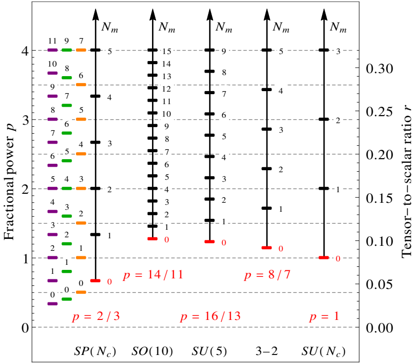
DCI: Viable and Simple Large-Field Model in Field Theory

Precise determination of p would allow to identify the dynamics of the inflaton sector!

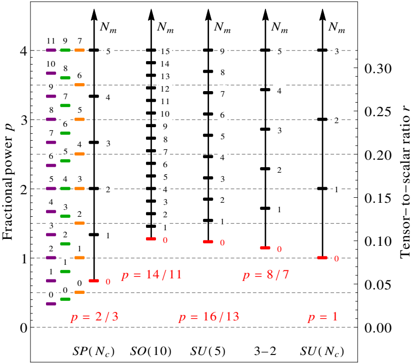


Virtues of dynamical chaotic inflation:

- Conformally invariant at the classical level in its simplest form (no input M_a).
- Energy scale of inflation generated via dimensional transmutation; thus, natural reason why $V^{1/4} \ll M_{\rm Pl}$.
- Fractional power-law in field theory!

DCI: Viable and Simple Large-Field Model in Field Theory

Precise determination of *p* would allow to identify the dynamics of the inflaton sector!



Virtues of dynamical chaotic inflation:

- Conformally invariant at the classical level in its simplest form (no input M_a).
- Energy scale of inflation generated via dimensional transmutation; thus, natural reason why $V^{1/4} \ll M_{\rm Pl}$.
- Fractional power-law in field theory!

Next steps towards a theory of DCI:

- Conformal window or DSB as alternatives to s-confinement.
- DSB during inflation in meta-stable vacuum or *in the conformal window*.
  [Intriligator, Seiberg & Shih '06] [Yanagida et al. '09]
- Embed DCI into string theory / explain origin of the shift symmetry for Φ.

Fascinating picture: Inflation as a mere consequence of strong supersymmetric gauge dynamics shortly below the Planck scale! Calls for further exploration!

Thank you for your attention!

Supplementary Material

Currently Most Attractive Models: Chaotic Inflation

Fractional power-law potential:

- Axion monodromy in string theory.
  [Silverstein & Westphal '08]

$$V(\phi) \propto \phi^{2/3},\ \phi^{2/5}$$

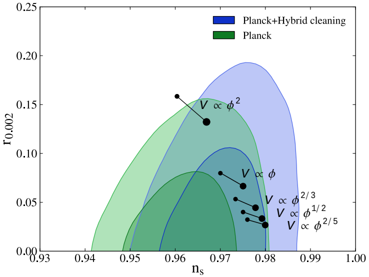
- Strong gauge dynamics in field theory.
  [Harigaya, Ibe, K.S. & Yanagdida '13]

$$V(\phi) \propto \phi^{2/(N_c+1)}, \quad G = Sp(N_c)$$

- Will be probed this year by PLANCK polarization data.

Re-analysed PLANCK data corrected for systematics in the 217 GHz map:

[Spergel, Flauger & Hlozek '13]



Common features of Starobinsky & chaotic Inflation:

- Large-field models of inflation. ⇒ Very sensitive to higher-dim. SUGRA corrections.
- Particular value of n_s singled out due to particularly shaped scalar potential.

Hybrid inflation is a small-field model, in which n_s is *a priori* undetermined!