Effective theories and dark matter

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based on work with M. Solon,

1111.0016,1309.4092(PRL),
1401.3339, and work to appear



If WIMP mass $M >> m_W$, isolation (M'-M >> m_W) becomes generic. Expand in m_W/M, m_W/(M'-M)

This regime is a focus of future experiments in direct, indirect and collider probes



basic problem in SM physics: scattering of nucleon from SU(2)xU(1) source

- discovery of SM-like higgs boson + necessary hadronic matrix elements: complete answer to this problem in principle, but a loopmediated, multi-scale problem

- M_{WIMP} >> m_W:WIMP phenomenology highly constrained and universal (analog of heavy quark spin-flavor symmetries). Interesting features, e.g., heavy WIMP "transparency" to nucleon scattering

- complications in addition to QED/QCD analogs: EWSB, and SU(3)xSU(2)xU(1) vs. U(1) or SU(3)

DM-nucleus scattering: multi-scale problem

Ε



Standard model anatomy well studied in quark flavor and EDM problems. [cf. morning talks of Buras, Lee]

Dark matter still in a relatively nascent stage. Naively subleading corrections can have large effects, e.g. determining observability of motivated candidates.



HQET, NRQCD, SCET

Very active field. Focus here on physics of direct detection above nuclear scale. Other applications of effective theories:

HQET

- Derivative suppressed single nucleon operators and nuclear responses. Lorentz invariance constraints
- Collider production via contact interactions and extensions. Relate constraints in high scale theory (n_f =5 or 6) to low scale theory (n_f =3 or 4) where direct detection and other observables are evaluated

- Annihilation, indirect detection. Large logs from SCET and consistent merging with nonperturbative enhancments

[work with M. Bauer, T. Cohen and M. Solon (not this talk)] [cf. Thurs. talk of Hellmann]

<u>SM + X</u>

- consider one or two SU(2)xU(1)
 multiplets
- expand in $m_W/(m_{X'}-m_X)$

<i>M</i> ·····
$m_W \sim m_h \sim m_t \dots \dots$
m_b , m_c
$\Lambda_{\rm QCD}$
$E_{ m nuclear}$

- convergence may be good (cf. Λ/m_b) or less good (cf. Λ/m_c), but a powerful handle on unknown dynamics

- hydrogen spectroscopy

$$E_n(H) = -\frac{1}{2}m_e(Z\alpha)^2 + \dots \qquad (m_eZ\alpha) \ll m_e$$
- heavy meson transitions

$$E^{B \to D}(\alpha' - \alpha) = 1 + \Lambda_{\rm QCD} \ll m_{b,c}$$

$$F^{B \to D}(v'=v) = 1 + \dots$$
 DM interactions

$$\sigma(\chi N \to \chi N) = ?$$

 $m_W \ll m_\chi$

<u>Setup</u>



- consider I or 2 multiplets, lightest neutral component stabilized by Z₂ symmetry (e.g., R parity, G parity)

$$\mathcal{L} = \bar{h} \left[iv \cdot \partial + eQv \cdot A + \frac{g_2}{c_W} v \cdot Z(T^3 - s_W^2 Q) + \frac{g_2}{\sqrt{2}} (v \cdot W^+ T^+ + v \cdot W^- T^-) - \delta M(v_{wk}) - f(\phi) \right] h + \mathcal{O}(1/M)$$

Low energy theory

Operator basis

$$\mathcal{L} = \mathcal{L}_{\phi_0} + \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\phi_0,\mathrm{SM}} + \dots,$$

Heavy neutral scalar:

$$\mathcal{L}_{\phi_0} = \phi_{v,Q=0}^* \left\{ iv \cdot \partial - \frac{\partial_{\perp}^2}{2M_{(Q=0)}} + \mathcal{O}(1/m_W^3) \right\} \phi_{v,Q=0}$$

SM interactions:

c_D=0 (reality constraint)

$$\mathcal{L}_{\phi_0,\text{SM}} = \frac{1}{m_W^3} \phi_v^* \phi_v \left\{ \sum_q \left[c_{1q}^{(0)} O_{1q}^{(0)} + c_{1q}^{(2)} v_\mu v_\nu O_{1q}^{(2)\mu\nu} \right] + c_2^{(0)} O_2^{(0)} + c_2^{(2)} v_\mu v_\nu O_2^{(2)\mu\nu} \right\} + \dots$$

Convenient to choose basis of definite spin

$$\begin{split} O_{1q}^{(0)} &= m_q \bar{q} q \,, \\ O_{2}^{(0)} &= (G_{\mu\nu}^A)^2 \,, \\ O_{1q}^{(2)\mu\nu} &= \bar{q} \left(\gamma^{\{\mu} i D^{\nu\}} - \frac{1}{d} g^{\mu\nu} i D \right) q \,, \\ O_{2}^{(2)\mu\nu} &= -G^{A\mu\lambda} G^{A\nu}{}_{\lambda} + \frac{1}{d} g^{\mu\nu} (G^A_{\alpha\beta})^2 \,. \end{split}$$

Universal mass shift induced by EWSB

heavy particle Feynman rules

$$\delta M = \Sigma (v \cdot p = 0) = \alpha_2 m_W \left[-\frac{1}{2} J^2 + \sin^2 \frac{\theta_W}{2} J_3^2 \right]$$

$$M_{(Q)} - M_{(Q=0)} = \alpha_2 Q^2 m_W \sin^2 \frac{\theta_W}{2} + \mathcal{O}(1/M) \approx (170 \,\mathrm{MeV})Q^2$$

Different pole masses for each charge eigenstate in low-energy theory (residual mass terms)

Matching (µ≈mw)

$$\mathcal{L} - \mathcal{L}_{SM} \rightarrow \bar{h}(iv \cdot \partial - \delta M + g_2 t^a v \cdot W^a + g_1 Y v \cdot B - f(H))h + \mathcal{O}(1/M) \xrightarrow{M \cdots} m_W \sim m_h \sim m_t \cdots m_W \sim m_h \sim m_t \cdots m_H \sim m_t \sim m_t \sim m_H \sim m_t \sim m_t \sim m_H \sim m_t \sim m_t \sim m_H \sim m_H \sim m_t \sim m_H \sim m_H \sim m_t \sim m_H \sim m_H$$

gluon operators

$$\begin{split} c_{2}^{(0)}(\mu_{t}) &= \mathcal{C}\frac{\alpha_{s}(\mu_{t})}{4\pi} \left[\frac{1}{3x_{h}^{2}} + \frac{3 + 4x_{t} + 2x_{t}^{2}}{6(1 + x_{t})^{2}} \right], \\ c_{2}^{(2)}(\mu_{t}) &= \mathcal{C}\frac{\alpha_{s}(\mu_{t})}{4\pi} \left[-\frac{32}{9} \log \frac{\mu_{t}}{m_{W}} - 4 - \frac{4(2 + 3x_{t})}{9(1 + x_{t})^{3}} \log \frac{\mu_{t}}{m_{W}(1 + x_{t})} - \frac{4(12x_{t}^{5} - 36x_{t}^{4} + 36x_{t}^{3} - 12x_{t}^{2} + 3x_{t} - 2)}{9(x_{t} - 1)^{3}} \log \frac{x_{t}}{1 + x_{t}} - \frac{8x_{t}(-3 + 7x_{t}^{2})}{9(x_{t}^{2} - 1)^{3}} \log 2 \\ - \frac{48x_{t}^{6} + 24x_{t}^{5} - 104x_{t}^{4} - 35x_{t}^{3} + 20x_{t}^{2} + 13x_{t} + 18}{9(x_{t}^{2} - 1)^{2}(1 + x_{t})} \right]. \end{split}$$

E.g. full theory:

$$i\mathcal{M} = -g_2^2 \int (dL) \left[\frac{1}{-v \cdot L + i0} + \frac{1}{v \cdot L + i0} \right] \frac{1}{(L^2 - m_W^2 + i0)^2} v_\mu v_\nu \Pi^{\mu\nu}(L)$$
electroweak polarizability tensor
in background gluon field

Electroweak gauge invariance is immediate:

$$v^{\mu} \left[g_{\mu\mu'} - (1 - \xi) \frac{L_{\mu} L_{\mu'}}{L^2 - \xi m_W^2} \right] = v_{\mu'} + \mathcal{O}(v \cdot L)$$

crossed and uncrossed diagrams cancel

Background gluon and Fock-Schwinger gauge (x.A=0):

$$\begin{split} iS(p) &= \frac{i}{\not p - m} + g \int (dq) \frac{i}{\not p - m} i \mathcal{A}(q) \frac{i}{\not p - \not q - m} & \mathcal{A}(q) = t^a \gamma^\alpha \left[\frac{-i}{2} \frac{\partial}{\partial q_\rho} G^a_{\rho\alpha}(0) (2\pi)^d \delta^d(q) + \dots \right] \\ &+ g^2 \int (dq_1) (dq_2) \frac{i}{\not p - m} i \mathcal{A}(q_1) \frac{i}{\not p - \not q_1 - m} i \mathcal{A}(q_2) \frac{i}{\not p - \not q_1 - m} + \dots \end{split}$$

QCD evolution and threshold matching



M ·····

Solution to RG equations

$$O_{1q}^{(0)} = m_q \bar{q}q, \qquad O_2^{(0)} = (G_{\mu\nu}^A)^2,$$

$$O_{1q}^{(2)\mu\nu} = \bar{q} \left(\gamma^{\{\mu} i D^{\nu\}} - \frac{1}{d} g^{\mu\nu} i D \right) q, \qquad O_2^{(2)\mu\nu} = -G^{A\mu\lambda} G^{A\nu}{}_{\lambda} + \frac{1}{d} g^{\mu\nu} (G_{\alpha\beta}^A)^2.$$

$$\frac{d}{d\log\mu} O_i^{(S)} = -\sum_j \gamma_{ij}^{(S)} O_j \qquad \frac{d}{d\log\mu} c_i^{(S)} = \sum_j \gamma_{ji}^{(S)} c_j^{(S)}$$
Spin 0: $c_2^{(0)}(\mu) = c_2^{(0)}(\mu_t) \frac{\frac{\beta}{g} [\alpha_s(\mu)]}{\frac{\beta}{g} [\alpha_s(\mu_t)]} \qquad \hat{\gamma}^{(0)} = \begin{pmatrix} 0 & | & 0 \\ & \ddots & | & \frac{1}{2} \\ 0 & | & 0 \\ & -2\gamma'_m \cdots -2\gamma'_m | (\beta/g)' \end{pmatrix}$

$$c_1^{(0)}(\mu) = c_1^{(0)}(\mu_t) - 2[\gamma_m(\mu) - \gamma_m(\mu_t)] \frac{c_2^{(0)}(\mu_t)}{\frac{\beta}{g} [\alpha_s(\mu_t)]}$$

Spin 2:

Diagonalize anomalous dimension matrix (familiar from PDF analysis)

As check, can evaluated spin-2 matrix elements at high scale (spin-0 and spin-2 decoupled)



Integrate out heavy quarks



$$\begin{aligned} c_{2}^{(0)}(\mu_{b}) &= \tilde{c}_{2}^{(0)}(\mu_{b}) \left(1 + \frac{4\tilde{a}}{3} \log \frac{m_{b}}{\mu_{b}} \right) - \frac{\tilde{a}}{3} \tilde{c}_{1b}^{(0)}(\mu_{b}) \left[1 + \tilde{a} \left(11 + \frac{4}{3} \log \frac{m_{b}}{\mu_{b}} \right) \right] + \mathcal{O}(\tilde{a}^{3}) \\ c_{1q}^{(0)}(\mu_{b}) &= \tilde{c}_{1q}^{(0)}(\mu_{b}) + \mathcal{O}(\tilde{a}^{2}), \\ c_{2}^{(2)}(\mu_{b}) &= \tilde{c}_{2}^{(2)}(\mu_{b}) - \frac{4\tilde{a}}{3} \log \frac{m_{b}}{\mu_{b}} \tilde{c}_{1b}^{(2)}(\mu_{b}) + \mathcal{O}(\tilde{a}^{2}), \\ c_{1q}^{(2)}(\mu_{b}) &= \tilde{c}_{1q}^{(2)}(\mu_{b}) + \mathcal{O}(\tilde{a}), \end{aligned}$$

Contribution to gluon operators familiar from $h \rightarrow gg$ Heavy quark mass scheme enters at higher order Charm quark treated similarly (after running to m_c)

Nucleon matrix elements

- having evolved to 3-flavor QCD, appeal to lattice QCD or SU(3) chiral perturbation theory for matrix elements

$$m_N = (1 - \gamma_m) \sum_q \langle N | m_q \bar{q} q | N \rangle + \frac{\beta}{2g} \langle N | (G^a_{\mu\nu})^2 | N \rangle$$

$$\Sigma_{\pi N} = \frac{m_u + m_d}{2} \langle p | (\bar{u}u + \bar{d}d) | p \rangle$$

$$\Sigma_0 = \frac{m_u + m_d}{2} \langle p | (\bar{u}u + \bar{d}d - 2\bar{s}s) | p \rangle$$

$$m_N(f_{u,N}^{(0)} + f_{d,N}^{(0)}) \approx \Sigma_{\pi N}$$

$$m_N f_{s,N}^{(0)} = \frac{m_s}{m_u + m_d} (\Sigma_{\pi N} - \Sigma_0) = \Sigma_s$$

$\mu({\rm GeV})$	$f_{u,p}^{(2)}(\mu)$	$f_{d,p}^{(2)}(\mu)$	$f_{s,p}^{(2)}(\mu)$	$f_{G,p}^{(2)}(\mu)$
1.0	0.404(6)	0.217(4)	0.024(3)	0.36(1)
1.2	0.383(6)	0.208(4)	0.027(2)	0.38(1)
1.4	0.370(5)	0.202(4)	0.030(2)	0.40(1)

$$f_{u,n}^{(2)} = f_{d,p}^{(2)} , \quad f_{d,n}^{(2)} = f_{u,p}^{(2)} , \quad f_{s,n}^{(2)} = f_{s,p}^{(2)}$$

MSTW 0901.0002

- strange quark scalar matrix element overestimated (central Mvalue) by SU(3) ch.p.t. $m_W \sim m_h \sim m_t$



- important impact on cross sections



- lattice still converging, but indicate small value



summary plot: Junnarkar and Walker-Loud, 1301.1114



- upper-bound \leftrightarrow prediction for some cross sections

Some results

 \mathbf{T}_{i}

(benchmark zero-velocity, heavy particle limit of single-nucleon cross section)



 $\alpha_2(m_Z)$

0.0338

Triplet (e.g., wino)			Value
$\sigma_{\rm SI}^T = 1.3^{+1.2}_{-0.5} \times 10^{-47} \rm cm^2$			~ 0
			~ 0
			~ 1
			0.49(13)
			19.5(2.5)
	$\mathbf{\lambda}$	$\Sigma_{\pi N}^{\mathrm{lat}}$	$0.047(9){ m GeV}$
			$0.050(8)\mathrm{GeV}$
		$\Sigma_{\pi N}$	$0.064(7){ m GeV}$
perturbative uncertainty	hadronic uncertainty	Σ_0	$0.036(7)\mathrm{GeV}$
	$\sigma_{\rm SI} \sim \frac{\alpha_2^4 m_N^4}{m_W^2} \left(\frac{1}{m_W^2}, \frac{1}{m_h^2}\right)^2 \sim 10^{-45} \rm cm^2$	m_W	$80.4{ m GeV}$
		m_t	$172 {\rm GeV}$
cf. dimensional estimate:		m_b	$4.75 {\rm GeV}$
		m_c	$1.4 \mathrm{GeV}$
		m_N	$0.94~{\rm GeV}$
		$\alpha_s(m_Z)$	0.118

By heavy WIMP spin symmetry, same result for self-conjugate fermion (wino) as real scalar (e.g. weakly interacting stable pion)

Doublet (e.g. higgsino)





By heavy WIMP spin symmetry, same result for fermion (higgsino) as scalar (e.g. "inert higgs")

Mixed cases

With the scale separation $M \gg m_W$, "pure states" are generic: effects of higher states suppressed by $m_W/(M'-M)$

Intricate interplay of $m_{W}/(M'-M)$ suppressed higgs exchange versus pure-state loop corrections

Again, analyze in the $M,M' \gg m_W$ limit.

Inclusion of additional multiplets allows nontrivial residual mass and direct Higgs coupling

$$\mathcal{L} = \bar{h}_v \left[iv \cdot D - \delta m - f(H) \right] h_v + \mathcal{O}(1/M)$$

$$T^{a} = \begin{pmatrix} 0 \cdot \cdot \cdot \\ \cdot \frac{\tau^{a}}{4} & \frac{-i\tau^{a}}{4} \\ \cdot \frac{i\tau^{a}}{4} & \frac{\tau^{a}}{4} \end{pmatrix} - \text{c.c.}, \quad Y = \begin{pmatrix} 0 \cdot \cdot \cdot \\ \cdot \mathbb{O}_{2} & \frac{-i\mathbb{I}_{2}}{2} \\ \cdot \frac{i\mathbb{I}_{2}}{2} & \mathbb{O}_{2} \end{pmatrix}, \quad f(H) = \frac{g_{2}\kappa_{1}}{\sqrt{2}} \begin{pmatrix} 0 & H^{T} & iH^{T} \\ H & \mathbb{O}_{2} & \mathbb{O}_{2} \\ iH & \mathbb{O}_{2} & \mathbb{O}_{2} \end{pmatrix} + \begin{bmatrix} iH \to H \\ \kappa_{1} \to \kappa_{2} \end{bmatrix} + \text{h.c.}$$

Tree level mass diagonalization

$$\begin{split} \delta m(v_{wk}) &= \delta m + v_{wk} \begin{pmatrix} 0 & a_1(0,1) & a_2(0,1) \\ a_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} & 0_2 & 0_2 \\ a_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} & 0_2 & 0_2 \end{pmatrix} \\ \text{mass eigenstate basis:} & \frac{h_{\lambda_1^{(0)}} & h_{\lambda_2^{(0)}}}{\frac{1}{2}} - 0 & \frac{h_{\lambda_1} & h_{\lambda_1^{(0)}}}{\frac{1}{2}} - \frac{h_{kv}^{(0)}}{\frac{1}{2}} \frac{h_{\lambda_1^{(0)}}}{\frac{1}{2}} \frac{h_{\lambda_1^{($$

Onshell renormalization

$$\begin{split} \mathcal{L} &= \bar{h}^{\text{bare}} \left[iv \cdot D - \mu^{\text{bare}} - f^{\text{bare}}(H) \right] h^{\text{bare}} \\ &= \bar{h} \left[iv \cdot D - \mu - f(H) \right] h + \bar{h} \left[\delta Z_h iv \cdot D - \delta \mu - \delta f(H) \right] h \\ \mu^{\text{bare}} &= \text{diag}(\mu_S^{\text{bare}}, \mu_D^{\text{bare}}, \mu_D^{\text{bare}}, \mu_D^{\text{bare}}, \mu_D^{\text{bare}}), \\ f^{\text{bare}} &= \frac{a_1^{\text{bare}}}{\sqrt{2}} \begin{pmatrix} 0 & H^{\dagger} + H^T i(H^T - H^{\dagger}) \\ H + H^* & \mathbb{Q}_2 & \mathbb{Q}_2 \\ i(H - H^*) & \mathbb{Q}_2 & \mathbb{Q}_2 \end{pmatrix} \end{split}$$

Compute loop corrections $\Sigma_2(v.k)$

$$\begin{split} [\delta\mu]_{22} + \operatorname{Re}[\Sigma_2(\delta m(v)_0^{(+)})]_{22} - \delta m(v)_0^{(+)}[\delta Z_h]_{22} = 0 \\ [\delta\mu]_{33} + \operatorname{Re}[\Sigma_2(0)]_{33} = 0 \\ [\delta\mu]_{11} + \operatorname{Re}[\Sigma_2(\delta m(v)_{\pm})]_{11} - \delta m(v)_0^{(0)}[\delta Z_h]_{11} = 0 \end{split} \begin{cases} \operatorname{renormalized lagrangian} \\ \operatorname{parameters} = \operatorname{physical} \\ \operatorname{masses} \end{cases} \\ \\ [\delta M]_{13} + \operatorname{Re}[\Sigma_2(0)]_{13} = 0 \\ [\delta M]_{23} + \operatorname{Re}[\Sigma_2(0)]_{23} = 0 \end{cases} \begin{cases} \operatorname{lightest state remains} \\ \operatorname{eigenstate} \end{cases} \end{split}$$



loop crossover occurs at ~I

1

interplay of mass-suppressed (tree level) and loop suppressed contributions

-0.5 0.0 0.5 1.0 1.5 2.0 2.5 Additional states in the Higgs sector

can the amplitude cancellation be avoided ?



cf. Carena et al. 1310.2248

Summary

• Indications of $M_{New Physics} >> m_W$ imply constrained but challenging WIMP phenomenology

• Heavy WIMP effective theory used to give first complete matching onto full basis of operators at the weak scale at leading order in perturbation theory: application to SM extensions consisting of I and 2 electroweak multiplets

• Systematic treatment of QCD perturbation theory and scale uncertainties (significant residual perturbative uncertainty)

• Important inputs from lattice (strange scalar matrix element), and potential impact of charm scalar matrix element

• Much work to do: power corrections, detailed nuclear modeling, interface with indirect detection