Complete two-loop QCD corrections to the neutral MSSM Higgs masses from the top/stop sector

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Outline

1. Background
2. Computation
3. Mass shifts in some benchmark scenarios

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3. Mass shifts in some benchmark scenarios

A very compact introduction (1)

- the Higgs mass is a prediction of SUSY models
- loop corrections are crucial (e.g. $m_h \leq m_Z \Rightarrow m_h \lesssim 130$ GeV in the MSSM)
- $m_H \sim 125$ GeV is an important constraint for MSSM!
- Public codes implementing full 1-loop and all the known analytic 2-loop results (fixed order / RGI) CPsuperH, FeynHiggs, SoftSusy, Spheno, Suspect . . .
- Parametrization in terms of physical observables (when possible!)
- 2-loop self-energies at $p^2 = 0$ easy and fast to evaluate ☺
  all codes implement $p^2 = 0$ (Effective Potential Approximation)

A very compact introduction (2)

- $p^2 \neq 0$ (pseudo)scalar 2-loop $\mathcal{O}(\alpha_t\alpha_s + \alpha_t^2)$ self-energies available, numerical [Martin 2004]
  - only $\overline{\mathrm{DR}}'$ and in terms of soft parameters

  cannot be directly used in the existing codes ☹

- $H_i - \tilde{t}_a - \tilde{t}_b$ and $H_i - H_j - \tilde{t}_a - \tilde{t}_b$ coupling structure $\Rightarrow$
  1. Yukawa-strong $\mathcal{O}(\alpha_t\alpha_s)$ [Borowka et al 14], see also Borowka's talk + [this talk]
  2. D-term induced gauge-strong $\mathcal{O}(\alpha\alpha_s)$ [not in EPA, this talk]

- Need also the $\mathcal{O}(\alpha\alpha_S)$ contrib's to the $m_z \leftrightarrow \overline{m_z}(\mu)$ relation, i.e. vector (ZZ) 2-loop self-energies [not in EPA, this talk]

  including b & light q [$\sim$ this talk]

Some notation

- requiring v_i min of V_{eff} allows to express V_{eff} in terms of physical parameters:

$$\tan\beta = v_2/v_1, \quad m_A^2 = -2m_3^2/(s_{2\beta}), \quad m_Z = Gv/2$$

- A, G mass matrix for bare fields

$$\mathcal{M}_{AG}^2 = \begin{pmatrix} m_A^2 & 0 \\ 0 & 0 \end{pmatrix} + \text{ct} + \begin{pmatrix} s_\beta^2 \frac{T_1}{v_1} + c_\beta^2 \frac{T_2}{v_2} & -\frac{s_{2\beta}}{2}\left(\frac{T_1}{v_1} - \frac{T_2}{v_2}\right) \\ -\frac{s_{2\beta}}{2}\left(\frac{T_1}{v_1} - \frac{T_2}{v_2}\right) & c_\beta^2 \frac{T_1}{v_1} + s_\beta^2 \frac{T_2}{v_2} \end{pmatrix} + \Pi_P(p^2)$$

- S_1, S_2 mass matrix for bare fields

$$\mathcal{M}_S^2 = \begin{pmatrix} m_Z^2 c_\beta^2 + m_A^2 s_\beta^2 & -\frac{s_{2\beta}}{2}(m_Z^2 + m_A^2) \\ -\frac{s_{2\beta}}{2}(mz^2 + m_A^2) & m_Z^2 s_\beta^2 + m_A^2 c_\beta^2 \end{pmatrix} + \text{ct} + \begin{pmatrix} \frac{T_1}{v_1} & 0 \\ 0 & 0\,\frac{T_2}{v_2} \end{pmatrix} + \Pi_S(p^2)$$

- EPA computation $\leftrightarrow$ eigenvalues of $\overline{\mathcal{M}}_S^2 = \mathcal{M}_S^2\big|_{\text{only }\Pi_{S,P}(0)}$

Mass shifts due to p^2 effects

1-loop and 2-loop QCD corrections [Brignole 1992; Brignole, Degrassi, Slavich, Zwirner 2002]

$$\left(\Delta\mathcal{M}^2_{11,p^2}\right)^{(i)} = s^2_\beta \mathrm{Re}\Delta\Pi^{(i)}_{AA}(m^2_A) - c^2_\beta \mathrm{Re}\Pi^{(i)}_{ZZ}(m^2_Z) - \Delta\Pi^{(i)}_{11}(p^2)$$

$$\left(\Delta\mathcal{M}^2_{12,p^2}\right)^{(i)} = -s_\beta c_\beta(\mathrm{Re}\Delta\Pi^{(i)}_{AA}(m^2_A) + c^2_\beta \mathrm{Re}\Pi^{(i)}_{ZZ}(m^2_Z)) - \Delta\Pi^{(i)}_{12}(p^2)$$

$$\left(\Delta\mathcal{M}^2_{22,p^2}\right)^{(i)} = c^2_\beta \mathrm{Re}\Delta\Pi^{(i)}_{AA}(m^2_A) - s^2_\beta \mathrm{Re}\Pi^{(i)}_{ZZ}(m^2_Z) - \Delta\Pi^{(i)}_{22}(p^2)$$

a perturbative treatment of the pole equation

$$\det(p^2 - \overline{\mathcal{M}}^2_S - \Delta\mathcal{M}^2_{S,p^2}) = 0$$

yelds the following shifts ($\overline{\alpha}$ diagonalizes the EPA $\overline{\mathcal{M}}^2_S$ and rotates S_1, S_2 to H, h)

$$\delta m^{2(i)}_h = c^2_{\beta-\overline{\alpha}} \mathrm{Re}\Delta\Pi^{(i)}_{AA}(m^2_A) + s^2_{\beta+\overline{\alpha}} \mathrm{Re}\Pi^{(i)}_{ZZ}(m^2_Z) - \Delta\Pi^{(i)}_{hh}(\overline{m_h}^2)$$

$$\delta m^{2(i)}_H = s^2_{\beta-\overline{\alpha}} \mathrm{Re}\Delta\Pi^{(i)}_{AA}(m^2_A) + c^2_{\beta+\overline{\alpha}} \mathrm{Re}\Pi^{(i)}_{ZZ}(m^2_Z) - \Delta\Pi^{(i)}_{HH}(\overline{m_H}^2)$$

Something about renormalization

- $\overline{\rm DR}$ wfr in order to make $\Delta\mathcal{M}^2_{S,p^2}$ finite, $\mathcal{Z}_i = 1 + \delta\mathcal{Z}_i^{(1)} + \delta\mathcal{Z}_i^{(2)}$

$$\delta\mathcal{Z}_i^{(1)} = -\left.\frac{d\Pi_{ii}^{(1)}(p^2)}{dp^2}\right|_{\rm pole} \Rightarrow \delta\mathcal{Z}_1^{(1)} = 0\,, \delta\mathcal{Z}_2^{(1)} = -\frac{\alpha_t}{4\pi}\frac{N_c}{\epsilon}$$

$$\delta\mathcal{Z}_i^{(2)} = -\left.\frac{d\Pi_{ii}^{(2)}(p^2)}{dp^2}\right|_{\rm pole} \Rightarrow \delta\mathcal{Z}_1^{(2)} = 0\,, \delta\mathcal{Z}_2^{(2)} = \frac{\alpha_t\alpha_s}{(4\pi)^2}N_c\,C_F\Big(\frac{2}{\epsilon^2} - \frac{2}{\epsilon}\Big)$$

- we choose $\overline{\rm DR}$ also for $\tan\beta$, which inherits the prescription:

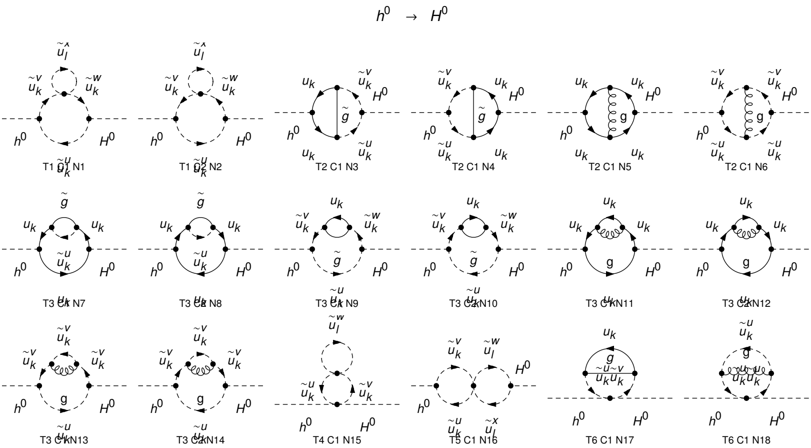
$$\frac{\delta\tan\beta^{(k)}}{\tan\beta} = \frac{1}{2}(\delta\mathcal{Z}_2^{(k)} - \delta\mathcal{Z}_1^{(k)}) \qquad k = 1, 2$$

- the strong renormalization of the $t/\tilde{t}$ sector $(m_t, m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}, \theta_{\tilde{q}}, A_t)$ in the 1-loop induces 2-loop $\mathcal{O}(\alpha_t\alpha_s + \alpha\alpha_s)$ effects

we work in $\overline{\rm DR}$ for the 1-loop insertions, obtaining a finite $\overline{\rm DR}$ result. then we translate our result to the mixed $\overline{\rm DR}$-OS scheme with the usual shifts for the top/stop parameters $\hat{\delta}x = \overline{x}(\mu) - x^{\rm OS}$

Outline

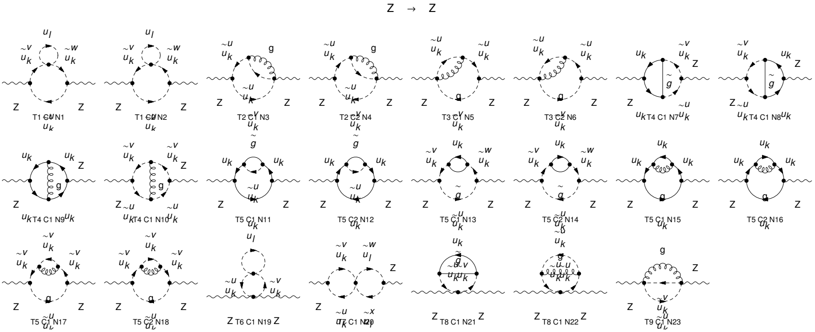
Sample Feynman diagrams: self-energies (1)



drawn with FeynArts

both massive and massless q

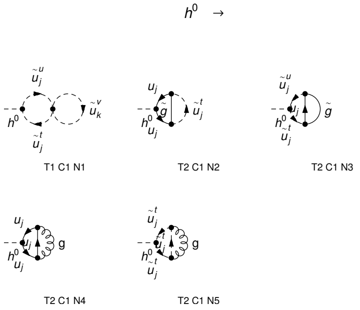
Sample Feynman diagrams: self-energies (2)

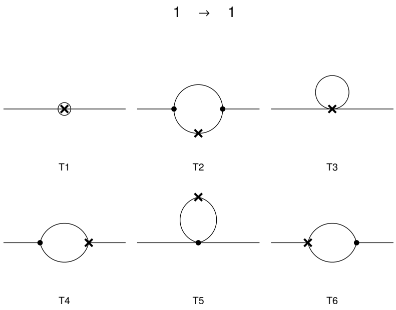


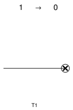
drawn with FeynArts

both massive and massless q

Sample Feynman diagrams: tadpoles and ct's







both massive and massless q

drawn with FeynArts

Reduction to Master Integrals

- $\mathcal{O}(100)$ diagrams for each self-energy (with **FeynArts** [Hahn 01])
- IBP reduction to linear combinations of few scalar master integrals [Chetyrkin and Tkachov 81; Laporta 01] with **REDUZE** [Studerus 09, + von Manteuffel 12]
- such MI's are known analytically only for special cases ☹
- numerical approach [Caffo, Czyz, Laporta, and Remiddi 98-02, Martin 03] based on the differential eq's method [Kotikov 91, Remiddi 97]
- evaluate the MI's with the public code **TSIL** [Martin and Robertson 05] combines Analytic + DiffEq, fast, written in C with built-in Fortran interface, maintained

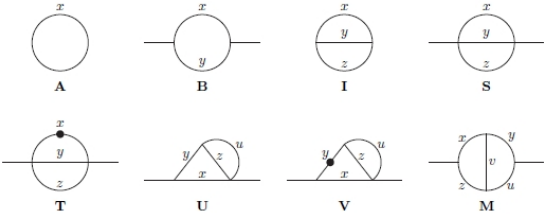


Figure 1: Feynman diagram topologies for the one- and two-loop vacuum and self-energy integrals as defined in this paper.

What we actually computed [Degrassi, Slavich and DV, to appear]

- top/stop: diagrams with many internal masses $\mathcal{O}(\alpha_t\alpha_s + \alpha\alpha_s)$
- bottom/sbottom in the approximation $h_b = m_b = 0$, $\mathcal{O}(\alpha\alpha_s)$
- first and second generation $q/\tilde{q}$ (massless) just like $\tilde{b/b}$, $\mathcal{O}(\alpha\alpha_s)$

- Higgs mass $\mathcal{O}(\alpha_t\alpha_s)$ p^2 shifts wrt EPA
- Higgs mass completely **new** $\mathcal{O}(\alpha\alpha_s)$ contrib's
- everything in terms of phys. parameters (all OS with $\overline{\mathrm{DR}}$ wfr and t_β)

- use **FeynHiggs** output as *unperturbed* result $\overline{\mathcal{M}}_S^2$

  [Heinemeyer, Hollik, Weiglein + Hahn, Frank, Rzehak + Degrassi, Slavich]
- compute MSSM benchmark scenarios [Carena et al 13] for their "default" μ, for $0 < m_A < 1.5$ TeV, $m_{\tilde{g}} = 1.5$ TeV and $\tan\beta = 5, 10, 20$
- add on top the new mass shifts

  here $t/\tilde{t}$ only, $b/\tilde{b}$ and $1^{st}, 2^{nd}$ gen. $q/\tilde{q}$ small corrections but not yet included in the shifts

Checks

- the poles of our $\Pi(p^2)$'s T's are *local* in p^2
- $\overline{\mathrm{DR}}$ is a mass independent scheme, no ratios of masses (and μ_R) can appear in the poles (no logs of m^2/μ_R^2!)
- our OS result does not depend on the unphysical $\mathcal{O}(\epsilon)$ coefficients of the loop function (e.g. B_0^ϵ)
- the poles of the counterterm for $\tan\beta$ satisfy the usual 1- and 2-loop relations with $\beta_{t_\beta}(\mu) = d\tan\beta/d\log\mu^2$

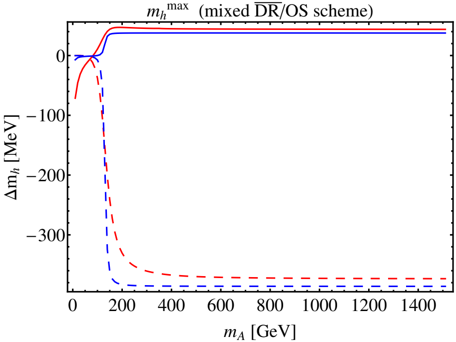
$p^2 \to 0$ result in perfect agreement with literature [Degrassi,Slavich,Zwirner 01]

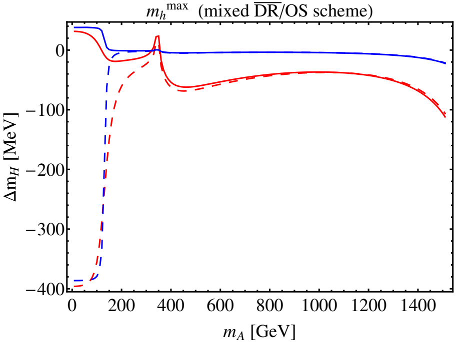
(pseudo)scalar $\Pi(p^2)$'s in perfect agreement with literature [Martin 04, thanks!]

1PI (pseudo)scalar $\Pi(p^2)$'s in perfect agreement with another independent $\mathcal{O}(\alpha_t\alpha_s)$ computation [Borowka et al 14, thanks!]

Outline

preliminary $\Delta m_{h,H}$ in the m_h^{max} scenario mixed $\overline{\text{DR}}$/OS scheme





$\Delta m_{h,H} = m_{h,H} - m_{h,H}^{\text{EPA}}$ (MeV) vs m_A up to 1.5 TeV

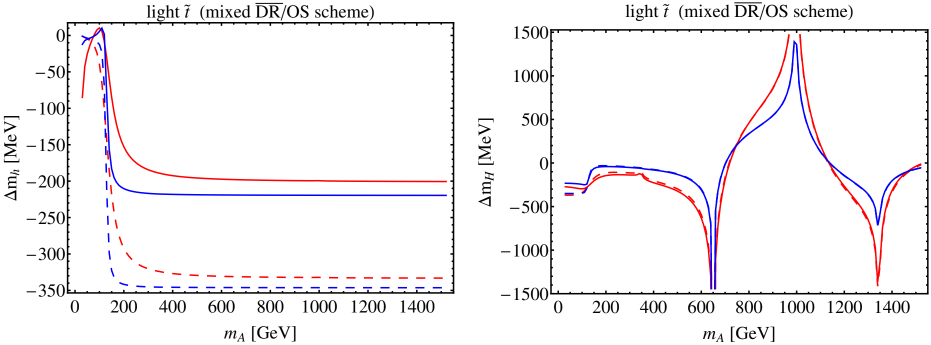
– – – only $\mathcal{O}(\alpha_t \alpha_s)$

——— full $t/\tilde{t}$ NLO QCD $\mathcal{O}(\alpha_t \alpha_s + \alpha \alpha_s)$

$\tan\beta = 5$ $\tan\beta = 20$

$\mu = 200$ GeV, $m_{\tilde{g}} = 1.5$ TeV

preliminary $\Delta m_{h,H}$ in the *light*-$\tilde{t}$ scenario mixed $\overline{\text{DR}}$/OS scheme



$\Delta m_{h,H} = m_{h,H} - m^{\text{EPA}}_{h,H}$ (MeV) vs m_A up to 1.5 TeV

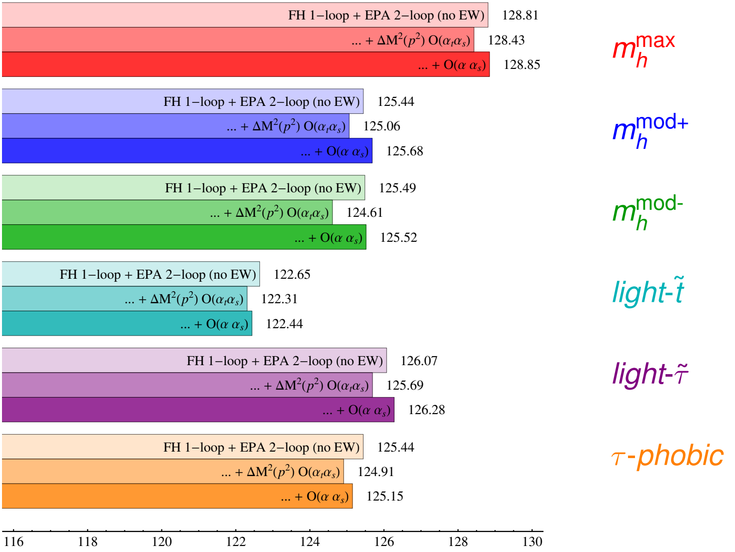
– – – only $\mathcal{O}(\alpha_t \alpha_s)$

—— full $t/\tilde{t}$ NLO QCD $\mathcal{O}(\alpha_t \alpha_s + \alpha \alpha_s)$

$\tan\beta = 5$ $\tan\beta = 20$

$\mu = 400$ GeV, $m_{\tilde{g}} = 1.5$ TeV

preliminary Δm_h for $m_A = 300$ GeV, $t_\beta = 10$, $m_{\tilde{g}} = 1.5$ TeV, mixed $\overline{\mathrm{DR}}$/OS scheme



Conclusions

NEW!

- MSSM Higgs mass p^2 shifts $\mathcal{O}(\alpha_t\alpha_s)$ and new $\mathcal{O}(\alpha\alpha_s)$ contrib's in terms of physical parameters (OS with $\overline{\mathrm{DR}}$ wfr and tan β)
- solid calculation, perfect agreement of the (pseudo)scalar self-energies with the literature for $p^2 \to 0$ and for the $\overline{\mathrm{DR}}$ scheme

Some plots

- top/stop: diagrams with many internal masses $\mathcal{O}(\alpha_t\alpha_s + \alpha\alpha_s)$
- corrections to m_h are small and seem to roughly compensate
- corrections to m_H are negligible (away from thresholds)

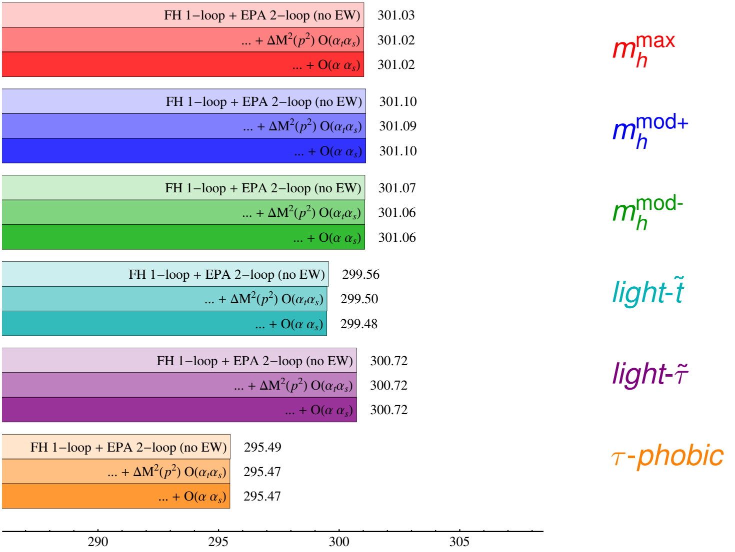
Already evaluated, soon to be included

- bottom/sbottom in the approximation $h_b = m_b = 0$, $\mathcal{O}(\alpha\alpha_s)$
- first and second generation $q/\tilde{q}$ (massless) just like $b\tilde{b}$, $\mathcal{O}(\alpha\alpha_s)$

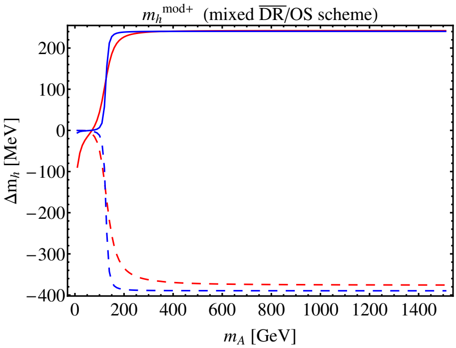
Thanks for your attention!

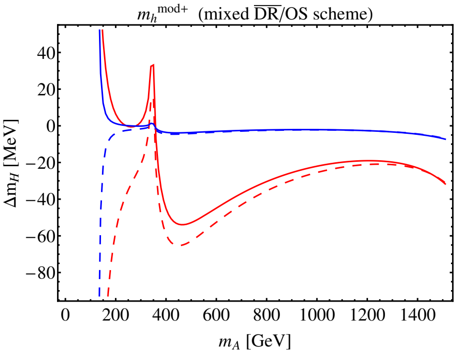
Backup

preliminary Δm_H for $m_A = 300$ GeV, $t_\beta = 10$, $m_{\tilde{g}} = 1.5$ TeV, mixed $\overline{\mathrm{DR}}$/OS scheme



preliminary $\Delta m_{h,H}$ in the $m_h^{\text{mod+}}$ scenario mixed $\overline{\text{DR}}$/OS scheme





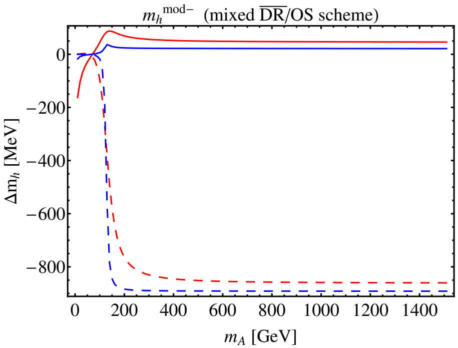
$\Delta m_{h,H} = m_{h,H} - m_{h,H}^{\text{EPA}}$ (MeV) vs m_A up to 1.5 TeV

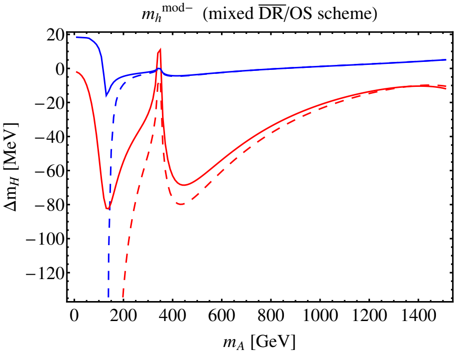
– – – only $\mathcal{O}(\alpha_t \alpha_s)$

—— full $t/\tilde{t}$ NLO QCD $\mathcal{O}(\alpha_t \alpha_s + \alpha \alpha_s)$

$\tan\beta = 5$ $\tan\beta = 20$

$\mu = 200$ GeV, $m_{\tilde{g}} = 1.5$ TeV





$\Delta m_{h,H} = m_{h,H} - m_{h,H}^{\text{EPA}}$ (MeV) vs m_A up to 1.5 TeV

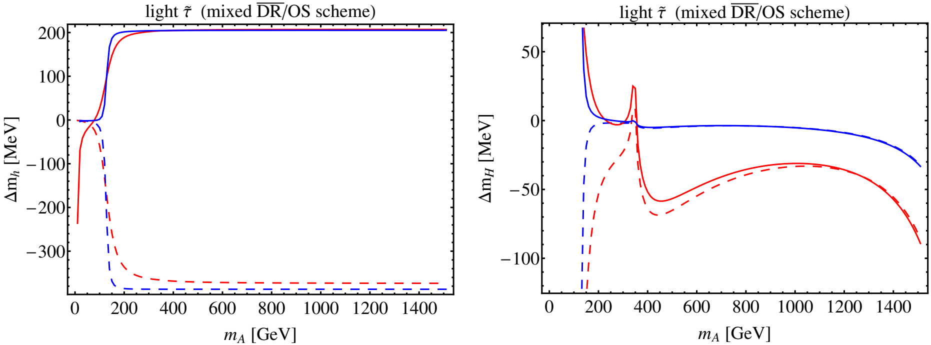
– – – only $\mathcal{O}(\alpha_t \alpha_s)$

—— full $t/\tilde{t}$ NLO QCD $\mathcal{O}(\alpha_t \alpha_s + \alpha \alpha_s)$

$\tan\beta = 5$ $\tan\beta = 20$

$\mu = 200$ GeV, $m_{\tilde{g}} = 1.5$ TeV

preliminary $\Delta m_{h,H}$ in the *light*-$\tilde{\tau}$ scenario mixed $\overline{\text{DR}}$/OS scheme



$\Delta m_{h,H} = m_{h,H} - m_{h,H}^{\text{EPA}}$ (MeV) vs m_A up to 1.5 TeV

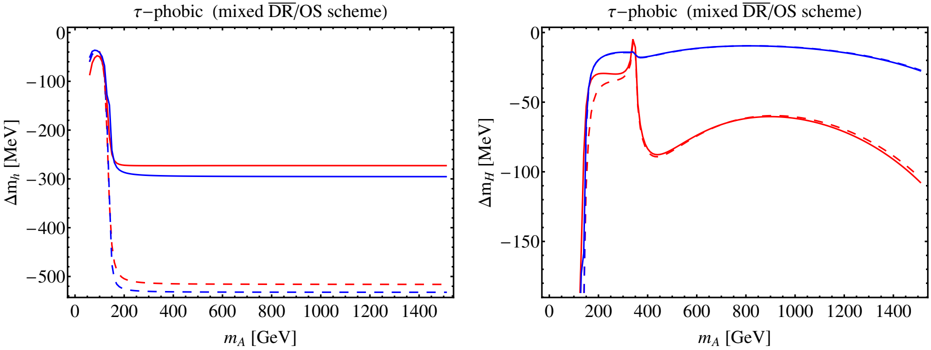
– – – only $\mathcal{O}(\alpha_t \alpha_s)$

—— full $t/\tilde{t}$ NLO QCD $\mathcal{O}(\alpha_t \alpha_s + \alpha \alpha_s)$

$\tan\beta = 5$ $\tan\beta = 20$

$\mu = 500$ GeV, $m_{\tilde{g}} = 1.5$ TeV

preliminary $\Delta m_{h,H}$ in the τ-*phobic* scenario mixed $\overline{\mathrm{DR}}$/OS scheme



$\Delta m_{h,H} = m_{h,H} - m_{h,H}^{\mathrm{EPA}}$ (MeV) vs m_A up to 1.5 TeV

– – – only $\mathcal{O}(\alpha_t \alpha_s)$

——— full $t/\tilde{t}$ NLO QCD $\mathcal{O}(\alpha_t \alpha_s + \alpha \alpha_s)$

$\tan\beta = 5$ $\tan\beta = 20$

$\mu = 2000$ GeV, $m_{\tilde{g}} = 1.5$ TeV

Renormalization (1)

- CP-odd: OS m_A from a scalar equation at this order no wfr induced mixing

$$\Gamma^{(2)}_{AA}(p^2) = p^2 - m_A^2 \underbrace{-\delta m_A^2 - s_\beta^2 \frac{T_1}{v_1} - c_\beta^2 \frac{T_2}{v_2} + \Pi_{AA}(p^2)}_{=0}$$

- Z: OS m_Z from a scalar equation at this order no wfr induced mixing

$$\Gamma^{(2)}_{ZZ}(p^2) = p^2 - m_Z^2 \underbrace{-\delta m_Z^2 + \Pi_{ZZ}(p^2)}_{=0}$$

- CP-even (matrix equation) ($\mathcal{M}^2_{S,\mathrm{ct}} \leftrightarrow \delta m_A^2, \delta m_Z^2, \delta \tan\beta$; wfr diagonal):

$$\begin{aligned}\Gamma^{(2)}_{S,\mathrm{tree}}(p^2) &= p^2 - \mathcal{M}^2_{S,\mathrm{tree}} \\ &\to \sqrt{\mathcal{Z}}\,\left(p^2 - \mathcal{M}^2_{S,\mathrm{tree}} - \mathcal{M}^2_{S,\mathrm{ct}} - \mathcal{M}^2_{S,\mathrm{tad}} + \Pi_S(p^2)\right)\,\sqrt{\mathcal{Z}} \\ &= \sqrt{\mathcal{Z}}\,\Big[p^2 - \mathcal{M}^2_{S,\mathrm{tree}} \\ &\qquad - \left(\mathcal{M}^2_{S,\mathrm{tad}} + \mathcal{M}^2_{S,\mathrm{ct}} + \delta_{\mathrm{EPA}} - \delta_{\mathrm{EPA}}\right) \\ &\qquad + \left(\Pi_S(0) + \Delta\Pi_S(p^2)\right)\Big]\,\sqrt{\mathcal{Z}}\end{aligned}$$

Renormalization (2)

- cast renormalized inverse propagator matrix (pole m_A, m_Z) as

$$\Gamma_S^{(2)}(p^2) = p^2 - \underbrace{\mathcal{M}^2_{S,\text{tree}} - \Delta\mathcal{M}^2_{S,\text{EPA}}}_{\overline{\mathcal{M}_S}^2} - \Delta\mathcal{M}^2_{S,p^2}(p^2)$$

- where we recovered the *finite* EPA result (nice counterterm interplay)

$$\Delta\mathcal{M}^2_{\text{EPA}} \equiv \sqrt{\mathcal{Z}}\left(\mathcal{M}^2_{S,\text{tree}} + \mathcal{M}^2_{S,\text{tad}} + \mathcal{M}^2_{S,\text{ct}} - \delta_{\text{EPA}} - \Pi_S(0)\right)\sqrt{\mathcal{Z}} - \mathcal{M}^2_{S,\text{tree}}\,,$$

- and defined the *finite* contribution due to $p^2 \neq 0$

$$\Delta\mathcal{M}^2_{p^2}(p^2) \equiv p^2 + \sqrt{\mathcal{Z}}(\delta_{\text{EPA}} - p^2 - \Delta\Pi(p^2))\sqrt{\mathcal{Z}}$$

- we need to compute 2-loop tadpoles (easy!) and propagators with arbitrary external momentum

Two-loop self-energies: IBP reduction to MI's (1)

Problem: exact eval. of multi-scale two-loop self-energies

$$J(Q^2;\underbrace{r_1,\ldots,r_5;s_1\ldots,s_5}_{r_i,s_i\geq 0}) = \iint \mathrm{d}^4p\,\mathrm{d}^4k\,\frac{S_1^{s_1}\cdots S_5^{s_5}}{D_1^{r_1}\cdots D_5^{r_5}}$$

5 indep. scalar products $\rightarrow$ solve for the D_i's

$$D_1 = p^2 - m_1^2\,, \qquad S_1 = p^2 = D_1 - m_1^2\,,$$

$$D_2 = (p-Q)^2 - m_2^2\,, \quad S_2 = p_\mu Q^\mu = -(D_2 - m_2^2 - D_1 + m_1^2)/2\,,\ldots$$

By writing the S_i in terms of the D_i we get

$$J(Q^2;\vec{r};\vec{s}) = \sum_{\vec{n}} c_{\vec{n}}\, I(Q^2;\vec{n}) = \iint \frac{\mathrm{d}^4p\,\mathrm{d}^4k}{D_1^{n_1}\cdots D_5^{n_5}}\,, \quad n_i \in \mathbb{Z}$$

Two-loop self-energies: IBP reduction to MI's (2)

- in d dimensions, IBP id's $\int \mathrm{d}^d k \, \frac{\partial}{\partial l^\mu} \left[\frac{(k,p,q_1,q_2)^\mu}{D_1^{n_1} \cdots D_5^{n_5}} \right] = 0, \; (l = p, k)$
- In principle can be solved at "operator" level for arbitrary $\vec{n}$
- By recursive application, $I(Q^2, \vec{n}) \to \sum_i c_i(\vec{n}) \, M_i(Q^2)$
- Coefficients = rational functions of polynomials in Q^2 , m_i's and d
- In practice, Laporta algorithm: generate IBP's for "several" explicit values of $\vec{n}$, choose ordering, solve, (wait . . .), *store, reuse!*
- C++ public code REDUZE2 [Studerus 2010, Studerus and von Manteuffel 2012]

✓ rational coefficients $\leftrightarrow$ purely algebraic effort, CAS

☹ MI's $\leftrightarrow$ unsolved in the general case, Numerical evaluation.

Here $\to$ TSIL, Analytic/DiffEq, C++, *fast* (0m0.126s on my PC for all the MI's), built-in Fortran interface) [Martin and Robertson 2005]