

Lepton Mixing Predictions from (Generalised) CP and discrete Symmetry

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Outline

- Generalised CP Symmetries
- Example: $\Delta(6n^2)$ Flavour Groups
- Conclusions/Future Work

(Based on King,Stuart,TN:1305.3200; King,Neder:1403.1758 and work in progress)

Introduction

What happens if CP is promoted to a symmetry of the theory?

Canonical CP

- Imagine that ϕ_r is a multiplet under a symmetry of the theory, $G = G_{\text{gauge}} \times G_{\text{flavour}}$.
- Canonical CP: $\phi_r(x) \mapsto \phi_r^{CP} = e^{i\varphi_r} \phi_r^*(x^P)$
- $CP^{-1} \circ G \circ CP \rightarrow G^1$
- G and CP not always compatible in this way. ¹²
- Is CP violated? Not necessarily!

¹Grimus, Rebelo '95

²Holthausen, Lindner, Schmidt '12

Generalized CP (1)

- Define generalised CP: $\phi_r(x) \mapsto \phi_r^{gCP} = X_r \phi_r^*(x^P)$ with a unitary matrix X_r .
- If done “right”, generalised CP makes observables conserve CP, even if canonical CP is violated³
- \Rightarrow Generalised CP and canonical CP transformations are physically indistinguishable!

³Chen, Fallbacher, Mahanthappa, Ratz, Trautner '14 

Generalised CP (2)

- Consistency Equation: $CP^{-1} \circ G \circ CP \rightarrow G \Rightarrow$

$$X_r^\dagger \rho_r(g)^* X_r = \rho_r(g')$$

- Which matrices X_r are allowed? \rightarrow Next slide
- What happens if G is broken? Unbroken subgroup and CP have to be consistent: CP can be broken.
- (Broken) Invariance under generalised CP will allow to predict Majorana phases purely from symmetry.


Classification of Groups

CP-properties of G depend on existence of⁴:

- w_r that conjugates-inverses: $w_r^\dagger \rho_r^*(g) w_r \in C(g^{-1})$
- a class-conserving outer automorphism $\nu : G \rightarrow G$ (rare)

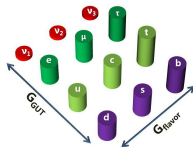
r is a faithful representation of G .

	$\nexists \nu$	$\exists \nu$
$\exists w \in G$	$\{X_r\} = \rho_r(G)$ $\Delta(6n^2)$ for $3 \nmid n$	$\{X_r\} = \rho_r(G), X_\nu \rho_r(G)$
$\exists w \notin G$	$\{X_r\} = w_r \rho_r(G)$	$\{X_r\} = w_r \rho_r(G), X_\nu w_r \rho_r(G)$
$\nexists w$	$\{X_r\} = \{\}$	$\{X_r\} = \{\}$

⁴Chen et al '14; King, TN '14; King, TN, in preparation 

Flavour Symmetries (1)

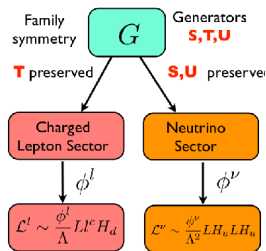
- Flavour symmetries extend the symmetry group of the Standard Model by an additional (“horizontal”) symmetry that connects different flavours of particles: $G_{\text{gauge}} \times G_{\text{Flavour}}$.
- The invariance of the Lagrangian under G_{Flavour} restricts the possible couplings and thus the possible mixing matrices.



Picture from

<http://theophys.kth.se/tepp/Flavor.jpg>

Direct Models



(From Steve F. King, Christoph Luhn, Neutrino Mass and Mixing with Discrete Symmetry, 1303.6180)

- In direct models, the mixing matrix is fixed just from the choice of the $Z_2 \times Z_2$ subgroup of G_{Flavour} .
- One can systematically list all mixing matrices allowed in a direct model by a flavour group via its subgroups.
- In direct models, only $\Delta(6n^2)$ groups remain viable^a
- Majorana phases are not predicted by the flavour symmetry \Rightarrow Enhance symmetry by generalised CP.

^aFonseca, Grimus '14

$\Delta(6n^2)$ Groups

- The groups $\Delta(6n^2)$ are non-abelian discrete subgroups of $U(3)$ of order $6n^2$ and are isomorphic to a semidirect product: $\Delta(6n^2) \cong (Z_n \times Z_n) \rtimes S_3$
- The left-handed leptons transform (without loss of generality) under a 3-dimensional representation with the generators a, b, c, d (where $\eta = e^{2\pi i/n}$).
- $U(3) \supset \Delta(6n^2) \supset \Delta(3n^2)$
- $\Delta(6 \times 1^2) = S_3$, $\Delta(6 \times 2^2) = S_4$,
 $\Delta(6 \times 3^2) = \Delta(54)$,
 $\Delta(6 \times 4^2) = \Delta(96)$, $\Delta(3 \times 1^2) = Z_3$,
 $\Delta(3 \times 2^2) = A_4$, $\Delta(3 \times 3^2) = \Delta(27)$

$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},$$
$$b = - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$
$$c = \begin{pmatrix} \eta & 0 & 0 \\ 0 & \eta^{-1} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$
$$d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \eta^{-1} \end{pmatrix}$$

Mixing Results from $\Delta(6n^2)$ and generalised CP⁵

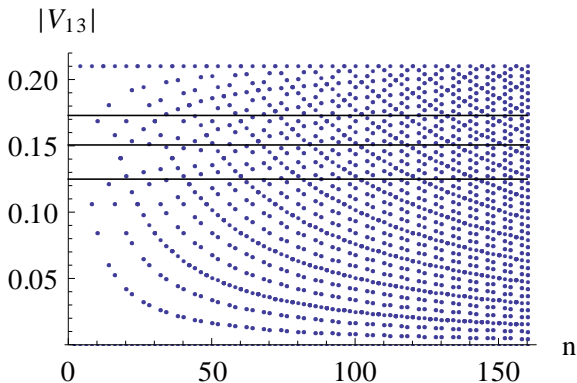
$$U_{\text{PMNS}}^{\pm} = \begin{pmatrix} \sqrt{\frac{2}{3}} \cos\left(\frac{\pi\gamma}{n}\right) & \frac{[i]e^{\frac{1}{2}i(\varphi_1 - \varphi_3)}}{\sqrt{3}} & [i]i\sqrt{\frac{2}{3}} \sin\left(\frac{\pi\gamma}{n}\right) \\ -\sqrt{\frac{2}{3}} \sin\left(\pi\left(\frac{\gamma}{n} + \frac{1}{6}\right)\right) & \frac{[i]e^{\frac{1}{2}i(\varphi_1 - \varphi_3)}}{\sqrt{3}} & [i]i\sqrt{\frac{2}{3}} \cos\left(\pi\left(\frac{\gamma}{n} + \frac{1}{6}\right)\right) \\ \sqrt{\frac{2}{3}} \sin\left(\pi\left(\frac{1}{6} - \frac{\gamma}{n}\right)\right) & -\frac{[i]e^{\frac{1}{2}i(\varphi_1 - \varphi_3)}}{\sqrt{3}} & [i]i\sqrt{\frac{2}{3}} \cos\left(\pi\left(\frac{1}{6} - \frac{\gamma}{n}\right)\right) \end{pmatrix}$$

All phases and mixing angles are fixed from symmetry and generalised CP.

⁵King, TN, Stuart '13; King, TN '14

Mixing Results (contd.)

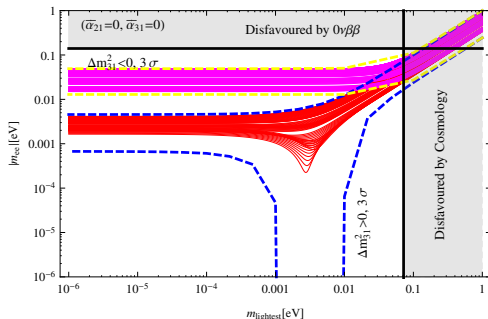
One can plot the possible values of $|V_{13}|$, the lines denote the present approximate 3σ range of $|V_{13}|$ (from Fogli et al,1205.5254):



Ex.: $|V_{13}| = 0.211, 0.170, 0.160, 0.154$ for $n = 4, 10, 16, 22$

Neutrinoless double-beta decay⁶

$$|m_{ee}| = \left| \frac{2}{3} m_1 \cos^2\left(\frac{\pi\gamma}{n}\right) + \frac{1}{3} m_2 e^{i\alpha_{21}} + \frac{2}{3} m_3 \sin^2\left(\frac{\pi\gamma}{n}\right) e^{i(\alpha_{31}-2\delta)} \right|$$



⁶King, TN '14

Summary and Outlook

- We examine lepton mixing patterns in direct models with $\Delta(6n^2)$ groups and consistent generalised CP.
- We classify groups according to their generalised CP properties.
- In direct flavour models only $\Delta(6n^2)$ groups remain viable and can be analysed simultaneously in n .
- All mixing angles and phases are predicted purely from symmetry and are accessible to future experiments.

BACKUP

Generalised CP (1)

- In a direct model, all mixing angles and the Dirac phase are purely predicted from symmetry.
- In the Standard Model, violation of CP occurs in the flavour sector.
- Promoting CP to a symmetry at high energies which is then broken allows to impose further constraints on low energy mass matrices.
- For direct models and especially with $\Delta(6n^2)$ groups, CP symmetries had not been studied in detail yet.

Generalised CP (2)

- Examples for models employing generalised CP:
 - I. Girardi, A. Meroni, S. T. Petcov and M. Spinrath, JHEP **1402**, 050 (2014)
 - G. -J. Ding and Y. -L. Zhou, arXiv:1312.5222
 - F. Feruglio, C. Hagedorn and R. Ziegler, Eur. Phys. J. C **74**, 2753 (2014)
 - C. Luhn, Nucl. Phys. B **875**, 80 (2013)
 - G. -J. Ding, S. F. King and A. J. Stuart, JHEP **1312**, 006 (2013)
 - G. -J. Ding, S. F. King, C. Luhn and A. J. Stuart, JHEP **1305**, 084 (2013)
 - M. S. Boucenna, S. Morisi, E. Peinado, Y. Shimizu and J. W. F. Valle, Phys. Rev. D **86**, 073008 (2012)
- The interplay of flavour and CP must be carefully discussed, this will make up the remainder of the talk.

Generalised CP (3)

- For fields that transform as $\varphi_r \mapsto \rho_r(g)\varphi_r$ under G_{Flavour} , define generalised CP as

$$\varphi_r \mapsto X_r \left(\varphi_r^*(x^P) \right).$$

X_R is a unitary matrix.

- Only gCP transformations that map each representation on itself make observables conserve CP (M. -C. Chen, M. Fallbacher, K. T. Mahanthappa, M. Ratz and A. Trautner, arXiv:1402.0507).
- The mass matrices will be constrained by

$$X_r^{e\dagger} M^e (M^e)^\dagger X_r^e = (M^e)^* (M^e)^T \quad \text{and} \quad X_r^{\nu T} M^\nu X_r^\nu = (M^\nu)^*$$

- If both flavour and gCP symmetries are present, they have to fulfill the consistency equation: $X_r \rho^*(\sigma) X_r^\dagger = \rho_r(\sigma')$

Brief Digression: Group automorphisms

- Inner automorphisms: A single group element h_u exists such that $\forall g : u(g) = h_u^{-1}gh_u$
- Outer automorphisms: All other automorphisms
- $(u \text{ inner}) \Rightarrow$ (All group elements are mapped into their original class)
- $(u \text{ outer}) \Rightarrow (u \text{ inner}) \Rightarrow$ (Not all group elements are mapped into their original class)
- (All group elements are mapped into their original class) $\Rightarrow (u \text{ inner})$
- (Not all group elements are mapped into their original class) $\Rightarrow (u \text{ outer})$
- This proves
 $(u \text{ inner}) \Leftrightarrow$ (All group elements are mapped into their original class)

Group automorphisms and generalised CP:

$X_r \in e^{i\alpha} G$ for real representations

- For $u_X(g) := \rho_r^{-1}(X_r \rho_r^*(g) X_r^\dagger)$, if $\rho_r(g)$ is real and $X_r \in e^{i\alpha} G$, u_X is an inner automorphism.
- Can there be a matrix \tilde{X}_r that is not in $e^{i\alpha} G$ but where $u_{\tilde{X}}$ maps every group element into its original class?
- $u_{\tilde{X}}$ is inner $\Rightarrow \exists! h_u : u_{\tilde{X}}(g) = h_u^{-1} g h_u$.
- From this follows that $\rho_r(h_u)^{-1} \tilde{X}_r \rho_r(g) = \rho_r(g) \rho_r(h_u^{-1}) \tilde{X}_r$, i.e. $\rho_r(h_u^{-1}) \tilde{X}_r$ commutes with every group element.
- With Schur's Lemma follows $\tilde{X}_r = \lambda \rho_r(h_u)$ with $|\lambda| = 1$ which contradicts $X_r \notin e^{i\alpha} G$.
- For real representations this proves that inner automorphisms correspond to $X_r \in e^{i\alpha} G$.

Group automorphisms and gCP:

$X_r \in e^{i\alpha} G$ for complex representations

- Assume a matrix w_r exists such that $\rho_r(g) \mapsto w_r^\dagger \rho_r(g)^* w_r$ is in the class of the inverse of g , $C(g^{-1})$.
- This can be seen as mapping g onto g^{-1} followed by an inner automorphism onto another element in $C(g^{-1})$.
- Are there $\tilde{X}_r \notin e^{i\alpha} G$ that map elements into a different class as g^{-1} ? - No, c.f. previous page.
- Only gCP transformations that map into the class of the inverse make observables conserve CP (M. -C. Chen, M. Fallbacher, K. T. Mahanthappa, M. Ratz and A. Trautner, arXiv:1402.0507).
- $\Delta(6n^2)$ contains an element that can be w_r , namely $w_r = \rho_r(b)$.
- $\Rightarrow \text{gCP}(\Delta(6n^2)) = e^{i\alpha} \Delta(6n^2)$.

Predictions for Majorana phases and $0\nu\beta\beta$ purely from flavour and CP symmetry (1)

- Residual flavour and residual gCP symmetries must still be consistent: For $K = \{1, c^{n/2}, abc^\gamma, abc^{\gamma+n/2}\}$, consistent gCP transformations are

$$X_r = \rho_r(e^{i\alpha} c^x d^{2x+2\gamma}), \rho_r(e^{i\alpha} abc^x d^{2x}) \text{ with } x = 0, \dots, n-1$$

- \Rightarrow Choice of residual flavour symmetry determines allowed residual gCP symmetries
- \Rightarrow For all $\Delta(6n^2)$ groups enhanced by CP invariance the mixing matrix is fixed up to a discrete choice and always has the form

$$U_{\text{PMNS}}^{(+)/[(-)]} = \begin{pmatrix} \sqrt{\frac{2}{3}} \cos\left(\frac{\pi\gamma}{n}\right) & \frac{e^{i(\varphi_1 - \varphi_3)/2}}{\sqrt{3}} & [i]i\sqrt{\frac{2}{3}} \sin\left(\frac{\pi\gamma}{n}\right) \\ -\sqrt{\frac{2}{3}} \sin\left(\pi\left(\frac{1}{6} + \frac{\gamma}{n}\right)\right) & \frac{e^{i(\varphi_1 - \varphi_3)/2}}{\sqrt{3}} & [i]i\sqrt{\frac{2}{3}} \cos\left(\pi\left(\frac{1}{6} + \frac{\gamma}{n}\right)\right) \end{pmatrix}$$

Predictions for Majorana phases and $0\nu\beta\beta$ purely from flavour and CP symmetry (2)

- The key observable for Majorana phases is neutrinoless double-beta decay.
- In this framework, the effective mass is given by

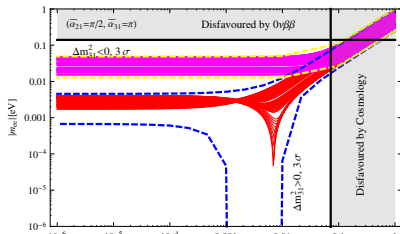
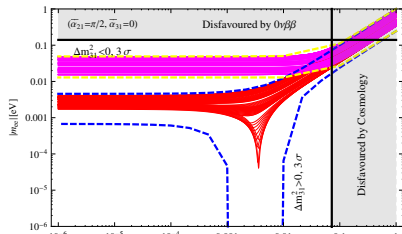
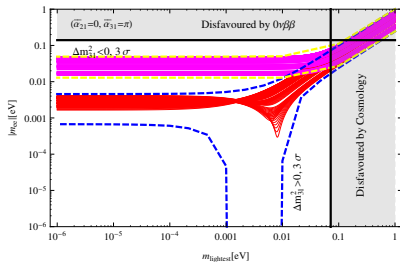
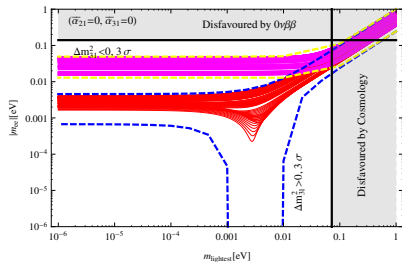
$$|m_{ee}| = \left| \frac{2}{3}m_1 \cos^2\left(\frac{\pi\gamma'}{n}\right) + \frac{1}{3}m_2 e^{i\alpha_{21}} + \frac{2}{3}m_3 \sin^2\left(\frac{\pi\gamma'}{n}\right) e^{i(\alpha_{31}-2\delta)} \right|$$

with $m_1 = m_l$, $m_2 = \sqrt{m_l^2 + \Delta m_{21}^2}$, $m_3 = \sqrt{m_l^2 + \Delta m_{31}^2}$ for normal ordering and $m_1 = \sqrt{m_l^2 + \Delta m_{31}^2}$, $m_2 = \sqrt{m_l^2 + \Delta m_{21}^2 + \Delta m_{31}^2}$, $m_3 = m_l$ for inverted ordering, where m_l is the mass of the lightest neutrino and $\gamma' = \gamma \bmod \frac{1}{6}$.

- There are 8 cases to distinguish for

$$\bar{\alpha}_{21} = \alpha_{21} + 6\pi \frac{\gamma+x}{n}, \quad \bar{\alpha}_{31} = \alpha_{31} - 2\delta.$$

$0\nu\beta\beta$ (1)



$0\nu\beta\beta$ (2)

- For inverted hierarchy there is no particular structure visible. Additionally, the predicted values for $|m_{ee}|$ are well within the reach of e.g. phase III of the GERDA experiment of $|m_{ee}^{\text{exp}}| \sim 0.02 \dots 0.03$ eV.
- For normal ordering, it follows that for the values of γ/n and x/n considered there always is a lower limit on $|m_{ee}|$ which means that these parameters are accessible to future experiments.
- Further for normal ordering, in the very low m_{lightest} region, predicted values of $|m_{ee}|$ are closer to the upper end of the blue three sigma range.
- With the current data, no combination of $\bar{\alpha}_{21}$ and $\bar{\alpha}_{31}$ is favoured. Only for values of $|m_{ee}| \lesssim 0.0001$ eV and $m_{\text{lightest}} \lesssim 0.01 \dots 0.001$ eV it would be possible to distinguish different values of $\bar{\alpha}_{21}$ and $\bar{\alpha}_{31}$.

Summary and Outlook

- We examine lepton mixing patterns in direct models with $\Delta(6n^2)$ groups and consistent generalised CP.
- For direct models, without corrections, $\Delta(6n^2)$ is the most (only?) promising class of flavour groups
- Further in direct flavour models, one can analyse $\Delta(6n^2)$ for all n simultaneously.
- This yields experimentally viable predictions for lepton mixing parameters and a sum rule.
- (Broken) invariance under consistent generalised CP transformations is the only framework that allows to predict Majorana phases purely from symmetry.
- We show, using a general argument, that in presence of $\Delta(6n^2)$, physical CP transformations are $X_r = e^{i\alpha} \Delta(6n^2)$.
- Predictions for neutrinoless double-beta decay are accessible to experiments in the (near) future.