Flavour Covariant Transport Equations for Resonant Leptogenesis

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based on

P. S. B. Dev, P. Millington, A. Pilaftsis and D. Teresi,
Nucl. Phys. B, in press [arXiv:1404.1003 [hep-ph]]

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Outline

Introduction: leptogenesis

- Lepton asymmetry from CP-violating decay of heavy Majorana neutrinos: [Fukugita, Yanagida, 1986]

$$-\mathcal{L}_N \;=\; h_l^{\alpha}\, \overline{L}^l\, \widetilde{\Phi}\, N_{\mathrm{R},\alpha} \;+\; \overline{N}^C_{\mathrm{R},\alpha}\, [M_N]^{\alpha\beta}\, N_{\mathrm{R},\beta} \;+\; \mathrm{H.c.}$$

- Resonantly enhanced if $\Delta m_N \sim \Gamma_N$ [Pilaftsis, 1997; Pilaftsis, Underwood, 2004]
- Flavour-diagonal rate equations:

$$\frac{n^{\gamma} H_N}{z} \frac{\mathrm{d}\eta^N_\alpha}{\mathrm{d}z} = \left(1 - \frac{\eta^N_\alpha}{\eta^N_{\mathrm{eq}}}\right) \sum_l \gamma^{N_\alpha}_{L_l \Phi}$$

$$\frac{n^{\gamma} H_N}{z} \frac{\mathrm{d}\delta\eta^L_l}{\mathrm{d}z} = \sum_\alpha \left(\frac{\eta^N_\alpha}{\eta^N_{\mathrm{eq}}} - 1\right) \delta\gamma^{N_\alpha}_{L_l \Phi} \;-\; \frac{2}{3}\delta\eta^L_l \sum_k \left(\gamma^{L_l \Phi}_{L^c_k \Phi^c} + \gamma^{L_l \Phi}_{L_k \Phi} + \delta\eta^L_k \left(\gamma^{L_k \Phi}_{L^c_l \Phi^c} - \gamma^{L_k \Phi}_{L_l \Phi}\right)\right)$$

- Flavour coherences play an important role [Abada et al., 2006; Asaka, Shaposhnikov, 2005; . . .] but so far only partially flavoured (not fully covariant) treatments
- Fully flavour-covariant formalism to capture consistently all flavour effects [Dev, Millington, Pilaftsis, Teresi, 2014] (this talk)

Flavour-covariant formalism

- Unitary flavour transformations:

$$L_l \to V_l^{\ m} L_m \qquad L^{\dagger,l} \to V^l_{\ m} L^{\dagger,m} \qquad N_{\mathrm{R},\alpha} \to U_\alpha^{\ \beta} N_{\mathrm{R},\beta} \qquad N_{\mathrm{R}}^{\dagger,\alpha} \to U^\alpha_{\ \beta} N_{\mathrm{R}}^{\dagger,\beta}$$

- $\mathcal{L}$ invariant if $\quad h_l^{\ \alpha} \to V_l^{\ m} U^\alpha_{\ \beta} h_m^{\ \beta} \qquad [M_N]^{\alpha\beta} \to U^\alpha_{\ \gamma} U^\beta_{\ \delta} [M_N]^{\gamma\delta}$
- Flavour-covariant quantization (see also P. Millington's talk)

$$L_l(x) = \int_{\mathbf{p},s} \left[(2E_L(\mathbf{p}))^{-\frac{1}{2}}\right]_l^{\ i} \left(\left[e^{-ip\cdot x}\right]_i^{\ j} [u(\mathbf{p},s)]_j^{\ k} b_k(\mathbf{p},s) + \left[e^{ip\cdot x}\right]_i^{\ j} [v(\mathbf{p},s)]_j^{\ k} d_k^\dagger(\mathbf{p},s) \right)$$

- Matrix number densities: $\quad [n^L]_l^{\ m} \propto \langle b^{\dagger,m} b_l \rangle \quad [\bar{n}^L]_l^{\ m} \propto \langle d_l^\dagger d^m \rangle \quad [n^N]_\alpha^{\ \beta} \propto \langle a^{\dagger,\beta} a_\alpha \rangle$
- Necessary to consider generalized discrete symmetries $\widetilde{C}, P, \widetilde{T}$, e.g.

$$d^l = (b_l)^{\widetilde{C}} \equiv \mathcal{G}^{\dagger,lm} (b_l)^C \qquad \text{with} \quad \mathcal{G} = V V^{\mathsf{T}}$$

Number densities transform as $(\boldsymbol{n}^L)^{\widetilde{C}} = (\bar{\boldsymbol{n}}^L)^{\mathsf{T}}$, $(\boldsymbol{n}^N)^{\widetilde{C}} = (\bar{\boldsymbol{n}}^N)^{\mathsf{T}}$ $\quad$ ($\boldsymbol{n}^N$, $\bar{\boldsymbol{n}}^N$ not independent)

- Define $\widetilde{C}P$-“even” and $\widetilde{C}P$-“odd” quantities:

$$\delta\boldsymbol{n}^L = \boldsymbol{n}^L - \bar{\boldsymbol{n}}^L \qquad \underline{\boldsymbol{n}}^N = \frac{1}{2}(\boldsymbol{n}^N + \bar{\boldsymbol{n}}^N) \qquad \delta\boldsymbol{n}^N = \boldsymbol{n}^N - \bar{\boldsymbol{n}}^N$$

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Flavour-covariant transport equations

Markovian master equation for number densities:

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{n}^X(\mathbf{k},t) \;\simeq\; i\langle\,[H_0^X,\ \check{\boldsymbol{n}}^X(\mathbf{k},t)]\,\rangle_t - \frac{1}{2}\int_{-\infty}^{+\infty}\mathrm{d}t'\ \langle\,[H_{\mathrm{int}}(t'),\ [H_{\mathrm{int}}(t),\ \check{\boldsymbol{n}}^X(\mathbf{k},t)]]\,\rangle_t$$

For charged-lepton and heavy-neutrino matrix number densities we find:

$$\frac{\mathrm{d}}{\mathrm{d}t}\,[n^L_{s_1 s_2}(\mathbf{p},t)]_l^{\ m} \;=\; -\,i\,\big[E_L(\mathbf{p}),\ n^L_{s_1 s_2}(\mathbf{p},t)\big]_l^{\ m} \;+\; [C^L_{s_1 s_2}(\mathbf{p},t)]_l^{\ m}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\,[n^N_{r_1 r_2}(\mathbf{k},t)]_\alpha^{\ \beta} \;=\; -\,i\,\big[E_N(\mathbf{k}),\ n^N_{r_1 r_2}(\mathbf{k},t)\big]_\alpha^{\ \beta} \;+\; [C^N_{r_1 r_2}(\mathbf{k},t)]_\alpha^{\ \beta} + \; G_{\alpha\lambda}\,[\overline{C}^N_{r_2 r_1}(\mathbf{k},t)]_\mu^{\ \lambda}\,G^{\mu\beta}$$

Collision terms of the form:

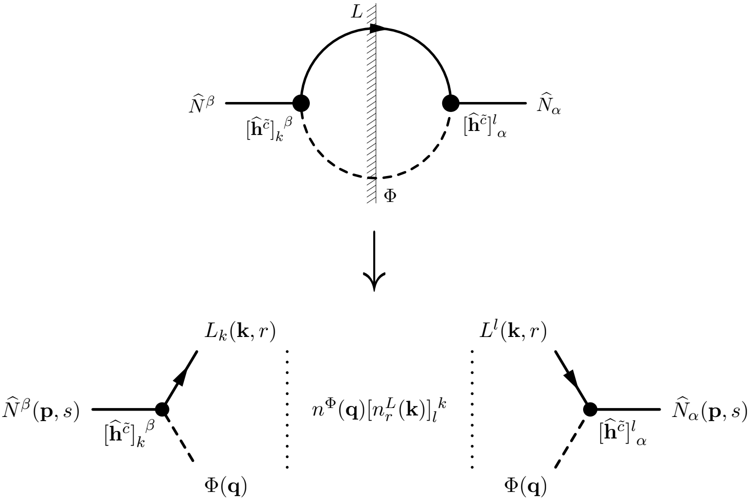
$$[C^L_{s_1 s_2}(\mathbf{p},t)]_l^{\ m} \;\supset\; -\frac{1}{2}\,\mathcal{F}_{s_1 s\, r_1 r_2}(\mathbf{p},\mathbf{q},\mathbf{k},t)_l^{\ n}{}_\alpha^{\ \beta}\ \Gamma_{s\, s_2 r_2 r_1}(\mathbf{p},\mathbf{q},\mathbf{k})_n^{\ m}{}_\beta^{\ \alpha}$$

Statistical tensors $\mathcal{F} \;=\; n^\Phi\,\boldsymbol{n}^L \otimes \big(\mathbf{1} - \boldsymbol{n}^N\big) \;-\; \big(1 + n^\Phi\big)\,\big(\mathbf{1} - \boldsymbol{n}^L\big) \otimes \boldsymbol{n}^N$

$\boldsymbol{\Gamma}$ novel absorptive rank-4 tensors describing decays and inverse decays

Rank-4 rates (see P. Millington's talk)

$n^\Phi \, [n^L]_l^{\;k} \, [\gamma(L\Phi \to N)]_{k\;\alpha}^{\;l\;\beta}$:

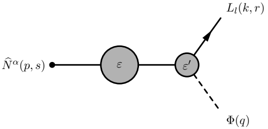


Application to Resonant Leptogenesis

- Classical statistics
- Kinetic equilibrium
- Degenerate spin degrees of freedom
- Small deviation from equilibrium: $[n^L]_l{}^m + [\bar{n}^L]_l{}^m \simeq 2\, n^L_{\rm eq}\, \delta_l{}^m$
- Take into account mixing by resummed Yukawa couplings: $h_l{}^\alpha \to \mathbf{h}_l{}^\alpha, [\mathbf{h}^{\tilde{c}}]_l{}^\alpha$

  [Pilaftsis, Underwood, 2004]

$\mathbf{h}_l{}^\alpha =$



$$[\gamma^N_{L\Phi}]_l{}^m{}_\alpha{}^\beta \propto \mathbf{h}^m{}_\alpha \mathbf{h}_l{}^\beta + [\mathbf{h}^{\tilde{c}}]^m{}_\alpha [\mathbf{h}^{\tilde{c}}]_l{}^\beta$$

$$[\delta\gamma^N_{L\Phi}]_l{}^m{}_\alpha{}^\beta \propto \mathbf{h}^m{}_\alpha \mathbf{h}_l{}^\beta - [\mathbf{h}^{\tilde{c}}]^m{}_\alpha [\mathbf{h}^{\tilde{c}}]_l{}^\beta$$

- RIS-subtracted scattering rates: $[\gamma'^{L\Phi}_{L\Phi}]_l{}^m{}_n{}^k, [\delta\gamma'^{L\Phi}_{L\Phi}]_l{}^m{}_n{}^k, [\gamma'^{L\Phi}_{L^{\tilde{c}}\Phi^{\tilde{c}}}]_l{}^m{}_n{}^k, [\delta\gamma'^{L\Phi}_{L^{\tilde{c}}\Phi^{\tilde{c}}}]_l{}^m{}_n{}^k$
- Charged-lepton decoherence interactions

Rate equations

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\underline{\eta}^N]_\alpha{}^\beta}{\mathrm{d}z} = -\,i\,\frac{n^\gamma}{2}\left[\mathcal{E}_N,\, \delta\eta^N\right]_\alpha{}^\beta + \left[\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\right]_\alpha{}^\beta - \frac{1}{2\,\eta^N_{\mathrm{eq}}}\left\{\underline{\eta}^N,\, \widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\right\}_\alpha{}^\beta$$

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\delta\eta^N]_\alpha{}^\beta}{\mathrm{d}z} = -\,2\,i\,n^\gamma\left[\mathcal{E}_N,\, \underline{\eta}^N\right]_\alpha{}^\beta + 2\,i\left[\widetilde{\mathrm{Im}}(\delta\gamma^N_{L\Phi})\right]_\alpha{}^\beta - \frac{i}{\eta^N_{\mathrm{eq}}}\left\{\underline{\eta}^N,\, \widetilde{\mathrm{Im}}(\delta\gamma^N_{L\Phi})\right\}_\alpha{}^\beta - \frac{1}{2\,\eta^N_{\mathrm{eq}}}\left\{\delta\eta^N,\, \widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\right\}_\alpha{}^\beta$$

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\delta\eta^L]_l{}^m}{\mathrm{d}z} = -\,[\delta\gamma^N_{L\Phi}]_l{}^m + \frac{[\underline{\eta}^N]_\beta{}^\alpha}{\eta^N_{\mathrm{eq}}}\,[\delta\gamma^N_{L\Phi}]_l{}^m{}_\alpha{}^\beta + \frac{[\delta\eta^N]_\beta{}^\alpha}{2\,\eta^N_{\mathrm{eq}}}\,[\gamma^N_{L\Phi}]_l{}^m{}_\alpha{}^\beta - \frac{1}{3}\left\{\delta\eta^L,\, \gamma^{L\Phi}_{L^{\tilde{c}}\Phi^{\tilde{c}}} + \gamma^{L\Phi}_{L\Phi}\right\}_l{}^m - \frac{2}{3}\,[\delta\eta^L]_k{}^n\left([\gamma^{L\Phi}_{L^{\tilde{c}}\Phi^{\tilde{c}}}]_n{}^k{}_l{}^m - [\gamma^{L\Phi}_{L\Phi}]_n{}^k{}_l{}^m\right) - \frac{2}{3}\left\{\delta\eta^L,\, \gamma_{\mathrm{dec}}\right\}_l{}^m + [\delta\gamma^{\mathrm{back}}_{\mathrm{dec}}]_l{}^m$$

Rate equations: mixing

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\underline{\eta}^N]_\alpha^{\ \beta}}{\mathrm{d}z} = -\,i\,\frac{n^\gamma}{2}\left[\mathcal{E}_N,\,\delta\eta^N\right]_\alpha^{\ \beta} + \left[\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\right]_\alpha^{\ \beta} - \frac{1}{2\,\eta^N_{\mathrm{eq}}}\left\{\underline{\eta}^N,\,\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\right\}_\alpha^{\ \beta}$$

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\delta\eta^N]_\alpha^{\ \beta}}{\mathrm{d}z} = -\,2\,i\,n^\gamma\left[\mathcal{E}_N,\,\underline{\eta}^N\right]_\alpha^{\ \beta} + 2\,i\left[\widetilde{\mathrm{Im}}(\delta\gamma^N_{L\Phi})\right]_\alpha^{\ \beta} - \frac{i}{\eta^N_{\mathrm{eq}}}\left\{\underline{\eta}^N,\,\widetilde{\mathrm{Im}}(\delta\gamma^N_{L\Phi})\right\}_\alpha^{\ \beta} - \frac{1}{2\,\eta^N_{\mathrm{eq}}}\left\{\delta\eta^N,\,\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\right\}_\alpha^{\ \beta}$$

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\delta\eta^L]_l^{\ m}}{\mathrm{d}z} = -\,[\delta\gamma^N_{L\Phi}]_l^{\ m} + \frac{[\underline{\eta}^N]_\beta^{\ \alpha}}{\eta^N_{\mathrm{eq}}}\,[\delta\gamma^N_{L\Phi}]_{l\ \alpha}^{\ m\ \beta} + \frac{[\delta\eta^N]_\beta^{\ \alpha}}{2\,\eta^N_{\mathrm{eq}}}\,[\gamma^N_{L\Phi}]_{l\ \alpha}^{\ m\ \beta} - \frac{1}{3}\left\{\delta\eta^L,\,\gamma^{L\Phi}_{L^{\tilde c}\Phi^{\tilde c}} + \gamma^{L\Phi}_{L\Phi}\right\}_l^{\ m} - \frac{2}{3}\,[\delta\eta^L]_k^{\ n}\left([\gamma^{L\Phi}_{L^{\tilde c}\Phi^{\tilde c}}]_{n\ l}^{\ k\ m} - [\gamma^{L\Phi}_{L\Phi}]_{n\ l}^{\ k\ m}\right) - \frac{2}{3}\left\{\delta\eta^L,\,\gamma_{\mathrm{dec}}\right\}_l^{\ m} + [\delta\gamma^{\mathrm{back}}_{\mathrm{dec}}]_l^{\ m}$$

Rate equations: oscillations

$$\frac{H_N\, n^\gamma}{z}\,\frac{\mathrm{d}[\underline{\eta}^N]_\alpha{}^\beta}{\mathrm{d}z} = -\,i\,\frac{n^\gamma}{2}\left[\mathcal{E}_N,\,\delta\eta^N\right]_\alpha{}^\beta + \left[\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\right]_\alpha{}^\beta - \frac{1}{2\,\eta^N_{\mathrm{eq}}}\left\{\underline{\eta}^N,\,\widetilde{\mathrm{Re}}(\gamma^N_{L\Phi})\right\}_\alpha{}^\beta$$

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$$\cdots\ \frac{2}{3}\,[\delta\eta^L]_k^{\ n}\Big([\gamma^{L\Phi}_{L^{\tilde c}\Phi^{\tilde c}}]_{n\ \ l}^{\ k\,m} - [\gamma^{L\Phi}_{L\Phi}]_{n\ \ l}^{\ k\,m}\Big)$$

$$-\,\frac{2}{3}\,\Big\{\delta\eta^L,\,\gamma_{\mathrm{dec}}\Big\}_l^{\ \cdots} + [\delta\gamma_{\mathrm{dec}}]_l^{\ \cdots}$$

Notice:

Different from ARS mechanism!

[Akhmedov, Rubakov, Smirnov, 1998; Asaka, Shaposhnikov, 2005]

Rate equations: charged-lepton decoherence

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Minimal Radiative Resonant τ-genesis [Deppisch, Pilaftsis, 2011]

- $\mathbb{O}(3)$ symmetric heavy-neutrino sector at $\mu_X \sim m_{GUT}$
- Mass splitting by RG evolution to m_N:

$$\boldsymbol{M}_N = m_N \mathbf{1} - \frac{m_N}{8\pi^2} \ln\left(\frac{\mu_X}{m_N}\right) \mathrm{Re}\left[\boldsymbol{h}^\dagger(\mu_X)\boldsymbol{h}(\mu_X)\right]$$

- Yukawa couplings break to almost exact $U(1)_{L_e+L_\mu} \times U(1)_{L_\tau}$: [Pilaftsis, 2005]

$$\boldsymbol{h} = \begin{pmatrix} 0 & ae^{-i\pi/4} & ae^{i\pi/4} \\ 0 & be^{-i\pi/4} & be^{i\pi/4} \\ 0 & 0 & 0 \end{pmatrix} + \boldsymbol{\delta h}, \qquad \boldsymbol{\delta h} = \begin{pmatrix} \epsilon_e & 0 & 0 \\ \epsilon_\mu & 0 & 0 \\ \epsilon_\tau & \kappa_1 e^{-i\left(\frac{\pi}{4}-\gamma_1\right)} & \kappa_2 e^{i\left(\frac{\pi}{4}-\gamma_2\right)} \end{pmatrix}$$

- Seesaw: $\boldsymbol{M}_\nu \simeq -\frac{v^2}{2}\boldsymbol{h}\boldsymbol{M}_N^{-1}\boldsymbol{h}^\mathsf{T}$
- Light neutrinos massless for $\boldsymbol{\delta h} = 0$
- $\mathbb{O}(3)$ guarantees Resonant Leptogenesis, $U(1)_{L_e+L_\mu} \times U(1)_{L_\tau}$ allows $m_N \sim 10^2$ GeV to satisfy neutrino data
- Input parameters: $m_N, \kappa_1, \kappa_2, \gamma_1, \gamma_2$

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$$\boldsymbol{M}_N = m_N \mathbf{1} - \frac{m_N}{8\pi^2} \ln\left(\frac{\mu_X}{m_N}\right) \mathrm{Re}\left[\boldsymbol{h}^\dagger(\mu_X)\boldsymbol{h}(\mu_X)\right]$$

- Yukawa couplings break to almost exact $U(1)_{L_e+L_\mu} \times U(1)_{L_\tau}$: [Pilaftsis, 2005]

$$\boldsymbol{h} = \begin{pmatrix} 0 & ae^{-i\pi/4} & ae^{i\pi/4} \\ 0 & be^{-i\pi/4} & be^{i\pi/4} \\ 0 & 0 & 0 \end{pmatrix} + \boldsymbol{\delta h}\,, \qquad \boldsymbol{\delta h} = \begin{pmatrix} \epsilon_e & 0 & 0 \\ \epsilon_\mu & 0 & 0 \\ \epsilon_\tau & \kappa_1 e^{-i\left(\frac{\pi}{4}-\gamma_1\right)} & \kappa_2 e^{i\left(\frac{\pi}{4}-\gamma_2\right)} \end{pmatrix}$$

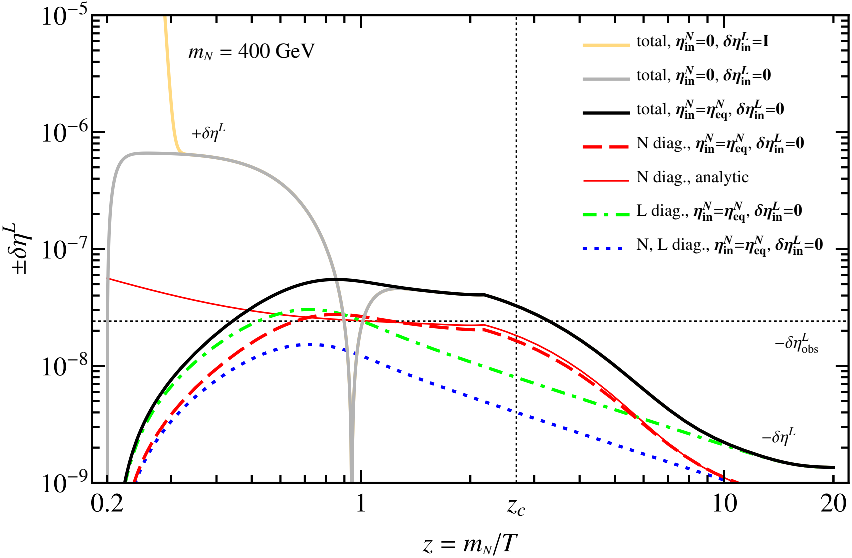
- Seesaw: $\boldsymbol{M}_\nu \simeq -\frac{v^2}{2}\boldsymbol{h}\,\boldsymbol{M}_N^{-1}\boldsymbol{h}^{\mathsf{T}}$
- Light neutrinos massless for $\boldsymbol{\delta h} = 0$
- $\mathbb{O}(3)$ guarantees Resonant Leptogenesis,
  $U(1)_{L_e+L_\mu} \times U(1)_{L_\tau}$ allows $m_N \sim 10^2\,\mathrm{GeV}$ to satisfy neutrino data
- Input parameters: $m_N, \kappa_1, \kappa_2, \gamma_1, \gamma_2$

Benchmark points

Parameter	BP1	BP2	BP3
m_N	120 GeV	400 GeV	5 TeV
κ_1	4×10^{-5}	2.4×10^{-5}	2×10^{-4}
κ_2	2×10^{-4}	6×10^{-5}	2×10^{-5}
a	$(7.41 - 5.54\,i) \times 10^{-4}$	$(4.93 - 2.32\,i) \times 10^{-3}$	$(4.67 + 4.33\,i) \times 10^{-3}$
b	$(1.19 - 0.89\,i) \times 10^{-3}$	$(8.04 - 3.79\,i) \times 10^{-3}$	$(7.53 + 6.97\,i) \times 10^{-3}$

Observable	BP1	BP2	BP3	Exp. Limit
BR($\mu \to e\gamma$)	4.5×10^{-15}	1.9×10^{-13}	2.3×10^{-17}	$< 5.7 \times 10^{-13}$
BR($\tau \to \mu\gamma$)	1.2×10^{-17}	1.6×10^{-18}	8.1×10^{-22}	$< 4.4 \times 10^{-8}$
BR($\tau \to e\gamma$)	4.6×10^{-18}	5.9×10^{-19}	3.1×10^{-22}	$< 3.3 \times 10^{-8}$
BR($\mu \to 3e$)	1.5×10^{-16}	9.3×10^{-15}	4.9×10^{-18}	$< 1.0 \times 10^{-12}$
$R^{Ti}_{\mu\to e}$	2.4×10^{-14}	2.9×10^{-13}	2.3×10^{-20}	$< 6.1 \times 10^{-13}$
$R^{Au}_{\mu\to e}$	3.1×10^{-14}	3.2×10^{-13}	5.0×10^{-18}	$< 7.0 \times 10^{-13}$
$R^{Pb}_{\mu\to e}$	2.3×10^{-14}	2.2×10^{-13}	4.3×10^{-18}	$< 4.6 \times 10^{-11}$
$\|\Omega\|_{e\mu}$	5.8×10^{-6}	1.8×10^{-5}	1.6×10^{-7}	$< 7.0 \times 10^{-5}$
$\langle m \rangle$ [eV]	3.8×10^{-3}	3.8×10^{-3}	3.8×10^{-3}	$<$ (0.11–0.25)

Benchmark Point 2

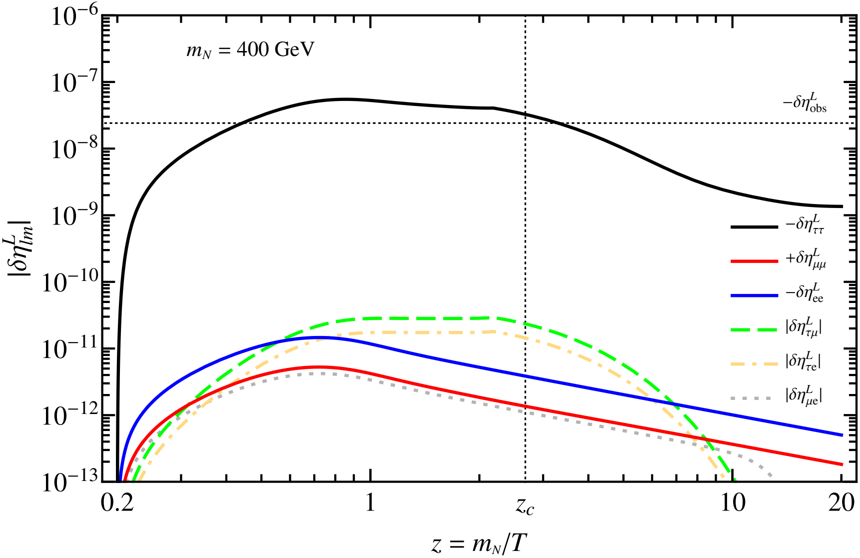


Conclusions

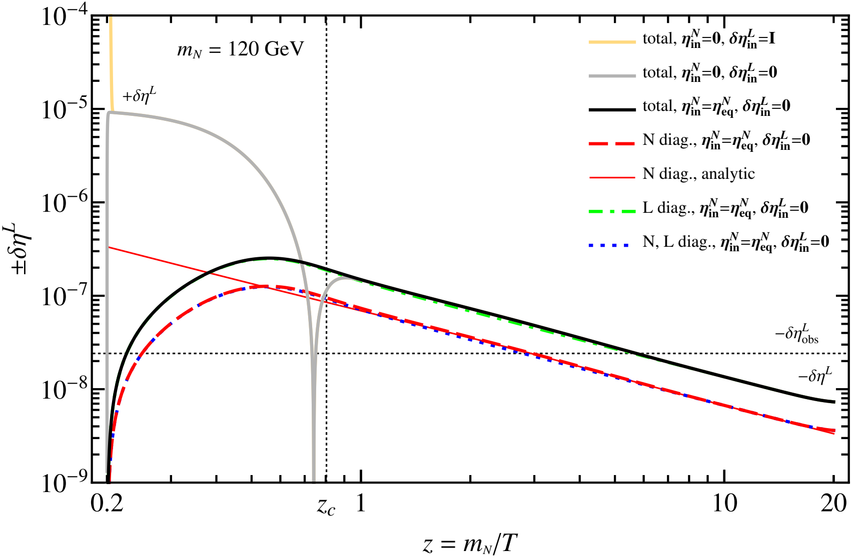
- Flavour covariant formalism to describe consistently all flavour effects
- Main new ingredients:
  - consistency requires one to consider generalized discrete symmetries
  - rank-4 rate tensors, interpreted as unitarity cuts of partial thermal self-energies
- The formalism provides a complete and unified description of Resonant Leptogenesis, capturing three distinct phenomena:
  - resonant mixing between heavy neutrinos
  - coherent oscillations between heavy-neutrino flavours
  - quantum decoherence effects in the charged-lepton sector
- We applied it explicitly to a minimal scenario of $\tau-$genesis testable in the near future
- Final asymmetry can be enhanced by even an order of magnitude, for $m_N \sim 200 - 1000\,\mathrm{GeV}$
- Much more detail (109 pages!) in [Dev, Millington, Pilaftsis and Teresi, arXiv:1404.1003 [hep-ph]]

Backup slides

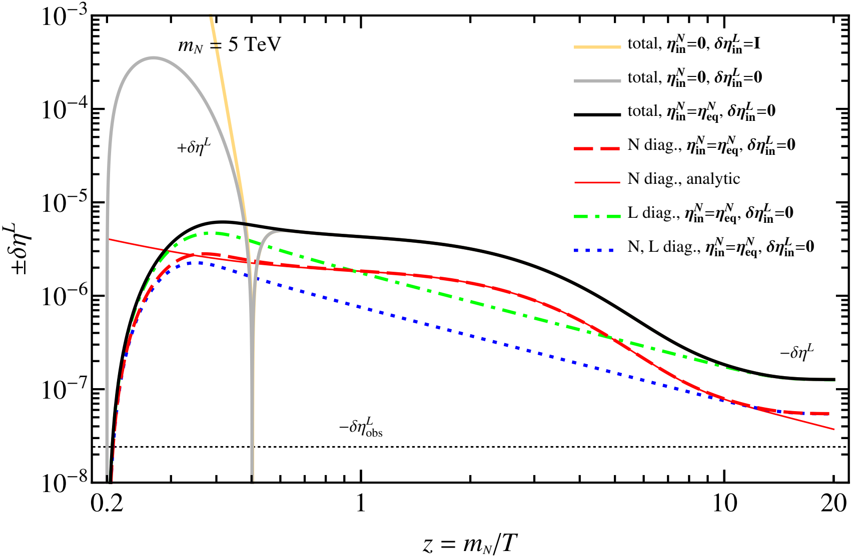
Benchmark Point 2: flavour content



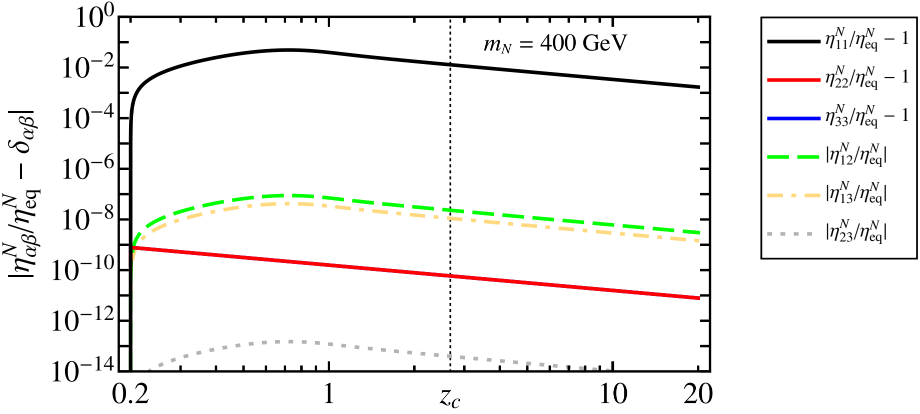
Benchmark Point 1



Benchmark Point 3



Benchmark Point 2: heavy neutrinos



Application to Resonant Leptogenesis

- Classical statistics
- Kinetic equilibrium
- Degenerate spin degrees of freedom
- Small deviation from equilibrium: $[n^L]_l^{\ m} + [\bar{n}^L]_l^{\ m} \simeq 2\, n^L_{\rm eq}\, \delta_l^{\ m}$

Due to flavour covariance:

$$\left.\begin{aligned} [\dot{n}^L]_l^{\ m} &\supset -\, i\, \left[\mathcal{E}_L, n^L\right]_l^{\ m} \\ [\dot{\bar{n}}^L]_l^{\ m} &\supset +\, i\, \left[\mathcal{E}_L, \bar{n}^L\right]_l^{\ m} \end{aligned}\right\} \quad \Longrightarrow \quad \text{no oscillation: } [\delta\dot{n}^L]_l^{\ m} \supset -\, i\, \left[\mathcal{E}_L, n^L + \bar{n}^L\right]_l^{\ m} \simeq 0$$

Had we chosen, **ignoring covariance**, $[n^L]_{lm} \propto \langle b_l^\dagger\, b_m \rangle$, $[\bar{n}^L]_{lm} \propto \langle d_l^\dagger\, d_m \rangle$,
$[n^L]_l^{\ m} + [\bar{n}^L]_l^{\ m} \simeq 2\, n^L_{\rm eq}\, \delta_l^{\ m}$ would be inconsistent under $\widetilde{C}P$