Moduli Identification in Heterotic Compactifications

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Based on work with: Lara Anderson and Eric Sharpe 1402.1532

and Lara Anderson, Andre Lukas and Burt Ovrut 1304.2704
1107.5076
1010.0255

See also Xenia de la Ossa and Eirik Svanes 1402.1725

Warm up: Bundles on Calabi-Yau

- The moduli space of a Calabi-Yau compactification in the presence of a gauge bundle is *not* described in terms of

$$H^1(\mathcal{TX}) \oplus H^1(\mathcal{TX}^\vee) \oplus H^1(\mathrm{End}^0(\mathcal{V}))$$

  – It is described in terms of a subspace of these cohomology groups determined by the kernel of certain maps

  – Those maps are determined by the supergravity data of the solution.

- To see this we can analyze the supersymmetry conditions.

- The conditions for the gauge field to be supersymmetric are the Hermitian Yang-Mills equations at zero slope:

$$F_{ab} = F_{\overline{a}\overline{b}} = 0 \qquad g^{a\overline{b}} F_{a\overline{b}} = 0$$

- Study perturbations obeying these equations:

Perturb the complex structure: $\mathcal{J} = \mathcal{J}^{(0)} + \delta\mathcal{J}$

$$\mathcal{J}^2 = -1$$
$$N(\mathcal{J}) = 0$$

$$\delta\mathcal{J}_{\overline{a}}{}^{b} \in H^1(TX)$$

and the gauge field: $A = A^{(0)} + \delta A$

- Define

$$\overline{P}_I^{\ J} = \frac{1}{2}(1 + i\mathcal{J})_I^{\ J}$$

and rewrite our equation in a more usable form

$$F_{\overline{a}\overline{b}} = 0 \quad \Rightarrow \quad \overline{P}_I^{\ I'}\overline{P}_J^{\ J'}F_{I'J'} = 0$$

- And work out the perturbed equation to first order:

$$i\delta\mathcal{J}_{[\overline{a}}^{\ d}F_{\overline{b}]d} = 2D_{[\overline{a}}\delta A_{\overline{b}]}$$

This equation is not of much practical use…

The Atiyah class:

- There is a description of this in terms of cohomology of a certain bundle:

Define: $$0 \to \mathrm{End}^0(\mathcal{V}) \to \mathcal{Q} \to \mathcal{TX} \to 0$$

Atiyah states that the moduli are not

$$H^1(\mathcal{TX}) \oplus H^1(\mathrm{End}^0(\mathcal{V}))$$

But rather: $$H^1(\mathcal{Q})$$

How do we tie this in with our field theory analysis?

- Take $H^0(\mathcal{TX})$ to vanish for simplicity
- Look at the long exact sequence in cohomology

$$0 \to H^1(\mathrm{End}^0(V)) \to H^1(\mathcal{Q}) \to H^1(\mathcal{TX}) \xrightarrow{\alpha} H^2(\mathrm{End}^0(V))$$

where $\alpha = [F]$

- Thus we see Atiyah claims the moduli are given by

$$H^1(\mathcal{Q}) = \left\{ \begin{array}{c} H^1(\mathrm{End}^0(\mathcal{V})) \\ \bigoplus \\ \ker(H^1(\mathcal{TX}) \to H^2(\mathrm{End}^0(\mathcal{V})) \end{array} \right.$$

Non-Kähler Compactifications

Hull, Strominger

- The most general $\mathcal{N}=1$ heterotic compactification with maximally symmetric 4d space:
  - Complex manifold

$$F_{ab} = F_{\overline{a}\overline{b}} = 0 \qquad H = i/2(\overline{\partial} - \partial)J$$

$$dH = -\frac{1}{30}\alpha' \mathrm{tr} F \wedge F + \alpha' \mathrm{tr} R \wedge R$$

$$g^{a\overline{b}} F_{a\overline{b}} = 0 \qquad H_{\overline{b}c\overline{a}} g^{\overline{b}c} = -6 \overline{\partial}_{\overline{a}} \phi$$

Gillard, Papadopoulos and Tsimpis

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- Perturb all of the fields just as we did in the Calabi-Yau case:

$$\mathcal{J} = \mathcal{J}^{(0)} + \delta\mathcal{J} \quad A = A^{(0)} + \delta A$$

$$J = J^{(0)} + \delta J$$

$$H = H^{(0)} + \delta H^{\text{closed}} - \frac{1}{30}\alpha'\delta\omega_3^{\text{YM}} + \alpha'\delta\omega_3^{\text{L}}$$

- And look at what the first order perturbation to the supersymmetry relations looks like…

In what follows I consider manifolds obeying the $\partial\bar{\partial}$-lemma

Lemma: Let X be a compact Kähler manifold. For A a d-closed (p,q) form, the following statements are equivalent.

$$A=\overline{\partial}C \Leftrightarrow A=\partial C' \Leftrightarrow A=dC''$$

$$\Leftrightarrow A=\partial\overline{\partial}\tilde{C} \Leftrightarrow A=\partial\hat{C}+\overline{\partial}\check{C}$$

For some $C, C', C'', \tilde{C}$ and $\check{C}$.

- For the perturbation analysis the Atiyah computation goes through unchanged.
- The other equations are somewhat more messy:

- Atiyah analysis:

$$i\delta\mathcal{J}_{[\overline{a}}^{\ d}F_{\overline{b}]d} = 2D_{[\overline{a}}\delta A_{\overline{b}]}$$

- Totally anti-holomorphic part of H eqn:

$$3\overline{\partial}_{[\overline{a}}\delta B_{\overline{b}\overline{c}]} - \frac{2}{10}\alpha'\left(\overline{\partial}_{[\overline{a}}\left(\delta A_{\overline{b}}^{y}A_{\overline{c}]}^{x}\delta_{xy}\right)\right) + 6\alpha'\left(\overline{\partial}_{[\overline{a}}\left(\delta W_{\overline{b}}^{\alpha\beta}W_{\overline{c}]}^{\beta\alpha}\right)\right) = -\frac{3}{2}i\overline{\partial}_{[\overline{a}}\delta J_{\overline{c}\overline{b}]}$$

$$\Rightarrow \qquad \delta B_{\overline{b}\overline{c}} = \frac{2}{30}\alpha'\left(\delta A_{[\overline{b}}^{y}A_{\overline{c}]}^{x}\delta_{xy}\right) - 2\alpha'\left(\delta W_{[\overline{b}}^{\alpha\beta}W_{\overline{c}]}^{\beta\alpha}\right) + \frac{i}{2}\delta J_{\overline{b}\overline{c}} + \delta B'_{\overline{b}\overline{c}}$$

- Remaining components:

$$2\overline{\partial}_{[\overline{a}}\delta B_{\overline{b}]c} + \partial_c\delta B_{\overline{a}\overline{b}} - \alpha'\frac{1}{30}\delta\omega^{\mathrm{YM}}_{3\overline{a}\overline{b}c} + \alpha'\delta\omega^{\mathrm{L}}_{3\overline{a}\overline{b}c} \;=\; i\overline{\partial}_{[\overline{a}}\delta J_{\overline{b}]c} - \delta J_{[\overline{a}}^{\ d}\partial J_{\overline{b}]cd} + \frac{1}{2}i\partial_c\delta J_{\overline{a}\overline{b}}$$

$$\Rightarrow \; \delta J_{[\overline{a}}^{\ d}\partial J_{\overline{b}]cd} - \frac{4}{30}\alpha'\delta_{xy}\delta A_{[\overline{a}}^{x}F_{\overline{b}]c}^{y} + 4\alpha'\delta W_{[\overline{a}}^{\alpha\beta}R_{\overline{b}]c}^{\beta\alpha} = i\overline{\partial}_{[\overline{a}}\delta J_{\overline{b}]c} - 2\overline{\partial}_{[\overline{a}}\delta B_{\overline{b}]c} - \overline{\partial}_{[\overline{a}}\Lambda_{\overline{b}]c}^{\alpha'}$$

- How do we interpret this result?
  - Proceed by analogy with the Atiyah case:

Define a bundle $\mathcal{Q}$:

$$0 \to \mathrm{End}_0(\mathcal{V}) \oplus \mathrm{End}_0(\mathcal{TX}) \to \mathcal{Q} \to \mathcal{TX} \to 0$$

and a bundle $\mathcal{H}$:

$$0 \to \mathcal{TX}^{\vee} \to \mathcal{H} \to \mathcal{Q} \to 0$$

Baraglia and Hekmati 1308.5159

We claim the cohomology $H^1(\mathcal{H})$ precisely encapsulates the allowed deformations.

- To make contact with the field theory we again look at the associated long exact sequences in cohomology.

$$H^0(\mathcal{Q}) \to H^1(\mathcal{TX}^\vee) \to H^1(\mathcal{H}) \to H^1(\mathcal{Q}) \to H^2(\mathcal{TX}^\vee)$$

and

$$H^1(\mathrm{End}_0(\mathcal{V})) \oplus H^1(\mathrm{End}_0(\mathcal{TX})) \to H^1(\mathcal{Q}) \to H^1(\mathcal{TX}) \to H^2(\mathrm{End}_0(\mathcal{V})) \oplus H^2(\mathrm{End}_0(\mathcal{TX}))$$

Do the sequence chasing and you find…

$$H^1(\mathcal{H}) = \begin{cases} \ker\left(\ker\{H^1(TX) \xrightarrow{[F],[R]} H^2(\mathrm{End}_0(V)) \oplus H^2(\mathrm{End}_0(TX))\} \xrightarrow{M} H^2(TX^\vee)\right) \\ \oplus \\ \ker\left(H^1(\mathrm{End}_0(V)) \xrightarrow{-\frac{4}{30}\alpha'[F]} H^2(TX^\vee)\right) \oplus \ker\left(H^1(\mathrm{End}_0(TX)) \xrightarrow{4\alpha'[R]} H^2(TX^\vee)\right) \\ \oplus \\ H^1(TX^\vee)\ . \end{cases}$$

- This is a subspace of

$$H^1(\mathcal{TX}^\vee) \oplus H^1(\mathcal{TX}) \oplus H^1(\mathrm{End}_0(\mathcal{V})) \oplus H^1(\mathrm{End}_0(\mathcal{TX}))$$

  defined by maps determined by the supergravity data.
- All maps are well defined, as are the extensions.
- This precisely matches the supergravity computation.

A few comments on the structure:

- One can easily generalize to the case where $H^0(\mathcal{TX}) \neq 0$.
- The overall volume is only a modulus in the CY case.
- Unlike in the Atiyah story, the bundle moduli are constrained by the map structure here.
- Matter can be included in the analysis, simply by thinking of it as the moduli of an E8 bundle.
- There is a nice mathematical interpretation of all of this…

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Garcia-Fernandez 1304.4294

Conclusions

- For the case where a non-Kähler heterotic compactification obeys the $\partial\overline{\partial}$-lemma:
  - The moduli are given by subgroups of the usual sheaf cohomology groups.
  - The subgroups of interest are determined by kernels and cokernels of maps determined by the supergravity data
  - This all has a nice mathematical interpretation in terms of Courant algebroids (transitive and exact) and generalized complex structures on the total space of certain bundles