F-theory on Spin(7) manifolds

Weak-coupling limit

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SUSY2014, Manchester, July 21

[FB, T. Grimm, T. Pugh, arXiv:1307.5858]

[FB, T. Grimm, E. Palti, T. Pugh, arXiv:1309.2287]

Outline

- Introduction
- Uplift of the effective action from three to four dimensions
- Weak-coupling limit
- Type IIB setups with five-planes
- Conclusions & outlook

Reminder: from M-theory to F-theory

- F-theory constructions extend Type IIB vacua beyond perturbation theory
- Rich phenomenology of 4d $\mathcal{N} = 1$ F-theory vacua
- The F-theory effective action is derived via duality with M-theory
- Relevant geometry: elliptically fibered Calabi-Yau fourfold

$$\begin{array}{ccc} T^2 & \hookrightarrow & Y_4 \\ & & \downarrow \pi \\ & & B_3 \end{array}$$

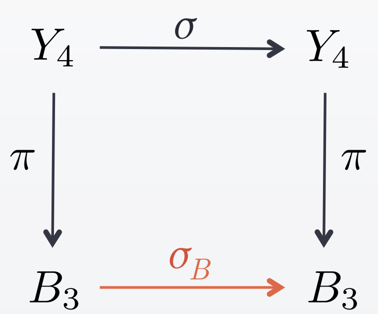
M-theory side	*F-theory side*
$\mathbb{R}^{1,2} \times Y_4$	$\mathbb{R}^{1,2} \times B_3 \times S^1$
$\mathrm{vol}(T^2) \to 0$	$\mathrm{vol}(S^1) \to \infty$

3d $\mathcal{N} = 2$ M-theory effective action $\xrightarrow[\text{uplift}]{\text{circle}}$ 4d $\mathcal{N} = 1$ F-theory effective action

F-theory and Spin(7) quotients

Can this construction be extended to Spin(7) geometries? [Vafa 96]

- We introduce an antiholomorphic quotient respecting the fibration structure



$$\sigma^* J = -J$$

$$\sigma_B^* J_B = -J_B$$

$$Z_8 = Y_4/\sigma$$

- ➢ Grading of 3d moduli into σ-even and σ-odd
- ➢ Truncation of the M-theory effective action to 3d $\mathcal{N} = 1$

Task: perform the uplift to four dimensions

Uplifting on an interval

- A Majorana spinor in 4d yields two Majorana spinors upon circle reduction
- A 3d $\mathcal{N}=1$ supergravity theory cannot come from circle reduction of a 4d theory
- The standard uplift of the M-theory action to the F-theory action is not possible

Proposal: uplift on the interval $S^1/\mathbb{Z}_2$

➢ Heuristic picture from the antiholomorphic quotient

the orientation of the torus is reversed → one of the two cycles is reflected → the T-duality circle is turned into an interval

➢ Allows to obtain 3d $\mathcal{N}=1$ from four dimensions

➢ Evidence for an interval in the weak-coupling limit

Interval and boundary conditions

- Reduction on $\mathbb{R}^{1,2} \times S^1/\mathbb{Z}_2$ (circle coordinate $x^3 \sim x^3 + 2\pi R$)
- Boundary conditions have to be specified for each field

4d field	allowed boundary conditions	massless 3d field
scalar	$\partial_3 \phi \big| = 0$	scalar
	$\phi \big| = 0$	no massless field
vector	$\partial_3 A_\mu \big| = 0 \quad \& \quad A_3 \big| = 0$	vector
	$A_\mu \big| = 0 \quad \& \quad \partial_3 A_3 \big| = 0$	scalar
Majorana spinor	$\frac{1}{2}(1 - \gamma_3)\chi \big| = 0$	Majorana spinor
	$\frac{1}{2}(1 + \gamma_3)\chi \big| = 0$	

- Dirichlet b.c. forbid constant non-zero profile: no massless degree of freedom in 3d

Ambiguities and minimal uplift

Ambiguities of the interval uplift

- F-theory data and symmetries fix the the uplift of spinors and vectors…

spinors and Cartan vectors in 3d —(Lorentz symmetry / gauge symmetry)→ reconstruction of 4d couplings

- … but we can add any 4d Dirichlet scalar in the uplift

Minimal uplift of 3d action S_3	=	4d Lorentz invariant action S_4 with minimal set of fields that yields S_3 on small interval

- S_3 has $\mathcal{N} = 1$ supersymmetry in 3d → S_4 is non-supersymmetric

scalar sector → "real slice" in the Kähler moduli space of a 4d $\mathcal{N} = 1$ theory

Supersymmetry restoration

Does the minimal uplift capture all light degrees of freedom?

- This question cannot be answered within supergravity
- In the F-theory limit some M2-brane states become light
- For a subclass of quotients complementary information comes from the analysis of the Type IIB weak-coupling limit

4d $\mathcal{N} = 1$ supersymmetry is restored in the infinite interval limit

- For any σ-odd Calabi-Yau modulus a Dirichlet scalar has to be included

σ-even scalar	σ-odd scalar
Neumann b.c.	Dirichlet b.c.
3d zeromodes + excited modes	3d excited modes only

- For infinite interval the 4d effective action is the Calabi-Yau effective action

Sen's limit and the Calabi-Yau threefold Y_3

- Recall Weierstrass form of the elliptic fibration

$$y^2 = x^3 + f(u)\, x\, z^4 + g(u)\, z^6$$

- Sen's parameterization [Sen 97]

$$f = C\,\eta - 3h^2$$

$$g = h(C\,\eta - 2h^2) + C^2\,\chi$$

$h\,,\,\eta\,,\,\chi \equiv$ functions of base coordinates

$C \equiv$ constant

- Engineered to have small coupling as $C \to 0$

- Resulting picture: orientifold of Type IIB on Calabi-Yau threefold Y_3
  - Y_3 is the double cover of the base B_3
  - conveniently described by an additional coordinate ξ

Calabi-Yau threefold Y_3 : $\xi^2 = h$

Two involutions on Y_3

- Orientifold of Type IIB on Y_3: holomorphic involution (u : coord. on B_3)

$$\sigma_{\mathrm{h}} : Y_3 \to Y_3 \qquad (u, \xi) \mapsto (u, -\xi)$$

- Fixed locus of σ_{h} ⟶ O7-plane located at $\xi = 0 \; \Leftrightarrow \; h = 0$

- $\sigma_B : B_3 \to B_3$ can be extended to Y_3 to give an antiholomorphic involution

$$\sigma_{\mathrm{ah}} : Y_3 \to Y_3 \qquad (u, \xi) \mapsto (\, \sigma_B(u) \, , \, \bar{\xi} \,)$$

Weak-coupling picture as Type IIB quotient

Weak-coupling picture

Type IIB on $\mathbb{R}^{1,2} \times S^1 \times Y_3$ modded out by the symmetry group generated by

$$\mathcal{O}_1 = \sigma_{\rm h}\, \Omega_p\, (-1)^{F_L}$$

$$\mathcal{O}_2 = R_3\, \sigma_{\rm ah}\, (-1)^{F_L}$$

where

$\Omega_p \equiv$ worldsheet orientation reversal

$F_L \equiv$ left-moving spacetime fermion number

$R_3 \equiv$ reflection in external spacetime $\mathbb{R}^{1,2} \times S^1$

$$(x^0, x^1, x^2, x^3) \mapsto (x^0, x^1, x^2, -x^3)$$

Weak-coupling picture as Type IIB quotient

Weak-coupling picture

Type IIB on $\mathbb{R}^{1,2} \times S^1 \times Y_3$ modded out by the symmetry group generated by

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Some comments:

- The S^1 factor is interpreted as the T-dual of the *B* cycle of the torus
- It grows macroscopically large in the F-theory limit
- The involution σ_{ah} induces a Pin-odd transformation on spinors
  - not a symmetry of chiral Type IIB!
- The reflection R_3 compensates σ_{ah} and induces the interval $S^1/\mathbb{Z}_2$

Weak-coupling picture as Type IIB quotient

Weak-coupling picture

Type IIB on $\mathbb{R}^{1,2} \times S^1 \times Y_3$ modded out by the symmetry group generated by

$$\mathcal{O}_1 = \sigma_{\rm h} \, \Omega_p \, (-1)^{F_L}$$

$$\mathcal{O}_2 = R_3 \, \sigma_{\rm ah} \, (-1)^{F_L}$$

Some further comments:

- The factors $\Omega_p \, (-1)^{F_L}$ and $R_3 \, (-1)^{F_L}$ in $\mathcal{O}_1$ and $\mathcal{O}_2$ are deduced from M-theory to F-theory duality
- Trace back to Type IIA and then M-theory $\longrightarrow$ geometric reflections of *A*, *B* cycles
- Consistency with the action of $\sigma : Y_4 \to Y_4$ on the fiber

Weak-coupling setups with five-planes

- Assumptions:
  - ➢ real three-dimensional fixed locus on the base
  - ➢ real one-dimensional fixed line on the fiber

symmetry	localized object	on the interval	inside Y_3 wraps	
$\mathcal{O}_1 = \sigma_{\mathrm{h}}\, \Omega_p\, (-1)^{F_L}$	O7	wrapping	$\mathcal{H}_4$	4-dim. holom. submanifold
$\mathcal{O}_2 = R_3\, \sigma_{\mathrm{ah}}\, (-1)^{F_L}$	X5	localized at endpoints	$\mathcal{L}_3$	3-dim. special Lagrangian subm.
$\mathcal{O}_1\mathcal{O}_2 = R_3\, \sigma_{\mathrm{h}}\, \sigma_{\mathrm{ah}}\, \Omega_p$	O5	localized at endpoints	$\mathcal{L}_3'$	3-dim special Lagrangian subm.

- Lagrangian subm. are calibrated in a way compatible with the holom. subm.
  - ➢ non-zero mutual supersymmetry ⟶ explicit check in toroidal model

Features of X5-planes

- Fixed locus of orbifold action with additional $(-1)^{F_L}$ factor
  [Kutasov 96] [Sen 96 98][Bergman, Gaberdiel 98] [Helllerman 05]
- S-duality: $(-1)^{F_L} \leftrightarrow \Omega_p$
- An X5-plane is S-dual to an O5-plane with one D5-brane (and its image D5')
- No net tadpole
- The twisted sector yields a massless U(1) ⟷ gauge symmetry on single D5
- Stable non-BPS states
  - decay is forbidden by charge under the U(1)
  - S-dual to strings stretching between D5 and D5' across O5

Large interval limit and supersymmetry restoration

- As the interval grows large
  - KK modes become light
  - all excitations are accessible at low-energies
  - the boundary sector decouples and 4d $\mathcal{N} = 1$ bulk supersymmetry is restored

Type IIB	Type IIA	M-theory
KK modes	winding strings	wrapped M2-branes

- Non-trivial enhancement in the quantum moduli space of M-theory

Spin(7) moduli space with vanishing fiber	$\cong$	Calabi-Yau moduli space with vanishing fiber

Conclusions

- Spin(7) manifolds from Calabi-Yau quotients
- ~~Circle uplift~~ → interval uplift
- Resulting picture

4d $\mathcal{N}=1$ bulk sector	coupled to	3d $\mathcal{N}=1$ localized objects at the boundaries of the interval

- Supersymmetry is restored in the limit of infinite interval
- Type IIB weak-coupling limit
  - five-planes (X5, O5) at the ends of the interval
  - exotic three-planes at the ends of the interval
  - … (other possible cases?)

Outlook

- Closer look at charged matter
  - massive in the Coulomb branch ⟶ not accessible from supergravity
  - weak-coupling: intersecting D7-branes ⟶ explicit string constructions
- Resolution of Spin(7) quotient singularities
  - weak-coupling picture can provide hints

    e.g. Type IIB: O5 Type IIA: O6 M-theory: Atiyah-Hitchin space
- Extend the weak-coupling to more exotic setups (e.g. Klein bottle fiber)
- Explore extensions to models with more phenomenological features
  - no dilution of susy breaking in F-theory limit?
  - high-scale susy breaking?
  - applications of stable non-BPS objects?

Thank you for your attention