The Bundle Moduli Space of Heterotic Standard Models

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based on: arXiv: 1404:2767, arXiv:1311.1941, arXiv:1307.4787, arXiv:1303.1832, arXiv:1106.4804, arXiv:1405:2073, arXiv:1202.1757
with Lara Anderson, Evgeny Buchbinder, Andrei Constantin, James Gray, Seung-Joo Lee and Eran Palti.

Overview

- A line bundle model on the tetra-quadric CY
- Exploring the bundle moduli space
  (focus on proton stability, Yukawa couplings and Higgs mass)

- GUT breaking with hypercharge flux in heterotic CY models
- Conclusion and outlook

A line bundle model on the tetra-quadric

(From standard model data base at:
http://www-thphys.physics.ox.ac.uk/projects/CalabiYau/linebundlemodels/index.html)

- tetra-quadric CY: $X = \left[\begin{array}{c|c} \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \\ \mathbb{P}^1 & 2 \end{array} \right]$ with $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry

- line bundle sum: $V = \bigoplus_{a=1}^{5} L_a$

$$L_1 = \mathcal{O}_X(-1,0,0,1) \quad , \quad L_2 = \mathcal{O}_X(-1,-3,2,2)$$
$$L_3 = \mathcal{O}_X(0,1,-1,0) \quad , \quad L_4 = \mathcal{O}_X(1,1,-1,-1)$$
$$L_1 = \mathcal{O}_X(1,1,0,-2)$$

- spectrum: $\mathbf{10}_2,\ \mathbf{10}_2,\ \mathbf{10}_5,\ \overline{\mathbf{5}}_{2,4},\ \overline{\mathbf{5}}_{4,5},\ \overline{\mathbf{5}}_{4,5},\ H_{2,5},\ \bar{H}_{2,5}$

$3\,\mathbf{1}_{2,1},\ 3\,\mathbf{1}_{5,1},\ 5\mathbf{1}_{2,3},\ 3\,\mathbf{1}_{2,4},\ \mathbf{1}_{5,3}$

- superpotential: $W = \lambda_i \overline{H}_{2,5}(Q_2^{(i)} u_5 + Q_5 u_2^{(i)}) + \rho_{\alpha i} \mathbf{1}_{2,4}^{(\alpha)} L_{4,5}^{(i)} \overline{H}_{2,5}$

At Abelian locus: - $V = \bigoplus_{a=1}^{5} L_a$, group $S(U(1)^5)$
- rank2 up Yukawa matrix
- proton stable
- massless pair of Higgs doublets

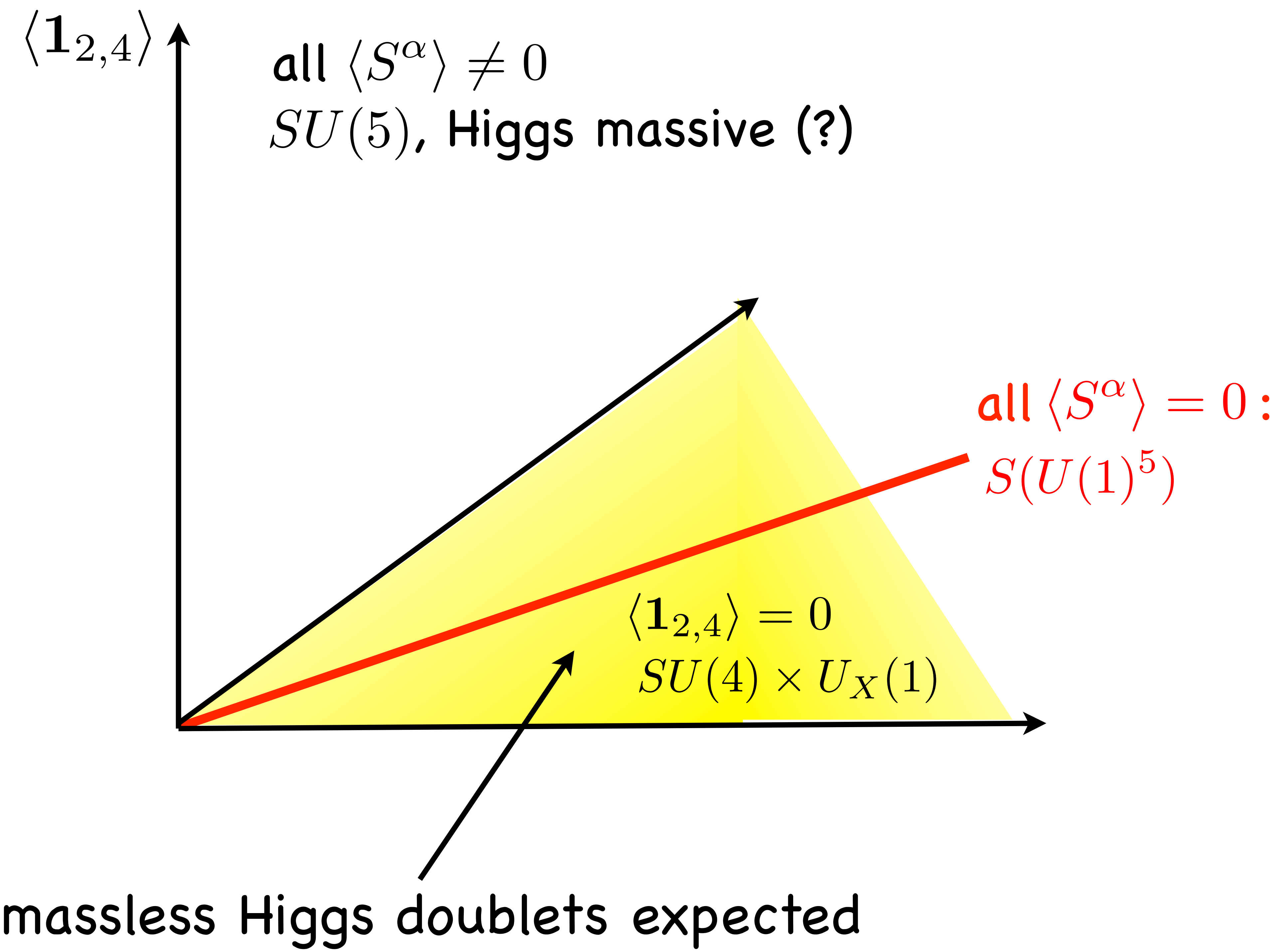
$U_{B-L}(1)$

Expectation away from Abelian locus:

Non-Abelian, $\langle \mathbf{1}_{2,4} \rangle = 0$: - $V \to \tilde{V} = U \oplus L_4$, group $S(U(4) \times U_X(1))$
- rank 2 up Yukawa matrix
- proton remains stable
- Higgs pair remains massless

Non-Abelian, $\langle \mathbf{1}_{2,4} \rangle \neq 0$: - $V \to \tilde{V}$, group $SU(5)$
- rank 2 up Yukawa matrix
- proton still stable
- Higgs becomes massive

Schematic structure of moduli space:



Proton stable (and structure of Yukawa matrix preserved) everywhere, due to symmetry enhancement at Abelian locus.

Exploring the bundle moduli space

Check fate of Higgs by constructing non-Abelian bundle

Recall:
$$\begin{aligned} L_1 &= \mathcal{O}_X(-1,0,0,1) \quad , \quad L_2 = \mathcal{O}_X(-1,-3,2,2) \quad , \quad L_3 = \mathcal{O}_X(0,1,-1,0) \\ L_4 &= \mathcal{O}_X(1,1,-1,-1) \quad , \quad L_5 = \mathcal{O}_X(1,1,0,-2) \end{aligned}$$

1) Extension bundles

For $V_1 = L_2 \oplus L_5$, $V_2 = L_1 \oplus L_3 \oplus L_4$ define extension

$$0 \longrightarrow V_1 \longrightarrow \tilde{V} \longrightarrow V_2 \longrightarrow 0$$

Compute $\#\mathbf{5} = h^2(X, \wedge^2 \tilde{V}) = \begin{cases} 3 & \text{for} \quad \langle \mathbf{1}_{2,4} \rangle = 0 \\ 0 & \text{for} \quad \langle \mathbf{1}_{2,4} \rangle \neq 0 \end{cases}$

2) Monads

$$0 \longrightarrow \widetilde{V} \longrightarrow B \stackrel{f}{\longrightarrow} C \longrightarrow 0$$

$$B \sim \left[\begin{array}{c|cc|c|c|cc} -1 & -1 & -1 & 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & -1 & -1 & 0 & 0 \\ 1 & 2 & 2 & 0 & -1 & 0 & 0 \end{array}\right] \qquad C \sim \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 2 & 0 \\ 2 & 2 \end{bmatrix}$$

$L_1 \quad L_2 \quad L_3 \, L_4 \quad L_5$

$$f \sim \begin{pmatrix} f_{(0,1,2,1)} & f_{(0,2,0,0)} & f'_{(0,2,0,0)} & 0 & 0 & 0 & 0 \\ f_{(2,1,0,1)} & 0 & 0 & f_{(1,0,1,2)} & f_{(0,0,1,3)} & f_{(0,0,0,2)} & f'_{(0,0,0,2)} \end{pmatrix}$$

bundle splits if zero

We can show for $\langle \mathbf{1}_{2,4} \rangle = 0$, $\tilde{V} = U \oplus L_4$:

- bundle $\tilde{V}$ is supersymmetric
- $\#\mathbf{5} = h^2(X, \wedge^2 \tilde{V}) = 3$

Summary of the $SU(4) \times U_X(1)$ model:

- $U_X(1) \longrightarrow U_{B-L}(1)$
- μ-term forbidden
- dangerous dim. 4 terms forbidden by $U_{B-L}(1)$
- $\overline{\mathbf{5}}\,\mathbf{10}\,\mathbf{10}\,\mathbf{10}$ operators still absent, due to symmetry enhancement at Abelian locus

Main messages:

- "Unexpected" absences of operators can help with proton stability.
- Finding models with a massless Higgs everywhere in moduli space is non-trivial -> examples in the data base.

GUT breaking with hypercharge flux

(Blumenhagen, Honecker, Weigand, 05
Blumenhagen, Moster, Weigand, 06)

Q: Can we construct a heterotic standard model without Wilson lines but "built-in" gauge unification?

embedding: $SU_W(2) \times SU_c(3) \times SU(6) \subset E_8$

bundle: $V = U_1 \oplus \cdots \oplus U_f$ with structure group

$$S(U(n_1) \times \cdots \times U(n_f)) \subset SU(6)$$

splitting types: $\mathbf{n} = (6),\ (5,1),\ (4,2),\ (3,3),\ (4,1,1),\ (3,2,1),\ (2,2,2),$
$(3,1,1,1),\ (2,2,1,1),\ (2,1,1,1,1),\ (1,1,1,1,1,1)\ .$

low-energy gauge group: $SU_W(2) \times SU_c(3) \times S(U(1)^f) \supset U_Y(1)$

hypercharge embedding: $\mathbf{y} = (y_1, \ldots, y_f)$

spectrum:

$(SU(2)\times SU(3))_{\mathbf{q}}$	$(\mathbf{1},\mathbf{1})_{\mathbf{e}_a-\mathbf{e}_b}$	$(\mathbf{1},\mathbf{3})_{-\mathbf{e}_a-\mathbf{e}_b}$	$(\mathbf{1},\overline{\mathbf{3}})_{\mathbf{e}_a+\mathbf{e}_b}$	$(\mathbf{2},\mathbf{3})_{\mathbf{e}_a}$	$(\mathbf{2},\overline{\mathbf{3}})_{-\mathbf{e}_a}$	$(\mathbf{2},\mathbf{1})_{\mathbf{e}_a+\mathbf{e}_b+\mathbf{e}_c}$
range	$a,b=1,\dots,6$	$a\le b$	$a\le b$	$a=1,\dots 6$	$a=1,\dots 6$	$a\le b\le c$
particle	$e_{a,b},\ S_{a,b}$	$\tilde{d}_{a,b},\ \tilde{u}_{a,b}$	$d_{a,b},\ u_{a,b}$	Q_a	$\tilde{Q}_a$	$L_{a,b,c},\ H_{a,b,c},\ \bar{H}_{a,b,c}$
bundle	$U_a\otimes U_b^*$	$U_a^*\otimes U_b^*$ $\wedge^2 U_a^*$	$U_a\otimes U_b$ $\wedge^2 U_a$	U_a	U_a^*	$U_a\otimes U_b\otimes U_c$ $\wedge^2 U_a\otimes U_b$, $U_a\otimes\wedge^2 U_b,\ \wedge^3 U_a$
contained in	$V\otimes V^*$	$\wedge^2 V^*$	$\wedge^2 V$	V	V^*	$\wedge^3 V$
hypercharge	y_a-y_b	$-y_a-y_b$	y_a+y_b	y_a	$-y_a$	$y_a+y_b+y_c$
phys. hypercharge	$2,\ 0$	$-2/3,\ 4/3$	$2/3,\ -4/3$	$1/3$	$-1/3$	$-1,\ -1,\ 1$

unification condition:

$$\sum_{a=1}^{f} n_a y_a^2 = \frac{10}{3}$$

1. step, group theory:

What are the allowed $\mathbf{y}$-vectors satisfying the unification condition and giving the correct hypercharges for a family (and at least one choice of $S(U(1)^f)$ charges)?

Answer:

splitting type $\mathbf{n}$	allowed $\mathbf{y}$ vectors
$(4,1,1)$	~~$(1/3,1/3,-5/3)$~~
$(3,2,1)$	~~$(1/3,1/3,-5/3)$~~, $(-2/3,1/3,4/3)$
$(2,2,2)$	no solution
$(3,1,1,1)$	~~$(1/3,1/3,1/3,-5/3)$~~, $(-2/3,1/3,1/3,4/3)$
$(2,2,1,1)$	~~$(1/3,1/3,1/3,-5/3)$~~, $(1/3,-2/3,-2/3,4/3)$
$(2,1,1,1,1)$	~~$(1/3,1/3,1/3,1/3,-5/3)$~~, $(1/3,-2/3,-2/3,-2/3,4/3)$, $(-2/3,-2/3,1/3,1/3,4/3)$ $(5/6,-7/6,-2/3,-1/6,1/3)$, $(-5/21,-17/21,-11/21,1/3,31/21)$
$(1,1,1,1,1,1)$	~~$(1/3,1/3,-5/3,1/3,1/3,1/3)$~~, $(1/3,4/3,-2/3,-2/3,-2/3,1/3)$ $(1/3,5/6,-7/6,-1/6,-2/3,5/6)$, $(1/3,7/12,-17/12,1/12,-5/12,5/6),\ldots$

2. step, geometry:

Can we find a CY and a bundle which leads to a standard model with hypercharge flux for one of the above $\mathbf{y}$-vectors?

No!

(Large computer scan for Abelian case finds no viable model.)

No-go theorem schematically, for Abelian case:

- choose y-vector from above classification
- write indices in terms of $X_{abc} = c_1(L_a)c_1(L_b)c_1(L_c)$ and $Z_a = c_1(L_a)c_2(TX)$, e.g. $\mathrm{ind}(L_a) = \frac{1}{6}X_{aaa} + \frac{1}{12}Z_a$
- impose physical constraints on indices, e.g. $\sum_{a:y_a=1/3} \mathrm{ind}(L_a) = -3$
- eliminate two $c_1(L_a)$ using $\sum_a c_1(L_a) = 0$ and $\sum_a y_a c_1(L_a) = 0$ (hypercharge massless).
- Solve the resulting linear system for X_{abc}, Z_a

This linear system has no solution for any $\mathbf{y}$- vector above!

What about approximate unification so that $\sum_{a=1}^{f} n_a y_a^2 \simeq \frac{10}{3}$ within 5% ?

More embeddings allowed but only one cannot be excluded by the no-go theorem:

$$\mathbf{y} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{5}{3}, \alpha, \frac{2}{3} - \alpha\right)$$

with a very specific pattern of indices

$$\begin{aligned}
\mathrm{ind}(L_1) = \mathrm{ind}(L_2) = \mathrm{ind}(L_3) &= -1 \quad \to \quad Q_1,\, Q_2,\, Q_3 \\
\mathrm{ind}(L_1 \otimes L_4) = \mathrm{ind}(L_2 \otimes L_4) = \mathrm{ind}(L_3 \otimes L_4) &= -1 \quad \to \quad u_{1,4},\, u_{2,4},\, u_{3,4} \\
\mathrm{ind}(L_5 \otimes L_6) &= -3 \quad \to 3\, d_{5,6} \\
\mathrm{ind}(L_4 \otimes L_5 \otimes L_6) = -\mathrm{ind}(L_1 \otimes L_2 \otimes L_3) &= -3 \quad \to \quad 3\, L_{4,5,6} \\
\mathrm{ind}(L_1 \otimes L_4^*) = \mathrm{ind}(L_2 \otimes L_4^*) = \mathrm{ind}(L_3 \otimes L_4^*) &= -1 \quad \to \quad e_{1,4},\, e_{2,4},\, e_{3,4}
\end{aligned}$$

We do not know if explicit models for this case can be found.

Conclusions and outlook

- We can continue line bundle models into the non-Abelian part of the moduli space, both by continuation along flat directions in the 4d theory and by explicit bundle constructions.

- Keeping a light Higgs pair everywhere in moduli space is non-trivial but we now have examples with this feature. Symmetry enhancement helps to control the mu-term.

- Additional symmetries at the Abelian locus can lead to ``unexpected" absences of operators and stabilize the proton.

- Heterotic CY models with hypercharge flux are over-constrained: Geometries with the right properties do not exist.

- GUTs are good.

Thanks