Fine Tuning in a Low Energy Exceptional Supersymmetric Standard Model

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Outline

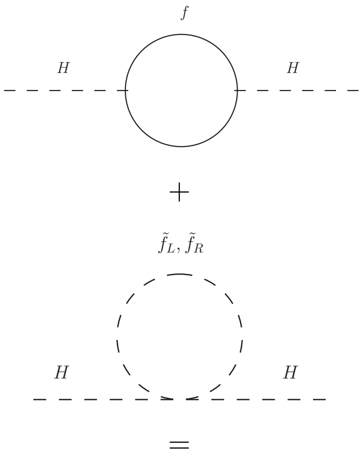
Motivation

- Higgs mechanism in the Standard Model (SM) requires fundamental scalar

  ⇒ Hierarchy Problem (HP)

  ⇒ requires **fine tuning** to avoid
- SUSY: natural solution to HP, since superpartner contributions cancel quadratic divergences

  ⇒ no need for fine tuning, provided SUSY is not too badly broken
- Other desirable features: natural dark matter candidates, gauge coupling unification, REWSB ...



non-quadratically divergent

The MSSM

Gauge group $G = SU(3)_C \times SU(2)_L \times U(1)_Y$

Chiral superfields			
Supermultiplet	Spin-0	Spin-1/2	$(SU(3)_C, SU(2)_L, U(1)_Y)$
$\hat{Q}_i$	$(\tilde{u}_L, \tilde{d}_L)_i$	$(u_L, d_L)_i$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
$\hat{u}_i^C$	$\tilde{u}_{iR}^*$	u_{iR}^C	$(\mathbf{3}, \mathbf{1}, -\frac{2}{3})$
$\hat{d}_i^C$	$\tilde{d}_{iR}^*$	d_{iR}^C	$(\mathbf{3}, \mathbf{1}, \frac{1}{3})$
$\hat{L}_i$	$(\tilde{\nu}_L, \tilde{e}_L)_i$	$(\nu_L, e_L)_i$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$\hat{e}_i^C$	$\tilde{e}_{iR}^*$	e_{iR}^C	$(\mathbf{1}, \mathbf{1}, 1)$
$\hat{H}_1$	(H_1^0, H_1^-)	$(\tilde{H}_1^0, \tilde{H}_1^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
$\hat{H}_2$	(H_2^+, H_2^0)	$(\tilde{H}_2^+, \tilde{H}_2^0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2})$

Superpotential (R-parity conserving)

$$W_{\mathrm{MSSM}} = \mu(\hat{H}_1 \cdot \hat{H}_2) + y_{ij}^e(\hat{L}_i \cdot \hat{H}_1)\hat{e}_j^C + y_{ij}^d(\hat{Q}_i \cdot \hat{H}_1)\hat{d}_j^C + y_{ij}^u(\hat{H}_2 \cdot \hat{Q}_i)\hat{u}_j^C$$

Little Hierarchy Problem

- Higgs mass is bounded at tree level in the MSSM: $m_{h_1}^2 \leq M_Z^2 \cos^2 2\beta$
- Can obtain $m_{h_1}^2 \approx 125$ GeV in the MSSM provided there are large loop contributions due to heavy stops,

$$m_{h_1}^2 \approx M_Z^2 \cos^2 2\beta \left(1 - \frac{3}{8\pi^2}\frac{m_t^2}{v^2}\log\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{M_t^2}\right) + \frac{3}{4\pi^2}\frac{m_t^4}{v^2}\log\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{M_t^2} + \dots$$

- But these also generate contributions to the Higgs VEV via the EWSB condition ($M_Z^2 = \bar{g}^2 v^2/4$)

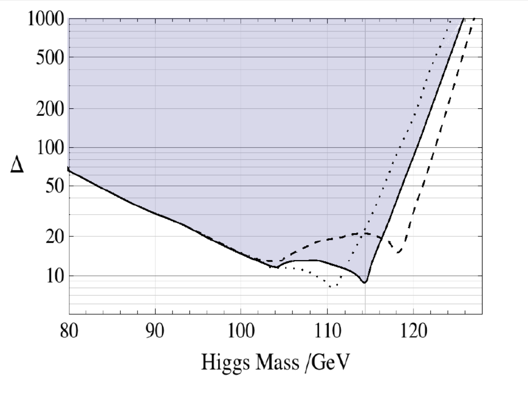
$$\frac{M_Z^2}{2} = -\mu^2 + \frac{\overbrace{m_{H_d}^2 - m_{H_u}^2}^{\text{RGE effects}} \tan^2\beta}{\tan^2\beta - 1} + \delta_{\text{loops}},$$

$$\text{1-loop } V_{\text{eff}}: \quad \delta_{\text{loops}} = \frac{3}{8\pi^2}\frac{m_t^2}{v^2\cos 2\beta}\left[m_{\tilde{t}_1}^2\left(\log\frac{m_{\tilde{t}_1}^2}{Q^2} - 1\right) + m_{\tilde{t}_2}^2\left(\log\frac{m_{\tilde{t}_2}^2}{Q^2} - 1\right)\right] + \dots$$

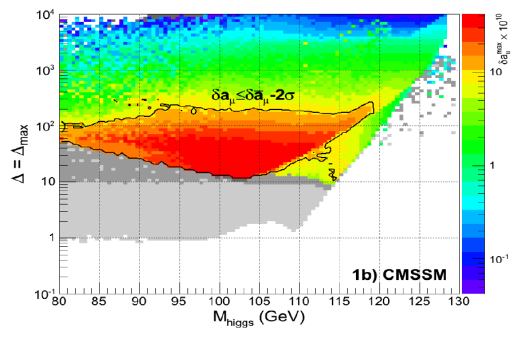
- Resulting tension between need for increased $m_{h_1}^2$ and light M_Z^2 $\Rightarrow$ a “Little Hierarchy Problem”
- With the discovery of the Higgs, many studies of MSSM models show large fine tuning Δ

Little Hierarchy Problem

- In the cMSSM, $m_{h_1} \approx 125$ GeV $\Rightarrow$ fine tuning is $\mathcal{O}(1000)$



S. Cassel and D. M. Ghilencea, Mod. Phys. Lett. A **27** (2012) 1230003



D. M. Ghilencea, H. M. Lee and M. Park, JHEP **1207** (2012) 046

$U(1)$ Extended Models

- Possible solution to the Little HP
- $U(1)$ extended models:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$$

- Additional F- and D- term contributions
  $\Rightarrow$ increased tree level upper bound on $m_{h_1}^2$
  $\Rightarrow$ **reduced fine tuning from heavy superpartners**
- Include a SM singlet charged under the $U(1)'$ coupling to $\hat{H}_1$, $\hat{H}_2$
  $\Rightarrow$ acquires a VEV $\langle S \rangle$
  $\Rightarrow$ dynamically generate an effective μ-term,

$$W \supset \lambda \underset{\substack{\uparrow \\ \text{SM singlet}}}{\hat{S}} (\hat{H}_1 \cdot \hat{H}_2) \longrightarrow \mu_{\text{eff}} (\hat{H}_1 \cdot \hat{H}_2), \quad \mu_{\text{eff}} = \lambda \langle S \rangle$$

  $\Rightarrow$ can avoid the domain wall problems of the NMSSM (no $\mathbb{Z}_3$ symmetry)
- In general need to be careful choosing $U(1)'$ charges to avoid introducing anomalies
- One way to do so is to appropriately embed within a larger gauge group

The E$_6$SSM

S. F. King, S. Moretti, and R. Nevzorov, Phys. Rev. D **73**, 035009 (2006)

- Example: the **Exceptional Supersymmetric Standard Model** (E$_6$SSM)
- Low energy effective model based on unified E_6 gauge group at the GUT scale:

$$\begin{aligned} E_6 &\longrightarrow SO(10) \times U(1)_\psi \\ &\longrightarrow SU(5) \times U(1)_\psi \times U(1)_\chi \\ &\longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\psi \times U(1)_\chi \end{aligned}$$

$$\boxed{E_6 \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'}$$

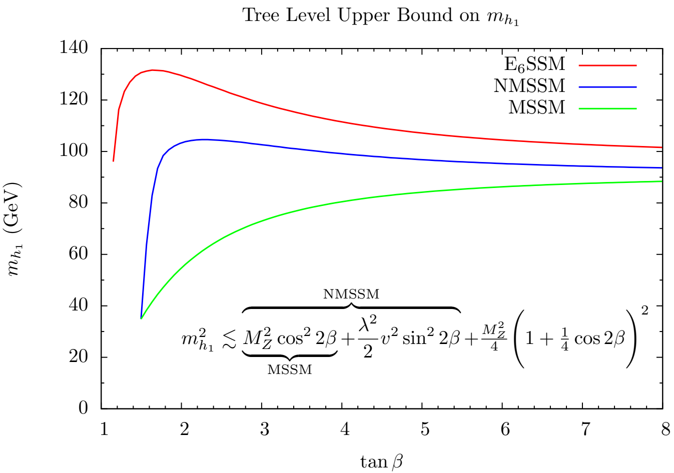
  where $U(1)' = U(1)_\chi \cos\theta_{E_6} + U(1)_\psi \sin\theta_{E_6}$
  $\Rightarrow$ choose θ_{E_6} so that right-handed neutrinos are uncharged: $U(1)' = U(1)_N$
  $\Rightarrow$ see-saw mechanism for neutrino masses
- Matter content $= 3 \times \mathbf{27}$-plets of E_6 + $SU(2)$ $\mathbf{2}$ and $\overline{\mathbf{2}}$ from incomplete $\mathbf{27}'$ and $\overline{\mathbf{27}'}$
  $\Rightarrow$ anomaly free
- Choose field basis so that Higgs doublets $\hat{H}_d$, $\hat{H}_u$ from one $\mathbf{27}$-plet get VEVs for EWSB
- Singlet from this $\mathbf{27}$-plet also gets a VEV and breaks $U(1)_N$
  $\Rightarrow$ massive Z' boson

Chiral superfields			
Supermultiplet	Spin-0	Spin-1/2	$(SU(3)_C, SU(2)_L, U(1)_Y, U(1)_N)$
$\hat{Q}_i$	$(\tilde{u}_L, \tilde{d}_L)_i$	$(u_L, d_L)_i$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6}, 1)$
$\hat{u}^C_i$	$\tilde{u}^*_{iR}$	u^C_{iR}	$(\mathbf{3}, \mathbf{1}, -\frac{2}{3}, 1)$
$\hat{d}^C_i$	$\tilde{d}^*_{iR}$	d^C_{iR}	$(\mathbf{3}, \mathbf{1}, \frac{1}{3}, 2)$
$\hat{L}_i$	$(\tilde{\nu}_L, \tilde{e}_L)_i$	$(\nu_L, e_L)_i$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2}, 2)$
$\hat{e}^C_i$	$\tilde{e}^*_{iR}$	e^C_{iR}	$(\mathbf{1}, \mathbf{1}, 1, 1)$
$\hat{N}^C_i$	$\tilde{N}^*_{iR}$	N^C_{iR}	$(\mathbf{1}, \mathbf{1}, 0, 0)$
$\hat{H}_{1i}$	(H^0_1, H^-_1)	$(\tilde{H}^0_1, \tilde{H}^-_1)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2}, -3)$
$\hat{H}_{2i}$	(H^+_2, H^0_2)	$(\tilde{H}^+_2, \tilde{H}^0_2)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2}, -2)$
$\hat{S}_i$	S_i	$\tilde{S}_i$	$(\mathbf{1}, \mathbf{1}, 0, -5)$
$\hat{D}_i$	$\tilde{D}_i$	D_i	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3}, -2)$
$\hat{\overline{D}}_i$	$\tilde{\overline{D}}_i$	$\overline{D}_i$	$(\bar{\mathbf{3}}, \mathbf{1}, \frac{1}{3}, -3)$
$\hat{H}'$	(H'^0, H'^-)	$(\tilde{H}'^0, \tilde{H}'^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2}, 2)$
$\hat{\overline{H}}'$	$(\overline{H}'^+, \overline{H}'^0)$	$(\tilde{\overline{H}}'^+, \tilde{\overline{H}}'^0)$	$(\mathbf{1}, \mathbf{2}, \frac{1}{2}, -2)$

$$W_{\mathrm{E_6SSM}} \approx \lambda \underbrace{\hat{S}(\hat{H}_d \cdot \hat{H}_u)}_{\hat{S}\equiv\hat{S}_3,\, \hat{H}_d\equiv\hat{H}_{13},\, \hat{H}_u\equiv\hat{H}_{23}} + \lambda_\alpha \hat{S}(\hat{H}_{1\alpha} \cdot \hat{H}_{2\alpha}) + \kappa_i \hat{S}(\hat{D}_i \hat{\overline{D}}_i) + \mu'(\hat{H}' \cdot \hat{\overline{H}}') + W_{\mathrm{MSSM}}(\mu = 0), \quad (i = 1, 2, 3,\ \alpha = 1, 2).$$

The E_6SSM

S. F. King, S. Moretti, and R. Nevzorov, Phys. Rev. D **73**, 035009 (2006)



$$m_{h_1}^2 \lesssim \overbrace{\underbrace{M_Z^2\cos^2 2\beta}_{\text{MSSM}} + \frac{\lambda^2}{2}v^2\sin^2 2\beta}^{\text{NMSSM}} + \frac{M_Z^2}{4}\left(1+\frac{1}{4}\cos 2\beta\right)^2$$

The E_6SSM

S. F. King, S. Moretti, and R. Nevzorov, Phys. Rev. D **73**, 035009 (2006)

- Increased m_{h_1} upper bound due to new F- and D-terms
  $\Rightarrow$ less fine tuning from heavy superpartners
  - the cE_6SSM has been found to be less fine tuned than the cMSSM [1]
- **But** extra D-terms also contribute to EWSB conditions, one of which can be written [1]

$$\underbrace{c(\tan\beta)}_{\mathcal{O}(1)} \frac{M_Z^2}{2} \approx -\underbrace{\frac{\lambda^2 s^2}{2}}_{\mu_{\text{eff}}^2} + \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} + \underbrace{d(\tan\beta)}_{\mathcal{O}(1)} \frac{M_{Z'}^2}{2}$$

- Heavy Z' contribution
  $\Rightarrow$ possible new *tree level* fine tuning
- For $M_{Z'}$ large enough, "cancels out" the benefits of having a higher Higgs mass upper bound.
  - for what values of $M_{Z'}$ does this occur?

[1] P. Athron, M. Binjonaid, S. F. King, Phys Rev D **87**, 115023 (2013)

Fine Tuning at Low Energies

$$c(\tan\beta)\frac{M_Z^2}{2} \approx -\mu_{\mathrm{eff}} + + \frac{m_{H_d}^2 - m_{H_u}^2\tan^2\beta}{\tan^2\beta - 1} + d(\tan\beta)\frac{M_{Z'}^2}{2} + \delta_{\mathrm{loops}}$$

- Constrained models e.g. cMSSM defined at Λ_{GUT}
  $\Rightarrow$ **very large** logarithms due to RG running contribute to tuning:

$$m_{H_u}^2(M_S) \supset m_{H_u}^2(\Lambda_{GUT}) + \frac{3y_t^2}{8\pi^2}(m_{H_u}^2 + m_{Q3}^2 + m_{u3}^2 + A_t^2)\underbrace{\log\frac{M_S}{\Lambda_{GUT}}}_{\text{big}} + \ldots \Rightarrow \text{ large tuning}$$

- Consequence of assumptions at the high scale $\Lambda_{GUT} \sim 10^{16}$ GeV
- Define model at low energies $M_X \approx 20$ TeV (e.g. pMSSM) $\Rightarrow$ no large $\log(M_S/\Lambda_{GUT})$ contribution
  $\Rightarrow$ significantly reduced tuning [2]
- Still have fine tuning due to tree level + Coleman-Weinberg δ_{loops} contributions + remaining RGE effects
- Aim of this study is to assess the impact of these contributions
  - when large logs are removed, is the “phenomenological” E_6SSM less tuned than its MSSM counterpart due to the increased Higgs mass bound, or does the Z' tuning negate this?

[2] M. Cahill-Rowley, J. L. Hewett, A. Ismail, T. Rizzo, Phys. Rev. D **86**, 075015 (2012)

Measuring the Fine Tuning

- Use the Ellis-Barbieri-Giudice [3] measure

$$\Delta = \max_{\{p\}} \Delta_p, \quad \Delta_p = \left| \frac{\partial \log M_Z^2}{\partial \log p} \right|, \quad \{p\} = \text{ parameters defined at cut-off scale } M_X,$$

  evaluated at the scale $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$

- Obtain in the MSSM (E_6SSM) by solving for $\partial v_i / \partial p$ the system

$$\sum_{i=1}^{2(3)} \frac{\partial f_k}{\partial v_i} \frac{\partial v_i}{\partial p} = - \sum_{\{q\}} \frac{\partial f_k}{\partial q} \frac{\partial q}{\partial p}$$

  where $\{f_k\} =$ EWSB conditions including 1-loop t, $\tilde{t}$ contributions to V_{eff}, $\{q\}$=running parameters appearing in $\{f_k\}$, evaluated at scale M_S

- Approximate solutions to RGEs (for speed) by ($t = \log(M_S/M_X)$, $\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\beta_p^{(1)}}{(4\pi)^2} + \frac{\beta_p^{(2)}}{(4\pi)^4}$),

$$p(M_S) \approx p(M_X) + \frac{t}{(4\pi)^2} \left(\beta_p^{(1)}(M_X) + \frac{1}{(4\pi)^2} \beta_p^{(2)}(M_X) \right) + \frac{t^2}{2(4\pi)^4} \sum_{\{q\}} \beta_q^{(1)}(M_X) \left. \frac{\partial \beta_p^{(1)}}{\partial q} \right|_{M_X}$$

[3] J. Ellis, K. Enqvist, D. Nanopoulos, F. Zwirner, Modern Physics Letters A **01**, 57 (1986); R. Barbieri and G. Giudice, Nuclear Physics B **306**, 63 (1988)

Parameter Space Scans

MSSM

$$p = \{\mu, B, m_{H_d}^2, m_{H_u}^2, m_{Q_3}^2, m_{u_3}^2, A_t, M_1, M_2, M_3\}$$

$2 \leq \tan\beta \leq 50$
-1 TeV $\leq \mu \leq 1$ TeV
-1 TeV $\leq B \leq 1$ TeV
200 GeV $\leq m_{Q_3} \leq 2000$ GeV
200 GeV $\leq m_{u_3} \leq 2000$ GeV
-10 TeV $\leq A_t \leq 10$ TeV
$M_2 = 100$ GeV, 1050 GeV, 2000 GeV

$m_{H_d}^2$, $m_{H_u}^2$ outputs of EWSB conditions

E_6SSM

$$p = \{\lambda, A_\lambda, m_{H_d}^2, m_{H_u}^2, m_S^2, m_{Q_3}^2, m_{u_3}^2, A_t, M_1, M_2, M_3, M_1'\}$$

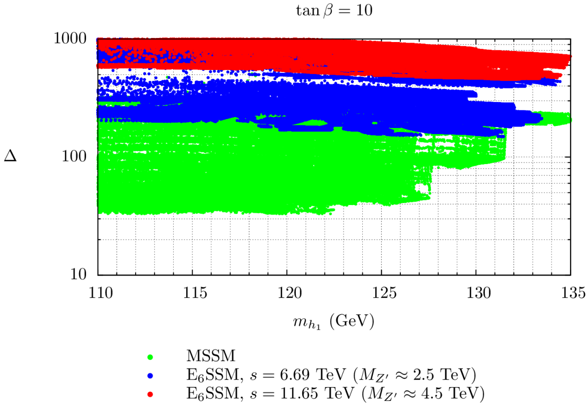
$2 \leq \tan\beta \leq 50$
$-3 \leq \lambda \leq 3$
-10 TeV $\leq A_\lambda \leq 10$ TeV
200 GeV $\leq m_{Q_3} \leq 2000$ GeV
200 GeV $\leq m_{u_3} \leq 2000$ GeV
-10 TeV $\leq A_t \leq 10$ TeV
$M_2 = 100$ GeV, 1050 GeV, 2000 GeV

$m_{H_d}^2$, $m_{H_u}^2$, m_S^2 outputs of EWSB conditions

- Linear scan in μ, λ, B, $m_{Q_3}^2$, $m_{u_3}^2$
- Log scan in A_λ and A_t
- Set singlet VEV $s = 6.69$ TeV ($M_{Z'} \approx 2.5$ TeV) and $s = 11.65$ TeV ($M_{Z'} \approx 4.5$ TeV)
- Use Softsusy for the MSSM and FlexibleSUSY (see talk by A. Voigt) for the E_6SSM to obtain spectrum

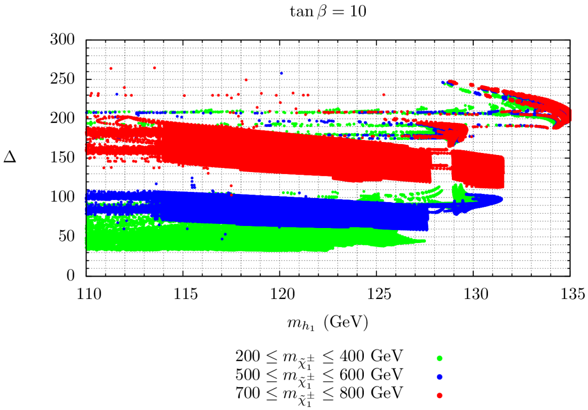
Overall Fine Tuning

- For $m_{h_1} \approx 125$ GeV, $\tan\beta = 10$, minimum fine tuning in the MSSM is ≈ 40
- Tuning increases in the MSSM for larger m_{h_1}, but is stable in the E_6SSM
- E_6SSM tuning is much larger, with minimum tuning ≈ 140 for $M_{Z'} \approx 2.5$ TeV
- Minimum tuning increases with $M_{Z'}$, i.e. $M_{Z'}$ sets a lower bound on the tuning
    - even for $M_{Z'} \approx 2.5$ TeV this is the dominant tuning contribution
- For $M_{Z'} \approx 4.5$ TeV, minimum tuning is ≈ 350 $\Rightarrow Z'$ searches in LHC Run II may significantly increase the current lower bound



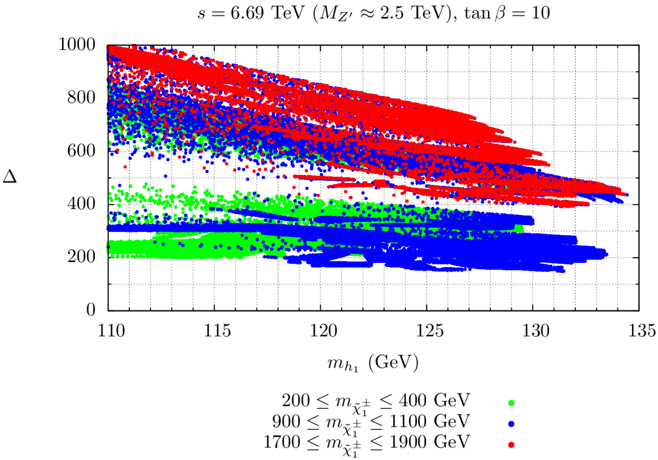
Impact of Chargino Bounds

- MSSM: lower bounds on $m_{\tilde{\chi}^{\pm}}$ significantly increase fine tuning
- As $m_{\tilde{\chi}^{\pm}}$ increases, lowest allowed value of μ increase
  $\Rightarrow$ increased tree level tuning contribution
- Increased M_2 has much smaller impact on tuning (appears solely through RG effects)
- For $m_{\tilde{\chi}^{\pm}} > 700$ GeV, $\tan\beta = 10$, MSSM tuning approaches minimum tuning of E_6SSM



Impact of Chargino Bounds

- E_6SSM: chargino bounds at first have a less marked impact
- For lower $m_{\tilde{\chi}^{\pm}}$, tuning is dominated by $M_{Z'}$ contribution
  $\Rightarrow$ increasing $m_{\tilde{\chi}^{\pm}}$ does not increase minimum tuning
- For large enough $m_{\tilde{\chi}^{\pm}}$, increased lower bound on λ ($\equiv \mu_{\text{eff}}$)
  $\Rightarrow$ $M_{Z'}$ contribution is no longer dominant effect
  $\Rightarrow$ minimum tuning is increased for $M_{Z'} \approx 2.5$ TeV



Summary

- $U(1)$ extended SUSY models offer a possible solution to the Little Hierarchy Problem
- But the presence of a massive Z' $\Rightarrow$ a new source of fine tuning
- We have compared the fine tuning in low energy constructions of the MSSM and E_6SSM
- Tuning in the E_6SSM has a lower bound imposed by $M_{Z'}$ limits
- Current $M_{Z'}$ limit ≈ 2.5 TeV
  $\Rightarrow$ tuning in the E_6SSM is already dominated by $M_{Z'}$ bound
  $\Rightarrow$ is already larger than in the MSSM
- Contrasts with the cE_6SSM, in which tuning is less than in the cMSSM
- Increasing $M_{Z'}$ to ≈ 4.5 TeV significantly increases minimum tuning
  $\Rightarrow$ future Z' limits will be important in determining tuning in the E_6SSM
- Coleman-Weinberg tuning contributions involving $m_{\tilde{t}}$ are not as significant in these models
- Chargino mass limits are important, especially in the MSSM
  - requiring $m_{\tilde{\chi}^{\pm}} > 700$ GeV $\Rightarrow$ level of tuning in MSSM approaches that in the E_6SSM

Back-up slides

MSSM Higgs Effective Potential

At leading 1-loop order, the effective Higgs potential is given by

$$V_{\text{eff}} = V_F + V_D + V_{\text{soft}} + \Delta V$$

where

$$V_F = F_I^* F_I = |\mu|^2(|H_1|^2 + |H_2|^2)$$

$$V_D = \frac{1}{2} D_i^a D_i^a = \frac{\bar{g}^2}{8}\left(|H_2|^2 - |H_1|^2\right)^2 + \frac{g_2^2}{2}|H_1^\dagger H_2|^2$$

$$V_{\text{soft}} = m_{H_d}^2 |H_1|^2 + m_{H_u}^2 |H_2|^2 + (B\mu H_1 \cdot H_2 + \text{h.c})$$

$$\Delta V = \frac{3}{32\pi^2}\left[m_{\tilde{t}_1}^4\left(\log\frac{m_{\tilde{t}_1}^2}{Q^2} - \frac{3}{2}\right) + m_{\tilde{t}_2}^4\left(\log\frac{m_{\tilde{t}_2}^2}{Q^2} - \frac{3}{2}\right) - 2m_t^4\left(\log\frac{m_t^2}{Q^2} - \frac{3}{2}\right)\right] + \ldots$$

E_6SSM Higgs Effective Potential

At leading 1-loop order, the effective Higgs potential is given by

$$V_{\mathrm{eff}} = V_F + V_D + V_{\mathrm{soft}} + \Delta V$$

where

$$V_F = \lambda^2 |S|^2 (|H_1|^2 + |H_2|^2) + \lambda^2 |(H_1 \cdot H_2)|^2$$

$$V_D = \frac{\bar{g}^2}{8} \left(|H_2|^2 - |H_1|^2\right)^2 + \frac{g_2^2}{2} |H_1^\dagger H_2|^2 + \frac{g_1'^2}{2} (\tilde{Q}_1 |H_1|^2 + \tilde{Q}_2 |H_2|^2 + \tilde{Q}_S |S|^2)^2$$

$$V_{\mathrm{soft}} = m_S^2 |S|^2 + m_{H_d}^2 |H_1|^2 + m_{H_u}^2 |H_2|^2 + \left[\lambda A_\lambda S (H_1 \cdot H_2) + \mathrm{h.c.}\right]$$

$$\begin{aligned}\Delta V = \frac{3}{32\pi^2} \Bigg[& m_{\tilde{t}_1}^4 \left(\log \frac{m_{\tilde{t}_1}^2}{Q^2} - \frac{3}{2} \right) + m_{\tilde{t}_2}^4 \left(\log \frac{m_{\tilde{t}_2}^2}{Q^2} - \frac{3}{2} \right) - 2 m_t^4 \left(\log \frac{m_t^2}{Q^2} - \frac{3}{2} \right) \\ & + \sum_{i=1}^{3} \left\{ m_{\tilde{D}_{1,i}}^4 \left(\log \frac{m_{\tilde{D}_{1,i}}^2}{Q^2} - \frac{3}{2} \right) + m_{\tilde{D}_{2,i}}^4 \left(\log \frac{m_{\tilde{D}_{2,i}}^2}{Q^2} - \frac{3}{2} \right) - 2 \mu_{D_i}^4 \left(\log \frac{\mu_{D_i}^2}{Q^2} - \frac{3}{2} \right) \right\} \Bigg] + \ldots\end{aligned}$$

The $\tilde{Q}_i$ are effective $U(1)_N$ charges arising due to $U(1)$ mixing effects, given by $\tilde{Q}_i = Q_i^N + Q_i^Y (g_{11}/g_1')$.

MSSM EWSB Conditions

We require that at the minimum of the Higgs effective potential, the fields H_1^0 and H_2^0 acquire non-zero VEVs of the form

$$\langle H_1^0 \rangle = \frac{v_1}{\sqrt{2}}, \qquad \langle H_2^0 \rangle = \frac{v_2}{\sqrt{2}},$$

leading to the two EWSB conditions ($\tan\beta = v_2/v_1$, $v^2 = v_1^2 + v_2^2$)

$$f_1 = |\mu|^2 + m_{H_d}^2 - B\mu \tan\beta + \frac{\bar{g}^2 v^2}{8}\cos 2\beta + \frac{1}{v_1}\frac{\partial \Delta V}{\partial v_1} = 0,$$

$$f_2 = |\mu|^2 + m_{H_u}^2 - B\mu \cot\beta - \frac{\bar{g}^2 v^2}{8}\cos 2\beta + \frac{1}{v_2}\frac{\partial \Delta V}{\partial v_2} = 0$$

These may be written in the form ($\tilde{m}_1^2 = m_{H_d}^2 + \frac{1}{v_1}\frac{\partial \Delta V}{\partial v_1}$, $\tilde{m}_2^2 = m_{H_u}^2 + \frac{1}{v_2}\frac{\partial \Delta V}{\partial v_2}$)

$$\frac{M_Z^2}{2} = -|\mu|^2 + \frac{\tilde{m}_1^2 - \tilde{m}_2^2 \tan^2\beta}{\tan^2\beta - 1},$$

$$\sin 2\beta = \frac{2B\mu}{2|\mu|^2 + \tilde{m}_1^2 + \tilde{m}_2^2},$$

E_6SSM EWSB Conditions

We require that at the minimum of the Higgs effective potential, the fields $H_d^0 \equiv H_{13}^0$, $H_u^0 \equiv H_{23}^0$ and $S \equiv S_3$ acquire non-zero VEVs of the form

$$\langle H_d^0 \rangle = \frac{v_1}{\sqrt{2}}, \qquad \langle H_u^0 \rangle = \frac{v_2}{\sqrt{2}}, \qquad \langle S \rangle = \frac{s}{\sqrt{2}},$$

leading to the three EWSB conditions

$$f_1 = m_{H_d}^2 + \frac{\lambda^2}{2}(v_2^2 + s^2) - \frac{\lambda A_\lambda}{\sqrt{2}} s \tan\beta + \frac{\bar{g}^2 v^2}{8} \cos 2\beta + \frac{g_1'^2}{2} \tilde{Q}_1 (\tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_S s^2) + \frac{1}{v_1} \frac{\partial \Delta V}{\partial v_1} = 0,$$

$$f_2 = m_{H_u}^2 + \frac{\lambda^2}{2}(v_1^2 + s^2) - \frac{\lambda A_\lambda}{\sqrt{2}} s \cot\beta - \frac{\bar{g}^2 v^2}{8} \cos 2\beta + \frac{g_1'^2}{2} \tilde{Q}_2 (\tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_S s^2) + \frac{1}{v_2} \frac{\partial \Delta V}{\partial v_2} = 0,$$

$$f_3 = m_S^2 + \frac{\lambda^2 v^2}{2} - \frac{\lambda A_\lambda}{\sqrt{2}} v \tan\varphi + \frac{g_1'^2}{2} \tilde{Q}_S (\tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_S s^2) + \frac{1}{s} \frac{\partial \Delta V}{\partial s} = 0, \quad \tan\varphi = \frac{v}{2s} \sin 2\beta.$$

E_6SSM EWSB Conditions

By taking linear combinations of f_1 and f_2, two of the EWSB conditions may be expressed as

$$c\frac{M_Z^2}{2} = -\frac{1}{2}\lambda^2 s^2 + \frac{\tilde{m}_1^2 - \tilde{m}_2^2 \tan^2\beta}{\tan^2\beta - 1} + \frac{g_1'^2}{2}\tilde{Q}_S s^2 \frac{(\tilde{Q}_1 - \tilde{Q}_2 \tan^2\beta)}{\tan^2\beta - 1},$$

$$\sin 2\beta = \frac{\sqrt{2}\lambda A_\lambda s}{\tilde{m}_2^2 + \tilde{m}_1^2 + \frac{\lambda^2}{2}(\frac{4M_Z^2}{\bar{g}^2} + 2s^2) + \frac{\tilde{Q}_1 + \tilde{Q}_2}{2} g_1'^2 (\frac{4M_Z^2}{\bar{g}^2}(\tilde{Q}_1 \cos^2\beta + \tilde{Q}_2 \sin^2\beta) + \tilde{Q}_S s^2)},$$

$$c = 1 - \frac{4}{(\tan^2\beta - 1)}\frac{g_1'^2}{\bar{g}^2}(\tilde{Q}_1 - \tilde{Q}_2 \tan^2\beta)(\tilde{Q}_1 \cos^2\beta + \tilde{Q}_2 \sin^2\beta)$$

It should be noted that, even at tree level, this does not allow to express M_Z and $\sin 2\beta$ solely in terms of the Lagrangian parameters, as occurs in the MSSM at tree level. For $M_{Z'} \approx g_1' \tilde{Q}_S s \gg M_Z$, the first of these reduces to the expression given previously.

Approximate RG Equation Solutions

For small values of $t \equiv \log(M_S/M_X)$, the solutions to the RGEs may be approximated by

$$p(M_S) \approx p(M_X) + \frac{t}{(4\pi)^2}\left(\beta_p^{(1)}(M_X) + \frac{1}{(4\pi)^2}\beta_p^{(2)}(M_X)\right) + \frac{t^2}{2(4\pi)^4}\sum_{\{q\}}\beta_q^{(1)}(M_X)\left.\frac{\partial\beta_p^{(1)}}{\partial q}\right|_{M_X}$$

For example, in the MSSM one may approximate $m_{H_u}^2(M_S)$ by

$$m_{H_u}^2(M_S) \approx m_{H_u}^2(M_X) + \frac{t}{(4\pi)^2}\left(\beta_{m_{H_u}^2}^{(1)}(M_X) + \frac{1}{(4\pi)^2}\beta_{m_{H_u}^2}^{(2)}(M_X)\right) + \frac{t^2}{(4\pi)^4}b_{m_{H_u}^2}^{(2)}(M_X)$$

Approximate RG Equation Solutions

where $\beta^{(1)}_{m^2_{H_u}}$ and $\beta^{(2)}_{m^2_{H_u}}$ are the 1- and 2-loop contributions to the β function, and

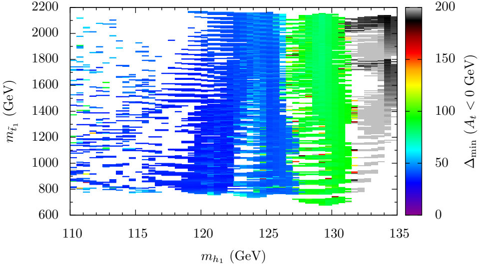
$$\begin{aligned} b^{(2)}_{m^2_{H_u}} &= 72y_t^4\left(m^2_{H_u} + m^2_{Q3} + m^2_{u3} + 2A_t^2\right) \\ &\quad + 6y_t^2y_b^2\left(m^2_{H_u} + m^2_{H_d} + 2m^2_{Q3} + m^2_{u3} + m^2_{d3} + (A_t + A_b)^2\right) \\ &\quad - 32g_3^2y_t^2\left(m^2_{H_u} + m^2_{Q3} + m^2_{u3} + A_t^2 - 2A_tM_3 + 2M_3^2\right) \\ &\quad - 18g_2^2y_t^2\left(m^2_{H_u} + m^2_{Q3} + m^2_{u3} + A_t^2 - 2A_tM_2 + 2M_2^2\right) \\ &\quad - \frac{26}{5}g_1^2y_t^2\left(m^2_{H_u} + m^2_{Q3} + m^2_{u3} + A_t^2 - 2A_tM_1 + 2M_1^2\right) \\ &\quad + \frac{198}{25}g_1^4\left(\mathcal{S} - 3M_1^2\right) - 18g_2^4M_2^2 \end{aligned}$$

is found by differentiating the 1-loop β function contribution, with

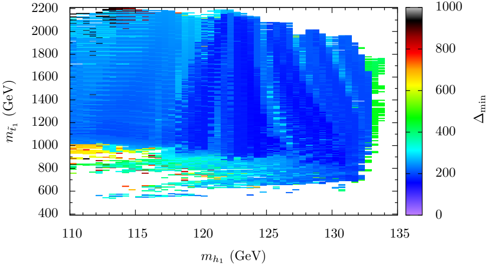
$$\mathcal{S} = m^2_{H_u} - m^2_{H_d} + \sum_{i=1}^{3}\left(m^2_{Qi} - m^2_{Li} - 2m^2_{ui} + m^2_{di} + m^2_{ei}\right)$$

Coleman-Weinberg Contributions

MSSM



E$_6$SSM



However, these are preliminary results and require further investigation.

Impact of Chargino Bounds