Karlsruher Institut für Technologie

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The $\mathcal{O}(\alpha_s\alpha_t)$ Corrections to the Higgs Masses in the Complex NMSSM

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Outline

1. Motivation
2. Calculation framework
3. Numerical analysis
4. Conclusions

Why the complex NMSSM?

1. Solve μ problem

$$\mu \hat{H}_u \hat{H}_d \to \lambda \hat{S} \hat{H}_u \hat{H}_d$$

   when $\langle S \rangle = \frac{v_S}{\sqrt{2}}$ then $\mu = \frac{\lambda v_S}{\sqrt{2}}$ about EW scale.

2. Higgs sector: 2 Higgs doublets + 1 Singlet $\rightsquigarrow$ 5 neutral Higgs bosons + $H^\pm$

$$\begin{aligned} V_H = & (|\lambda S|^2 + m_{H_d}^2) H_{d,i}^* H_{d,i} + (|\lambda S|^2 + m_{H_u}^2) H_{u,i}^* H_{u,i} \\ & + \frac{1}{8}(g_2^2 + g_1^2)(H_{d,i}^* H_{d,i} - H_{u,i}^* H_{u,i})^2 + \frac{1}{2} g_2^2 |H_{d,i}^* H_{u,i}|^2 \\ & + m_S^2 |S|^2 + | - \epsilon^{ij} \lambda H_{d,i} H_{u,j} + \kappa S^2 |^2 + \left[- \epsilon^{ij} \lambda A_\lambda S H_{d,i} H_{u,j} + \frac{1}{3} \kappa A_\kappa S^3 + \text{H.c} \right], \end{aligned}$$

   $\lambda, \kappa, A_\lambda, A_\kappa$ are in general complex. CP-odd and CP-even Higgs bosons can already mix at tree-level.

3. Provide additional CP violation which can be useful for electroweak baryogenesis.

4. Tree-level light Higgs boson: $m_h^2 < M_Z^2 \cos^2(2\beta) + \lambda^2 v^2 \sin^2(2\beta) \leftarrow \lambda S H_u H_d$

   less fine tune, large λ and moderate $\tan\beta$ are favoured

Why $\mathcal{O}(\alpha_s\alpha_t)$ correction to Higgs masses?

1. The Higgs mass has been measured with good accuracy:
   $125.36 \pm 0.38(\text{stat.}) \pm 0.17(\text{sys.})$ GeV at ATLAS,
   $125.03^{+0.26}_{-0.27}(\text{stat.})^{+0.13}_{-0.15}(\text{syst.})$ GeV at CMS.
2. The $\mathcal{O}(\alpha_s\alpha_t)$ correction to the NMSSM Higgs masses is important to reduce large theoretical uncertainty and improve theoretical prediction.
   - One-loop: $\delta m_h \rightsquigarrow 50\%$ with uncertainty about 10% comming from undefined top mass.
   - Expected $\mathcal{O}(\alpha_s\alpha_t)$ correction contribute about 10% and reduce the uncertainty to about 3%.
3. Status of Higgs masses calculation in the NMSSM:
   - Full one-loop masses in the complex NMSSM, full momentum dependence, mixed renormalization scheme, using Feynman diagram approach, [Graf, Grober, Mühlleitner, Rzehak, Walz],
     in public code NMSSMCALC [Baglio, Gröber, Mühlleitner, DTN, Rzehak, Spira, Streicher and Walz]
   - Full one-loop masses in the real NMSSM (full momentum dependence) + $\mathcal{O}(\alpha_s\alpha_t + \alpha_s\alpha_b)$ correction ($\overline{\text{DR}}$ scheme, Effective potential approach), [Degrassi, P. Slavich]
     in public codes: NMSSMTOOLS [Ellwanger, Hugonie], SOFTSUSY [Allanach et. al.] and SPHENO [Porod, Staub]

The tree-level Higgs masses

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1. Higgs fields are expanded around their VEVs.

$$H_d = \begin{pmatrix} \frac{(v_d + h_d + i a_d)}{\sqrt{2}} \\ h_d^- \end{pmatrix}, \quad H_u = e^{i\phi_u} \begin{pmatrix} h_u^+ \\ \frac{(v_u + h_u + i a_u)}{\sqrt{2}} \end{pmatrix}, \quad H_s = \frac{e^{i\phi_s}(v_s + h_s + i a_s)}{\sqrt{2}}$$

2. The 6×6 Higgs masses matrix: $h = (h_d, h_u, h_s)^T$, $a = (a_d, a_u, a_s)^T$

$$M_{\Phi\Phi} = \begin{pmatrix} M_{hh} & M_{ha} \\ M_{ha} & M_{aa} \end{pmatrix}$$

$$M_{ha} = \begin{pmatrix} 0 & 0 & 3v_s \sin\beta \\ 0 & 0 & 3v_s \cos\beta \\ -v_s \sin\beta & -v_s \cos\beta & -2v \sin 2\beta \end{pmatrix} \frac{v|\kappa||\lambda| \sin\phi_y}{2}$$

only one CP-violating phase: $\phi_y = \phi_k - \phi_\lambda + 2\phi_s - \phi_u$

3. Tree-level Higgs boson masses determined by

$$\tan\beta, M_{H^\pm}, v_s, |\lambda|, |\kappa|, |A_\kappa|, \phi_y$$

The loop-corrected Higgs masses

- Loop-corrected Higgs mass matrices, Goldstone component has been singled out. h_1, h_2, h_3, h_4, h_5 mass eigenstates at tree level.

$$M^2(p^2) = \begin{pmatrix} m^2_{h_1} - \hat{\Sigma}_{h_1 h_1} & -\hat{\Sigma}_{h_1 h_2} & -\hat{\Sigma}_{h_1 h_3} & -\hat{\Sigma}_{h_1 h_4} & -\hat{\Sigma}_{h_1 h_5} \\ -\hat{\Sigma}_{h_2 h_1} & m^2_{h_2} - \hat{\Sigma}_{h_2 h_2} & -\hat{\Sigma}_{h_2 h_3} & -\hat{\Sigma}_{h_2 h_5} & -\hat{\Sigma}_{h_2 h_5} \\ -\hat{\Sigma}_{h_3 h_1} & -\hat{\Sigma}_{h_3 h_2} & m^2_{h_3} - \hat{\Sigma}_{h_3 h_3} & -\hat{\Sigma}_{h_3 h_4} & -\hat{\Sigma}_{h_3 h_5} \\ -\hat{\Sigma}_{h_1 h_4} & -\hat{\Sigma}_{h_2 h_4} & -\hat{\Sigma}_{h_3 a} & m^2_{h_4} - \hat{\Sigma}_{h_4 h_4} & -\hat{\Sigma}_{h_4 h_5} \\ -\hat{\Sigma}_{h_1 h_5} & -\hat{\Sigma}_{h_2 h_5} & -\hat{\Sigma}_{h_3 h_5} & -\hat{\Sigma}_{h_4 h_5} & m^2_{h_5} - \hat{\Sigma}_{h_5 h_5} \end{pmatrix},$$

  - $\hat{\Sigma}_{h_i h_j}(p^2)$ is renormalized self-energy of $h_i \to h_j$ transition

$$\hat{\Sigma}_{h_i h_j}(p^2) = \hat{\Sigma}^{\alpha}_{h_i h_j}(p^2) + \hat{\Sigma}^{\alpha_s \alpha_t}_{h_i h_j}(0)$$

  - Mixings with G,Z are negligible and are not taken into account
- Loop corrected Higgs boson masses are obtained by diagonalizing mass matrix iteratively

$$M^2_{H_i} = M^2_{h_i}(\text{tree}) + \Delta M^2_{H_i}(\text{loop}), \quad i = 1, 5,$$

$$M_{H_1} < M_{H_2} < M_{H_3} < M_{H_4} < M_{H_5}$$

- Loop corrected Higgs boson mixing: $H_i = Z^S_{ik} h_k$
  - Z^S is not unitary if $p^2 \neq 0$, wave function renormalization factor
  - $p^2 = 0$, Z^S is orthogonal and equal to the rotation matrix

One-loop Higgs boson self-energies: $\hat{\Sigma}^{\alpha}_{h_i h_j}(p^2)$

- One-loop corrected Higgs masses: $\mathcal{O}(\alpha)$ including full momentum dependence in our code. See Graf, Grober, Mühlleitner, Rzehak, Walz [hep-ph/1206.6806, hep-ph/1111.4952]
- Two renormalization schemes are possible:

$$\underbrace{e, M_Z, M_W, M_{H^\pm}, t_{h_u}, t_{h_d}, t_{h_s}, t_{a_d}, t_{a_s}}_{\text{on-shell scheme}}, \underbrace{\tan\beta, v_s, \phi_u, \phi_s, \phi_\lambda, \phi_\kappa, |\lambda|, |\kappa|, |A_\kappa|}_{\overline{\text{DR}}\text{ scheme}}.$$

OR

$$\underbrace{e, M_Z, M_W, t_{h_u}, t_{h_d}, t_{h_s}, t_{a_d}, t_{a_s}}_{\text{on-shell scheme}}, \underbrace{\tan\beta, v_s, \phi_u, \phi_s, \phi_\lambda, \phi_\kappa, |\lambda|, |\kappa|, |A_\kappa|, |A_\lambda|}_{\overline{\text{DR}}\text{ scheme}}.$$

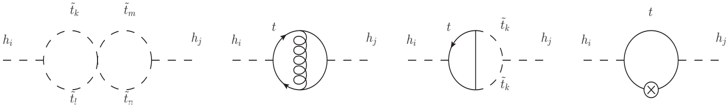
Higgs fields are renormalized in $\overline{\text{DR}}$ scheme

The two-loop correction: $\hat{\Sigma}^{\alpha_s\alpha_t}_{h_ih_j}(0)$

Using Feynman Diagram Approach, gauge less limit $e \to 0$ but $M_W/M_Z \neq 0$

$$\hat{\Sigma}^{\alpha_s\alpha_t}_{h_ih_j}(0) = \Sigma^{\alpha_s\alpha_t}_{h_ih_j}(0) - \frac{m_i^2}{2} dZ_{h_ih_j} - \frac{m_j^2}{2} dZ_{h_jh_i} - dM^2_{h_ih_j}$$

- The phases involved in the $\mathcal{O}(\alpha_s\alpha_t)$ correction are: $\phi_u, \overbrace{\phi_s, \phi_\lambda}^{\phi_{\mu_{\text{eff}}}}, \phi_{M_3}, \phi_{A_t}$
- The unrenormalized selfenergies: $\Sigma^{\alpha_s\alpha_t}_{h_ih_j}(0)$



- The wave function renormalization: $dZ_{h_ih_j}$

  Using $\overline{\text{DR}}$, necessary for UV-finiteness. This contribution is not present in MSSM ($p^2 = 0$).

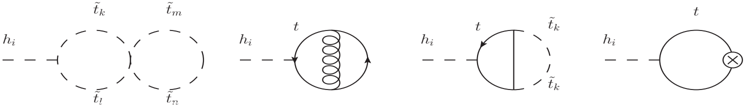
  $dZ_{h_uh_u} \propto \alpha_s\alpha_t$; $d\tan\beta = \frac{1}{2} dZ_{h_uh_u}$

The two-loop correction: $\hat{\Sigma}^{\alpha_s\alpha_t}_{h_ih_j}(0)$

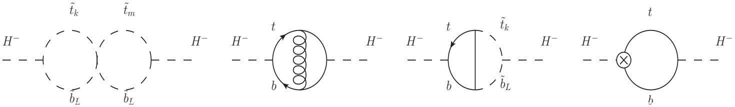
- The Two-loop Higgs mass countertem matrix $dM^2_{h_ih_j}$. Parameters need to be renormalized.

$$\underbrace{v, M_{H^\pm}, t_{h_u}, t_{h_d}, t_{h_s}, t_{a_d}, t_{a_s}}_{\text{on-shell scheme}}, \underbrace{\tan\beta, |\lambda|}_{\overline{\text{DR}}\text{ scheme}} .$$

  - Tadpole contributions: δt_{h_i}

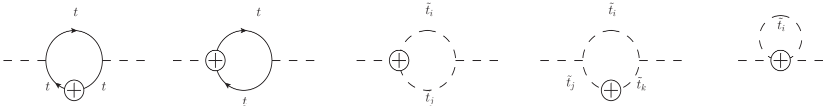


  - Charged Higgs selfenergy: $\delta M_{H^\pm} = \Sigma_{H^+H^+}(0) - \frac{M^2_{H^\pm}}{2}\cos\beta^2 dZ_{h_uh_u}$, $m_b = 0$



  - W, Z boson selfenergies: contribute to $\frac{\delta v}{v} = \frac{c_W^2}{2s_W^2}\left(\frac{\Sigma^T_{ZZ}(0)}{M_Z^2} - \frac{\Sigma^T_{WW}(0)}{M_W^2}\right) + \frac{1}{2}\frac{\Sigma^T_{WW}(0)}{M_W^2}$
    This contribution is not present in the MSSM

Renormalization of top/stop sector with complex parameters



- Parameters need to be renormalized: $m_t, M_{\tilde{t}_1}, M_{\tilde{t}_2}, A_t$.
- Two renormalization schemes are possible: On-shell or $\overline{\text{DR}}$

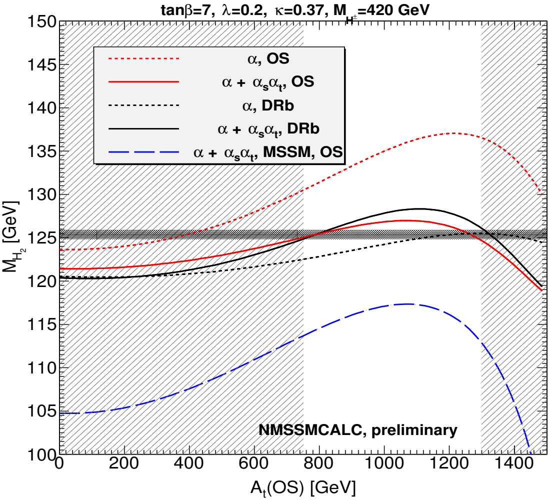
$$
\begin{aligned}
\delta M_t^{\text{OS}} &= \frac{1}{2}\widetilde{\text{Re}}\left\{M_t\left[\Sigma_{t,L}(M_t^2)+\Sigma_{t,R}(M_t^2)\right]+\Sigma_{t,l}(M_t^2)+\Sigma_{t,r}(M_t^2)\right\},\\
\delta M_{\tilde{t}_1}^{\text{OS}} &= \Sigma_{\tilde{t}_1\tilde{t}_1}(M_{\tilde{t}_1}), \qquad \delta M_{\tilde{t}_2}^{\text{OS}} = \Sigma_{\tilde{t}_2\tilde{t}_2}(M_{\tilde{t}_2}),\\
\delta A_T^{\text{OS}} &= \frac{e^{-i\phi_u}}{M_t}(-X_T\delta M_t+\delta M_{\tilde{t}_1}U_{12}^{\tilde{t}*}U_{11}^{\tilde{t}}+\delta M_{\tilde{t}_2}U_{22}^{\tilde{t}*}U_{21}^{\tilde{t}}\\
&+ \Sigma_{\tilde{t}_1\tilde{t}_2}(M_{\tilde{t}_1})U_{22}^{\tilde{t}*}U_{11}^{\tilde{t}}+\Sigma_{\tilde{t}_2\tilde{t}_1}(M_{\tilde{t}_2})U_{12}^{\tilde{t}*}U_{21}^{\tilde{t}}\\
X_T &= A_t e^{i\phi_u}-\frac{\lambda^* v_S e^{-i\phi_s}}{\sqrt{2}\tan\beta}
\end{aligned}
$$

For the $\overline{\text{DR}}$ scheme: $\delta X^{\overline{\text{DR}}} = \delta X^{\text{OS}}\big|_{\text{div}}$

Tools and checking

- FeynArts-3.6, FeynCalc-8.1 and Tarcer for two-loop tensor reduction. Using Dimensional reduction [Siegel, 1979]
  SUSY preserved for $\mathcal{O}(\alpha_s \alpha_t)$ Higgs masses [Hollik, Stöckinger, hep-ph/0509298]
- Codes are implemented in Fortan 77.
- Two independent calculations are in perfect agreement.
- Check MSSM components $\hat{\Sigma}_{h_d h_d}, \hat{\Sigma}_{h_d h_u}, \hat{\Sigma}_{h_u h_u}, \hat{\Sigma}_{h_d a}, \hat{\Sigma}_{h_u a}, \hat{\Sigma}_{aa}$ with complex phases. Perfect agreement.
- Check with the Slavich's code for $\alpha_s \alpha_t$ in the real NMSSM, using $\overline{\text{DR}}$ scheme for top/stop sector, A_λ. perfect agreement.

$\overline{\text{DR}}$ vs OS schemes



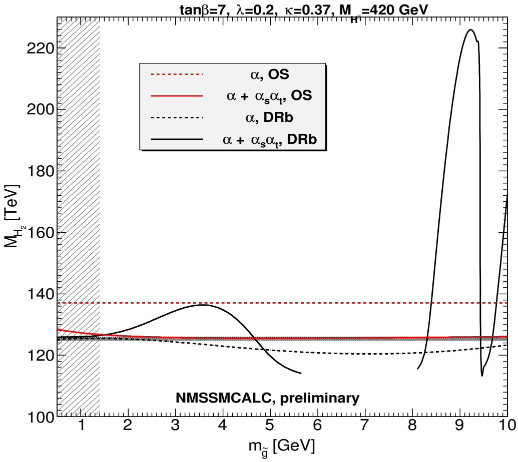
Uncertainty:

$\delta = \frac{|\text{M(OS)} - \text{M}(\overline{\text{DR}})|}{\text{M(OS)}}$

- at $\mathcal{O}(\alpha)$
  $\delta_{\text{max}} \sim 8.5\%$
- at $\mathcal{O}(\alpha + \alpha_s \alpha_t)$
  $\delta_{\text{max}} \sim 3\%$

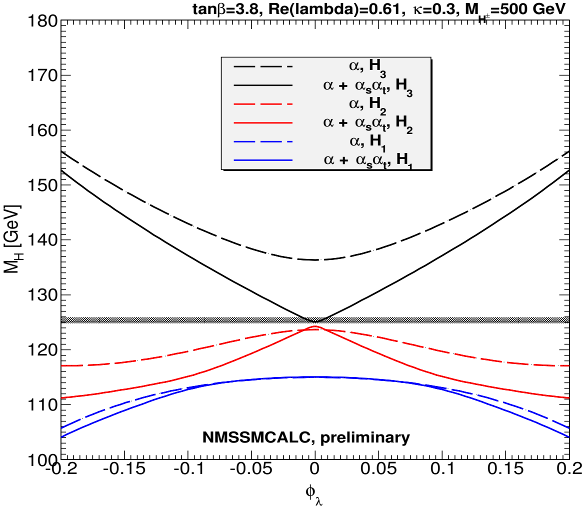
HiggsBounds and HiggsSignals have been used to validate all the points

$\overline{\text{DR}}$ vs OS schemes



- OS-scheme: well behavior at large gluino mass
- $\overline{\text{DR}}$: can be very large [Degrassi, Slavich, Zwirner, 2001]

Two-loop Higgs masses in OS scheme with complex parameters



Conclusions

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- The $\mathcal{O}(\alpha_s\alpha_t)$ contribution to the Higgs masses in the NMSSM with complex parameters has been calculated and studied. It increases precision of the predicted Higgs masses in this model.
- Two renormalization schemes: OS and $\overline{\text{DR}}$ have been used and studied.
- The theoretical uncertainty on the Higgs masses is significantly reduced.
- New version of NMSSMCALC which includes this correction will appear soon, [Baglio, Gröber, Mühlleitner, DTN, Rzehak, Spira, Streicher and Walz]

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THANK YOU FOR YOUR ATTENTION