MANCHESTER 1824

The University of Manchester

Based on PSB Dev, PM, A Pilaftsis & D Teresi (cf. previous talk), arXiv: 1404.1003 [hep-ph]; accepted for publication in Nucl. Phys. B

IP3

Durham University

Flavour Covariance in Semi-Classical and Field-Theoretic Transport Phenomena

Peter Millington
Consortium for Fundamental Physics, University of Manchester
Institute for Particle Physics Phenomenology, Durham University

SUSY 2014, University of Manchester
Monday, 21st July, 2014

Outline

- Introduction
- Flavour-Covariant Quantization
- Generalized Optical Theorem
- Implications for Resonant Leptogenesis
- Conclusions

Introduction

Why flavour?

- Observed mixing in
  - SM quark sector (**C**abibbo-**K**obayashi-**M**askawa)
  - SM neutrino sector (**P**ontecorvo-**M**aki-**N**akagawa-**S**akata)
  - ⇒ global $U(3)_Q x U(3)_L$ structure
- Predicted mixing
  - BSM right-handed (Majorana) neutrino sector
  - Squark and slepton sector

⇒ **complex couplings** permit **CP violating phases**

Introduction

Why transport phenomena?

- Development of the observed structure of the Universe requires processes occurring **out-of-equilibrium**

$$\Gamma(X \to Y) \; < \; H \; = \; \left(\frac{4\pi^3}{45}\right)^{1/2} g_*^{1/2} \frac{T^2}{M_{\mathrm{Pl}}} \qquad t \; = \; \frac{M_{\mathrm{Pl}}}{34T^2}$$

- Core collapse supernovae
  [Zhang and Burrows, arXiv:1310.2164 [hep-ph].]

- QCD phase diagram; jet modification in Quark Gluon Plasma
  [Blaizot, Dominguez, Iancu and Mehtar-Tani, arXiv:1311.5823 [hep-ph].]

- CPT-violation from neutrino propagating in gravitational backgrounds
  [Mavromatos and Sarkar, Eur. Phys. J. C 73, 2359 (2013).]

+

Goal

- Quest for a "first-principles" description of **quantum transport phenomena** that includes
  - non-Markovianity (memory effects)
  - spatial inhomogeneity
  - finite particle widths
  - spin coherence
  - 'interspecial' coherence (not discussed here)
  - **flavour effects: oscillations, mixing** and **coherence** [*see D. Teresi's talk, Flavour Covariant Transport Equations for Resonant Leptogenesis*]

- **Non-equilibrium QFT/Kadanoff-Baym equations** vs. **semi-classical Boltzmann equations** [see e.g. extensive list of references in Dev, Millington, Pilaftsis, Teresi arXiv: 1404.1003 [hep-ph].]

Flavour-Covariant Quantization

Creation and Annihilation Operators

- $\mathcal{N}$**-flavour** model of **Dirac fermions**

$$\psi_k(x) \;=\; \begin{pmatrix} \xi_{\mathfrak{a},\,k}(x) \\ \bar{\eta}^{\dot{\mathfrak{a}}}_{k}(x) \end{pmatrix}$$

(Weyl basis)

Flavour-Covariant Quantization

Creation and Annihilation Operators

- $\mathcal{N}$**-flavour** model of **Dirac fermions**

$$\psi_k(x) \;=\; \begin{pmatrix} \xi_{\mathfrak{a},\,k}(x) \\ \bar{\eta}_k^{\dot{\mathfrak{a}}}(x) \end{pmatrix}$$ **(Weyl basis)**

$$\psi_k(x) \;=\; \sum_s \int_{\mathbf{p}} [(2E(\mathbf{p}))^{-1}]_k{}^l \Big([e^{-ip\cdot x}]_l{}^m [u(\mathbf{p},s)]_m{}^n b_n(\mathbf{p},s,0) + [e^{ip\cdot x}]_l{}^m [v(\mathbf{p},s)]_m{}^n d_n^\dagger(\mathbf{p},s,0)\Big)$$ **(Dirac basis)**

$$\{b_k(\mathbf{p},s,\tilde{t}),\; b^l(\mathbf{p}',s',\tilde{t})\} \;=\; (2\pi)^3[2E(\mathbf{p})]_k{}^l\delta^{(3)}(\mathbf{p}-\mathbf{p}')\delta_{ss'}$$

Flavour-Covariant Quantization

Spinorial Structure (Helicity Amplitude Formalism)

- **Flavour rank-2 spinors**

  [cf. Bouchiat and Michel, Nucl. Phys. 5, 416 (1958); Michel, Suppl. Nuovo Cim. 14, 95 (1959); see also Pokorski, Gauge field theories, 2° ed., Cambridge University Press (2000) and Haber, in *Proceedings of the 21st SLAC Summer Institute on Particle Physics*, p. 231 (1993).]

$$[u(\mathbf{p},s)]_k{}^l \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} [(E(\mathbf{p})+m)^{1/2}]_k{}^l u_s(\mathbf{s}) \\ s[(E(\mathbf{p})-m)^{1/2}]_k{}^l u_s(\mathbf{s}) \end{pmatrix}$$

+

Flavour-Covariant Quantization

Spinorial Structure (Helicity Amplitude Formalism)

- **Flavour rank-2 spinors**

  [cf. Bouchiat and Michel, Nucl. Phys. 5, 416 (1958); Michel, Suppl. Nuovo Cim. 14, 95 (1959); see also Pokorski, Gauge field theories, 2° ed., Cambridge University Press (2000) and Haber, in *Proceedings of the 21st SLAC Summer Institute on Particle Physics*, p. 231 (1993).]

$$[u(\mathbf{p},s)]_k{}^l \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix}[(E(\mathbf{p})+m)^{1/2}]_k{}^l u_s(\mathbf{s})\\ s[(E(\mathbf{p})-m)^{1/2}]_k{}^l u_s(\mathbf{s})\end{pmatrix}$$

$$s^\mu \;=\; (0,\;\mathbf{s})$$

$$[s'^\mu]_k{}^l \;=\; \Lambda^\mu{}_\nu(\beta_k{}^l)s^\nu \;=\; \Big(|\mathbf{p}|[m^{-1}]_k{}^l,\;[E(\mathbf{p})]_k{}^m[m^{-1}]_m{}^l\frac{\mathbf{p}}{|\mathbf{p}|}\Big)$$

Flavour-Covariant Quantization

Spinorial Structure (Helicity Amplitude Formalism)

- **Flavour rank-2 spinors**
  [cf. Bouchiat and Michel, Nucl. Phys. 5, 416 (1958); Michel, Suppl. Nuovo Cim. 14, 95 (1959); see also Pokorski, Gauge field theories, 2° ed., Cambridge University Press (2000) and Haber, in *Proceedings of the 21st SLAC Summer Institute on Particle Physics*, p. 231 (1993).]

$$[u(\mathbf{p}, s)]_k{}^l \;=\; \frac{1}{\sqrt{2}}\begin{pmatrix} [(E(\mathbf{p})+m)^{1/2}]_k{}^l u_s(\mathbf{s}) \\ s[(E(\mathbf{p})-m)^{1/2}]_k{}^l u_s(\mathbf{s}) \end{pmatrix}$$

$$s^\mu \;=\; (0,\;\mathbf{s})$$

$$[s'^\mu]_k{}^l \;=\; \Lambda^\mu{}_\nu(\beta_k{}^l)s^\nu \;=\; \Big(|\mathbf{p}|[m^{-1}]_k{}^l,\; [E(\mathbf{p})]_k{}^m[m^{-1}]_m{}^l\frac{\mathbf{p}}{|\mathbf{p}|}\Big)$$

- And we cannot resist writing:

$$[E(\mathbf{p})]_k{}^l \;=\; [\gamma(\mathbf{p})]_k{}^m m_m{}^l \qquad [\gamma^2(\mathbf{p})]_k{}^l \;=\; \delta_k{}^l + \mathbf{p}^2[m^{-2}]_k{}^l$$

Flavour-Covariant Quantization

Non-Homogeneous Flavour-Covariant CTP Propagators

- **Schwinger-Keldysh CTP formalism** in a nut shell: replace **vacuum expectation values (VEVs)** with **ensemble expectation values (EEVs)**:
  [see e.g. Schwinger, J. Math. Phys. **2**, 407 (1961); Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964); Calzetta and Hu, Phys. Rev. D 35, 495 (1987); Phys. Rev. D. 37, 2878 (1988); Berges, AIP Conf. Proc. 739 (2005) 3-62; Millington and Pilaftsis, Phys. Rev. **D** 88, 085009 (2013).]

$$\left\langle \psi_k(x;\tilde{t}_i)\overline{\psi}^{\,l}(y;\tilde{t}_i)\right\rangle_t \;=\; \mathrm{Tr}\Big[\rho(\tilde{t};\tilde{t}_i)\psi_k(x;\tilde{t}_i)\overline{\psi}^{\,l}(y;\tilde{t}_i)\Big]$$

$$t \;=\; \tilde{t} - \tilde{t}_i$$

$$\langle b^l(\mathbf{p}',s',\tilde{t})b_k(\mathbf{p},s,\tilde{t})\rangle_t \;=\; [(2E)^{1/2}]_k{}^m[f_{ss'}(\mathbf{p},\mathbf{p}',t)]_m{}^n[(2E')^{1/2}]^l{}_n$$

+

Flavour-Covariant Quantization

Non-Homogeneous Flavour-Covariant CTP Propagators

$$[iS_{\gtrless}(p,p',\tilde{t})]_k{}^l \;=\; \sum_{s,\,s'} 2\pi|2p_0|^{1/2}[\delta(p^2-m^2)]_k{}^i \times\; 2\pi|2p_0'|^{1/2}[\delta(p'^2\;-\;m^2)]^l{}_m \times\; e^{i(p_0\,-\,p_0')\tilde{t}} \times\; [\mathscr{P}(p,s;p',s')]_i{}^{jm}{}_n \times\; \Big(\theta(\pm p_0)\theta(\pm p_0')(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')\delta_{ss'}\delta_j{}^n \;-\; [\tilde{f}_{ss'}(p,p',t)]_j{}^n\Big)$$

Flavour-Covariant Quantization

Non-Homogeneous Flavour-Covariant CTP Propagators

$$[iS_{\gtrless}(p,p',\tilde{t})]_k^{\ l} \;=\; \sum_{s,\,s'} 2\pi|2p_0|^{1/2}[\delta(p^2-m^2)]_k^{\ i}$$
$$\times\; 2\pi|2p_0'|^{1/2}[\delta(p'^2\;-\;m^2)]^l_{\ m}$$
$$\times\; e^{i(p_0\,-\,p_0')\tilde{t}}$$
$$\times\; [\mathscr{P}(p,s;p',s')]_i^{\ jm}{}_n$$
$$\times\; \Big(\theta(\pm p_0)\theta(\pm p_0')(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')\delta_{ss'}\delta_j^{\ n}$$
$$-\;[\tilde{f}_{ss'}(p,p',t)]_j^{\ n}\Big)$$

Spectral – *time-translational invariance violated*

Flavour-Covariant Quantization

Non-Homogeneous Flavour-Covariant CTP Propagators

$$[iS_{\gtrless}(p,p',\tilde{t})]_k{}^l \; = \; \sum_{s,\,s'} 2\pi|2p_0|^{1/2}[\delta(p^2-m^2)]_k{}^i$$
$$\times\; 2\pi|2p_0'|^{1/2}[\delta(p'^2 \;-\; m^2)]^l{}_m$$

Spectral – ***time-translational invariance violated***

$$\times\; e^{i(p_0\,-\,p_0')\tilde{t}}$$

Free phase – ***controls memory effects***

$$\times\; [\mathscr{P}(p,s;p',s')]_i{}^{jm}{}_n$$
$$\times\; \Big(\theta(\pm p_0)\theta(\pm p_0')(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')\delta_{ss'}\delta_j{}^n$$
$$-\; [\tilde{f}_{ss'}(p,p',t)]_j{}^n\Big)$$

Flavour-Covariant Quantization

Non-Homogeneous Flavour-Covariant CTP Propagators

$$[iS_{\gtrless}(p,p',\tilde{t})]_k{}^l \;=\; \sum_{s,\,s'} 2\pi|2p_0|^{1/2}[\delta(p^2-m^2)]_k{}^i$$

$$\times\; 2\pi|2p_0'|^{1/2}[\delta(p'^2\;-\;m^2)]^l{}_m$$

Spectral – ***time-translational invariance violated***

$$\times\; e^{i(p_0\,-\,p_0')\tilde{t}}$$

Free phase – ***controls memory effects***

$$\times\; [\mathscr{P}(p,s;p',s')]_i{}^{jm}{}_n$$

Spinorial – ***interdependence of spin and flavour***

$$\times\; \Big(\theta(\pm p_0)\theta(\pm p_0')(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')\delta_{ss'}\delta_j{}^n$$

$$-\;[\tilde{f}_{ss'}(p,p',t)]_j{}^n\Big)$$

Flavour-Covariant Quantization

Non-Homogeneous Flavour-Covariant CTP Propagators

$$[iS_{\gtrless}(p,p',\tilde{t})]_k^{\ l} \;=\; \sum_{s,\,s'} 2\pi|2p_0|^{1/2}[\delta(p^2-m^2)]_k^{\ i}$$

$$\times\; 2\pi|2p_0'|^{1/2}[\delta(p'^2\;-\;m^2)]^l_{\ m}$$

Spectral – ***time-translational invariance violated***

$$\times\; e^{i(p_0\,-\,p_0')\tilde{t}}$$

Free phase – ***controls memory effects***

$$\times\; [\mathscr{P}(p,s;p',s')]_i^{\ jm}{}_n$$

Spinorial – ***interdependence of spin and flavour***

$$\times\; \Big(\theta(\pm p_0)\theta(\pm p_0')(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')\delta_{ss'}\delta_j^{\ n}$$

Vacuum contribution – ***as normal***

$$-\;[\tilde{f}_{ss'}(p,p',t)]_j^{\ n}\Big)$$

Flavour-Covariant Quantization

Non-Homogeneous Flavour-Covariant CTP Propagators

$$[iS_{\gtrless}(p,p',\tilde{t})]_k{}^l \;=\; \sum_{s,\,s'} 2\pi|2p_0|^{1/2}[\delta(p^2-m^2)]_k{}^i$$

$$\times\; 2\pi|2p_0'|^{1/2}[\delta(p'^2\;-\;m^2)]^l{}_m$$

Spectral – ***time-translational invariance violated***

$$\times\; e^{i(p_0\,-\,p_0')\tilde{t}}$$

Free phase – ***controls memory effects***

$$\times\; [\mathscr{P}(p,s;p',s')]_i{}^{jm}{}_n$$

Spinorial – ***interdependence of spin and flavour***

$$\times\; \Big(\theta(\pm p_0)\theta(\pm p_0')(2\pi)^3\delta^{(3)}(\mathbf{p}-\mathbf{p}')\delta_{ss'}\delta_j{}^n$$

Vacuum contribution – ***as normal***

$$-\;[\tilde{f}_{ss'}(p,p',t)]_j{}^n\Big)$$

Statistical – ***spin and flavour coherences; three-momentum non-conservation***

Flavour-Covariant Quantization

Resummation and Quasi-Particle Approximation

- **Violation of time-translation invariance** controlled by the phase:

$$(p_0 - p_0')t \approx \frac{m_k^2 - m_l^2}{2|\mathbf{p}|}$$

- **Infeasibility** of **exact inversion of the Schwinger-Dyson equation**:
  - violation of time-translational invariance means infinite nesting of convolution integrals over zeroth-component momenta.
  - energy non-conservation at interaction vertices due to finite time effects.
- How are we to **extract number densities** from **resummed propagators**?

Generalized Optical Theorem

Schematics

- **Optical theorem**:

$$S^\dagger S = S S^\dagger = \mathbb{I} \qquad S = \mathbb{I} + i\mathcal{T} \qquad 2\,\mathrm{Im}\,\mathcal{T} = \mathcal{T}^\dagger \mathcal{T}$$

- Completeness of the **Fock space** is a **flavour, spin** and **isospin singlet** and a **Lorentz scalar**.

  ⇒ From where does all the non-trivial structure arise?

Generalized Optical Theorem

Schematics

- **Optical theorem**:

$$S^\dagger S = S S^\dagger = \mathbb{I} \qquad S = \mathbb{I} + i\mathcal{T} \qquad 2\,\mathrm{Im}\,\mathcal{T} = \mathcal{T}^\dagger \mathcal{T}$$

- Completeness of the **Fock space** is a **flavour, spin** and **isospin singlet** and a **Lorentz scalar**.
  - ⇒ From where does all the non-trivial structure arise?

$$\langle 2\,\mathrm{Im}\,\mathcal{T}\rangle = \langle \mathcal{T}^\dagger \mathcal{T}\rangle = \mathrm{Tr}\,\mathcal{T}\rho\,\mathcal{T}^\dagger$$

$$\langle 2\,\mathrm{Im}\,\mathcal{T}\rangle = \langle \mathcal{T}^\dagger \mathcal{T}\rangle = \sum_{A,B,C} \langle A\,|\mathcal{T}|\,B\rangle\,\langle B\,|\rho|\,C\rangle\,\langle C\,|\mathcal{T}^\dagger|\,A\rangle$$

Generalized Optical Theorem

Example: Heavy-Neutrino Production Rate

$$-\mathcal{L}_N \;=\; h_l{}^{\alpha}\overline{L}^{l}\tilde{\Phi}N_{\mathrm{R},\,\alpha} \;+\; \mathrm{H.c.}$$

- Choose from the inserted completeness relations the term containing **an ingoing charged-lepton, Higgs state** and **an outgoing heavy Neutrino state**:

+

Generalized Optical Theorem

Example: Heavy-Neutrino Production Rate

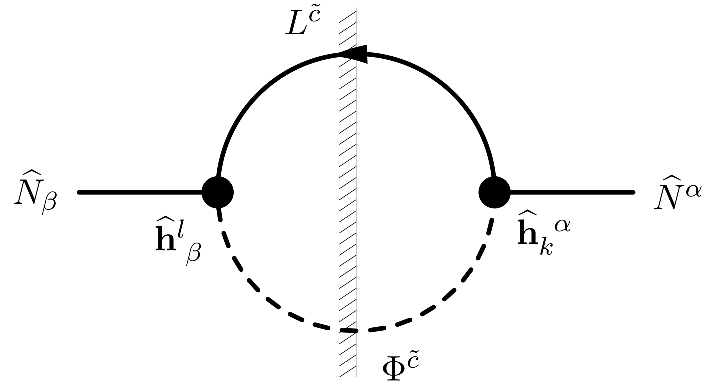
$$-\mathcal{L}_N \;=\; h_l{}^{\alpha}\overline{L}^l\tilde{\Phi}N_{\mathrm{R},\,\alpha} \;+\; \mathrm{H.c.}$$

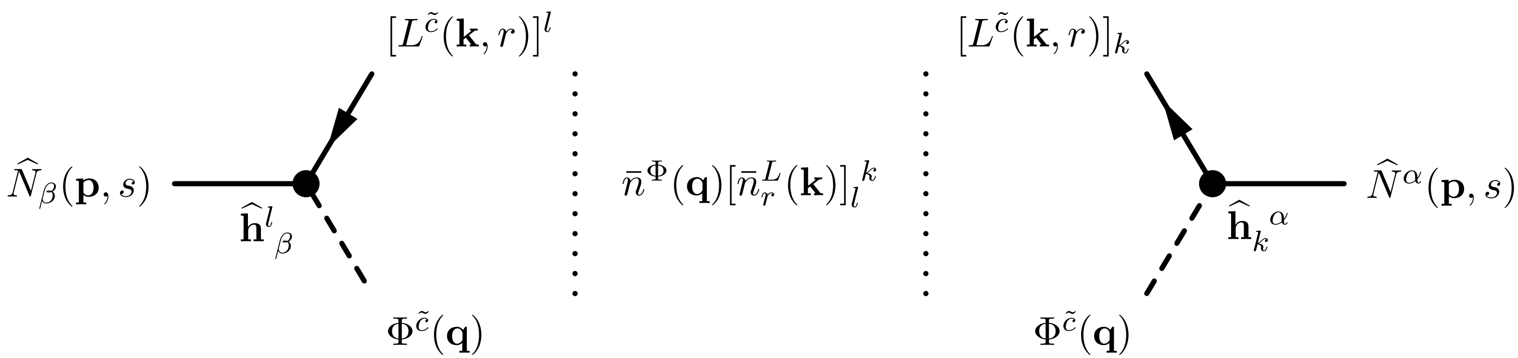
- Choose from the inserted completeness relations the term containing **an ingoing charged-lepton, Higgs state** and **an outgoing heavy Neutrino state**:

$$[\mathcal{T}^0_{rs'}(L\Phi\to N;\mathbf{k},\mathbf{p}',\mathbf{q}')]^l{}_{\alpha} \;\equiv\; \langle\mathbf{k},r,N_\alpha|\,\mathcal{T}^0\,|\mathbf{p}',s',L^l;\mathbf{q}',\Phi^\dagger\rangle$$
$$=\; [\mathcal{M}^0(L\Phi\to N)]^{i'}{}_{\gamma'}[(2E_L(\mathbf{p}'))^{-1/2}]_{i'}{}^{j'}$$
$$\times\; [(2E_N(\mathbf{k}))^{-1/2}]^{\gamma'}{}_{\delta'}(2E_\Phi(\mathbf{q}'))^{-1/2}$$
$$\times\; (2\pi)^4[\delta^{(4)}(k-p'-q')]_{j'}{}^{m'\delta'}{}_{\epsilon'}[\bar{u}(\mathbf{k},r)]^{\epsilon'}{}_{\alpha}\mathrm{P_L}[u(\mathbf{p}',s')]_{m'}{}^{l}$$

Generalized Optical Theorem

Diagrammatically

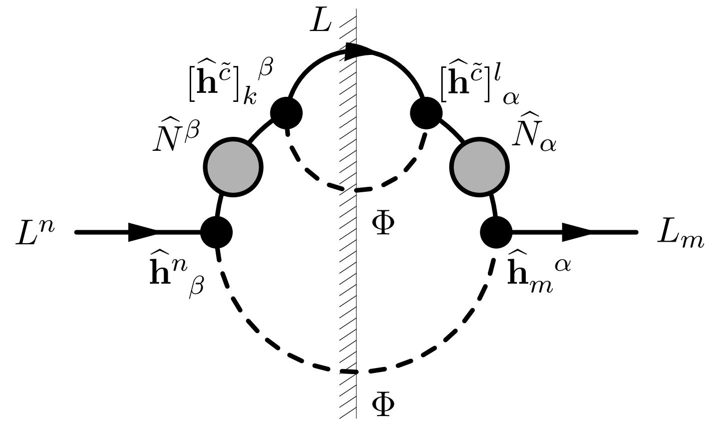


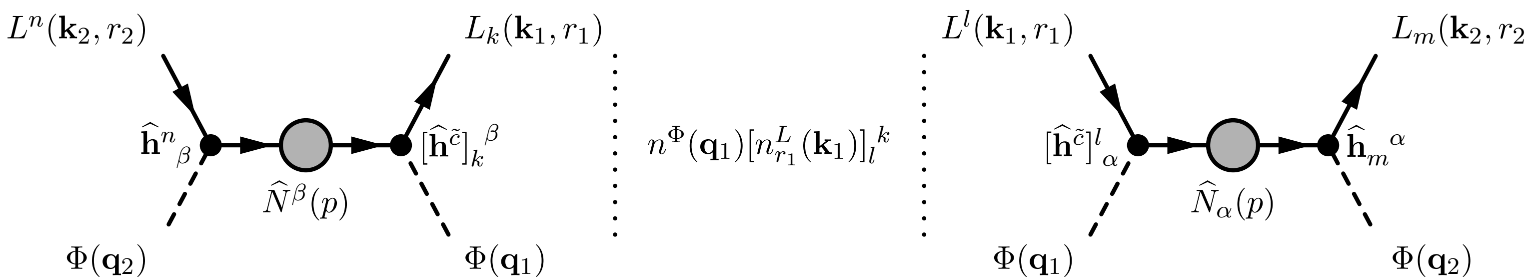


⇒ **Cuts of partial self-energies leads to rank-4 tensor rates** $[\gamma(L\Phi \to N)]_k{}^l{}_\alpha{}^\beta$

Generalized Optical Theorem

Diagrammatically

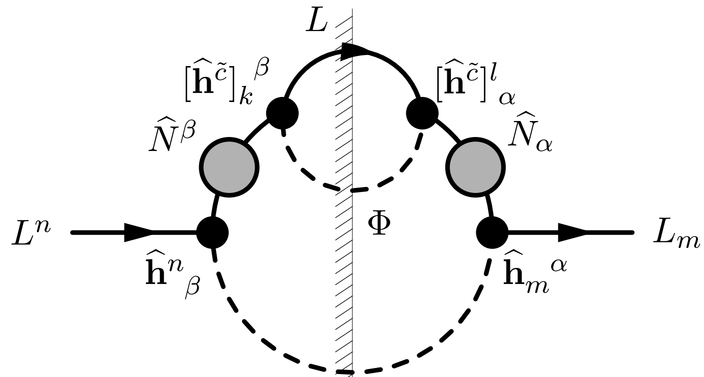




⇒ **Cuts of partial self-energies leads to rank-4 tensor rates** $[\gamma(L\Phi \to L\Phi)]_k{}^l{}_m{}^n$

Generalized Optical Theorem

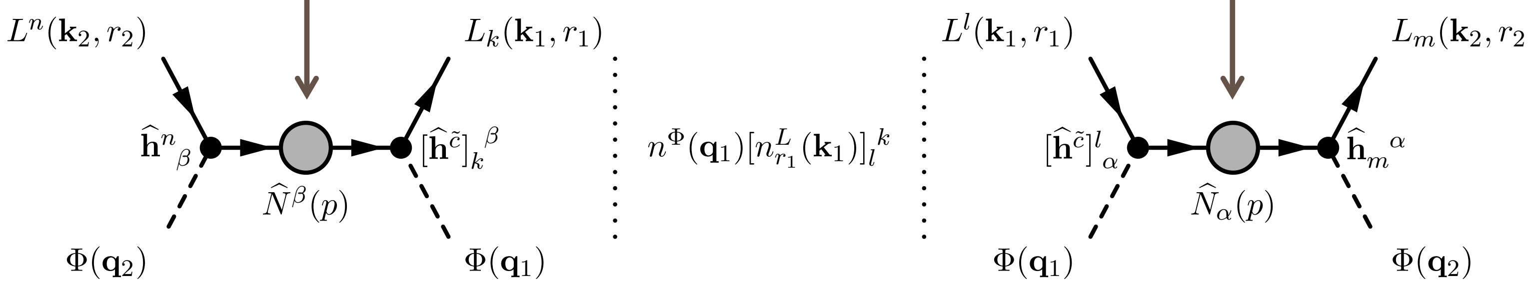
Diagrammatically



Φ [RIS: Kolb and S. Wolfram, Nucl. Phys. B172 (1980) 224.]

Thermal Real Intermediate State Subtraction

[see Dev, Millington, Pilaftsis and Teresi, arXiv:1404.1003.]



⇒ **Cuts of partial self-energies leads to rank-4 tensor rates** $[\gamma(L\Phi \to L\Phi)]_k{}^l{}_m{}^n$

Implications for Resonant Leptogenesis

[see e.g. Pliaftsis and Underwood, Nucl. Phys. B692 (2004) 303-345]

- Inclusion of **higher-rank tensor rates in flavour space** allows us to account for processes that depend upon

1. **flavour oscillations**
2. **flavour mixing**
3. **flavour coherences**

- Consistently accounting for these effects can lead to **enhancement** of CP asymmetry in RL by **up to an order of magnitude** [*see D. Teresi's talk, Flavour Covariant Transport Equations for Resonant Leptogenesis*]

Conclusions

- **Interdependence** of **flavour, spinorial and momentum structure** in the case of non-equilibrium statistical backgrounds.
- Potential importance for "first-principle" treatments of quantum transport phenomena, including **quasi-particle approximations**.
- Emergence of **higher-rank tensor rates** from a generalization of the optical theorem.
- Potential applications: in early Universe particle cosmology, astro-particle physics and high-temperature QCD.
- **Why still use semi-classical approaches?**