The flavour of natural SUSY

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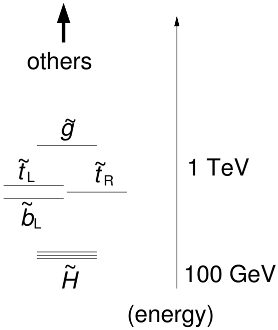
Natural SUSY (working definition)

- higgsinos $\lesssim$ 300 GeV
- stops $\lesssim$ 1 TeV
- gluinos $\lesssim$ 2 TeV
- mediation scale can be high



→ Barbieri/Giudice '87, Cohen/Kaplan/Nelson '96, Wells '03, Barbieri/Pappadopulo '09, Papucci/Ruderman/Weiler '11, various others. . .

Large inverted squark mass hierarchy (ISMH)

3rd generation squarks $\lesssim$ 1 TeV

1st two generation squarks $\gtrsim$ 10 TeV

- consistent with squark searches
  (bounds on 3rd generation squarks still much weaker)
- consistent with naturalness
  (only 3rd gen. strongly coupled to Higgs sector $\Rightarrow$ affects EW scale)
- radiatively induced maximal stop mixing contributions to Higgs mass in MSSM $\rightarrow$ FB/Kraml/Kulkarni '12
  $\Rightarrow$ "minimal" realization of $m_h = 126$ GeV
- suggested by flavour constraints
  (most stringent bounds from 1st two generations)

Radiatively induced flavour violation?

Inverted squark mass hierarchy at high scale M_{GUT}	$\xrightarrow{\text{RG running}}$	ISMH at low scale M_{soft}
Generic squark masses at high scale	$\longrightarrow$	flavour violation at low scale
?	$\longrightarrow$	flavour-safe spectrum with ISMH

Aim: find a sufficient condition on the GUT-scale soft terms to obtain an inverted squark mass hierarchy with FCNCs under control

Flavour basis dependence of soft terms

Spurious non-abelian flavour symmetry of SSM matter sector:

$G_F = \mathrm{SU}(3)_Q \times \mathrm{SU}(3)_U \times \mathrm{SU}(3)_D \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_E$

Soft terms are basis dependent (unless they're universal)

A prescription like $\mathbf{m}_Q^2 = m_0^2 \begin{pmatrix} 1 & & \\ & 1 & \\ & & \epsilon \end{pmatrix}$ only makes sense when fixing the flavour basis, e.g. SCKM

First step: Find a basis-independent parametrization of soft terms

Covariant expansion of soft terms

Under $G_F = \mathrm{SU}(3)_Q \times \mathrm{SU}(3)_U \times \mathrm{SU}(3)_D \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_E$: $\mathbf{m}_Q^2$ transforms as bifundamental of $\mathrm{SU}(3)_Q$. So do $\mathbf{A} \equiv \mathbf{Y}_d \mathbf{Y}_d^\dagger$ and $\mathbf{B} \equiv \mathbf{Y}_u \mathbf{Y}_u^\dagger$.

Thus the following expansion is G_F-covariant:

$$\left(\mathbf{m}_Q^2\right)^{\mathrm{T}} = m_0^2 \Big(a_1^q \, \mathbf{1} + a_2^q \, \mathbf{A} + a_3^q \, \mathbf{B} + a_4^q \, \mathbf{A}^2 + a_5^q \, \mathbf{B}^2 + a_6^q \, \{\mathbf{A}, \mathbf{B}\} + i\, b_1^q \, [\mathbf{A}, \mathbf{B}] + i\, b_2^q \, [\mathbf{A}, \mathbf{B}^2] + i\, b_3^q \, [\mathbf{B}, \mathbf{A}^2] \Big) \,,$$

- Basis matrices are linearly independent for generic **A** and **B** ($\Rightarrow$ no loss of generality: can expand any hermitian 3×3 like this)
- This basis choice is not unique (but the simplest option)
- These combinations of **A** and **B** are what enters the $\mathbf{m}_Q^2$ RGE
- Relation to Minimal Flavour Violation: $\rightarrow$ D'Ambrosio/Giudice/Isidori/Strumia '02
  all coefficients $a_i^q, b_i^q \lesssim \mathcal{O}(1) \quad :\Leftrightarrow \quad$ MFV

Covariant expansion of soft terms

$$\mathbf{m}_U^2 = m_0^2\Big(a_1^u\,\mathbf{1} + \mathbf{Y}_u^\dagger\big(a_2^u\,\mathbf{1} + a_3^u\,\mathbf{A} + a_4^u\,\mathbf{B} + a_5^u\mathbf{A}^2 + a_6^u\,\{\mathbf{A},\mathbf{B}\}$$
$$+ i\,b_1^u\,[\mathbf{A},\mathbf{B}] + i\,b_2^u\,[\mathbf{A},\mathbf{B}^2] + i\,b_3^u\,[\mathbf{B},\mathbf{A}^2]\big)\mathbf{Y}_u\Big)\,,$$
$$\mathbf{m}_D^2 = m_0^2\Big(a_1^d\,\mathbf{1} + \mathbf{Y}_d^\dagger\big(a_2^d\,\mathbf{1} + a_3^d\,\mathbf{A} + a_4^d\,\mathbf{B} + a_5^d\,\mathbf{B}^2 + a_6^d\,\{\mathbf{A},\mathbf{B}\}$$
$$+ i\,b_1^d\,[\mathbf{A},\mathbf{B}] + i\,b_2^d\,[\mathbf{A},\mathbf{B}^2] + i\,b_3^d\,[\mathbf{B},\mathbf{A}^2]\big)\mathbf{Y}_d\Big)\,,$$

$$\mathbf{T}_{u,d} = A_0\Big(c_1^{u,d}\,\mathbf{1} + c_2^{u,d}\,\mathbf{A} + c_3^{u,d}\,\mathbf{B} + c_4^{u,d}\,\mathbf{A}^2 + c_5^{u,d}\,\mathbf{B}^2 + c_6^{u,d}\,\{\mathbf{A},\mathbf{B}\}$$
$$+ i\,c_7^{u,d}\,[\mathbf{A},\mathbf{B}] + i\,c_8^{u,d}\,[\mathbf{A},\mathbf{B}^2] + i\,c_9^{u,d}\,[\mathbf{B},\mathbf{A}^2]\Big)\mathbf{Y}_{u,d}\,.$$

Note $\mathbf{m}^2_{Q,U,D}$ hermitian $\Rightarrow a_i^x, b_i^x$ real
but $\mathbf{T}_{u,d}$ general complex $3\times 3 \Rightarrow c_i^x$ generally complex

RG evolution

Can write RGEs for flavour coefficients a_i^x, b_i^x, c_i^x, at least in principle
$\rightarrow$ Paradisi/Ratz/Schieren/Simonetto '08, Colangelo/Nikolidakis/Smith '08

e.g.

$$\begin{aligned} 16\pi^2 \frac{d}{dt} a_1^q = & -\frac{32}{3} g_3^2 \left|M_3\right|^2 - 6 g_2^2 \left|M_2\right|^2 - \frac{2}{15} g_1^2 \left|M_1\right|^2 + \frac{1}{5} g_1^2 \, S \\ & + a_6^q \left(2 \operatorname{tr} \mathbf{BAB} - 2 \operatorname{tr} \mathbf{AB} \operatorname{tr} \mathbf{B} + \operatorname{tr} \mathbf{A} (\operatorname{tr} \mathbf{B})^2 - \operatorname{tr} \mathbf{A} \operatorname{tr} \mathbf{B}^2\right) \\ & + \text{(lots more unwieldy terms)} \end{aligned}$$

Not too useful in practice

- Obvious: MFV condition (all a_i^x, b_i^x, c_i^x at most $\mathcal{O}(1)$) is stable under the RG since logarithmic running won't induce large hierarchies
- Less obvious: MFV condition is IR attractive

Realizing natural SUSY / ISMH

As above $\mathbf{B} \equiv \mathbf{Y}_u \mathbf{Y}_u^\dagger \approx \begin{pmatrix} 0 & & \\ & 0 & \\ & & y_t^2 \end{pmatrix}$. Set

$$\mathbf{m}_Q^2(M_{\mathrm{GUT}}) = m_0^2 \left(\mathbf{1} - \frac{\alpha_q}{\operatorname{tr} \mathbf{B}} \mathbf{B} \right)^{\mathrm{T}}$$
$$\mathbf{m}_U^2(M_{\mathrm{GUT}}) = m_0^2 \left(\mathbf{1} - \frac{\alpha_u}{\operatorname{tr} \mathbf{B}} \mathbf{Y}_u^\dagger \mathbf{Y}_u \right)$$

with $\alpha_q \approx 1$ and $\alpha_u \approx 1$.

- Sups and scharms heavy $\approx m_0^2$
- Stops light $\approx m_0^2(1 - \alpha_{q,u})$
- Minimally flavour violating since $\frac{\alpha_{q,u}}{\operatorname{tr} \mathbf{B}} \approx \frac{1}{y_t^2} \sim \mathcal{O}(1)$

What about the RH sbottom?

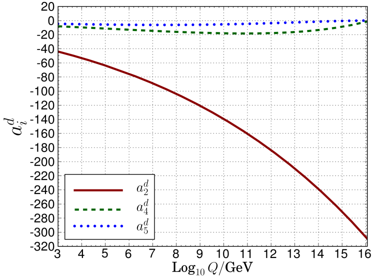
Recall $\mathbf{m}_D^2 = m_0^2 \Big(a_1^d\, \mathbf{1} + \mathbf{Y}_d^\dagger (a_2^d\, \mathbf{1} + a_3^d\, \mathbf{A} + a_4^d\, \mathbf{B} + a_5^d\, \mathbf{B}^2 + \ldots) \mathbf{Y}_d + \ldots \Big)$

- Imposing

$$\mathbf{m}_D^2(M_{\mathrm{GUT}}) = m_0^2 \left(\mathbf{1} - \frac{\alpha_d}{\mathrm{tr}\,\mathbf{A}} \mathbf{Y}_d^\dagger \mathbf{Y}_d \right)$$

  with $\alpha_d \approx 1$ violates MFV:
  at least for moderate $\tan\beta$, $\frac{1}{\mathrm{tr}\,\mathbf{A}} \approx \frac{1}{y_b^2} \gg \mathcal{O}(1)$

- RG running comes to the rescue:



- RG running drives flavour violating coefficients small (MFV = IR attractor)
  In this case, small enough to be safe from flavour constraints

What have we gained?

Conceptual:

- Used quark mass hierarchy (+ tuning) to get inverted squark mass hierarchy
- Speculative: High-scale dynamics accounting for the tuning of $\alpha_{q,u,d}$?

Practical:

- Approximate RG evolution of soft terms in simplifying limits inaccurate
  Example: Switch off CKM CP phase
  $\Rightarrow$ can get $\mathcal{O}(10\%)$ deviations in off-diagonal soft terms, depending on how CP conserving limit is taken
- By contrast: Computing RG evolution of coefficients in simplifying limit, then restoring exact soft terms using full CKM matrix much more accurate

Summary

- Proposed a flavour-covariant reparametrization of SSM soft terms
- Allows a clear definition of “Minimal Flavour Violation”
- Allows to realize inverted squark mass hierarchy within MFV
- Should be useful for (flavour-)model independent parameter scans