

**Doublet-Triplet Mass Splitting and Proton Decay in  
SU(5) SUSY GUT with Horizontal Symmetry SU(1,1)**

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## Table of Contents

1. **Motivation**
2.  **$SU(5)$  SUSY GUT with  $SU(1, 1)$** 
  - 2-1 Doublet-Triplet Mass Splitting
  - 2-2 Proton Decay
3. **Summary**

# 1. Motivation

## How to understand Nature

Hints of Physics beyond the Standard Model

Matter sector: Quarks and Leptons

Three Chiral Generations

Hierarchical Mass Structures

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Charge Quantization

Anomaly Cancellations

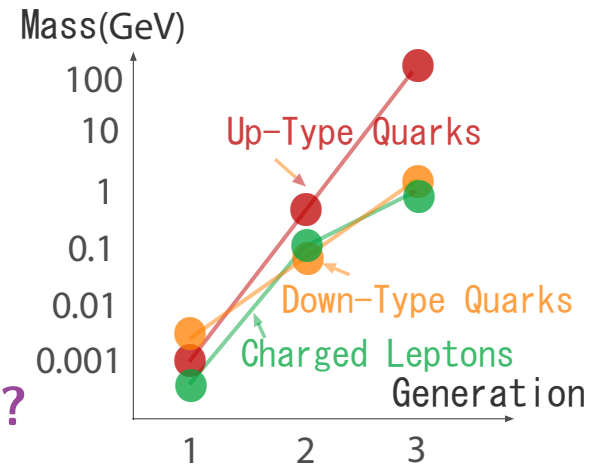
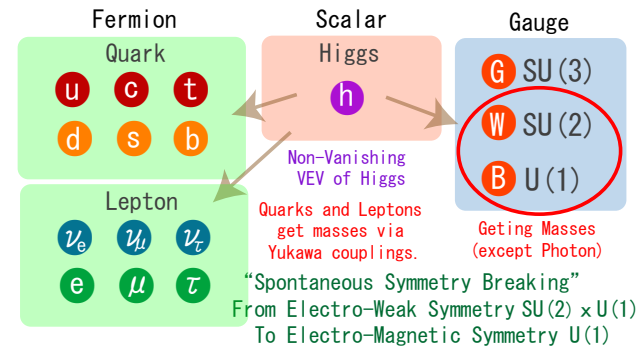
} ⇒ ②

Only Fundamental Representations ⇒ ③

⇒ How can we understand these questions in a four dimensional effective field theory?

① Horizontal Symmetry, ② GUT, ③ Unknown.

## The Standard Model



## Approaches to understand generations using horizontal symmetry

### Non-Abelian (e.g., $SU(3)$ , $A_4$ )

Naturally explaining three generations.

[2, 3, S.F.King, G.G.Ross, PLB'01; H. Ishimori et al.; ...]

### Abelian $U(1)$

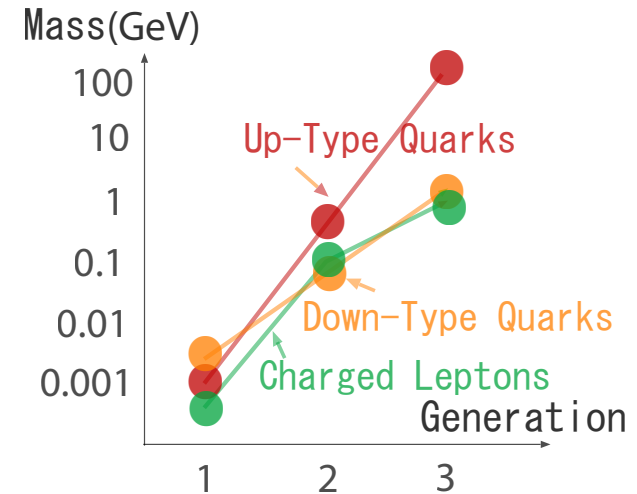
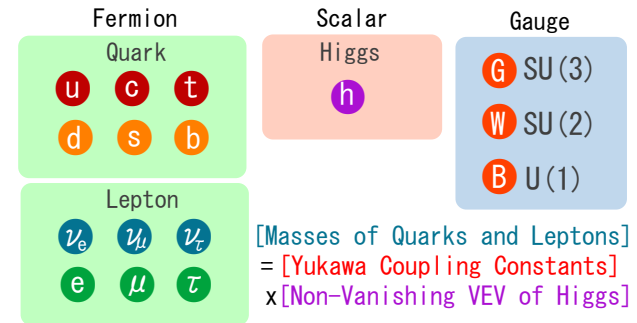
Naturally explaining the mass hierarchies.  
Froggatt-Nielsen mechanism

[4, 5, C.D.Froggatt, H.B.Nielsen, NPB'79; N.Maekawa, PTP'01; ...]

### Noncompact Non-Abelian $SU(1, 1)$

[6-8, K.Inoue, PTP'95; K.Inoue, N.Y., PTP'08; N.Yamatsu, PTEP'12; ...]

### The Standard Model



## Noncompact Non-Abelian Group (minimal $SU(1, 1)$ )

### Noncompact Model $\sim$ A Vectorlike MSSM with Horizontal Symmetry

[6–8, K.Inoue,PTP'95; K.Inoue, N.Y.,PTP'08; N.Y., PTEP'13]

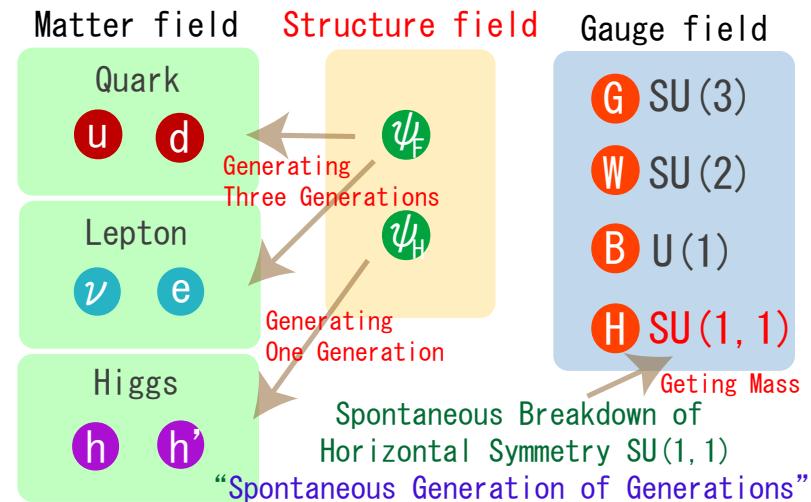
#### Three Chiral Generations

Chiral matters produced from Vectorlike matters via  
 “Spontaneous Generation of Generations”

#### Hierarchical Mass Structures

Hierarchical structure of Clebsch-Gordan coefficients

#### Noncompact Model



$\mathcal{N} = 1$  Supersymmetry [9, K.Inoue, H.Kubo & N.Y.,NPB'10]

Chirality of Scalar Fields and Stability of Vacuum Structures

## Definition of terms in the Noncompact Model

**Matter field** = infinite-dim. unitary reps. of the noncompact group

$$\begin{cases} \hat{F} = \{\hat{f}_{+\alpha}, \hat{f}_{+\alpha+1}, \hat{f}_{+\alpha+2}, \dots\}, \\ \hat{F}^c = \{\hat{f}_{-\alpha}^c, \hat{f}_{-\alpha-1}^c, \hat{f}_{-\alpha-2}^c, \dots\}, \end{cases}$$

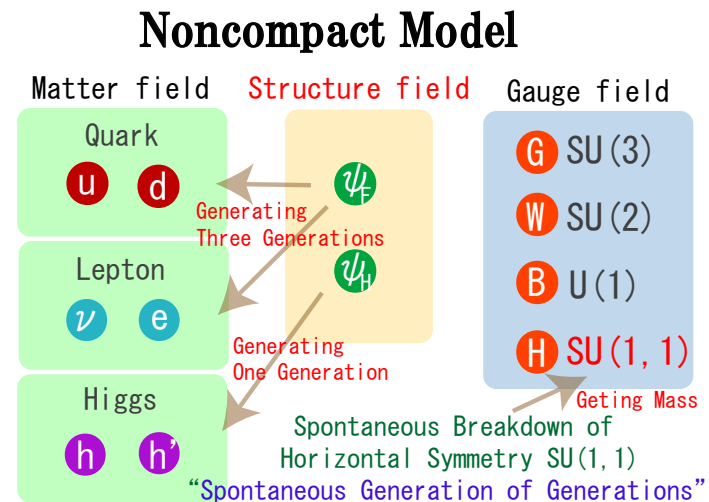
$\alpha = [\text{real and positive}]$ .

**Structure field** = finite-dim. reps.

$$\hat{\Psi} = \{\hat{\psi}_{-S}, \hat{\psi}_{-S+1}, \dots, \hat{\psi}_{S-1}, \hat{\psi}_S\},$$

$S = [\text{Integer or half-integer}], \text{ called } SU(1, 1) \text{ spin.}$

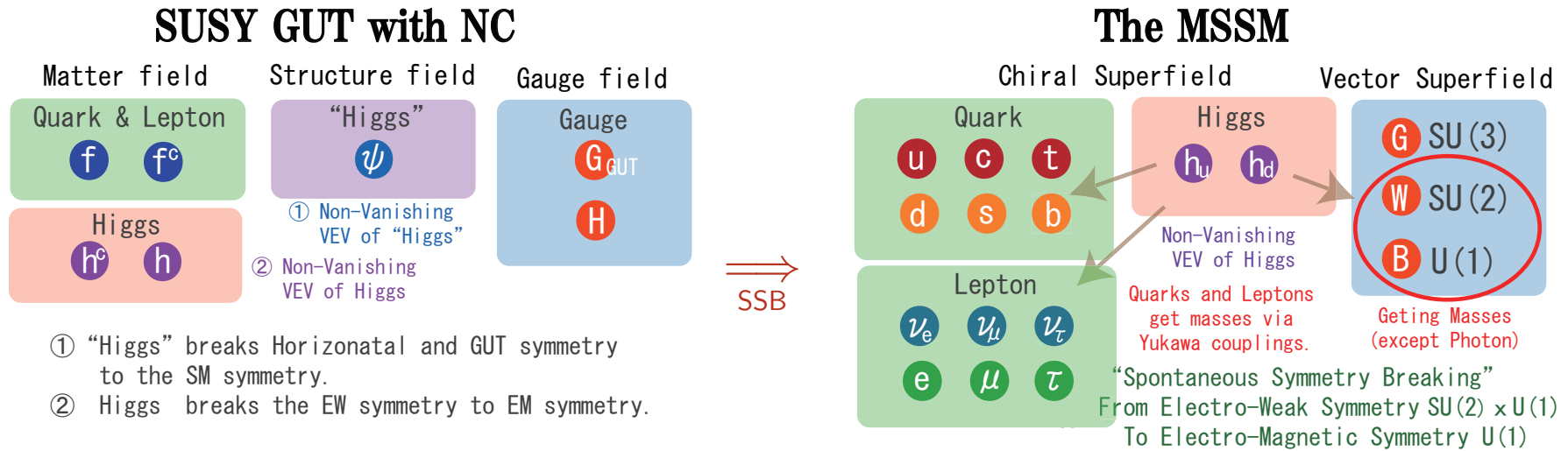
Subscripts, such as  $\alpha, S$ , stand for the eigenvalue of  $\tau_3$ .



All Matter and Structure fields are chiral superfields in  $\mathcal{N} = 1$  SUSY.

[8, N.Y., PTEP'13]

## 2. $SU(5)$ SUSY GUT with $SU(1, 1)$



Assuming the following:

- The structure fields get nonvanishing VEVs  $M_{GUT} \sim O(10^{16})$  GeV.
  - The structure fields break  $SU(5) \times SU(1, 1)$  to  $G_{SM}$ .
- $\Rightarrow$  Through spontaneous generation of generations, chiral matter fields appear.

## 2-1 Doublet-Triplet Mass Splitting [7,10, K.Inoue, N.Yamashita, PTP'00;K.Inoue, N.Y., PTP'08]

- (1) Matter fields  $SU(5)$   $\mathbf{5}^{(*)}$ -plets  $\hat{H}_5 + \hat{H}_{5^*}^c$

$$\hat{H}_5 = \{\hat{h}_{-\gamma}, \hat{h}_{-\gamma-1}, \dots\}, \quad \hat{H}_{5^*}^c = \{\hat{h}_\gamma^c, \hat{h}_{\gamma+1}^c, \dots\}$$

Structure fields  $\hat{\Phi}_1, \hat{\Phi}'_{24}$  ( $S$ : a highest weight of  $SU(1, 1)$ )

$$\hat{\Phi}_1 = \{\hat{\phi}_{-S}, \hat{\phi}_{-S+1}, \dots, \hat{\phi}_S\}, \quad \hat{\Phi}'_{24} = \{\hat{\phi}'_{-S}, \hat{\phi}'_{-S+1}, \dots, \hat{\phi}'_S\}.$$

- (2) Superpotential of Matter-Structure Couplings ( $x, x'$ : coupling constant)

$$W = x \hat{H}_5 \hat{H}_{5^*}^c \hat{\Phi}_1 + x' \hat{H}_5 \hat{H}_{5^*}^c \hat{\Phi}'_{24}.$$

- (3) Nonvanishing VEV of the structure fields  $\hat{\Phi}_1, \hat{\Phi}'_{24}$

$$\langle \hat{\Phi}_1 \rangle = \{\dots, 0, \langle \phi_0 \rangle, 0, \dots\}, \quad \langle \hat{\Phi}'_{24} \rangle = \{\dots, 0, \langle \phi'_{+1} \rangle_{SU(5) \rightarrow G_{SM}}, \dots\}.$$

To determine the massless modes, we need to calculate mixing between states.

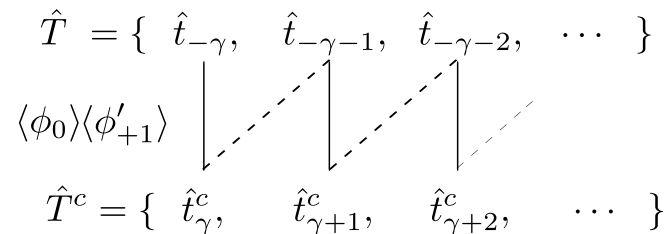
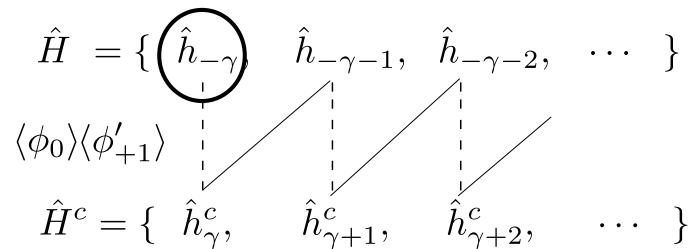


Normalizable condition for massless modes leads to the following:

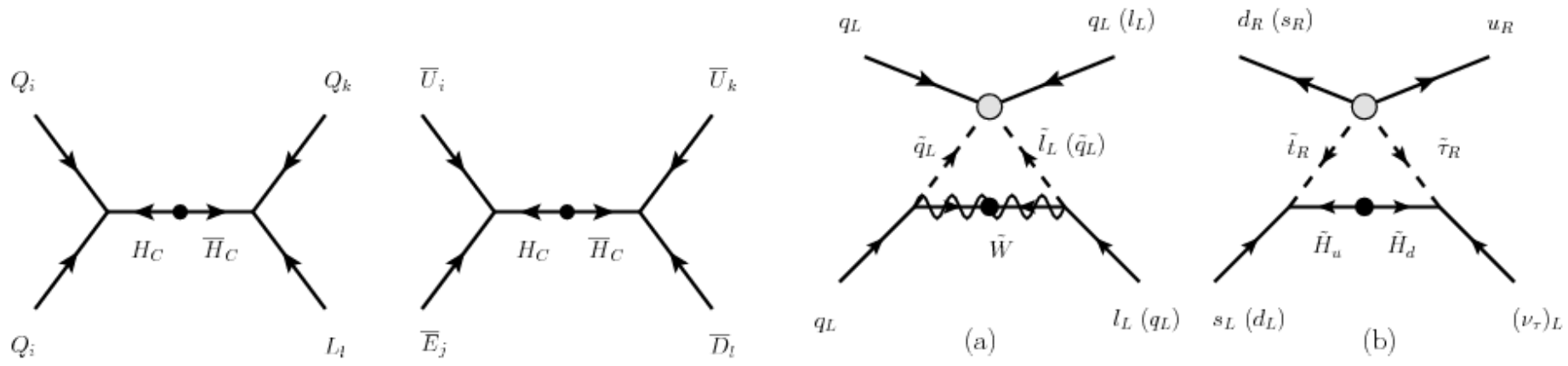
	Condition for $\epsilon$ 's	Massless Modes
①	$\epsilon_{cr} <  \epsilon_2  <  \epsilon_3 $	<del>doublet <math>\hat{H}</math>, triplet <math>\hat{T}</math></del>
②	$ \epsilon_2  < \epsilon_{cr} <  \epsilon_3 $	doublet $\hat{H}$ , triplet $\hat{T}$
③	$ \epsilon_2  <  \epsilon_3  < \epsilon_{cr}$	doublet $\hat{H}$ , triplet $\hat{T}$

$$\epsilon_3 = (-3/2)\epsilon_2, \quad \epsilon_2 := x\langle\phi_0\rangle/x'\langle\phi'_{+1}\rangle, \quad \epsilon_{cr} := \sqrt{S!S!/(S+1)!(S-1)!}$$

For the case ②, the lightest doublet higgs has no Dirac mass, while the lightest colored higgs has a Dirac mass  $O(M_{GUT}) \sim O(10^{16})$  GeV.



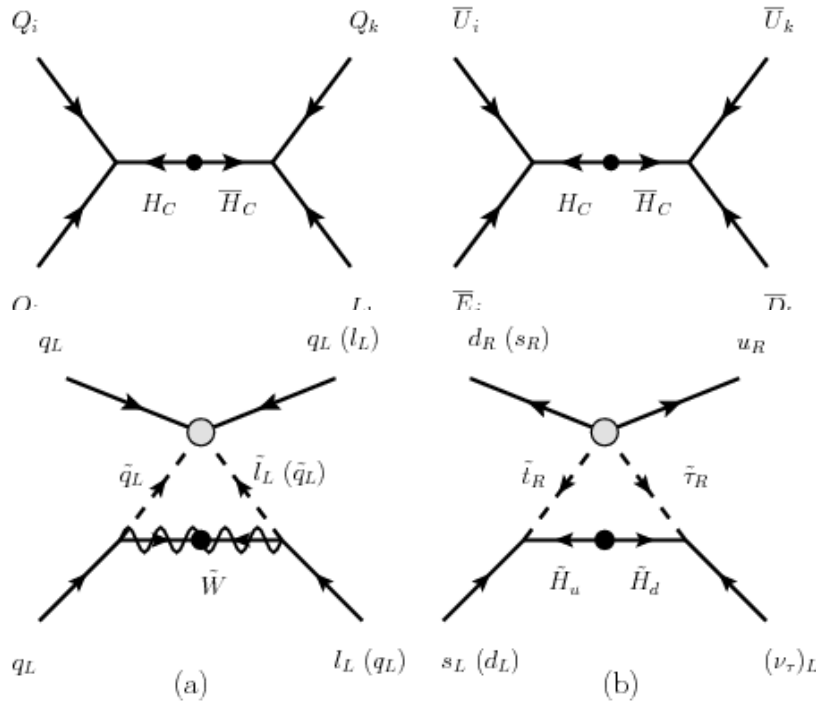
## 2-2 Proton Decay (via colored higgses)



From Ref. [11, J.Hisano et al.,'13]

Superpotential quartic terms  $\hat{Q}\hat{Q}\hat{Q}\hat{L}$  and  $\hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c$  are generated by colored higgses, and lead to dimension-6 operators causing proton decays.

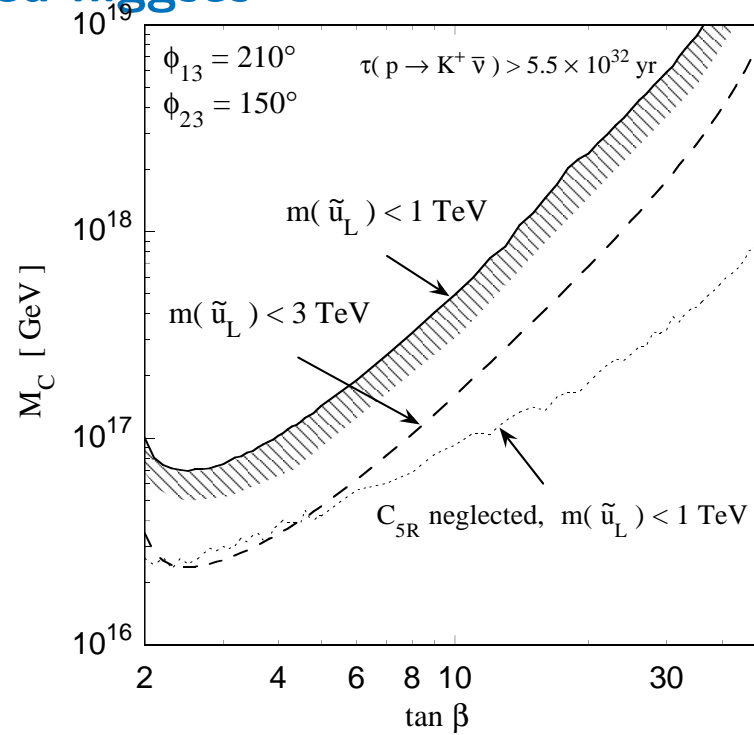
## Proton decays mediated by colored higgses



From Ref. [11, J.Hisano et al., '13]

$$W \sim -\frac{1}{M_C} \left( \hat{Q}\hat{Q}\hat{Q}\hat{L} + \hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c \right).$$

(Recent Super-Kamiokande result  $\tau(p \rightarrow K^+ \bar{\nu}) > 3.3 \times 10^{33}$  yr [13, M.Miura, PoS'2010])



From Ref. [12, T.Goto, T.Nihei, PRD'99]

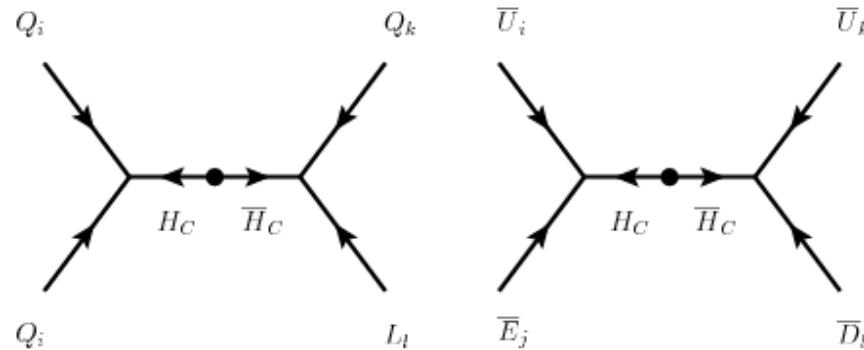
## Some known methods suppressing proton decays via colored higgses

- # SUSY particle's masses are much heavier than 1 TeV. [11, J.Hisano, et al.'arXiv'13; . . .]
- # In an orbifold extra dimension model  $S^1/Z^2 \times Z^2$ , when only doublet higgses have zero modes and colored higgses have no zero modes [14, Y.Kawamura,PTP'01], the colored higgses have Dirac mass terms. The contribution to proton decay via colored higgses is strongly suppressed [15, L.J.Hall, Y.Nomura,PRD'01;. . .].

## The $SU(5) \times SU(1, 1)$ Model

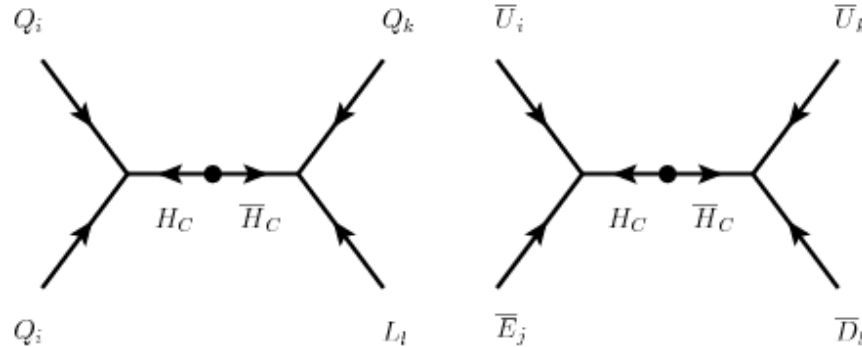
- # When only doublet higgses are chiral and colored higgses are vectorlike via spontaneous generation of generations [7, 10, K.Inoue,N.Yamashita,PTP'00;K.Inoue,N.Y.,PTP'08], the colored higgses have Dirac mass terms. The contribution to proton decay via colored higgses is strongly suppressed [1, N.Y.,PTEP'13].

## The $SU(5) \times SU(1, 1)$ Model



- $\mu$ -term  $\hat{H}_u \hat{H}_d$ ,  $\hat{Q} \hat{Q} \hat{Q} \hat{L}$  and  $\hat{U}^c \hat{U}^c \hat{D}^c \hat{E}^c$  are forbidden by the horizontal symmetry  $SU(1, 1)$ . (They are allowed by ordinary  $R$ -parity.)
- Spontaneous horizontal symmetry breaking can produce nonzero  $\mu$ -parameter  $O(m_{\text{SUSY}})$ , and then  $\hat{Q} \hat{Q} \hat{Q} \hat{L}$  and  $\hat{U}^c \hat{U}^c \hat{D}^c \hat{E}^c$  are also generated.
- The masses of the colored higgses come from mass terms  $O(M_{\text{GUT}})$  between the colored higgses and their conjugate fields as well as the effective  $\mu$ -term  $O(m_{\text{SUSY}})$  between up- and down-type colored higgses.

## The $SU(5) \times SU(1, 1)$ Model



$$W \sim -\frac{\mu}{M_{T_u}M_{T_d}} \left( \hat{Q}\hat{Q}\hat{Q}\hat{L} + \hat{U}^c\hat{U}^c\hat{D}^c\hat{E}^c \right), \quad "M_C" := \frac{M_{T_u}M_{T_d}}{\mu},$$

where  $\mu = O(m_{\text{SUSY}})$  and  $M_{T_u} \simeq M_{T_d} = O(M_{\text{GUT}})$ .

$$\Rightarrow "M_C" = O(M_{\text{GUT}}^2/m_{\text{SUSY}}) \sim O(10^{29}) \text{ GeV}.$$

- The contribution to proton decay via colored higgses ( $p \rightarrow K^+\bar{\nu}$ ) is highly suppressed. Thus the main contribution comes from the GUT gauge bosons ( $p \rightarrow \pi^0 e^+$ ).

### 3. Summary

- $SU(5)$  SUSY GUT with  $SU(1, 1)$

- # Through the spontaneous generation of generations, the doublet higgses can appear and the colored higgses can disappear naturally.

- # We found that once the doublet-triplet mass splitting is realized naturally, proton decay via colored higgses are highly suppressed.

- Other Topics

- # How to realize three chiral generations of quarks and leptons

- # How to explain hierarchical structure of Yukawa coupling constants

- # How and When we identify  $R$ -parity as a discrete subgroup of  $SU(1, 1)$ .

If you are interested in them, please read a paper [1, N.Y., PTEP'13]

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