Higgs Spectra from Maximally Symmetric Two Higgs Doublet Model Potential

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PSBD and A. Pilaftsis, arXiv:1407.xxxx.

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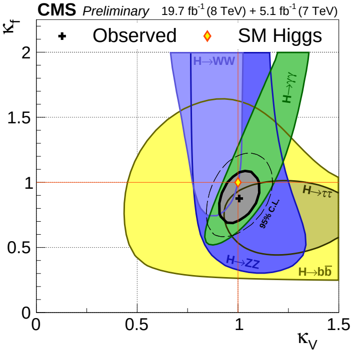
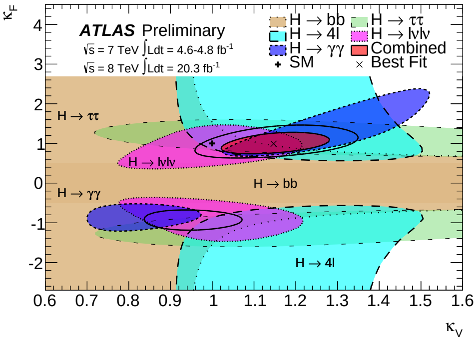
Outline

- Introduction
- Symmetry classifications of the 2HDM Potential
- Spectrum Analysis for the Maximally Symmetric Potential
- Some Collider Phenomenology
- Conclusion

Introduction





- Discovery of a Higgs boson with $m_H = 125 \pm 2$ GeV and couplings within $\mathcal{O}(10\%)$ of the SM predictions.
- Opportunity in the search of (or constraining) BSM physics through Higgs portal.
  - Precision Higgs Study (Higgcision).
  - Search for additional Higgses.

Two Higgs Doublet Model

- Several theoretical reasons to go beyond the SM Higgs sector.
- Any scalar sector in a local $SU(2) \times U(1)$ gauge theory must be consistent with $\rho_{\rm exp} \simeq 1$.
- With n Higgs multiplets, $\rho_{\rm tree} = \frac{\sum_{i=1}^{n}[T_i(T_i+1) - Y_i^2]v_i}{2\sum_{i=1}^{n} Y_i^2 v_i}$.
- Simplest choice: Add multiplets with $T(T+1) = 3Y^2$.
- SM: One $SU(2)_L$ doublet ϕ with $Y = \pm\frac{1}{2}$.
- 2HDM: Two $SU(2)_L$ doublets $\phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix}$ (with $i = 1, 2$).
- General 2HDM potential:

$$\begin{aligned} V(\phi_1, \phi_2) &= -\mu_1^2(\phi_1^\dagger\phi_1) - \mu_2^2(\phi_2^\dagger\phi_2) - \left[m_{12}^2(\phi_1^\dagger\phi_2) + \mathrm{H.c.}\right] \\ &\quad +\lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &\quad + \left[\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_2) + \mathrm{H.c.}\right]. \end{aligned}$$

- Four real mass parameters $\mu_{1,2}^2$, $\mathrm{Re}(m_{12}^2)$, $\mathrm{Im}(m_{12}^2)$, and 10 real quartic couplings $\lambda_{1,2,3,4}$, $\mathrm{Re}(\lambda_{5,6,7})$, $\mathrm{Im}(\lambda_{5,6,7})$.
- Explore possible symmetries relating the quartic couplings.

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&\quad + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
&\quad + \left[\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_2) + \text{H.c.}\right].
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An Alternative Formulation of the 2HDM Potential

- Introduce an 8-dimensional complex multiplet: [Battye, Brawn, Pilaftsis '11; Nishi '11]

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ i\sigma^2\phi_1^* \\ i\sigma^2\phi_2^* \end{pmatrix} .$$

- Convenient to go over to a 6-dimensional bilinear field space [Pilaftsis '12]

$$R^A = \Phi^\dagger \Sigma^A \Phi \qquad (A = 0, 1, 2, 3, 4, 5),$$

where the 8×8 matrices Σ^A can be expressed in terms of the Pauli matrices $\sigma^{1,2,3}$ and $\sigma^0 = \mathbf{1}_2$:

$$\Sigma^0 = \frac{1}{2}\sigma^0 \otimes \sigma^0 \otimes \sigma^0 \equiv \frac{1}{2}\mathbf{1}_8, \quad \Sigma^1 = \frac{1}{2}\sigma^0 \otimes \sigma^1 \otimes \sigma^0, \quad \Sigma^2 = \frac{1}{2}\sigma^3 \otimes \sigma^2 \otimes \sigma^0,$$
$$\Sigma^3 = \frac{1}{2}\sigma^0 \otimes \sigma^3 \otimes \sigma^0, \quad \Sigma^4 = -\frac{1}{2}\sigma^2 \otimes \sigma^2 \otimes \sigma^0, \quad \Sigma^5 = -\frac{1}{2}\sigma^1 \otimes \sigma^2 \otimes \sigma^0.$$

- Realizes an $SO(1,5)$ symmetry group.

An Alternative Formulation of the 2HDM Potential

$$V = -\frac{1}{2}M_A R^A + \frac{1}{4}R_A L^A_B R^B,$$

where

$$M_A = \Big(\mu_1^2+\mu_2^2, \quad 2\mathrm{Re}(m_{12}^2), \quad -2\mathrm{Im}(m_{12}^2), \quad \mu_1^2-\mu_2^2, \quad 0, \quad 0\Big),$$

$$R^A = \begin{pmatrix} \phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 \\ \phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1 \\ -i(\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_1) \\ \phi_1^\dagger\phi_1 - \phi_2^\dagger\phi_2 \\ \phi_1^{\mathsf{T}} i\sigma^2\phi_2 - \phi_2^\dagger i\sigma^2\phi_1^* \\ -i(\phi_1^{\mathsf{T}} i\sigma^2\phi_2 + \phi_2^\dagger i\sigma^2\phi_1^*) \end{pmatrix},$$

$$L^A_B = \begin{pmatrix} \lambda_1+\lambda_2+\lambda_3 & \mathrm{Re}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_6+\lambda_7) & \lambda_1-\lambda_2 & 0 & 0 \\ \mathrm{Re}(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_5) & \mathrm{Re}(\lambda_6-\lambda_7) & 0 & 0 \\ -\mathrm{Im}(\lambda_6+\lambda_7) & -\mathrm{Im}(\lambda_5) & \lambda_4-\mathrm{Re}(\lambda_5) & -\mathrm{Im}(\lambda_6-\lambda_7) & 0 & 0 \\ \lambda_1-\lambda_2 & \mathrm{Re}(\lambda_6-\lambda_7) & -\mathrm{Im}(\lambda_6-\lambda_7) & \lambda_1+\lambda_2-\lambda_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Symmetry Classifications of the 2HDM Potential

- Three classes of accidental symmetries of the 2HDM potential:
  - Higgs Family (HF) Symmetries involving transformations of $\phi_{1,2}$ only (but not $\phi^*_{1,2}$), e.g. Z_2 [Glashow, Weinberg '58], $U(1)_{\rm PQ}$ [Peccei, Quinn '77], $SU(2)_{\rm HF}$ [Deshpande, Ma '78].
  - *CP* Symmetries relating $\phi_{1,2}$ to $\phi^*_{1,2}$, e.g. $\phi_{1(2)} \to \phi^*_{1(2)}$ (CP1) [Lee '73], $\phi_{1(2)} \to (-)\phi^*_{2(1)}$ (CP2) [Davidson, Haber '05], CP1 combined with $SO(2)_{\rm HF}$ (CP3) [Ivanov '08; Ferreira, Haber, Silva '09].
  - Mixed HF and *CP* transformations that leave the gauge-kinetic terms of $\phi_{1,2}$ invariant [Battye, Brawn, Pilaftsis '11], e.g. $O(8)$ and $O(4) \otimes O(4)$ in real field space [Deshpande, Ma '78].
- Maximum of 13 *distinct* accidental symmetries. [Battye, Brawn, Pilaftsis '11]
- Can derive explicit transformation relations based on the bilinear scalar field formalism. [Pilaftsis '12]

Symmetry Classifications of the 2HDM Potential

Table 1
Parameter relations for the 13 accidental symmetries [1] related to the $U(1)_Y$-invariant 2HDM potential in the diagonally reduced basis, where $\mathrm{Im}\,\lambda_5 = 0$ and $\lambda_6 = \lambda_7$. A dash signifies the absence of a constraint.

No.	Symmetry	μ_1^2	μ_2^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	$\mathrm{Re}\,\lambda_5$	$\lambda_6 = \lambda_7$
1	$Z_2 \times O(2)$	–	–	Real	–	–	–	–	–	Real
2	$(Z_2)^2 \times SO(2)$	–	–	0	–	–	–	–	–	0
3	$(Z_2)^3 \times O(2)$	–	μ_1^2	0	–	λ_1	–	–	–	0
4	$O(2) \times O(2)$	–	–	0	–	–	–	–	0	0
5	$Z_2 \times [O(2)]^2$	–	μ_1^2	0	–	λ_1	–	–	$2\lambda_1 - \lambda_{34}$	0
6	$O(3) \times O(2)$	–	μ_1^2	0	–	λ_1	–	$2\lambda_1 - \lambda_3$	0	0
7	$SO(3)$	–	–	Real	–	–	–	–	λ_4	Real
8	$Z_2 \times O(3)$	–	μ_1^2	Real	–	λ_1	–	–	λ_4	Real
9	$(Z_2)^2 \times SO(3)$	–	μ_1^2	0	–	λ_1	–	–	$\pm\lambda_4$	0
10	$O(2) \times O(3)$	–	μ_1^2	0	–	λ_1	$2\lambda_1$	–	0	0
11	$SO(4)$	–	–	0	–	–	–	0	0	0
12	$Z_2 \times O(4)$	–	μ_1^2	0	–	λ_1	–	0	0	0
13	$SO(5)$	–	μ_1^2	0	–	λ_1	$2\lambda_1$	0	0	0

[Pilaftsis '12]

- $SO(5)$ is the *maximal* symmetry group in the bilinear field space which leaves R^0 invariant.
- In a specific bilinear basis [Gunion, Haber '05], where L_{IJ} is made diagonal by an $SO(3) \subset SO(5)$ rotation, $\mathrm{Im}(\lambda_5) = 0$ and $\lambda_6 = \lambda_7$.
- 7 independent quartic couplings for the $U(1)_Y$-invariant 2HDM potential.
- $SO(5)$ is isomorphic to $Sp(4)/Z_2$, which gives a one-to-one correspondence between the generators of the maximal reparametrization groups $G^R_{\mathrm{2HDM}} = SO(5)$ and $G^\Phi_{\mathrm{2HDM}} = Sp(4)$. [PSBD, Pilaftsis '14]

Symmetry Generators

Table 2
Symmetry generators [cf. (10), (14)] and discrete group elements [cf. (17)] for the 13 accidental symmetries of the $U(1)_Y$-invariant 2HDM potential. For each symmetry, the maximally broken SO(5) generators compatible with a neutral vacuum are displayed, along with the pseudo-Goldstone bosons (given in parentheses) that result from the Goldstone theorem.

No.	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete group elements	Maximally broken SO(5) generators	Number of pseudo-Goldstone bosons
1	$Z_2 \times O(2)$	T^0	D_{CP1}	–	0
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	–	0
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	–	0
4	$O(2) \times O(2)$	T^3, T^0	–	T^3	1 (a)
5	$Z_2 \times [O(2)]^2$	T^2, T^0	D_{CP1}	T^2	1 (h)
6	$O(3) \times O(2)$	$T^{1,2,3}, T^0$	–	$T^{1,2}$	2 (h, a)
7	$SO(3)$	$T^{0,4,6}$	–	$T^{4,6}$	2 $(h^\pm)$
8	$Z_2 \times O(3)$	$T^{0,4,6}$	$D_{Z_2} \cdot D_{CP2}$	$T^{4,6}$	2 $(h^\pm)$
9	$(Z_2)^2 \times SO(3)$	$T^{0,5,7}$	$D_{CP1} \cdot D_{CP2}$	$T^{5,7}$	2 $(h^\pm)$
10	$O(2) \times O(3)$	$T^3, T^{0,8,9}$	–	T^3	1 (a)
11	$SO(4)$	$T^{0,3,4,5,6,7}$	–	$T^{3,5,7}$	3 $(a, h^\pm)$
12	$Z_2 \times O(4)$	$T^{0,3,4,5,6,7}$	$D_{Z_2} \cdot D_{CP2}$	$T^{3,5,7}$	3 $(a, h^\pm)$
13	$SO(5)$	$T^{0,1,2,\ldots,9}$	–	$T^{1,2,8,9}$	4 $(h, a, h^\pm)$

[Pilaftsis '12]

- T^a and K^a are the generators of $SO(5)$ and $Sp(4)$ respectively ($a = 0, \ldots, 9$).
- T^0 is the hypercharge generator in R-space, which is equivalent to the electromagnetic generator $Q_{\rm em} = \frac{1}{2}\sigma^0 \otimes \sigma^0 \otimes \sigma^3 + K^0$ in Φ-space.
- $Sp(4)$ contains the custodial symmetry group $SU(2)_C$.
- Three *independent* realizations of custodial symmetry induced by (i) $K^{0,4,6}$, (ii) $K^{0,5,7}$, (iii) $K^{0,8,9}$.

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Higgs Spectra

- Consider normal vacua with real vevs $v_{1,2}$, where $\sqrt{v_1^2 + v_2^2} = v_{\rm SM}$ and $\tan\beta = v_2/v_1$.
- Eight real scalar fields: $\phi_j = \begin{pmatrix} \phi_j^+ \\ \frac{1}{\sqrt{2}}(v_j + \rho_j + i\eta_j) \end{pmatrix}$ (with $j = 1, 2$).
- After EWSB, three Goldstone bosons ($G^\pm$, G^0), which are eaten by $W^\pm$ and Z, and five physical scalar fields: two CP-even (h, H), one CP-odd (a) and two charged ($h^\pm$).
- In the charged sector,

$$\begin{pmatrix} G^\pm \\ h^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}.$$

with $M_{h^\pm}^2 = \frac{1}{s_\beta c_\beta}\left[{\rm Re}(m_{12}^2) - \frac{1}{2}\left(\{\lambda_4 + {\rm Re}(\lambda_5)\}\, s_\beta c_\beta + {\rm Re}(\lambda_6)c_\beta^2 + {\rm Re}(\lambda_7)s_\beta^2\right)\right].$

- In the CP-odd sector,

$$\begin{pmatrix} G^0 \\ a \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}.$$

$$\text{with} \quad M_a^2 = \frac{1}{s_\beta c_\beta}\left[{\rm Re}(m_{12}^2) - v^2\left({\rm Re}(\lambda_5)s_\beta c_\beta + \frac{1}{2}\left\{{\rm Re}(\lambda_6)c_\beta^2 + {\rm Re}(\lambda_7)s_\beta^2\right\}\right)\right]$$

$$= M_{h^\pm}^2 + \frac{1}{2}\left[\lambda_4 - {\rm Re}(\lambda_5)\right] v^2.$$

Higgs Spectra

- In the *CP*-even sector,

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} .$$

$$\left(M_S^2\right)_{ij} \equiv \begin{pmatrix} A & C \\ C & B \end{pmatrix} = M_A^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix}$$

$$+ v^2 \begin{pmatrix} 2\lambda_1 c_\beta^2 + \mathrm{Re}(\lambda_5) s_\beta^2 + 2\mathrm{Re}(\lambda_6) s_\beta c_\beta & \lambda_{34} s_\beta c_\beta + \mathrm{Re}(\lambda_6) c_\beta^2 + \mathrm{Re}(\lambda_7) s_\beta^2 \\ \lambda_{34} s_\beta c_\beta + \mathrm{Re}(\lambda_6) c_\beta^2 + \mathrm{Re}(\lambda_7) s_\beta^2 & 2\lambda_2 s_\beta^2 + \mathrm{Re}(\lambda_5) c_\beta^2 + 2\mathrm{Re}(\lambda_7) s_\beta c_\beta \end{pmatrix}$$

with $\lambda_{34} = \lambda_3 + \lambda_4$ and $\tan 2\alpha = \frac{2C}{A-B}$. [Pilaftsis, Wagner '99]

- The SM Higgs boson is given by

$$H_{\mathrm{SM}} = \rho_1 \cos\beta + \rho_2 \sin\beta = H \cos(\beta-\alpha) + h \sin(\beta-\alpha) .$$

- With respect to the SM Higgs couplings $H_{\mathrm{SM}} VV$ ($V = W^\pm, Z$),

$$g_{hVV} = \sin(\beta-\alpha) , \qquad g_{HVV} = \cos(\beta-\alpha) .$$

Unitarity constraints uniquely fix other V-Higgs-Higgs couplings [Gunion, Haber, Kane, Dawson '90]

$$g_{haZ} = \frac{g}{2\cos\theta_w} \cos(\beta-\alpha) , \qquad g_{HaZ} = \frac{g}{2\cos\theta_w} \sin(\beta-\alpha) ,$$

$$g_{h^+hW^-} = \frac{g}{2} \cos(\beta-\alpha) , \qquad g_{h^+HW^-} = \frac{g}{2} \sin(\beta-\alpha) .$$

Quark Yukawa Couplings

$$
\begin{aligned}
-\mathcal{L}_Y^q &= \bar{Q}_L(h_1^u\phi_1 + h_2^u\phi_2)u_R + \bar{Q}_L(h_1^d\widetilde{\phi}_1 + h_2^d\widetilde{\phi}_2)d_R \\
&= (\bar{u}_L\ ,\ \bar{d}_L)\left(\phi_1\ ,\ \phi_2\ ,\ \widetilde{\phi}_1\ ,\ \widetilde{\phi}_2\right)\begin{pmatrix} h_1^u & 0 \\ h_2^u & 0 \\ 0 & h_1^d \\ 0 & h_2^d \end{pmatrix}\begin{pmatrix} u_R \\ d_R \end{pmatrix}.
\end{aligned}
$$

- Introduced a non-square Yukawa coupling matrix $\mathcal{H}$.
- The three independent realizations of the custodial symmetry can be identified as those satisfying $[\mathcal{U}_C^a, \mathcal{H}] = \mathbf{0}_{4\times 2}$, where the $Sp(4)$ generators in Φ-space are given by $K^a = \mathcal{U}_C^a \otimes \sigma^0$. [PSBD, Pilaftsis '14]

- By convention, choose $h_1^u = 0$. For Type-I (Type-II) 2HDM, $h_1^d (h_2^d) = 0$.
- Quark yukawa couplings w.r.t. the SM are given by

Coupling	Type-I	Type-II
$g_{ht\bar{t}}$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$
$g_{hb\bar{b}}$	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$
$g_{Ht\bar{t}}$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$
$g_{Hb\bar{b}}$	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$
$g_{at\bar{t}}$	$\cot\beta$	$\cot\beta$
$g_{ab\bar{b}}$	$-\cot\beta$	$\tan\beta$

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&= (\bar{u}_L\ ,\ \bar{d}_L)\left(\phi_1\ ,\ \phi_2\ ,\ \widetilde{\phi}_1\ ,\ \widetilde{\phi}_2\right)\begin{pmatrix} h_1^u & 0 \\ h_2^u & 0 \\ 0 & h_1^d \\ 0 & h_2^d \end{pmatrix}\begin{pmatrix} u_R \\ d_R \end{pmatrix} .
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Maximally Symmetric 2HDM

- In the $SO(5)$-symmetric limit, $\lambda_2 = \lambda_1,\ \lambda_3 = 2\lambda_1,\ \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0$.
- A single quartic coupling λ:

$$V \ = \ -\mu^2(|\phi_1|^2 + |\phi_2|^2) + \lambda(|\phi_1|^2 + |\phi_2|^2)^2.$$

- Four Goldstone bosons $(h, a, h^{\pm})$, while $M_H^2 = 2\lambda_2 v^2$ and $\alpha = \beta$.
- **Natural alignment limit.**

- Custodial symmetry broken by g' and Yukawa couplings, as in the SM.

$$SO(5) \xrightarrow{g' \neq 0} O(3) \otimes O(2) \xrightarrow{y_t \neq y_b} O(2) \otimes O(2)$$

- Not enough for a Higgs spectrum satisfying the experimental constraints.
- Must include soft breaking by $\mathrm{Re}(m_{12}^2) \neq 0$.

Maximally Symmetric 2HDM

- In the $SO(5)$-symmetric limit, $\lambda_2 = \lambda_1,\ \lambda_3 = 2\lambda_1,\ \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0$.
- A single quartic coupling λ:

$$V \;=\; -\mu^2(|\phi_1|^2 + |\phi_2|^2) + \lambda(|\phi_1|^2 + |\phi_2|^2)^2.$$

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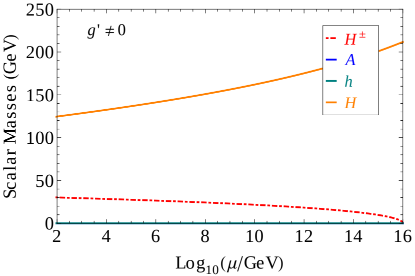
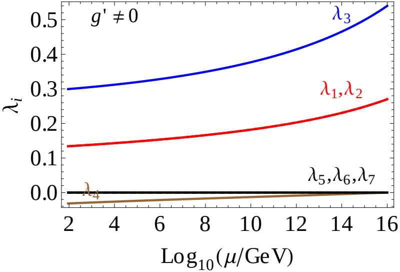
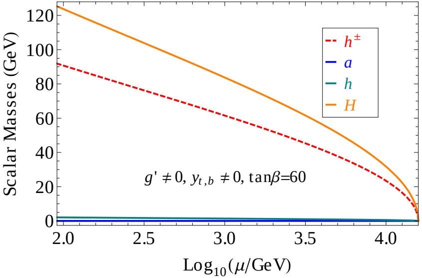
g' Effect

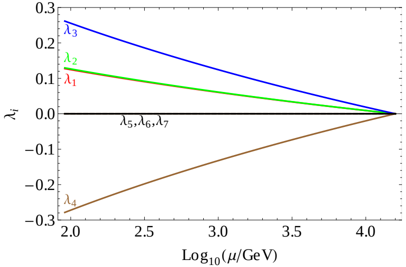




No.	Symmetry	Generators $T^a \leftrightarrow K^a$	Discrete group elements	Maximally broken SO(5) generators	Number of pseudo-Goldstone bosons
1	$Z_2 \times O(2)$	T^0	D_{CP1}	–	0
2	$(Z_2)^2 \times SO(2)$	T^0	D_{Z_2}	–	0
3	$(Z_2)^3 \times O(2)$	T^0	D_{CP2}	–	0
4	$O(2) \times O(2)$	T^3, T^0	–	T^3	1 (a)
5	$Z_2 \times [O(2)]^2$	T^2, T^0	D_{CP1}	T^2	1 (h)
6	$O(3) \times O(2)$	$T^{1,2,3}, T^0$	–	$T^{1,2}$	2 (h, a)
7	$SO(3)$	$T^{0,4,6}$	–	$T^{4,6}$	2 $(h^\pm)$
8	$Z_2 \times O(3)$	$T^{0,4,6}$	$D_{Z_2} \cdot D_{CP2}$	$T^{4,6}$	2 $(h^\pm)$
9	$(Z_2)^2 \times SO(3)$	$T^{0,5,7}$	$D_{CP1} \cdot D_{CP2}$	$T^{5,7}$	2 $(h^\pm)$
10	$O(2) \times O(3)$	$T^3, T^{0,8,9}$	–	T^3	1 (a)
11	$SO(4)$	$T^{0,3,4,5,6,7}$	–	$T^{3,5,7}$	3 $(a, h^\pm)$
12	$Z_2 \times O(4)$	$T^{0,3,4,5,6,7}$	$D_{Z_2} \cdot D_{CP2}$	$T^{3,5,7}$	3 $(a, h^\pm)$
13	$SO(5)$	$T^{0,1,2,\ldots,9}$	–	$T^{1,2,8,9}$	4 $(h, a, h^\pm)$

Yukawa Coupling Effects





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Soft Breaking Effects

- In the $SO(5)$ limit for quartic couplings, but with $\mathrm{Re}(m_{12}^2) \neq 0$,

$$
\begin{aligned}
M_S^2 &= M_a^2 \begin{pmatrix} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{pmatrix} + 2\lambda_2 v^2 \begin{pmatrix} c_\beta^2 & s_\beta c_\beta \\ s_\beta c_\beta & s_\beta^2 \end{pmatrix} \\
&= \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} 2\lambda_2 v^2 & 0 \\ 0 & M_a^2 \end{pmatrix} \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \equiv O \widehat{M}_S^2 O^\mathsf{T} \,.
\end{aligned}
$$

$$
\boxed{M_H^2 = 2\lambda_2 v^2 \,, \quad \text{and} \quad M_h^2 = M_a^2 = M_{h^\pm}^2 = \frac{\mathrm{Re}(m_{12}^2)}{s_\beta c_\beta}}
$$

- For $\mathrm{Re}(m_{12}^2) \gg v^2$, obtain decoupling limit.
- For the general case,

$$
\widehat{M}_S^2 = \begin{pmatrix} 2v^2(\lambda_1 c_\beta^4 + \lambda_{34} s_\beta^2 c_\beta^2 + \lambda_2 s_\beta^4) & v^2 s_\beta c_\beta [s_\beta^2(2\lambda_2 - \lambda_{34}) - c_\beta^2(2\lambda_1 - \lambda_{34})] \\ v^2 s_\beta c_\beta [s_\beta^2(2\lambda_2 - \lambda_{34}) - c_\beta^2(2\lambda_1 - \lambda_{34})] & M_a^2 + 2v^2 s_\beta^2 c_\beta^2 (\lambda_1 + \lambda_2 - \lambda_{34}) \end{pmatrix}
$$

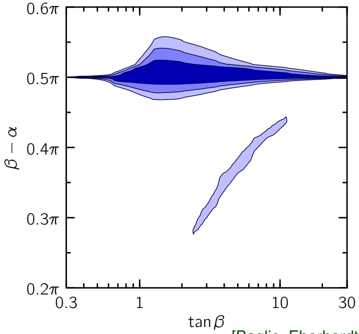
- Identify $\lambda_{\mathrm{SM}} = 2(\lambda_1 c_\beta^4 + \lambda_{34} s_\beta^2 c_\beta^2 + \lambda_2 s_\beta^4)$.
- Alignment obtained for $\tan^2\beta = \frac{2\lambda_1 - \lambda_{34}}{2\lambda_2 - \lambda_{34}}$, independent of M_a.
  (similar to [Gunion, Haber '03; Carena, Low, Shah, Wagner '13])

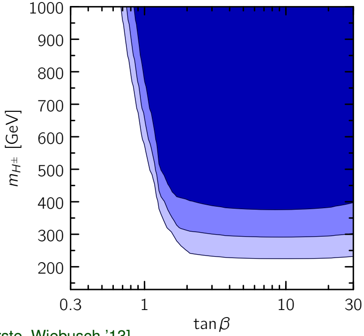
Theoretical and Experimental Constraints

- Stability of the potential: [Branco *et al* '12]

$$\lambda_{1,2} > 0, \quad \lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0, \quad \lambda_3 + \lambda_4 + \sqrt{\lambda_1 \lambda_2} - \mathrm{Re}(\lambda_5) > 0.$$

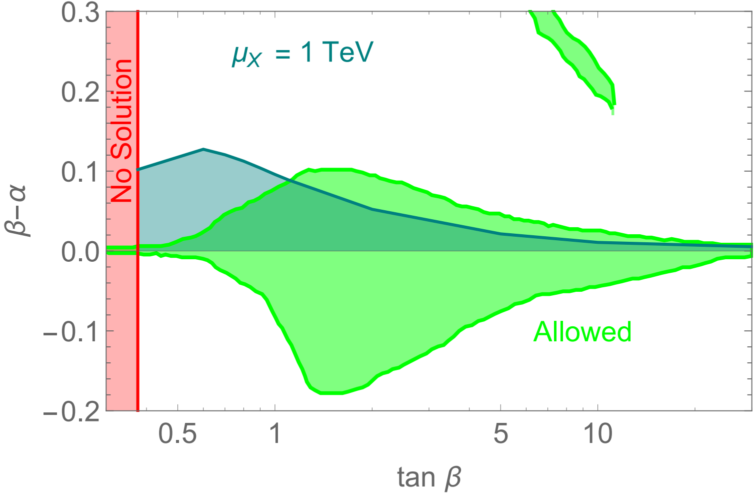
- Perturbativity of the Higgs self-couplings: $\|S_{\phi\phi\to\phi\phi}\| < \frac{1}{8}$.
- Electroweak precision observables.
- LHC signal strengths of the light *CP*-even Higgs boson.
- Limits on heavy *CP*-even scalar from $H \to WW, ZZ, \tau\tau$ searches.
- Flavor observables such as B_s mixing and $B \to X_s\gamma$.





[Baglio, Eberhardt, Nierste, Wiebusch '13]

With $SO(5)$ Boundary Conditions at μ_X



[PSBD, Pilaftsis (preliminary)]

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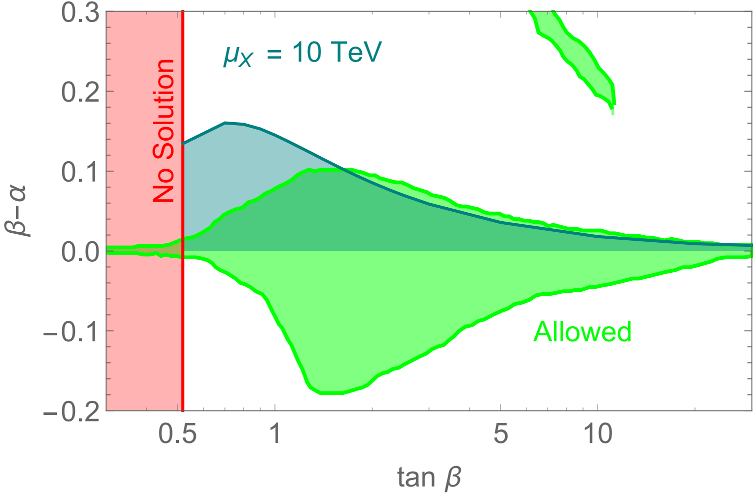

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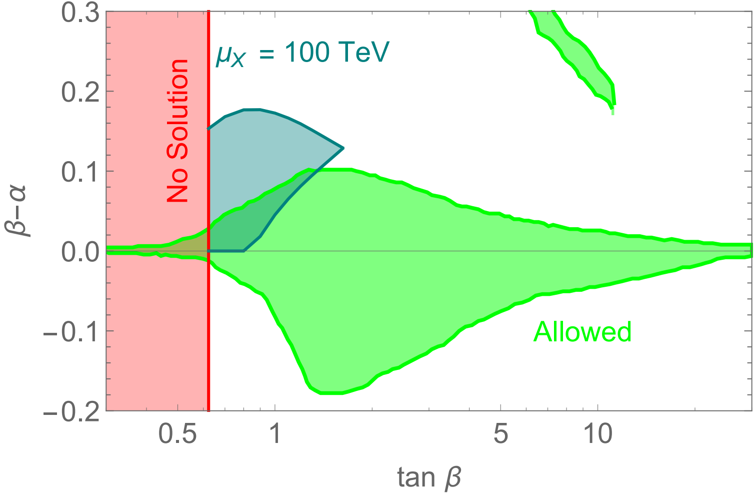
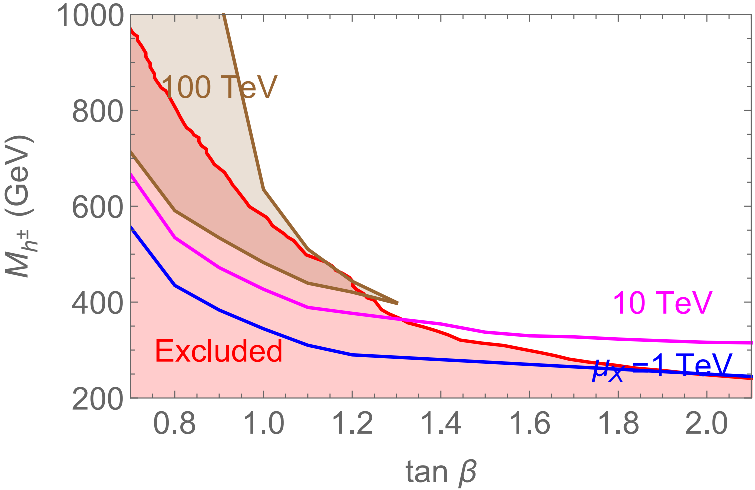


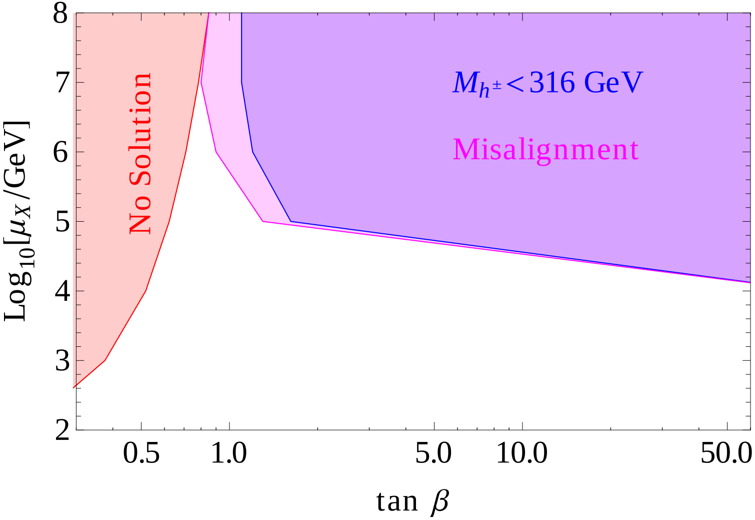
[PSBD, Pilaftsis (preliminary)]

Constraints on Higgs Sector



[PSBD, Pilaftsis (preliminary)]

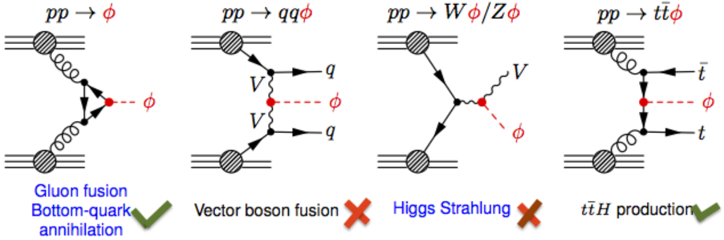
Constraints on tan β

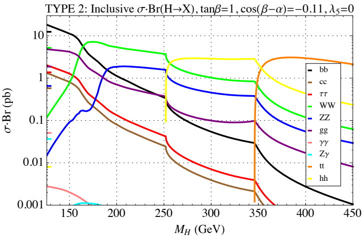


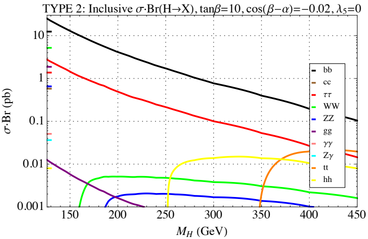
[PSBD, Pilaftsis (preliminary)]

Implications for LHC

Higgs production processes:



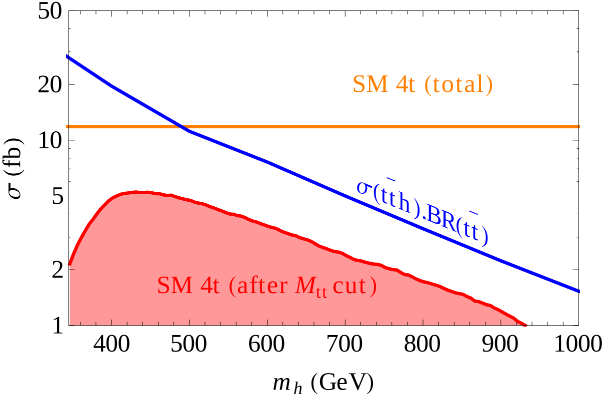




[Craig, Galloway, Thomas ’13]

Implications for LHC

- Promising Channels: Heavy Higgs $\to$ 2 Light Higgs, $t\bar{t}$ (low $\tan\beta$), $b\bar{b}$, $\tau\bar{\tau}$ (moderate-high $\tan\beta$).
- For $t\bar{t}$ mode, gluon fusion process not helpful (large background).
- $t\bar{t}h$ production mode, with $h \to t\bar{t}$ gives a unique $t\bar{t}t\bar{t}$ signal, with one $M_{t\bar{t}}$ around m_h.



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Conclusion

- 2HDM potential in the bilinear scalar field formalism.
- One-to-one correspondence between Φ-space and R-space.
- Maximal symmetric group is $SO(5)$.
- Alignment limit can be realized naturally, *independent* of other model parameters.
- Definite predictions for Higgs spectra.
- Interesting consequences at colliders.

THANK YOU.

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