The Weyl Consistency Conditions & Standard Model Vacuum Stability

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based on arXiv:1306.3234, in collaboration with

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Outline

- ◇ The local renormalisation group & Weyl consistency conditions
- ◇ Consequences: relations among the ß functions
- ◇ An example: stability of the Standard Model vacuum

The power of conformal symmetry

Conformal transformation ≡ local version of scale transformation

$$\gamma_{\mu\nu} \to \Omega(x)\gamma_{\mu\nu}$$

(obviously defined in curved space, but consequences in flat space)

The conformal symmetry is broken at the quantum level:
the renormalised couplings depend on a scale

$$g_i(\mu) \to g_i\left(\Omega(x)^{-1/2}\,\mu\right)$$

Must consider space-time dependent couplings $g_i(x)$

$\Longrightarrow$ they act as sources for the composite operators $\mathcal{O}^i$

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Renormalisation in curved space

In the presence of a curved background, additional counterterms are needed to make the theory finite:

$$W_{\text{flat}} = \log\left[\int \mathcal{D}\Phi\, e^{iS_{\text{renormalised}}+iS_{\text{counterterms}}}\right]$$

renormalised generating functional in flat space

$$W_{\text{curved}} = W_{\text{flat}} + \int \mathrm{d}^4x\sqrt{-\gamma}\,\left[Z_a\,E + Z_b\,R^2 + Z_c\,W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right]$$

4d curvature terms

Under a Weyl transformation: $\Delta_\sigma \equiv \int \mathrm{d}^4x\,\sigma(x)\left(2\gamma_{\mu\nu}\frac{\delta}{\delta\gamma_{\mu\nu}} - \beta_i\frac{\delta}{\delta g_i}\right)$

$$\Delta_\sigma W_{\text{curved}} = \int \mathrm{d}^4x\sqrt{-\gamma}\,\sigma\left[a\,E + b\,R^2 + c\,W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}\right]$$

Weyl anomaly

$$\Longleftrightarrow \quad T^\mu_\mu = \beta_i\mathcal{O}^i + a\,E + b\,R^2 + c\,W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}$$

Renormalisation with local couplings

H. Osborn (1987-1989)

With space-dependent couplings, even more counterterms are needed, proportional to $\partial_\mu g_i(x)$

Weyl anomaly:

Einstein tensor $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}\gamma^{\mu\nu} R$

$a,\ \chi^{ij},\ \omega^i, \ldots$: Functions of the couplings g_i

$$\Delta_\sigma W = \int \mathrm{d}^4x \sqrt{-\gamma}\, [\sigma\, a\, E + G^{\mu\nu} \left(\sigma\, \chi^{ij}\, \partial_\mu g_i\, \partial_\nu g_j + \partial_\mu \sigma\, \omega^i\, \partial_\nu g_i\right) + \ldots]$$

There are 16 diffeomorphism-invariant terms that include curvature tensors and derivatives of the couplings

We neglected here anomalous flavour currents that can lead to limit cycles

Fortin, Grinstein, Stergiou (2012)
Luty, Polchinski, Rattazzi (2012)

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The Weyl consistency conditions

I. Jack, H. Osborn (1990-1991)

The Weyl anomaly has to be abelian:

$$\Delta_\tau \Delta_\sigma W = \Delta_\sigma \Delta_\tau W$$

Gives a number of consistency relations among the functions $a,\ \chi^{ij},\ \omega^i,\ \ldots$

e.g. $$\frac{\partial \tilde{a}}{\partial g_i} = \beta_j \left(\chi^{ij} + \frac{\partial \omega^i}{\partial g_j} - \frac{\partial \omega^j}{\partial g_i} \right) \qquad \tilde{a} = a - \omega^i \beta_i$$

Note that $\frac{d\tilde{a}}{d\mu} > 0$ if $\chi^{ij} > 0$

$\Longrightarrow$ $\tilde{a}$ theorem

In general, ω^i is an exact one-form at the leading orders in perturbation theory

$$\frac{\partial \tilde{a}}{\partial g_i} \approx \chi^{ij} \beta_j \quad \Leftrightarrow \quad \beta_i \approx \chi_{ij} \frac{\partial \tilde{a}}{\partial g_j}$$

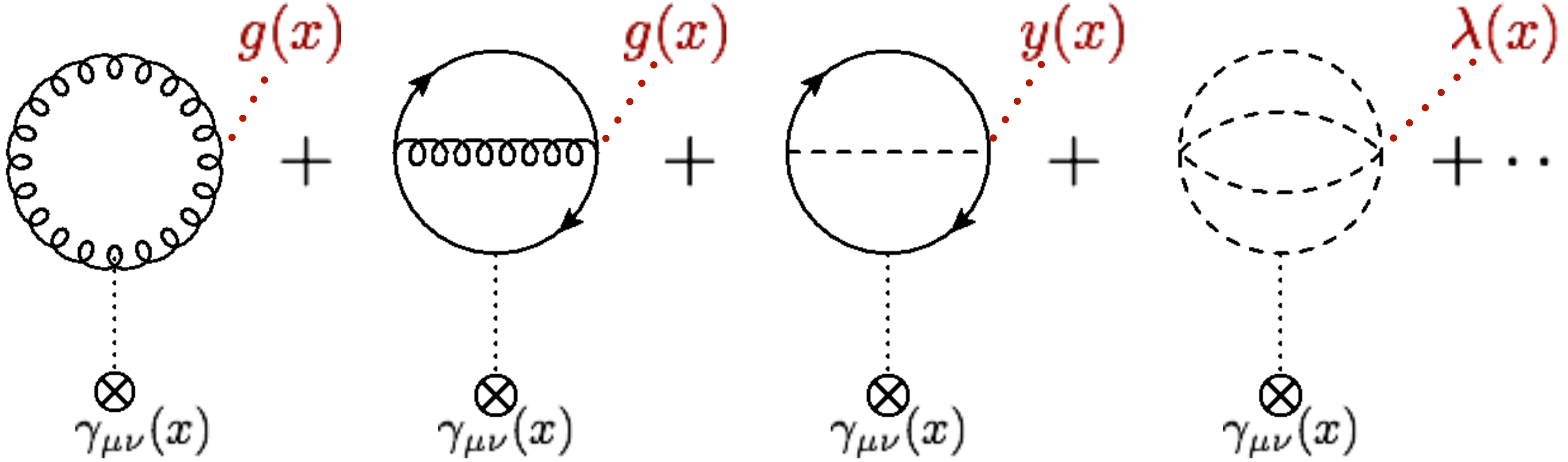
computable in flat space!

The RG flow is a gradient flow in a space with metric χ^{ij}

In terms of Feynman diagrams

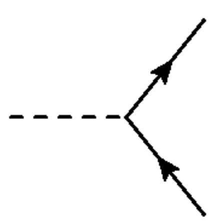
a is equal to the trace of the energy-momentum tensor on a 4-sphere:

$$a = \left\langle T^{\mu}_{\mu} \right\rangle_{S^4} =$$



Partial derivatives are equivalent to removing one interaction vertex

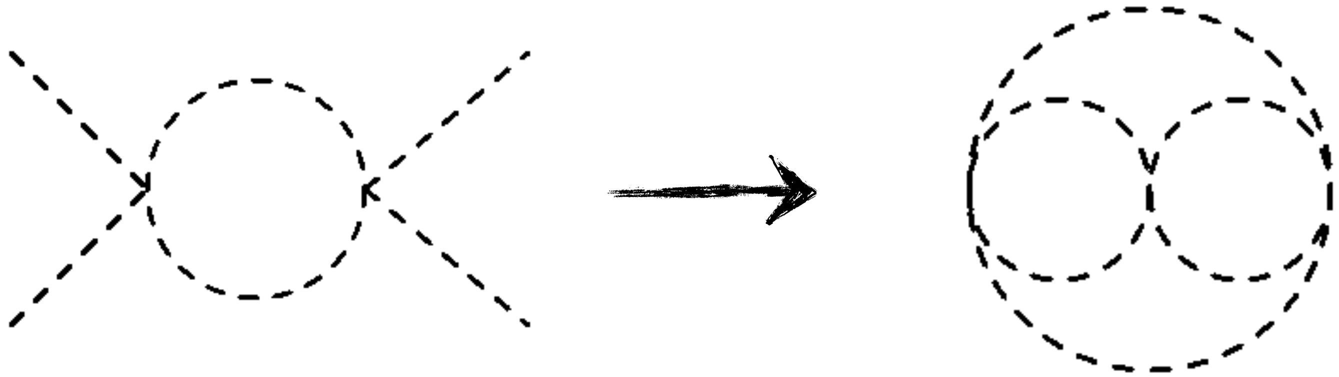
$$\beta_i \approx \chi_{ij} \frac{\partial \tilde{a}}{\partial g_j} \qquad \frac{\partial}{\partial g_i} \to \frac{\delta}{\delta g_i(x)}$$

$$\beta_y =$$  $$+ \cdots$$

$$\beta_\lambda =$$  $$+ \cdots$$

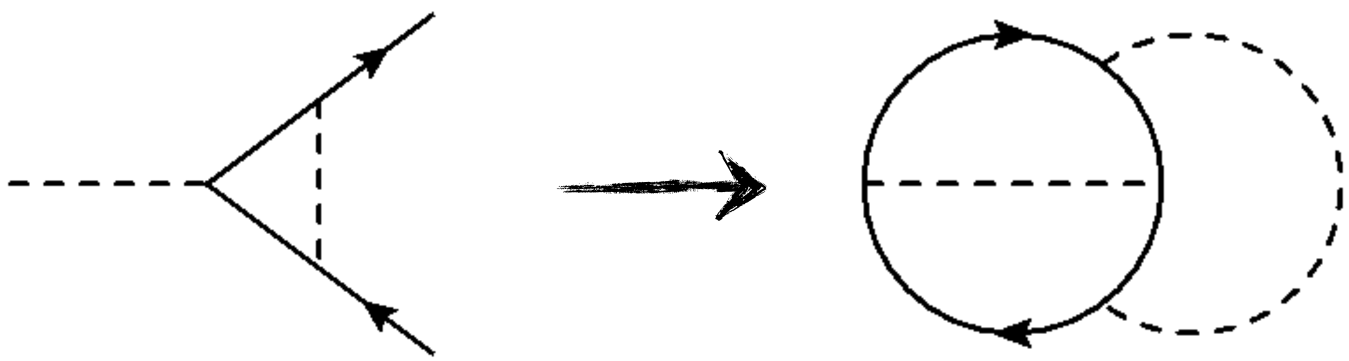
Counting loops

◇ One-loop β function of a scalar quartic interaction



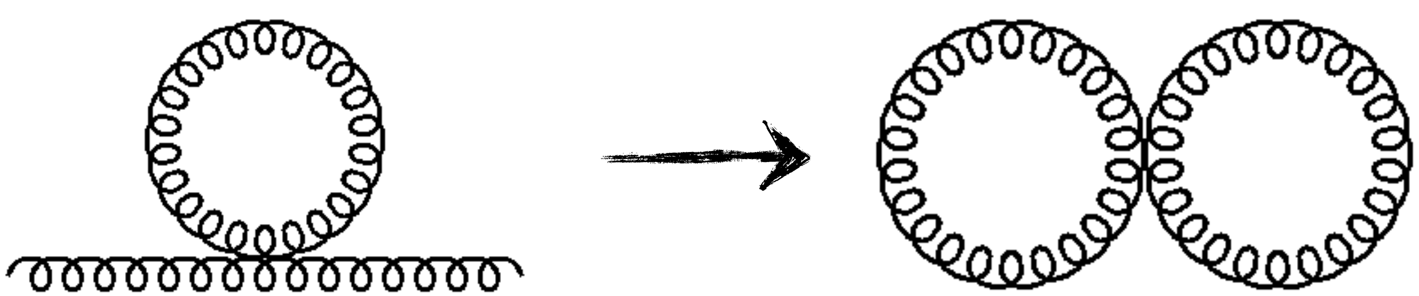
4-loops diagram

◇ One-loop β function of a Yukawa interaction



3-loops diagram

◇ One-loop β function of a gauge interaction



2-loops diagram

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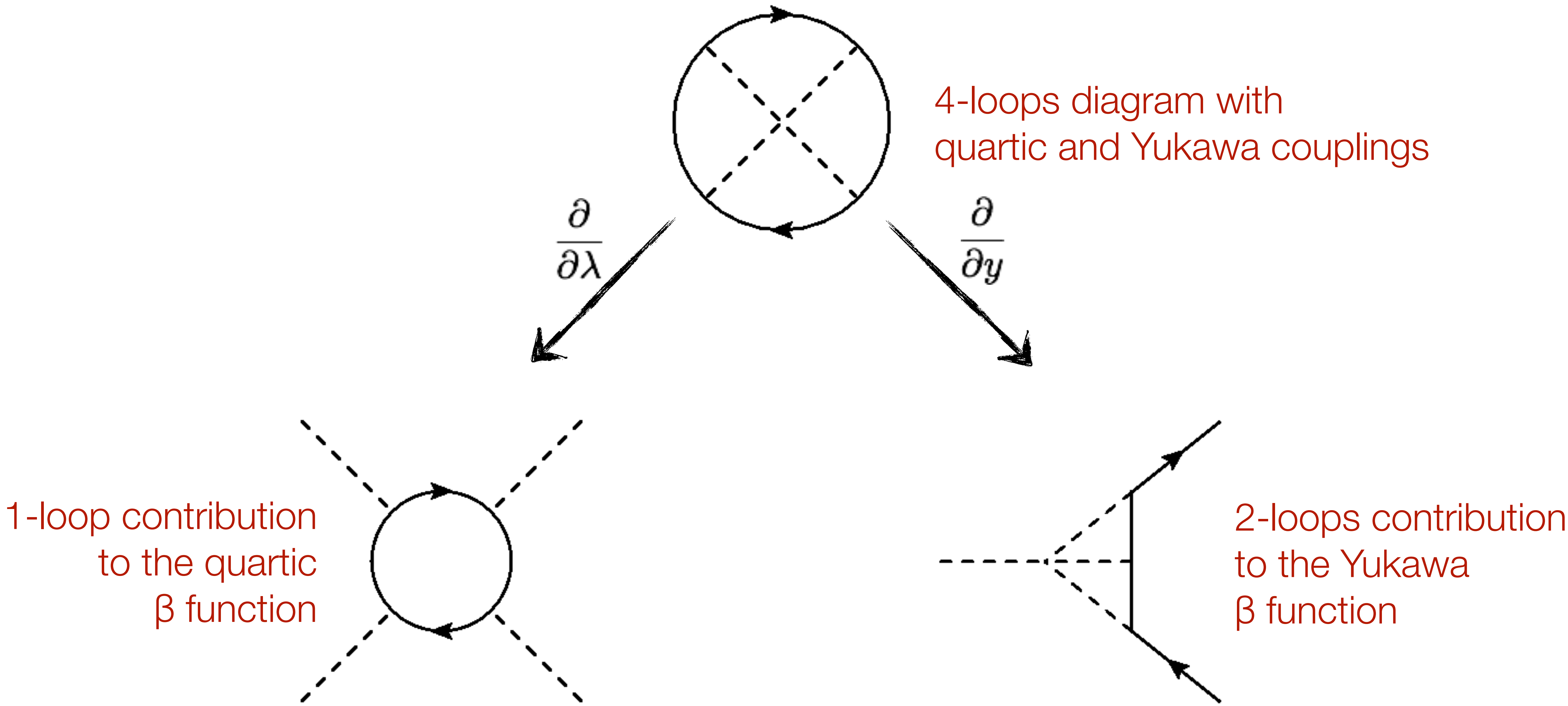
Multiple couplings

What about diagrams involving multiple couplings?



$$\frac{\partial^2 \tilde{a}}{\partial g_i \partial g_j} \approx \frac{\partial}{\partial g_i}\left(\chi^{jk}\beta_k\right) \approx \frac{\partial}{\partial g_j}\left(\chi^{ik}\beta_k\right)$$

An example: the Standard Model

Neglecting all Yukawa coupling apart from the top one, the theory has five couplings:

$$\left\{\alpha_1, \alpha_2, \alpha_3, \alpha_t, \alpha_\lambda\right\} \equiv \left\{\frac{g_1^2}{(4\pi)^2}, \frac{g_2^2}{(4\pi)^2}, \frac{g_3^2}{(4\pi)^2}, \frac{y_t^2}{(4\pi)^2}, \frac{\lambda}{(4\pi)^2}\right\}$$

The metric is diagonal at lowest order

Jack, Osborn (1990)

$$\chi^{ij} = \mathrm{diag}\left(\frac{1}{\alpha_1^2}, \frac{3}{\alpha_2^2}, \frac{8}{\alpha_3^2}, \frac{2}{\alpha_t}, 4\right)$$

Gives a set of relations among the β functions, e.g.

$$2\frac{\partial}{\partial\alpha_t}\beta_\lambda = \frac{\partial}{\partial\alpha_\lambda}\left(\frac{\beta_t}{\alpha_t}\right) + \mathcal{O}\left(\alpha_i^2\right),$$

1-loop

2-loop

$$\frac{3}{8}\frac{\partial}{\partial\alpha_3}\left(\frac{\beta_2}{\alpha_2^2}\right) = \frac{\partial}{\partial\alpha_2}\left(\frac{\beta_3}{\alpha_3^2}\right) + \mathcal{O}\left(\alpha_i^2\right),$$

$$\ldots$$

The Standard Model β functions

$$\beta_1 = 2\alpha_1^2\Big\{\frac{1}{12} + \frac{10n_G}{9} + \left(\frac{1}{4} + \frac{95n_G}{54}\right)\alpha_1 + \left(\frac{3}{4} + \frac{n_G}{2}\right)\alpha_2 + \frac{22n_G}{9}\alpha_3 + \left(\frac{163}{1152} - \frac{145n_G}{81} - \frac{5225n_G^2}{1458}\right)\alpha_1^2$$
$$+ \left(\frac{87}{64} - \frac{7n_G}{72}\right)\alpha_1\alpha_2 - \frac{137n_G}{162}\alpha_1\alpha_3 + \left(\frac{3401}{384} + \frac{83n_G}{36} - \frac{11n_G^2}{18}\right)\alpha_2^2 + \left(\frac{1375n_G}{54} - \frac{242n_G^2}{81}\right)\alpha_3^2 - \frac{n_G}{6}\alpha_2\alpha_3$$
$$+ \alpha_t\left[-\frac{17}{12} - \frac{2827}{576}\alpha_1 - \frac{785}{64}\alpha_2 - \frac{29}{6}\alpha_3 + \left(\frac{113}{32} + \frac{101n_t}{16}\right)\alpha_t\right] + \alpha_\lambda\left(\frac{3}{4}\alpha_1 + \frac{3}{4}\alpha_2 - \frac{3}{2}\alpha_\lambda\right)\Big\}$$

relations between the 2-loop gauge β functions

$$\beta_2 = 2\alpha_2^2\Big\{-\frac{43}{12} + \frac{2n_G}{3} + \left(\frac{1}{4} + \frac{n_G}{6}\right)\alpha_1 + \left(-\frac{259}{12} + \frac{49n_G}{6}\right)\alpha_2 + 2n_G\alpha_3 + \left(\frac{163}{1152} - \frac{35n_G}{54} - \frac{55n_G^2}{162}\right)\alpha_1^2$$
$$+ \left(\frac{187}{64} + \frac{13n_G}{24}\right)\alpha_1\alpha_2 - \frac{n_G}{18}\alpha_1\alpha_3 + \left(-\frac{667111}{3456} + \frac{3206n_G}{27} - \frac{415n_G^2}{54}\right)\alpha_2^2$$
$$+ \frac{13n_G}{2}\alpha_2\alpha_3 + \left(\frac{125n_G}{6} - \frac{22n_G^2}{9}\right)\alpha_3^2$$
$$+ \alpha_t\left[-\frac{3}{4} - \frac{593}{192}\alpha_1 - \frac{729}{64}\alpha_2 - \frac{7}{2}\alpha_3 + \left(\frac{57}{32} + \frac{45n_t}{16}\right)\alpha_t\right] + \alpha_\lambda\left(\frac{1}{4}\alpha_1 + \frac{3}{4}\alpha_2 - \frac{3}{2}\alpha_\lambda\right)\Big\}$$

$$\vdots$$

relations between the 3-loop gauge and 1-loop Higgs quartic β functions

$$\beta_\lambda = \frac{9}{16}\alpha_2^2 - \frac{9}{2}\alpha_\lambda\alpha_2 + \frac{3}{16}\alpha_1^2 - \frac{3}{2}\alpha_\lambda\alpha_1 + \frac{3}{8}\alpha_1\alpha_2 + 12\alpha_\lambda^2 + 6\alpha_\lambda\alpha_t - 3\alpha_t^2 + \ldots$$

Precision running in the Standard Model

Knowing the value of the Standard Model couplings at an arbitrary energy scale is crucial: vacuum stability, grand unification, cosmology...

The state-of-the-art computations make use of the gauge, top Yukawa and Higgs quartic β functions at 3-loops order

Degrassi et al. (2012), Buttazzo et al. (2013)

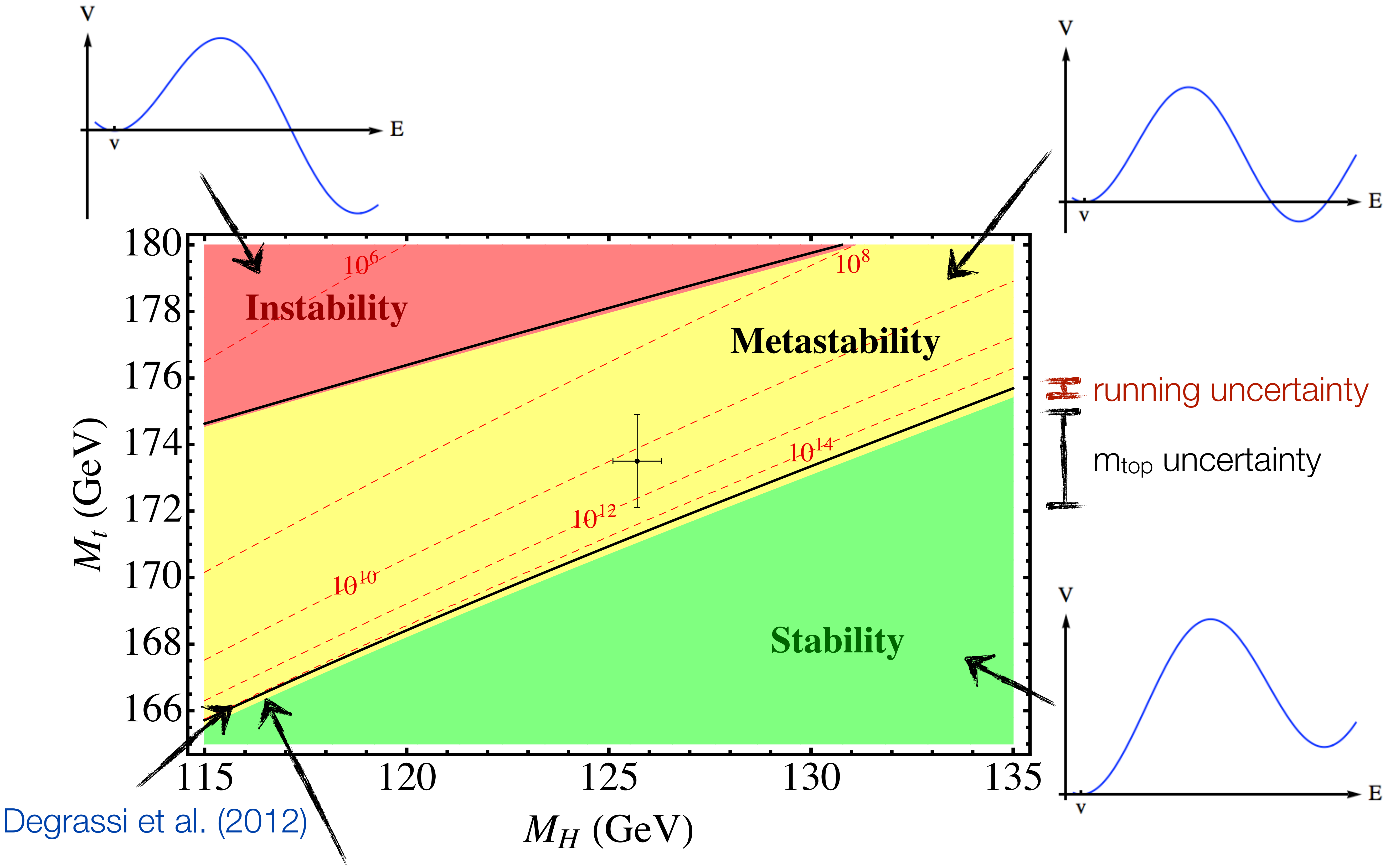
Inconsistent with the Weyl symmetry!

Already going to 2 loops in the Higgs quartic β functions means including diagrams that contributes to the 4-loop gauge β functions

The best Weyl-consistent running based on the existing computations:

◇ 3 loops in the gauge β functions
◇ 2 loops in the top Yukawa β function
◇ 1 loop in the Higgs quartic β function

Standard Model vacuum stability



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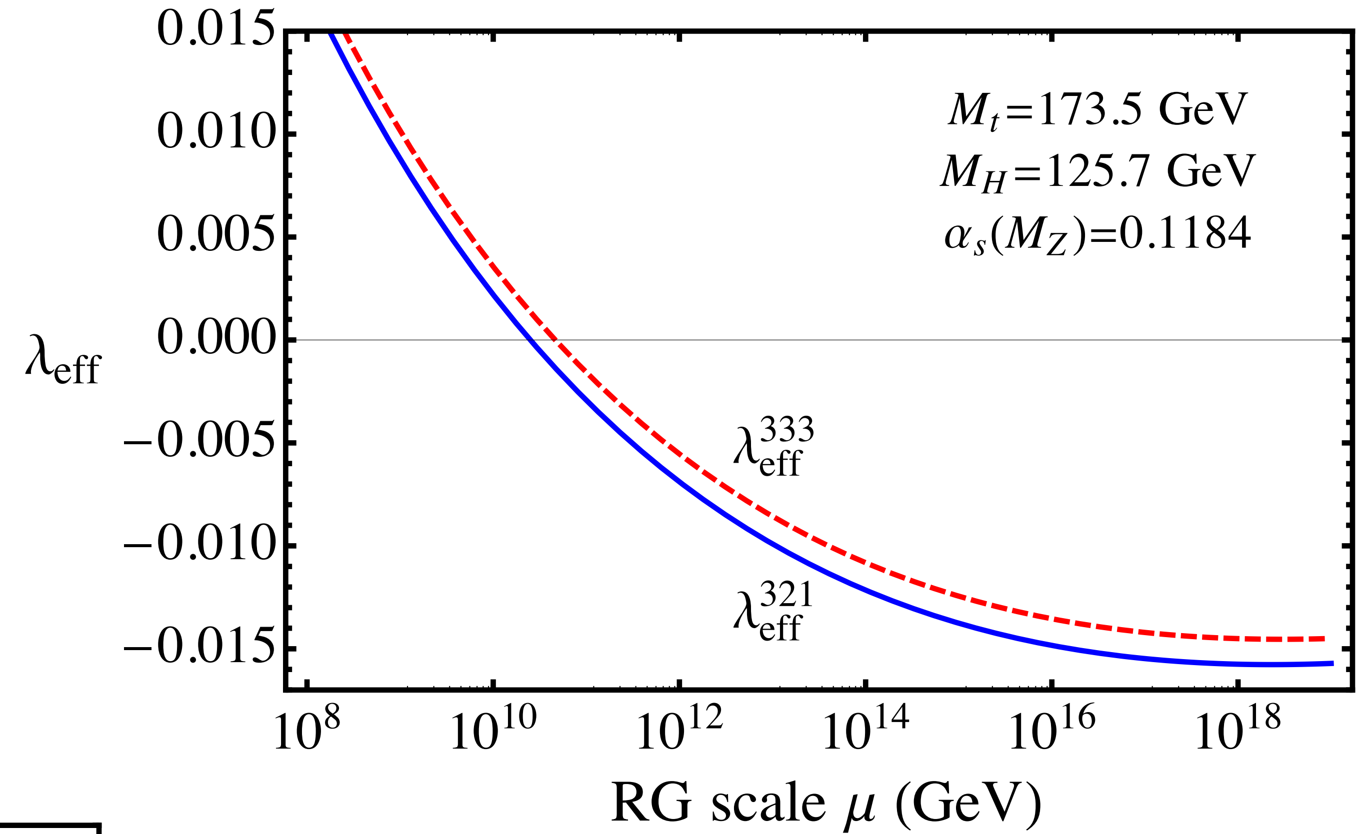
Importance of precision running

"Coincidence" in the SM:

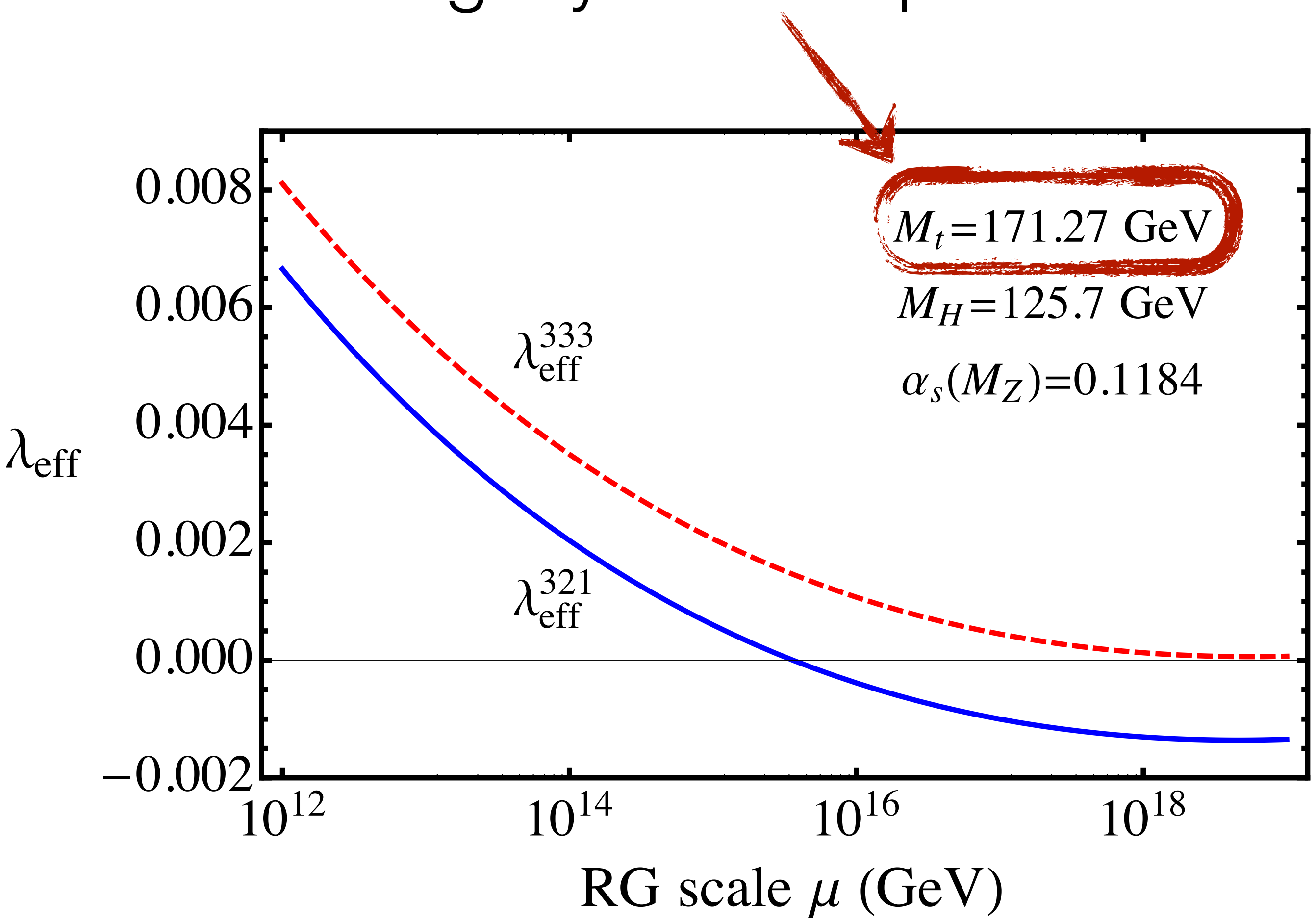
$$\lambda(\mu) = 0 \quad \text{and} \quad \frac{\mathrm{d}\lambda}{\mathrm{d}\mu}(\mu) = 0$$

happen around the same scale

→ Higgs inflation?



With a slightly lower top mass...



Important uncertainties also in:

- ◇ matching of $\overline{\text{MS}}$ parameters at the electroweak scale
- ◇ tunneling probability

Summary & Outlook

- The Weyl symmetry constrains the RG flow of any theory
- For theories with multiple couplings, it provides relations among the β functions at different loop order
- Precision computations should make use of a loop counting scheme consistent with the Weyl symmetry

- A new method to compute β functions? see next talk!
- Important for the search of perturbative fixed points in gauge-Yukawa theories Antipin, Gillioz, Mølgaard, Sannino (2013), …
- Ongoing work: Weyl consistency conditions for dim-6 operators in the Standard Model