Grand Unification with partial fine tuning

David J Miller
University of Glasgow

Based on work by DJM, António Morais JHEP 1310 (2013) 226 [arXiv:1307.1373] and in preparation.

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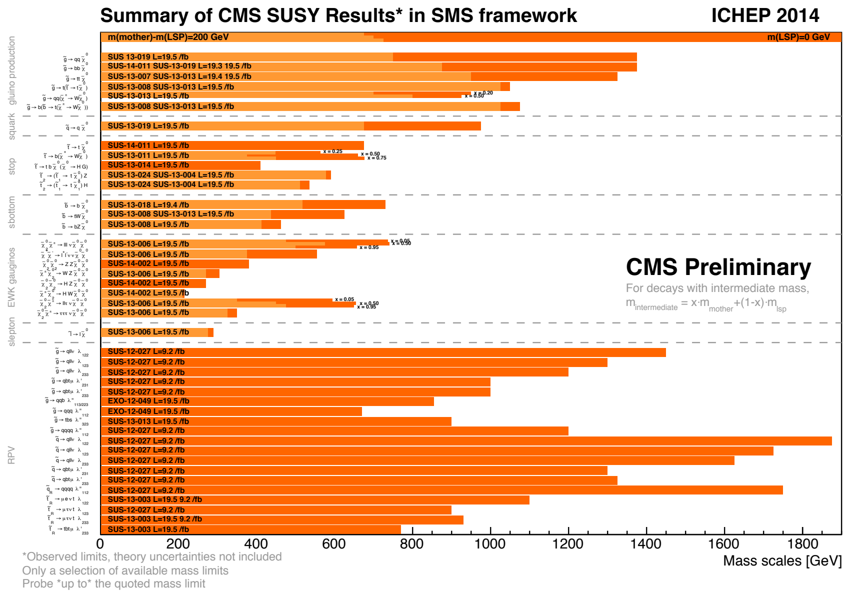
Introduction

LHC exclusions are pushing SUSY to higher energy.



There is still room for a lightish stop, but this is shrinking fast. What happens when it is gone?

Heavy mass spectrum → fine-tuning

What does this mean for GUT theories?

Fine-tuning in supersymmetry

At tree-level the Z-boson mass is given by

$$M_Z^2 = -2\left(m_{H_u}^2 + |\mu|^2\right) + \frac{2}{\tan^2\beta}\left(m_{H_d}^2 - m_{H_u}^2\right) + \mathcal{O}\left(1/\tan^4\beta\right)$$

If m_{H_u} or μ are large, natural fluctuations will give large fluctuations in M_Z.

Measure fine-tuning by $\Delta = \max\left\{\Delta_{\mathcal{P}_i}\right\}$ with $\Delta_{\mathcal{P}_i} = \left|\frac{\mathcal{P}_i}{M_Z^2}\frac{\partial M_Z^2}{\partial \mathcal{P}_i}\right|$

[Barbieri and Guidice, 1988]

Then $\Delta_\mu \approx \frac{4|\mu|^2}{M_Z^2}$

For $\Delta_\mu \lesssim 10$ we need to have $\mu \lesssim \sqrt{5/2}M_Z \approx 150\,\mathrm{GeV}$

Partial fine-tuning

But μ is an peculiar parameter anyway. It suffers from the **μ-problem**.

It is not a supersymmmetry breaking parameter like the other mass scales.

Could the susy fine-tuning problems be originating only from μ?

Note:

- I am not saying fine-tuning in μ is not a problem. It is. But maybe this problem is tied up with the μ-problem?
- I have no fix for this problem [neither Guidice-Masiero nor NMSSM help].
- This wouldn't work for the unconstrained MSSM since one would also have fluctuations in m_{H_u}. However, in **GUT models**, m_{H_u} is not a fundamental parameter either.

SO(10) GUTs

Breaking via SU(5)…

$$SO(10) \to SU(5) \times U(1)_X \to SU(3) \times SU(2) \times U(1)_Z \times U(1)_X \to G_{SM}$$

$$\mathbf{16} \to \mathbf{1}_{-5} \oplus \overline{\mathbf{5}}_3 \oplus \mathbf{10}_{-1},$$

$$\mathbf{10} \to \mathbf{5}_2 \oplus \overline{\mathbf{5}}_{-2},$$

$$\mathbf{1} \to (\mathbf{1},\mathbf{1})_0\,,$$

$$\mathbf{5} \to (\mathbf{1},\mathbf{2})_3 \oplus (\mathbf{3},\mathbf{1})_{-2}\,,$$

$$\overline{\mathbf{5}} \to (\mathbf{1},\mathbf{2})_{-3} \oplus (\overline{\mathbf{3}},\mathbf{1})_2\,,$$

$$\mathbf{10} \to (\mathbf{1},\mathbf{1})_6 \oplus (\overline{\mathbf{3}},\mathbf{1})_{-4} \oplus (\mathbf{3},\mathbf{2})_1\,,$$

…either “normal” or “flipped” ($e_R \leftrightarrow N_R$ and $u_R \leftrightarrow d_R$)

or via Pati-Salam…

$$SO(10) \to SU(4) \times SU(2)_L \times SU(2)_R \to SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_W \to G_{SM},$$

$$\mathbf{16} \to (\mathbf{4},\mathbf{2},\mathbf{1}) \oplus (\overline{\mathbf{4}},\mathbf{1},\mathbf{2})\,,$$

$$\mathbf{10} \to (\mathbf{1},\mathbf{2},\mathbf{2}) \oplus (\mathbf{6},\mathbf{1},\mathbf{1})\,,$$

$$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) = \begin{pmatrix} \hat{u}_x^\dagger & \hat{N}^\dagger \\ \hat{d}_x^\dagger & \hat{e}^\dagger \end{pmatrix}_R$$

$$(\mathbf{4},\mathbf{2},\mathbf{1}) = \begin{pmatrix} \hat{u}^x & \hat{\nu} \\ \hat{d}^x & \hat{e} \end{pmatrix}_L$$

$$(\mathbf{1},\mathbf{2},\mathbf{2}) = \begin{pmatrix} \hat{h}_u^+ & \hat{h}_d^0 \\ \hat{h}_u^0 & \hat{h}_d^- \end{pmatrix}$$

$$(\mathbf{4},\mathbf{2},\mathbf{1}) \to (\mathbf{1},\mathbf{2},\mathbf{1})_3 \oplus (\mathbf{3},\mathbf{2},\mathbf{1})_{-1}\,,$$

$$(\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) \to (\mathbf{1},\mathbf{1},\mathbf{2})_{-3} \oplus (\mathbf{3},\mathbf{1},\mathbf{2})_1\,,$$

$$(\mathbf{1},\mathbf{2},\mathbf{2}) \to (\mathbf{1},\mathbf{2},\mathbf{2})_0\,,$$

…again either “normal” or “flipped”

Boundary Conditions

Scalar masses:

for SU(5) only

$$m^2_{Q_{ij}}(0) = m^2_{u_{ij}}(0) = m^2_{e_{ij}}(0) = \begin{pmatrix} K_{\mathbf{16}} & 0 & 0 \\ 0 & K_{\mathbf{16}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \left(m^2_{\mathbf{16}} + g^2_{10} D\right), \quad \longleftarrow \quad m^2_{\mathbf{10}}$$

$$m^2_{L_{ij}}(0) = m^2_{d_{ij}}(0) = \begin{pmatrix} K_{\mathbf{16}} & 0 & 0 \\ 0 & K_{\mathbf{16}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \left(m^2_{\mathbf{16}} - 3g^2_{10} D\right), \quad \longleftarrow \quad m^2_{\overline{\mathbf{5}}}$$

$$m^2_{N_{ij}}(0) = \begin{pmatrix} K_{\mathbf{16}} & 0 & 0 \\ 0 & K_{\mathbf{16}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \left(m^2_{\mathbf{16}} + 5g^2_{10} D\right),$$

$$m^2_{H_u}(0) = m^2_{\mathbf{10}+\mathbf{126}} - 2g^2_{10} D, \quad \longleftarrow \quad m^2_{\mathbf{5}'}$$

$$m^2_{H_d}(0) = m^2_{\mathbf{10}+\mathbf{126}} + 2g^2_{10} D, \quad \longleftarrow \quad m^2_{\overline{\mathbf{5}}'}$$

Trilinear couplings: $a_t(0) = a_b(0) = a_\tau(0) = a_{\mathbf{10}}$ $\longleftarrow$ $a_t(0) = a_{\mathbf{5}'}$, $a_b(0) = a_\tau(0) = a_{\overline{\mathbf{5}}'}$.

The four different SO(10) embeddings give the same scalar masses and D-terms but give different gaugino masses. To quantify our non-universal Gaugino masses we set: $M_1/\rho_1 = M_2/\rho_2 = M_3 \equiv M_{1/2}$

Theoretical and Experimental Constraints

LHC susy constraints: $m_{\tilde{q}} > 1.7\,\mathrm{TeV}$, $m_{\tilde{g}} > 1.2\,\mathrm{TeV}$

$m_{\tilde{q}} > 1.4\,\mathrm{TeV}$

$m_{\tilde{g}} > 0.8\,\mathrm{TeV}$

for SU(5)

LHC Higgs mass constraint $m_H = 125.7 \pm 2.1\mathrm{GeV}$

Direct Dark Matter constraint from **LUX** (XENON100 for SU(5))

Relic Abundance $\Omega_c h^2 = 0.1157 \pm 0.0023$ (WMAP)

Other low energy constraints from $b \to s\gamma,\ B_s \to \mu^+\mu^-,\ B \to \tau\nu_\tau,\ a_\mu$

$$P_{\rm tot} = P_{m_h} \cdot P_{\Omega_c h} \cdot P_{b\to s\gamma} \cdot P_{\mathcal{R}_{\tau\nu_\tau}} \cdot P_{B_s\to\mu\mu} \cdot P_{a_\mu} > 10^{-3}$$

Only if deviation greater than in SM

We also implement **vacuum stability** as described by Casas, Lleyda, Munoz (1996)

$[0\,\mathrm{TeV}, 4\,\mathrm{TeV}]$ $[-4\,\mathrm{TeV}, 4\,\mathrm{TeV}]$ $[1, 60]$

Inputs: $m_{16},\ m_{10+126},\ M_{1/2},\ g_{10}^2 D,\ a_{10},\ K_{16},\ \rho_1,\ \rho_2,\ \tan\beta,\ \mathrm{sign}\,\mu$

$[-10\,\mathrm{TeV}, 10\,\mathrm{TeV}]$ $[0, 15]$

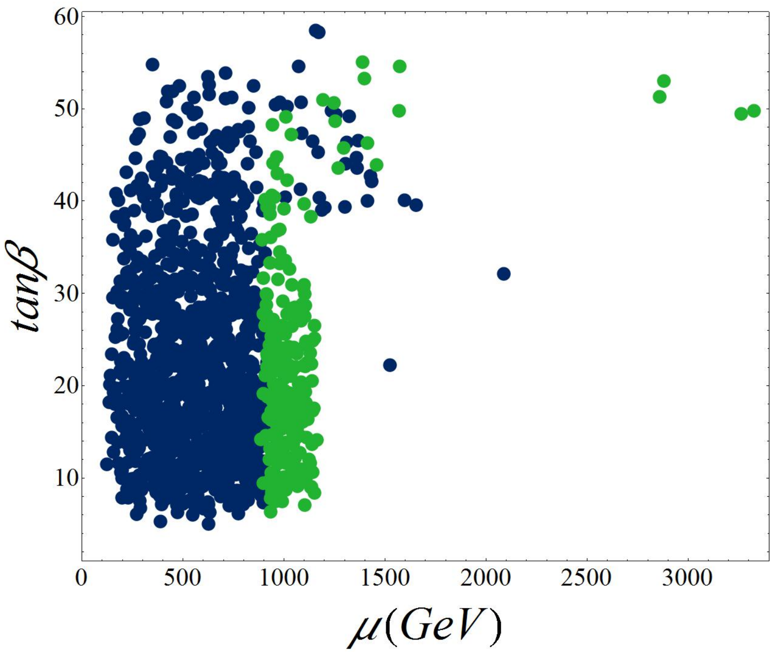
We used:

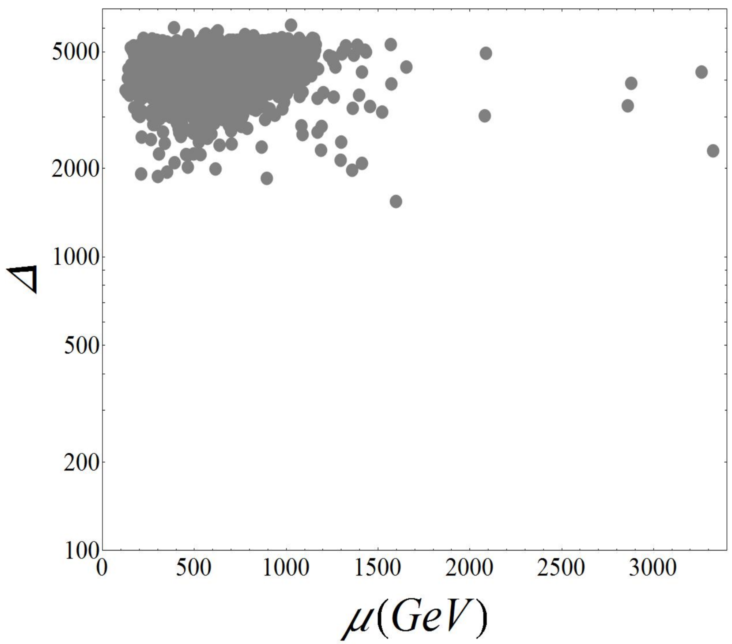
SOFTSUSY 3.3.0 (Allananch 2002) for the RGE running and fine-tuning measure.

micoOMEGAs 2.4.5 (Belanger et al 2006) for Relic density, Dark Matter nucleon cross-section and other low energy constraints.

Universal Gaugino Masses

First we looked at scenarios with universal gauging masses $\rho_1 = \rho_2 = 1$





Green points have the correct relic density, while **blue** points have too little.

Although there are plenty of viable points, we could only find ones that are fine-tuned, even neglecting fine-tuning from μ.

Non-Universal Gauginos

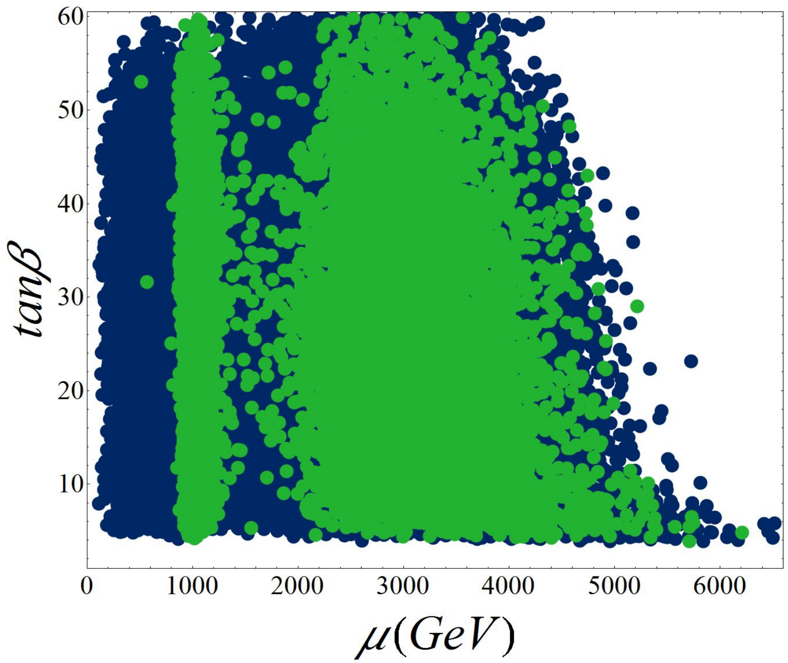
Generally one might expect the gauginos to have non-universal masses at the high scale. For example, if the symmetry is broken by some hidden sector field $\hat{X}$ with an F-term F_X then we generate masses of the form

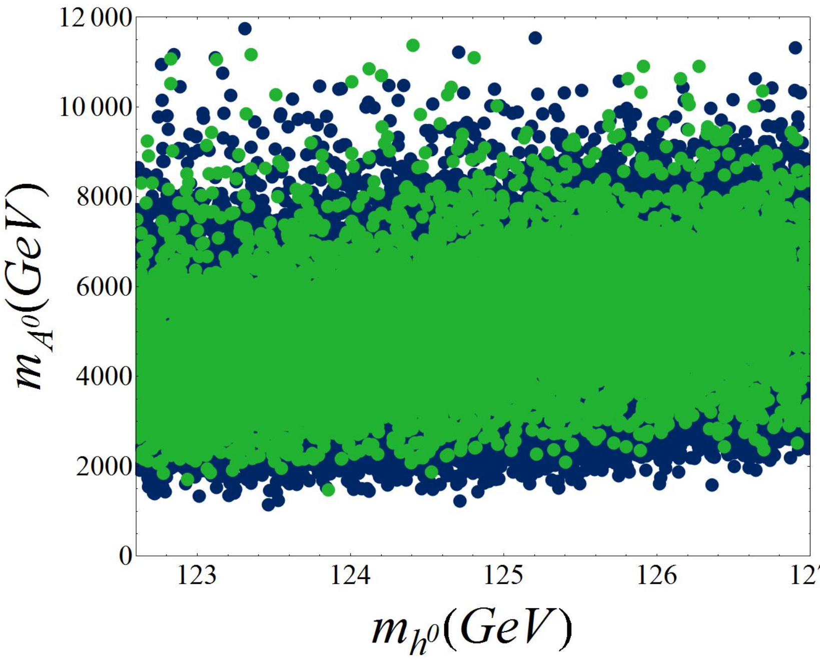
$$\frac{1}{2}\frac{\langle F_X^j \rangle}{\langle Ref_{\alpha\beta}\rangle}\left\langle \frac{\partial f_{\alpha\beta}^*}{\partial \varphi^{j*}}\right\rangle \tilde{\lambda}^\alpha \tilde{\lambda}^\beta$$

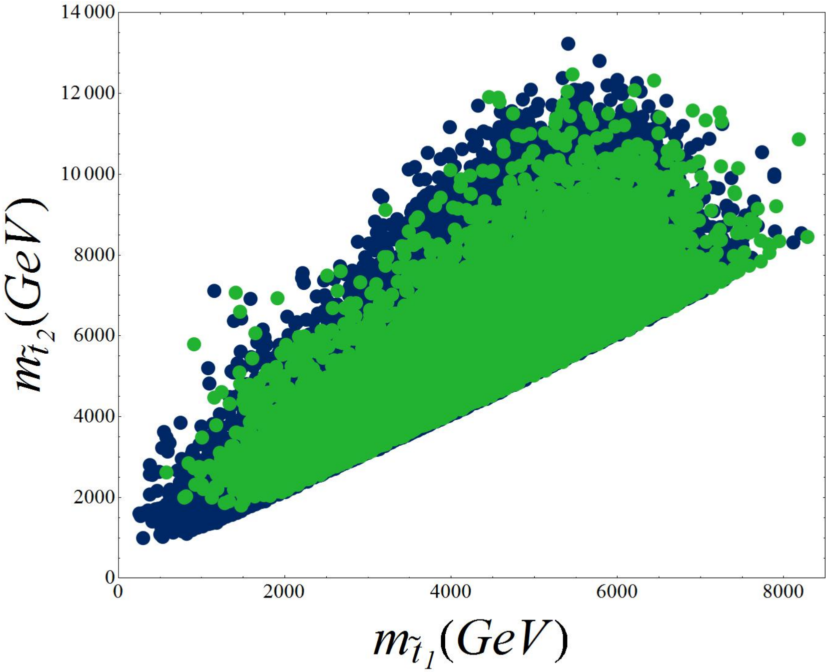
If $\hat{X}$ is a singlet, this gives **universal** gauginos, but if it is not we will find **non-universal** gaugino masses.

At the GUT scale we set

$$M_1/\rho_1 = M_2/\rho_2 = M_3 \equiv M_{1/2}$$



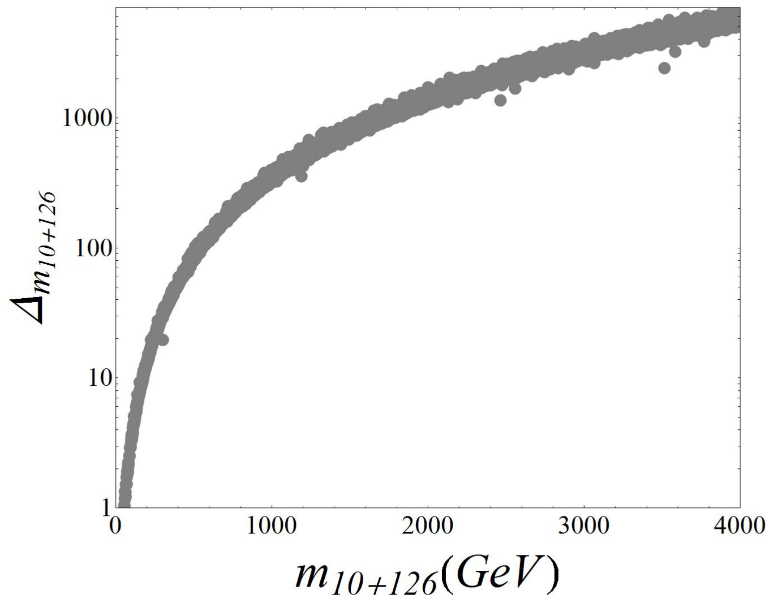


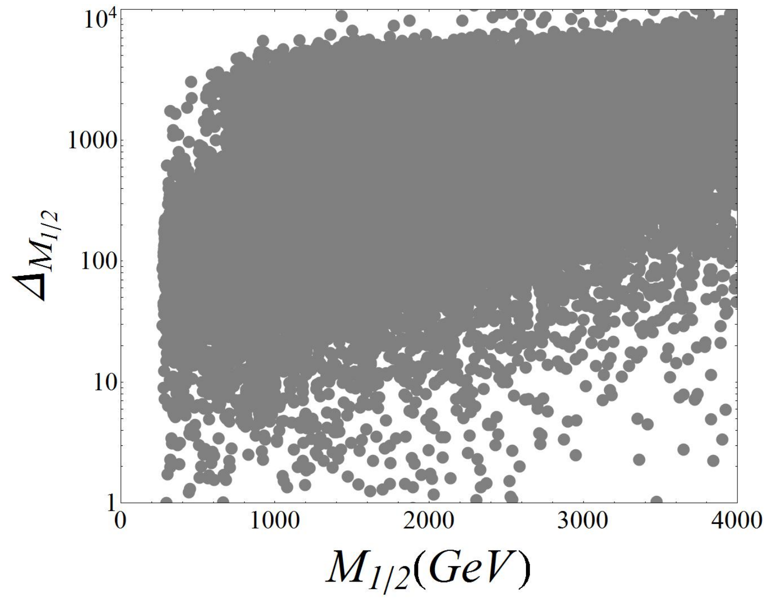


Lots of scenarios open up, some with quite light stops.

But it is very difficult to get a small μ and the correct relic density.

Fine-tuning



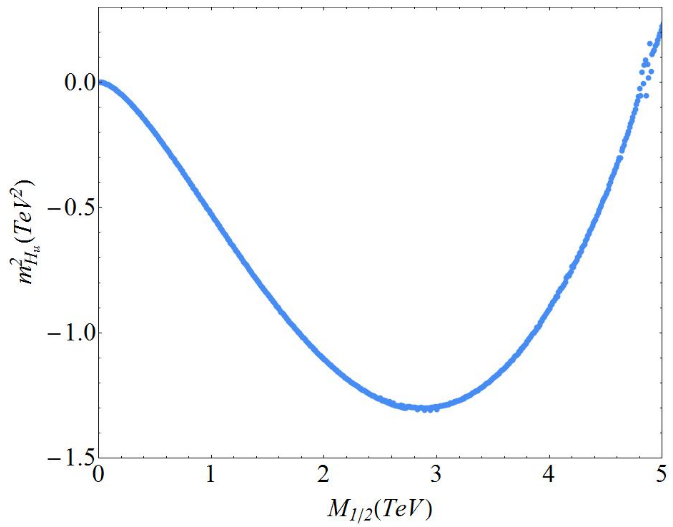


Fine-tuning arising from scalar masses (and D-terms, trilinears) grows with the mass but $M_{1/2}$ seems to allow low fine-tuning even for large values.

$$M_Z^2 = -2\left(m_{H_u}^2 + |\mu|^2\right) + \frac{2}{\tan^2\beta}\left(m_{H_d}^2 - m_{H_u}^2\right) + \mathcal{O}\left(1/\tan^4\beta\right)$$

m_{H_u} is not an input parameter. It is a complicated function of the other inputs.

If we set all the masses other than $M_{1/2}$ to zero then one expects $m_{H_u}^2 = aM_{1/2}^2$

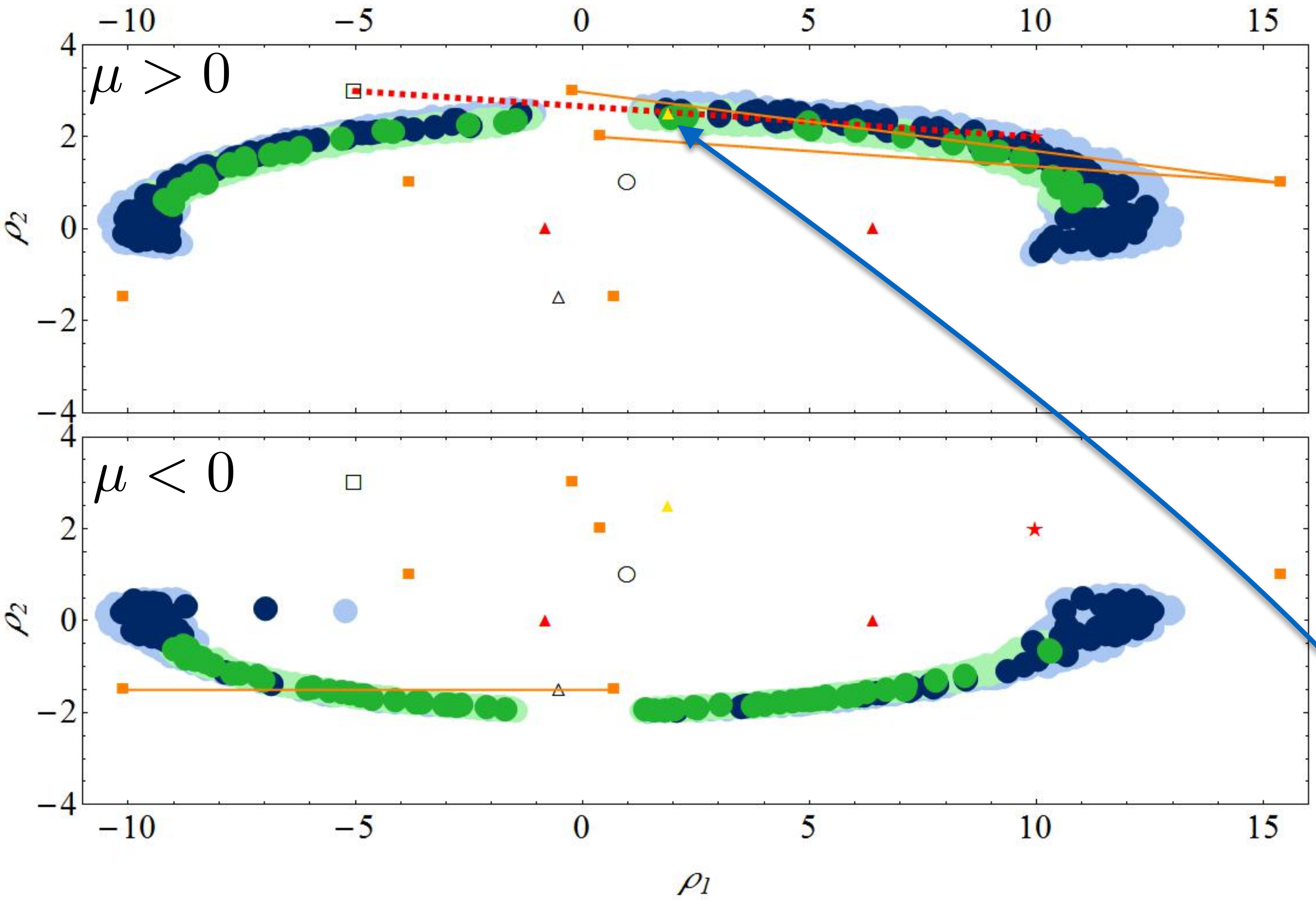


However, adding radiative corrections at the low scale, makes this more complicated and a also becomes $M_{1/2}$ dependent.

The dependence of m_{H_u} on $M_{1/2}$ gains a minimum.

This plot was made with SOFTSUSY. This behaviour persists also with Spheno, but the position of the minima moves.

Set the scalar masses and trilinear $< 150\,\mathrm{GeV}$ (they will fed by $M_{1/2}$ during running) and see what happens:



Light:

$10 < \Delta < 100$

Dark:

$\Delta < 10$

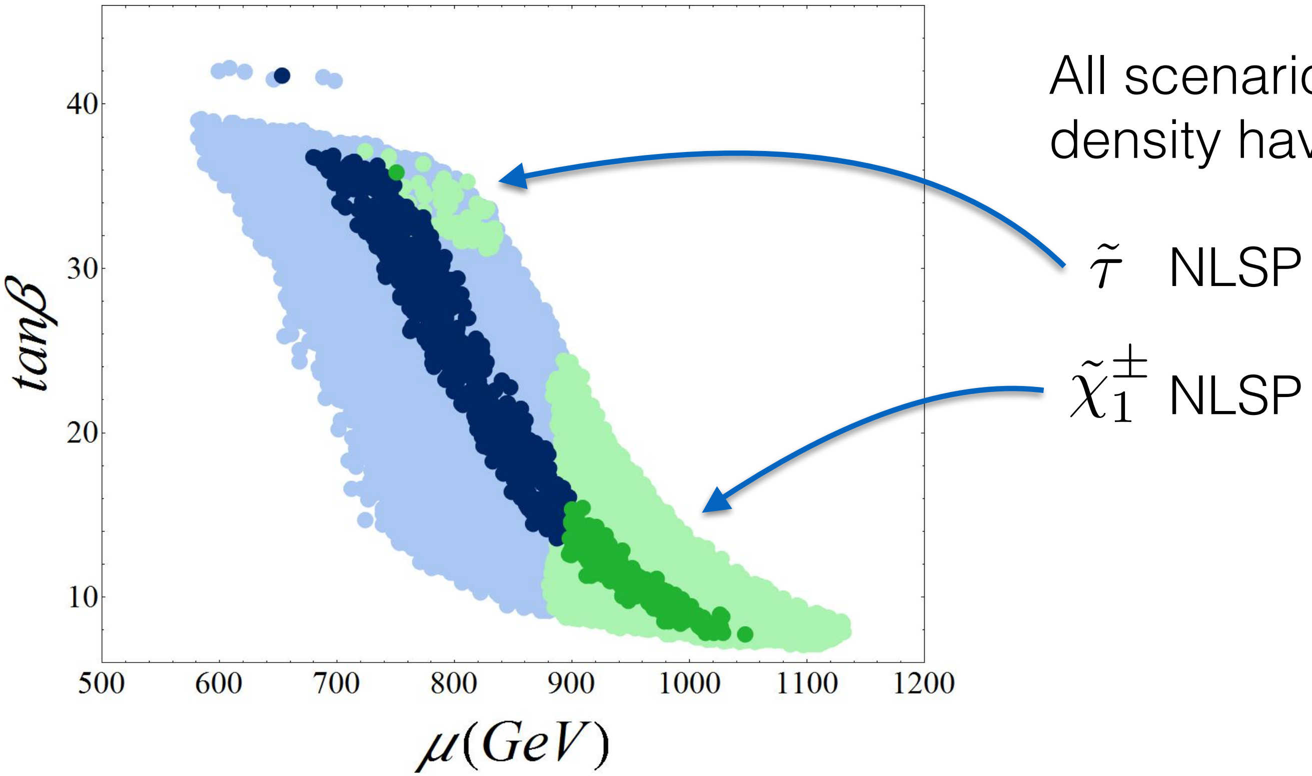
Each of the symbols is a different embedding at the GUT scale. For example, the yellow triangle is a PS embedding.

Pati-Salam Embedding

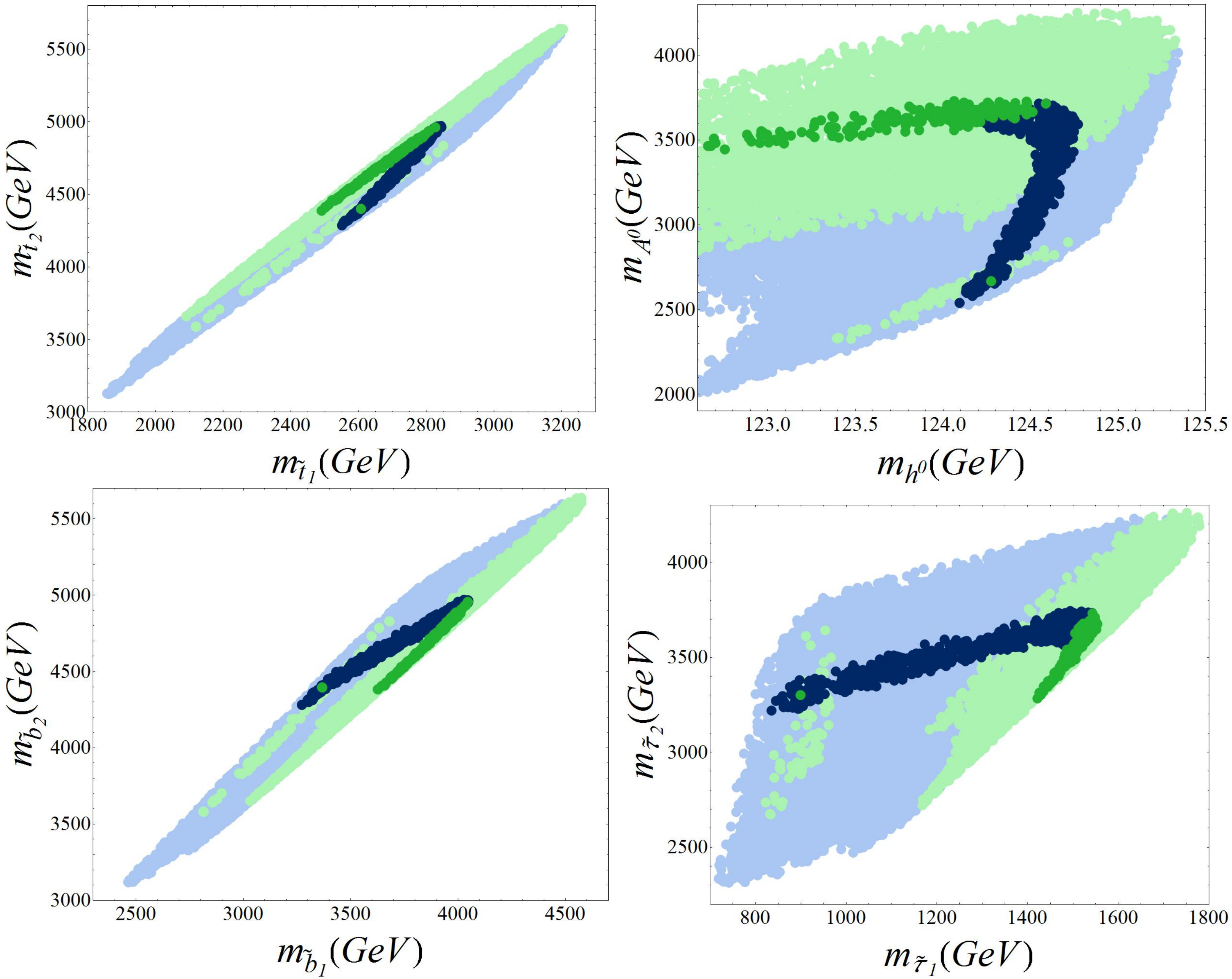
As an example, let's consider the PS breaking (the yellow triangle)

$$\begin{cases} SO(10) \to SU(4) \times SU(2)_R \\ \mathbf{770} \to (\mathbf{1}, \mathbf{1}) \end{cases}$$

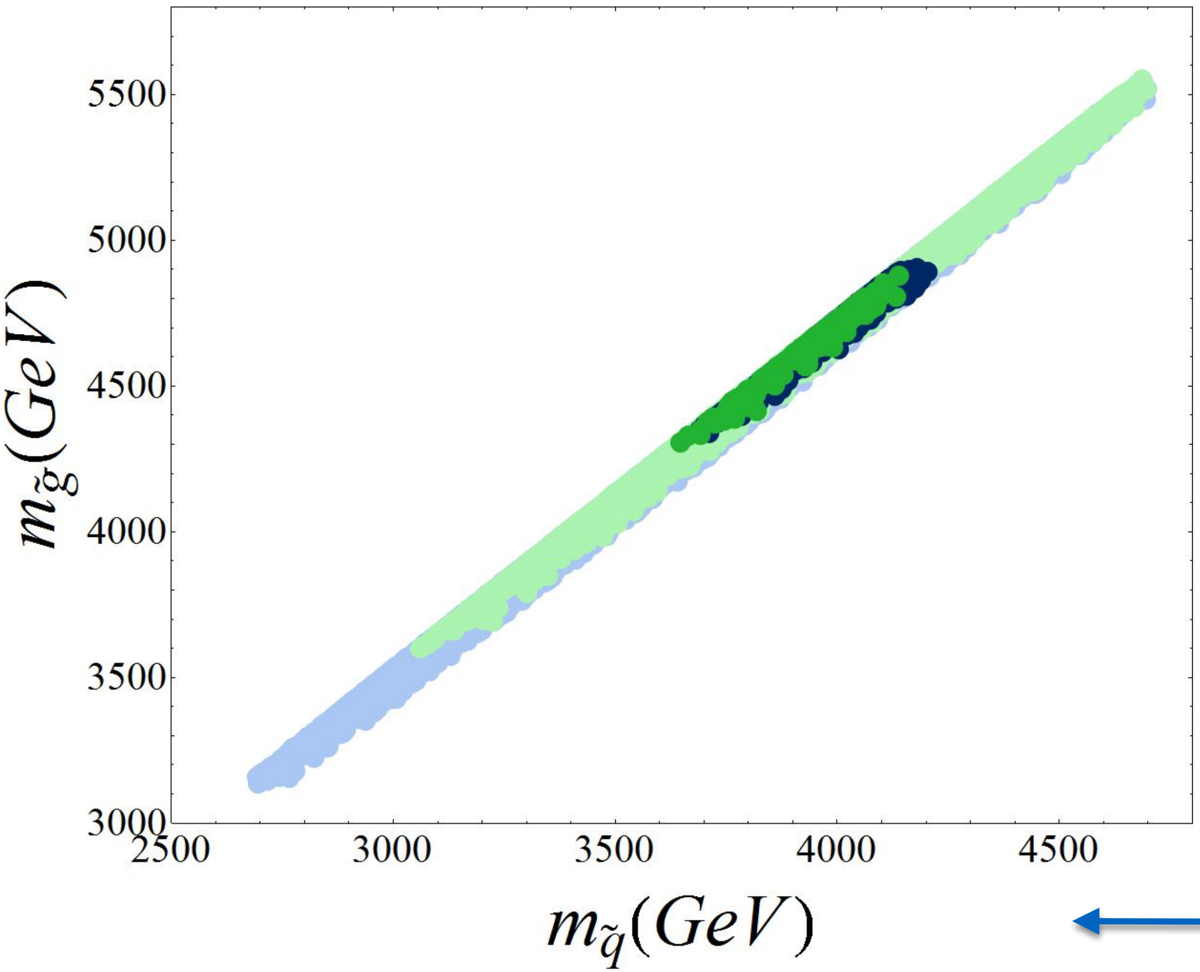
$$\rho_1 = \frac{19}{10}, \, \rho_2 = \frac{5}{2}$$



All scenarios with the correct relic density have higgsino LSP.



Unfortunately the mass spectrum is very heavy, so this is very challenging to see.



first/second gen. squarks

Since the scalar masses are generated by $M_{1/2}$ these models predict

$m_{\tilde{d}_R} \approx 0.9 m_{\tilde{g}}$

An example scenario:

$m_{\mathbf{16}}$	113.8	$m_{\tilde{u}_L}$	5785	$m_{\tilde{t}_1}$	2987	$M_{\tilde{g}}$	5175	
$K_{\mathbf{16}}$	12.3	$m_{\tilde{u}_R}$	4481	$m_{\tilde{t}_2}$	5243	$M_{\tilde{\chi}_1^0}$	949.4	(LSP)
$m_{\mathbf{10+126}}$	132.5	$m_{\tilde{d}_L}$	5786	$m_{\tilde{b}_1}$	4240	$M_{\tilde{\chi}_2^0}$	952.2	
$g_{10}^2 D$	-6674	$m_{\tilde{d}_R}$	4417	$m_{\tilde{b}_2}$	5239	$M_{\tilde{\chi}_3^0}$	2050	
$a_{\mathbf{10}}$	-116.7	$m_{\tilde{e}_L}$	4036	$m_{\tilde{\tau}_1}$	1577	$M_{\tilde{\chi}_4^0}$	5040	
$M_{1/2}$	2471	$m_{\tilde{e}_R}$	1765	$m_{\tilde{\tau}_2}$	3955	$M_{\tilde{\chi}_1^\pm}$	951.3	(NLSP)
ρ_1	1.90	$m_{\tilde{\nu}^1}$	4035	$m_{\tilde{\nu}^3}$	3954	$M_{\tilde{\chi}_2^\pm}$	5040	
ρ_2	2.50							

m_{h^0}	125.0	$R_{tb\tau}$	4.76
m_{A^0}	3842	$R_{b\tau}$	1.32
m_{H^0}	3842	Δ	33.62
$m_{H^\pm}$	3843	Δ_μ	453.5
μ	907.5	$\Omega_c h^2$	0.0934
$\tan\beta$	19.13		

Summary

The lack of SUSY at the LHC is forcing SUSY to higher energies.

If the LHC doesn't see light stops in Run II, we are forced to tolerate fine-tuning.

Here I advocated ignoring fine-tuning from μ and instead minimising fine-tuning from the soft susy breaking parameters.

I looked at SO(10) GUT models with various breaking mechanisms and embeddings that lead to non-universal gaugino masses.

We saw and example of a Pati-Salam embedding with low fine-tuning from soft parameters, that gives the correct Higgs mass, the correct relic density and evades all experimental constraints.

Unfortunately it has a rather heavy spectrum that will be difficult to see.