Deformed Wess-Zumino Model

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Outline

Non-anticommutativity

Non-anticommutativity of fermion coordinates of superspace
N Seiberg. *Journal of High Energy Physics*, 6:10, June 2003.

$$\{\theta^a, \theta^b\} = C^{ab} \tag{1}$$

The Susy generators

$$Q_a = \partial_a - i(\sigma_{a\dot{a}})^\mu \theta^{\dot{a}} \partial_\mu \qquad Q_{\dot{a}} = -\partial_{\dot{a}} + i\theta^a (\sigma_{a\dot{a}})^\mu \partial_\mu \tag{2}$$

do not satisfies the standard susy relation

$$\{Q_a, Q_b\} = 0, \qquad \{Q_{\dot{a}}, Q_{\dot{b}}\} = -4C^{ab} \sigma^\mu_{a\dot{a}} \sigma^\nu_{b\dot{b}} \frac{\partial^2}{\partial y^\mu \partial y^\nu}, \tag{3}$$

It is named Susy $\mathcal{N} = 1/2$.

Non-anticomutativity in three spacetime dimensions

A. F. Ferrari, M. Gomes, J. R. Nascimento, A. Yu. Petrov, and A. J. da Silva. *Phys. Rev. D*, 74:125016, Dec 2006.

$$\{\theta_a, \theta_b\} = \Sigma_{ab} \, , \tag{4}$$

but

$$\{Q_a, Q_b\} = 2P_{ab} - \Sigma^{ab} P_{ma} P_{nb} \, . \tag{5}$$

To preserved the supersymmetry

$$\tilde{Q}_a = Q_a + \frac{i}{2} \Sigma^{bc} \partial_b P_{ca} \tag{6}$$

The new generators satisfies

$$\left\{\tilde{Q}_a, \tilde{Q}_b\right\} = 2P_{ab}. \tag{7}$$

Hopf Algebra

The Hopf algebra $\mathcal{H}$ is a vector space

Product

$$\mu : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$$
$$a \otimes b \to \mu(a \otimes b) = a \cdot b$$

coproduct

$$\Delta : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}$$
$$c \to \Delta(h) = \sum_i (h_1)_i \otimes (h_2)_i$$

and exist one antihomomorfism, $S : \mathcal{H} \to \mathcal{H}$

Antipode

$$S(a \cdot b) = S(b) \cdot S(a), \tag{8}$$
$$S(\mathbf{1}) = \mathbf{1}, \tag{9}$$

Hopf algebra actions or *H*-modules

Let (N, m) be $\mathcal{H}$-Module

$$\begin{aligned} \alpha : \mathcal{H} \otimes N &\to N \\ h \otimes n &\to \alpha(h, n) = h \triangleright n \end{aligned} \tag{10}$$

compatibility between action and H-modulo N

$$h \triangleright (m\,(v \otimes w)) = m\,(\Delta(h) \triangleright (v \otimes w))\,, \quad v, w \in N \tag{11}$$

For a Hopf algebra $\mathcal{H}$, there is an action on itself

Adjoint action

$$\begin{aligned} ad : \mathcal{H} \otimes \mathcal{H} &\to \mathcal{H} \\ (h, x) &\to ad_h x = h \blacktriangleright x = \sum (h_1)_i \cdot x \cdot S\left((h_2)_i\right) \end{aligned} \tag{12}$$

Enveloping algebra $U(\mathfrak{g})$ has a natural Hopf algebra structure, if $\tau_i \in \mathfrak{g}$

$$\Delta(\tau_i) = \tau_i \otimes 1 + 1 \otimes \tau_i, \qquad \Delta(1) = 1, \tag{13}$$

$$S(\tau_i) = -\tau_i, \qquad S(1) = 1, \tag{14}$$

The adjoin action of $U(\mathfrak{g})$ is the Lie commutator

Lie bracket

$$\begin{aligned} ad_{\tau_i}\tau_j &= (\tau_i)_1 \cdot \tau_j \cdot S((\tau_i)_2) \\ &= \tau_i \cdot \tau_j - \tau_j \cdot \tau_i = [\tau_i, \tau_j] = C_{ij}^k \tau_k \in \mathfrak{g} \quad \forall \tau_j, \tau_i \in \mathfrak{g}. \end{aligned} \tag{15}$$

Deformation using Drinfel'd *twist*

A twist is an element $\mathcal{F} \in \mathcal{H} \otimes \mathcal{H}$

$$\mathcal{F} = f^a \otimes f_a. \qquad (16)$$

$$\mathcal{F}^{-1} = \bar{f}^a \otimes \bar{f}_a, \qquad (17)$$

it satisfies

2-cocycle

$$(\mathbf{1} \otimes \mathcal{F})\,(id \otimes \Delta)\,\mathcal{F} = (\mathcal{F} \otimes \mathbf{1})\,(\Delta \otimes id)\,\mathcal{F} \qquad (18)$$

Deformed Lie Algebra

The twist can modified product m of the N and μ of $U(\mathfrak{g})$

Produto $\star$ in N

$$u \star v = \mu\left(\mathcal{F}^{-1} \triangleright (u \otimes v)\right) = \left(\bar{f}^a \triangleright u\right) \cdot \left(\bar{f}_a \triangleright v\right) \; u, v \in N.$$

Produto $\star$ in $U(\mathfrak{g})$

$$\tau_i \star \tau_j = \mu\left(\mathcal{F}^{-1} \blacktriangleright (\tau_i \otimes \tau_j)\right) = \left(\bar{f}^a \blacktriangleright \tau_i\right) \cdot \left(\bar{f}_a \blacktriangleright \tau_j\right) \; \tau_i, \tau_j \in \mathfrak{g}. \quad (19)$$

These are new algebras and $N_\star = (N, \star)$ and $U^\star(\mathfrak{g}) = (U(\mathfrak{g}), \star)$

But $\star$-elements in $U^\star(\mathfrak{g})$ do not satisfy

$$[\tau_i \stackrel{\star}{,} \tau_j] = \tau_i \star \tau_j - \tau_j \star \tau_i \neq C_{ij}^k \tau_k \quad (20)$$

P. Aschieri, M. Dimitrijević, F. Meyer and J. Wess. *Class and Quantum grav,* **23** 1883, 2006.

deformed generators

Let $u \in \mathfrak{g}$

$$X_u = \bar{f}^a \cdot u \cdot \chi \cdot S\left(\bar{f}_a\right) \quad where \quad \chi = \mu\,(id \otimes S)\,\mathcal{F}. \tag{21}$$

$$\Delta_\star(X_u) = \mathcal{F}\,(X_u \otimes 1 + 1 \otimes X_u)\,\mathcal{F}^{-1} \tag{22}$$

Lie Algebra bracket

$$[X_u \overset{\star}{,} X_v] = X_u \star X_v - X_v \star X_u = X_{[u,v]} \quad \forall u, v \in \mathfrak{g} \tag{23}$$

Deformed Lie algebra actions

$$X_u \triangleright^\star n = u \triangleright n \quad \forall u \in \mathfrak{g},\ \forall n \in N_\star \tag{24}$$

Lie Superalgebras

The formulas above (8),(15),(19),(21),(23) can be extended to superalgebras case

Extension to $\mathbb{Z}_2$-Graded Lie

$$S(u \cdot v) = (-1)^{\kappa(v)\kappa(u)}\, S(v) \cdot S(u). \tag{25}$$

$$u \blacktriangleright v = u(v) = (-1)^{\kappa(v)\kappa(u_2)}\, u_1 \cdot v \cdot S(u_2). \tag{26}$$

$$u \star v = \sum_a (-1)^{\kappa(\bar{f}_a)\kappa(u)}\, \bar{f}^a(u) \cdot \bar{f}_a(v). \tag{27}$$

$$X_u = \sum_a (-1)^{\kappa(\bar{f}_a)\kappa(u)}\, \bar{f}^a \cdot u \cdot \chi \cdot S\left(\bar{f}_a\right). \tag{28}$$

$$[X_u \overset{\star}{,} X_v\} = X_u \star X_v - (-1)^{\kappa(u)\kappa(v)}\, X_v \star X_u = X_{[u,v\}}. \tag{29}$$

Susy in three dimensions and Hopf Algebra

The Lorentz group act on real two-component spinor $\psi^a = \left(\psi^1, \psi^2\right)$.

superespaço

$$z = (x^{ab}, \theta^c) \qquad where \qquad x^{ab} = (\sigma^\mu)^{ab}\, x_\mu \tag{30}$$

such that

$$[x^{mn}, x^{rs}] = [x^{mn}, \theta^a] = 0, \tag{31}$$

$$\left\{\theta^a, \theta^b\right\} = 0. \tag{32}$$

The SUSY properties can be written using the Hopf algebra language

coproduct

$$\Delta(A) = A \otimes 1 + 1 \otimes A$$

adjoint action

$$A \blacktriangleright B = [A, B\} = A \cdot B - (-1)^{\kappa(B)\kappa(A)} B \cdot A. \tag{33}$$

The Hopf algebra is defined by

SUSY generators

$$[P_{ab}, P_{cd}] = 0, \quad \{Q_a, Q_b\} = 2P_{ab}, \quad [Q_a, P_{cd}] = 0. \tag{34}$$

supercovariant Derivative

$$[D_a, Q_b\} = 0, \quad [D_a, P_{cd}\} = 0, \quad [D_a, D_b\} = 2P_{cd}. \tag{35}$$

this algebra is represented by differential operator

Differential operator

$$Q_a = i\left(\partial_a - \theta^c i\partial_{ca}\right), \qquad P_{ab} = i\partial_{ab}, \qquad D_a = \partial_a + i\,\theta^b\partial_{ba}. \tag{36}$$

and act on superfield algebra N

superfield

$$\Phi\left(x,\theta\right) = A\left(x\right) + \theta^a\psi_a\left(x\right) - \theta^2 F\left(x\right). \tag{37}$$

with product m

$$m\left(\Phi\otimes\Psi\right) = \Phi\left(z\right)\cdot\Psi\left(z\right) \qquad \Phi,\Psi\in N \tag{38}$$

the action is compatible with product m

compatibility

$$A\left(\Phi\cdot\Psi\right)=m\Big(\Delta\left(A\right)\left(\Phi\otimes\Psi\right)\Big)$$
$$=m\Big(A\left(\Phi\right)\otimes\Psi+\left(-1\right)^{\kappa(A)\kappa(\Psi)}\Phi\otimes A\left(\Psi\right)\Big)$$
$$=A\left(\Phi\right)\cdot\Psi+\left(-1\right)^{\kappa(A)\kappa(\Psi)}\Phi\cdot A\left(\Psi\right). \quad (39)$$

SUSY transformation law

tranformation law on the fields

$$\delta_{\xi}\Phi\left(x,\theta\right)\equiv i\xi^{a}Q_{a}\Phi\left(x,\theta\right). \quad (40)$$
$$\delta_{\xi}\left(\Phi\cdot\Psi\right)=\delta_{\xi}\left(\Phi\right)\cdot\Psi+\Phi\cdot\delta_{\xi}\left(\Psi\right) \quad (41)$$

Wess-Zumino Model

The Susy actions is defined by

Wess-Zumino Action

$$\mathcal{S} = \int d^3x\, d^2\theta\ \left[\frac{1}{2}\Phi D^2\Phi + \frac{1}{2}m\Phi^2 + \frac{1}{6}\lambda\,\Phi^3\right] \quad (42)$$

the Susy action can be rewritten as

Action in Components

$$\mathcal{S} = \int d^3x \left\{ \frac{1}{2}\left[F^2 + A\Box A + \psi^a i\partial_a^b\psi_b\right] + m\left(\psi^2 + AF\right) + \right.$$
$$\left. + \lambda\left(A\psi^2 + \frac{1}{2}A^2F\right)\right\} \quad (43)$$

Susy three dimensions deformation

The algebra $\Im$ to be deformed is

commutation relations $\Im$

$$\{Q_a, Q_b\} = 2P_{ab}, \tag{44a}$$
$$\{D_a, D_b\} = 2P_{cd}, \tag{44b}$$
$$\{D_a, \partial_b\} = P_{ab}, \tag{44c}$$
$$\{Q_a, \partial_b\} = -iP_{ab}, \tag{44d}$$
$$\{\partial_a, \partial_b\} = 0, \tag{44e}$$
$$[P_{ab}, P_{cd}] = [Q_a, P_{cd}] = [\partial_a, P_{cd}] = \{D_a, Q_b\} = 0. \tag{44f}$$

its enveloping algebra will be denoted by $\mathcal{U}(\Im)$.

In this case, we use the twist

Twist

$$\mathcal{F} = e^{\frac{1}{2}\,C^{ab}\,\partial_a \otimes \partial_b}$$
$$= 1 \otimes 1 + \frac{1}{2}\,C^{ab}\,\partial_a \otimes \partial_b - \frac{1}{8}\,C^{ab}C^{mn}\,\partial_a\partial_m \otimes \partial_b\partial_n. \qquad (45)$$

where we used the relations

$$(A \otimes B)\cdot(C \otimes D) = (-1)^{\kappa(B)\kappa(C)}\,(A \cdot C \otimes B \cdot D)$$
$$(\partial_a)^3 = 0 \qquad (46)$$

From this twist we can find the deformed algebra $\mathcal{U}^{\star}(\Im) = (\mathcal{U}(\Im), \star)$

$$\begin{aligned} \{Q_a \overset{\star}{,} Q_b\} &= Q_a \star Q_b + Q_b \star Q_a, \\ &= \{Q_a, Q_b\} + C^{mn} \{\partial_m, Q_a\} \{\partial_n, Q_b\}, \\ &= 2P_{ab} - C^{mn} P_{ma} P_{nb}. \end{aligned} \quad (47)$$

Following the prescription given in the deformation section

Deformed generator

$$\begin{aligned} X_{Q_a} &= \sum_c (-1)^{\kappa(\bar{f}_c)\kappa(Q_a)} \bar{f}^c Q_a S(\bar{f}_c), \\ &= Q_a - \frac{1}{2} C^{lm} \partial_m \{Q_a, \partial_l\}, \\ &= Q_a + \frac{i}{2} C^{lm} \partial_m P_{al}, \end{aligned} \quad (48)$$

the same way

Deformed generators

$$X_{D_a} = D_a - \frac{i}{2} C^{lm}\, \partial_m P_{al}, \quad (49)$$

$$X_{P_{ab}} = P_{ab}, \quad (50)$$

$$X_{\partial_a} = \partial_a. \quad (51)$$

Deformed coproducts

$$\Delta_\star (X_{Q_a}) = X_{Q_a} \otimes 1 + 1 \otimes X_{Q_a} - C^{mn}\, \partial_m \otimes \partial_{na}, \quad (52)$$

$$\Delta_\star (X_{D_a}) = X_{D_a} \otimes 1 + 1 \otimes X_{D_a} - iC^{mn}\, \partial_m \otimes \partial_{na}, \quad (53)$$

$$\Delta_\star (X_{P_{ab}}) = X_{P_{ab}} \otimes 1 + 1 \otimes X_{P_{ab}}, \quad (54)$$

$$\Delta_\star (X_{\partial_a}) = X_{\partial_a} \otimes 1 + 1 \otimes X_{\partial_a}, \quad (55)$$

The algebra $\Im^{\star}$ has the same Susy algebra commutation relation

commutation relation of $\Im^{\star}$

$$\{X_{Q_a} \stackrel{\star}{,} X_{Q_b}\} = 2X_{P_{ab}}, \quad (56)$$

$$\{X_{D_a} \stackrel{\star}{,} X_{D_b}\} = 2X_{P_{cd}}, \quad (57)$$

$$\{X_{D_a} \stackrel{\star}{,} X_{\partial_b}\} = X_{P_{ab}}, \quad (58)$$

$$\{X_{Q_a} \stackrel{\star}{,} X_{\partial_b}\} = -iX_{P_{ab}}, \quad (59)$$

$$\{X_{\partial_a} \stackrel{\star}{,} X_{\partial_b}\} = 0, \quad (60)$$

$$[X_{P_{ab}} \stackrel{\star}{,} X_{P_{cd}}] = [X_{Q_a} \stackrel{\star}{,} X_{P_{cd}}] = [X_{\partial_a} \stackrel{\star}{,} X_{P_{cd}}] = \{X_{Q_a} \stackrel{\star}{,} X_{D_b}\} = 0. \quad (61)$$

The product $\star$ in not commutative on the fields

$$\begin{aligned}\Phi(z)\star\Psi(z) &= m^{\mathcal{F}}(\Phi\otimes\Psi) = m(\mathcal{F}^{-1}\triangleright(\Phi\otimes\Psi))\\ &= (-1)^{\kappa(\Phi)\kappa(\bar{f}_a)}\left(\bar{f}^a\triangleright\Phi\right)\cdot\left(\bar{f}_a\triangleright\Psi\right)\\ &= \Phi(z)\cdot\Psi(z)-\frac{1}{2}(-1)^{\kappa(\Phi)}C^{ab}\left(\partial_a\Phi(z)\right)\cdot\left(\partial_b\Psi(z)\right)-\\ &-\frac{1}{8}C^{ab}C^{mn}\left(\partial_a\partial_m\Phi(z)\right)\cdot\left(\partial_b\partial_n\Psi(z)\right). \end{aligned} \tag{62}$$

deformed Superspace

$$\left[x^{mn}\overset{\star}{,}x^{rs}\right]=\left[x^{mn}\overset{\star}{,}\theta^a\right]=0,$$
$$\left\{\theta^a\overset{\star}{,}\theta^b\right\}=C^{ab}.$$

by construction the algebra $\Im^{\star}$ is compatible wit star product $\star$ above

$$X_A \triangleright (\Phi(z) \star \Psi(z)) = m^{\mathcal{F}}\left(\Delta_{\star}(X_A) \triangleright (\Phi \otimes \Psi)\right). \qquad (63)$$

deformed Susy transformation

$$\begin{aligned} \delta^{\star}_{\xi}\Phi(x,\theta) &= i\xi^{a}\; X_{Q_a} \triangleright \Phi(x,\theta) \\ &= i\xi^{a} Q_a \Phi(x,\theta). \end{aligned} \qquad (64)$$

Deformed Susy transformation law on field product

$$\delta^{\star}_{\xi}(\Phi \star \Psi) = \left(\delta^{\star}_{\xi}\Phi\right) \star \Psi + \Phi \star \left(\delta^{\star}_{\xi}\Psi\right) + C^{mn}\, \partial_m \Phi \star \xi^{a}\, \partial_{an}\Psi. \qquad (65)$$

The star product $\star$ can be rewritten as

$$\Phi \star \Psi = \Phi\left(z\right) \cdot \Psi\left(z\right) - \frac{1}{2}(-1)^{\kappa(\Phi)}\, C^{ab}\, \partial_a \Big(\Phi \cdot \partial_b \Psi\Big) - \frac{1}{8}\, C^{ab} C^{mn}\, \partial_a \partial_m \Big(\Phi \cdot \partial_b \partial_n \Psi\Big)\,.$$

therefore

$$\int d^3x\, d^2\theta\, \Phi \star \Psi = \int d^3x\, d^2\theta\, \Phi \cdot \Psi. \tag{66}$$

Then

there are not modifications

$$\mathcal{S}^{\star}_{cin} + \mathcal{S}^{\star}_{m} = -\frac{1}{4} \int d^3x\, d^2\theta\, (D^b\Phi) \cdot (D_b\Phi) + \frac{1}{2} \int d^3x\, d^2\theta\, m\Phi \cdot \Phi. \tag{67}$$

The deformation is not trivial to interaction terms

$$\mathcal{S}_I^\star = \alpha \int d^3x\, d^2\theta\ (\Phi)_\star^n \tag{68}$$

$$= \alpha \int d^3x\, d^2\theta\ \underbrace{\Phi \star \cdots\cdots \star \Phi}_{n-vezes}. \tag{69}$$

Cubic Interaction

$$\mathcal{S}_I^\star = \frac{\lambda}{6} \int d^3x\, d^2\theta\ (\Phi)_\star^3 \tag{70}$$

$$= \frac{\lambda}{6} \int d^3x\, d^2\theta\, \Phi \star \Phi \star \Phi. \tag{71}$$

Using the definition of $\star$ product

$$\mathcal{S}_I^\star = \frac{\lambda}{6}\int d^3x\, d^2\theta \left(\Phi^3 - \frac{1}{8}C^{lm}C^{nk}\,\Phi\partial_l\partial_n\Phi\partial_m\partial_k\Phi\right), \tag{72}$$

$$= \frac{\lambda}{6}\int d^3x\, d^2\theta\, \Phi^3 + \frac{\lambda}{14}\int d^3x\ \det C\, F^3 \tag{73}$$

therefore
N Seiberg. *Journal of High Energy Physics*, 6:10, June 2003.

Total action

$$\mathcal{S}^\star = \mathcal{S}_{cin}^\star + \mathcal{S}_m^\star + \mathcal{S}_I^\star, \tag{74}$$

$$= \mathcal{S} + \frac{\lambda}{14}\int d^3x\, det\,(C)\ F^3 \tag{75}$$

Conclusion

- Is possible to define one twist using Grassmannian derivatives.
- Find us the same deformed Susy generators of the literature the consistent way using the Hopf Algebra.
- Show us that the deformed algebra satisfies the same Susy commutation relations.
- The modification in the action only can be obtained of the interaction terms.

Perspectives

The perspectives are study the quantum correction to two loops and study the renormalization group equation for the non anticommutation parameter C^{ab}.

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Questions?

Quantum correction

The action can be expressed in superfield terms

Total action

$$\mathcal{S}^{\star} = \int d^3x\, d^2\theta\, \left[\frac{1}{2}\Phi D^2\Phi + \frac{1}{2}m\Phi^2 + \frac{1}{6}\lambda\,\Phi^3 + \frac{\lambda}{14}U\,(D^2\Phi)^3\right], \qquad (76)$$

where U is called the spurion field

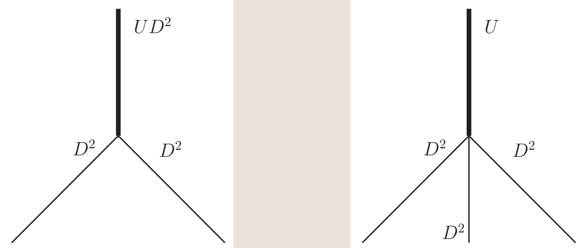
Spurion

$$U = det\,(C)\,\theta^2. \qquad (77)$$

M. T. Grisaru, S. Penati, A. Romagnoni. *Journal of High Energy Physics. 0308 (2003) 003*

Using the quantum-background splitting $\Phi \to \Phi + \Phi_q$

Extra vertices from U



The propagator is

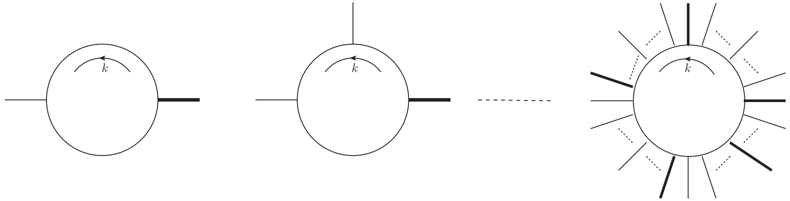
Propagator

$$\langle \Phi\Phi \rangle = \frac{D^2 - m}{k^2 + m^2}\,\delta\left(\theta - \theta'\right) \tag{78}$$

One Loop

The first vertex, we have additional diagrams

One loop diagrams



To process of high energy we can set that external momentums taking to zero

The kind of integral that appear in these diagrams are

Diagram's integral

$$\int \frac{d^3k}{(2\pi)^3}\frac{1}{(k^2+m^2)}, \int \frac{d^3k}{(2\pi)^3}\frac{\left(k^2\right)^2}{\left(k^2+m^2\right)^3}, \ldots, \int \frac{d^3k}{(2\pi)^3}\frac{\left(k^2\right)^a}{\left(k^2+m^2\right)^b}$$

Using

Dimensional regularization

$$\int \frac{d^nk}{(2\pi)^n}\frac{\left(k^2\right)^a}{\left(k^2+N\right)^b} = i^{n+3}\frac{\Gamma\left(b-a-\frac{1}{2}n\right)\Gamma\left(a+\frac{1}{2}n\right)}{\Gamma\left(b\right)\Gamma\left(\frac{1}{2}n\right)}N^{-\left(b-a-\frac{1}{2}n\right)}$$

The integral are finite.