Effect of interaction terms on particle production due to time-varying mass

[work in progress]

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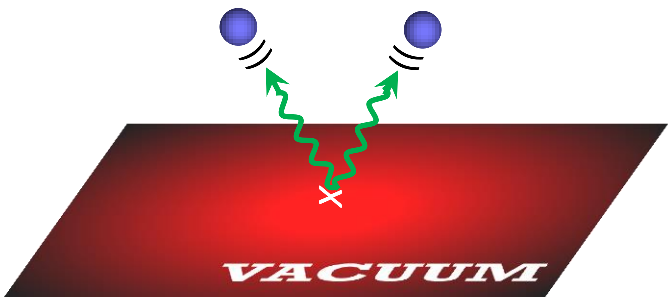
Outline

1. Introduction
2. How to calculate particle number
3. Calculation of particle number
4. Summary

1. Introduction



- Particle production from vacuum
  - It is known that a varying background causes production of particles
    - Oscillating Electric field → pair production of electrons
      [E. Brezin and C. Itzykson, *Phys. Rev.* **D 2**,1191 (1970)]
    - Changing metric → gravitational particle production
      [L. Parker, *Phys. Rev.* **183**, 1057 (1969)]
      [L. H. Ford, *Phys. Rev.* **D 35**, 2955 (1987)]
    - Oscillating inflaton → (p)reheating
      [L. Kofman, A. D. Linde, A. A. Starobinsky, *Phys. Rev. Lett.* **73**, 3195 (1994)]
      [L. Kofman, A. D. Linde, A. A. Starobinsky, *Phys. Rev.* **D 56**, 3258 (1997)]
    - etc…

Example of scalar particle production

g : coupling

- Let us consider : $\mathcal{L}_{int} = -\frac{1}{2} g^2 |\phi|^2 \chi^2$

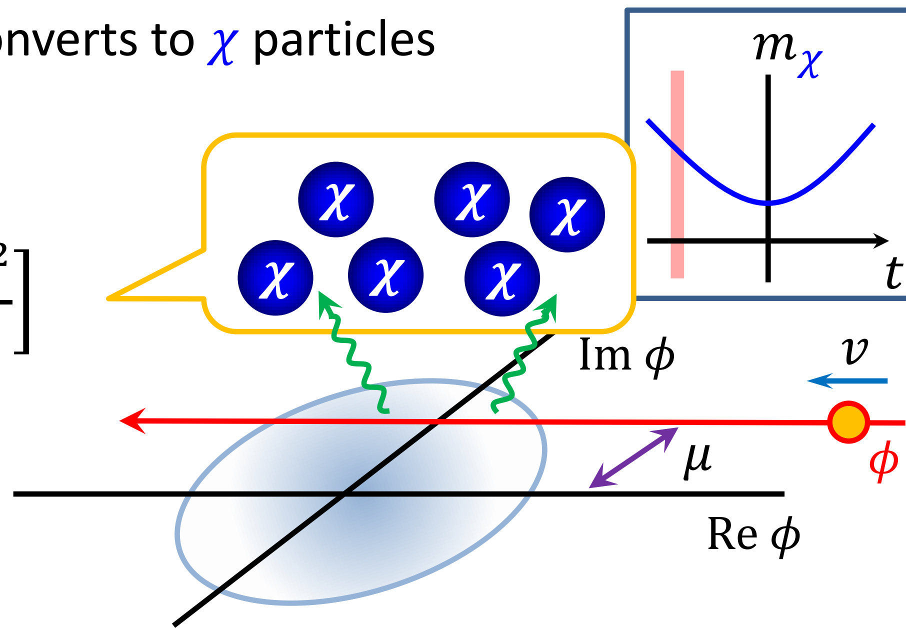
ϕ : complex scalar field (*classical*)

χ : real scalar particle (*quantum*)

- If ϕ goes near the origin, χ particles are produced
  - Because
    - → mass of χ ($m_\chi = g\phi$) becomes small around $|\phi| = 0$
    - → kinetic energy of ϕ converts to χ particles
  - produced occupation number :

$$n_{\chi k} = V \cdot \exp\left[-\pi \frac{k^2 + g^2 \mu^2}{g v}\right]$$

[L. Kofman, A. D. Linde, X. Liu, A. Maloney, L. McAllister and E. Silverstein, *JHEP* **0405**, 030 (2004)].

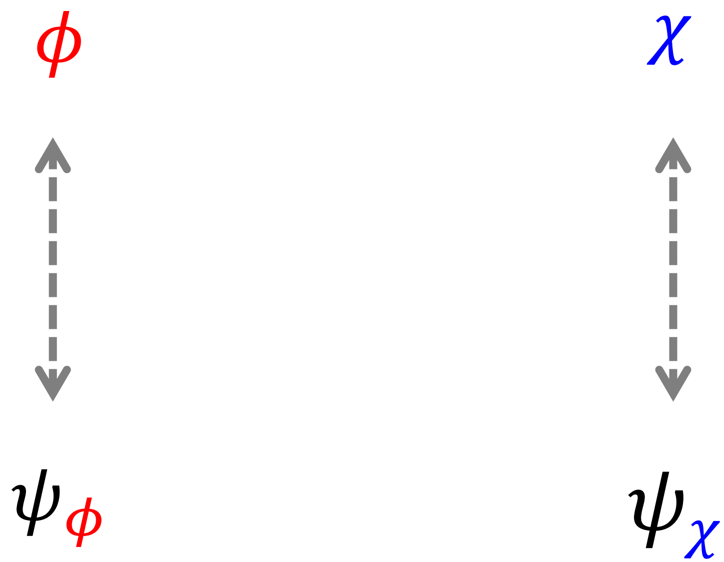


Our interests

1. How about supersymmetric model?
   - What is the role of the superpartner of the background field?

2. How do (quantum) interaction terms affect particle production?
   - Usually production rates are calculated in the purely classical background

→ We would like to estimate the contribution of the quantum interaction term



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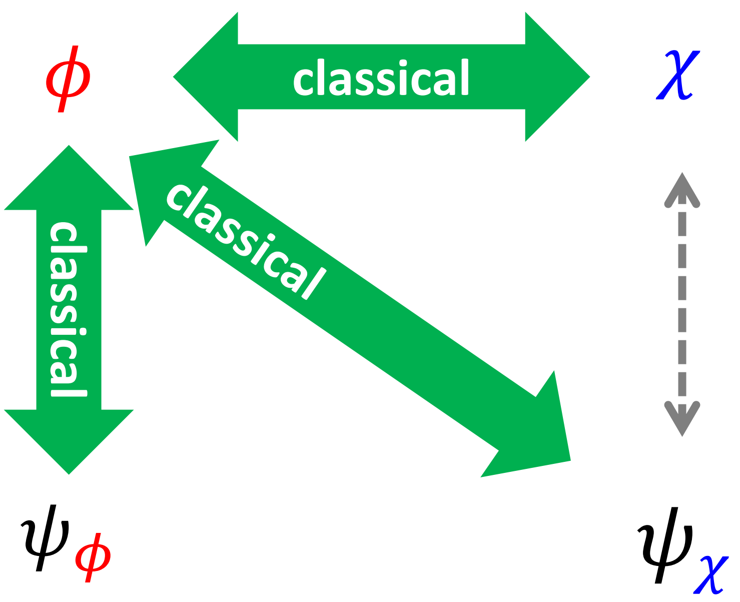

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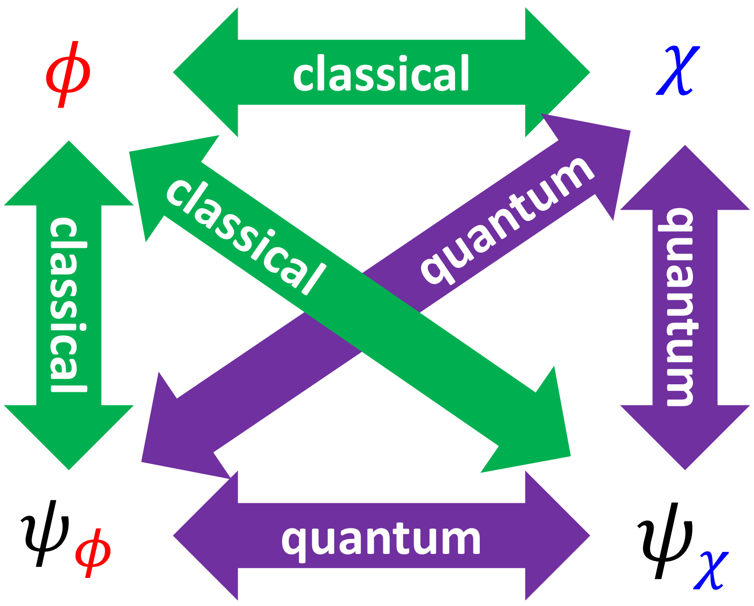


Model in this talk

- **Super potential :**

$$W = \frac{1}{2} g \Phi X^2$$

$\Phi = \phi + \sqrt{2}\theta\psi_\phi + \theta^2 F_\phi$

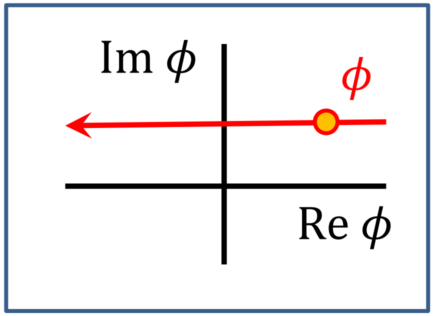
$X = \chi + \sqrt{2}\theta\psi_\chi + \theta^2 F_\chi$

g : coupling

→ Interaction terms in components :

$$\mathcal{L}_{int} = -g^2|\phi|^2|\chi|^2 - \frac{1}{4}g^2|\chi|^4 - g\left(\frac{1}{2}\phi\psi_\chi\psi_\chi + \psi_\phi\psi_\chi\chi + (h.c.)\right)$$

- **Stationary point :**
  - $\chi = \psi_\phi = \psi_\chi = 0$, but ϕ can have ***any*** value



- **Masses**

Production is possible

impossible

  - $\phi \neq 0$ → χ, ψ_χ: mass = $g\phi$, ψ_ϕ: massless
  - However, ψ_ϕ's mass may be influenced by ϕ through loop effects…?
  - Is quantum part of ϕ also influenced?

Equations of Motion for field operators :

$$\phi \;:\quad 0 = (\partial^2 + g^2|\chi|^2)\phi + \frac{1}{2} g \psi_\chi^\dagger \psi_\chi^\dagger$$

$$\chi \;:\quad 0 = \left(\partial^2 + g^2|\phi|^2 + \frac{1}{2} g^2 |\chi|^2\right)\chi + g \psi_\phi^\dagger \psi_\chi^\dagger$$

$$\psi_\phi \;:\quad 0 = \bar{\sigma}^\mu \partial_\mu \psi_\phi + i g \chi^* \psi_\chi^\dagger$$

$$\psi_\chi \;:\quad 0 = \bar{\sigma}^\mu \partial_\mu \psi_\chi + i g \phi^* \psi_\chi^\dagger + i g \chi^* \psi_\phi^\dagger$$

How do we calculate produced particle number including interaction term ?

2. How to calculate particle number

- Definition of (occupation) number :

$$n_k = \langle 0^{\text{in}} | a_{\mathbf{k}}^{\text{out}\dagger} a_{\mathbf{k}}^{\text{out}} | 0^{\text{in}} \rangle$$

→ Information about in-state (@ far past) and out-state (@ far future) of field needs for the calculation

- How are they related to each other?

(in-state) ⟺ (out-state)

→ Asymptotic field expansion

An example with a scalar field

(mass) (source term)

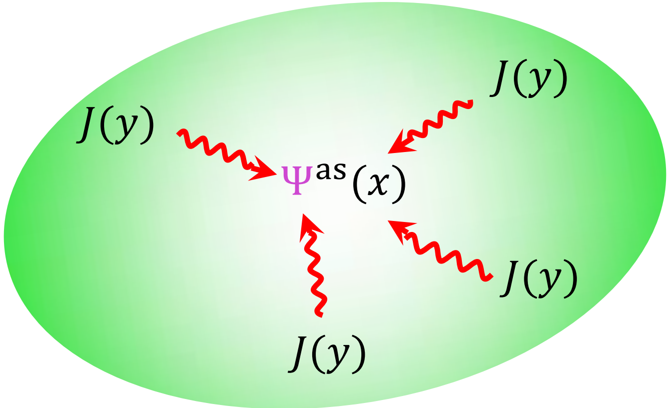
- Operator field equation : $0 = (\partial^2 + M^2)\Psi + J$
- Commutation relation : $\left[\Psi(\mathbf{x}), \dot{\Psi}^*(\mathbf{y})\right] = i\delta^3(\mathbf{x}-\mathbf{y})$

→ Formal solution (***Yang-Feldman equations***)

$$\Psi(x) = \sqrt{Z}\Psi^{\mathrm{as}}(x) - iZ\int_{t^{\mathrm{as}}}^{x^0} dy^0 \int d^3y\, [\Psi^{\mathrm{as}}(x), \Psi^{\mathrm{as},*}(y)]\, J(y)$$

Z : some const.

Ψ^{as} : asymptotic field
$0 = (\partial^2 + M^2)\Psi^{\mathrm{as}}$



- If we take $t^{as} = t^{\mathrm{in}} = -\infty$ or $t^{as} = t^{\mathrm{out}} = +\infty$,

$$\Psi^{\mathrm{out}}(x^{\mathrm{out}}) = \Psi^{\mathrm{in}}(x^{\mathrm{out}}) - i\sqrt{Z}\int d^4y \left[\Psi^{\mathrm{in}}(x^{\mathrm{out}}), \Psi^{\mathrm{in}}(y)\right] J(y)$$

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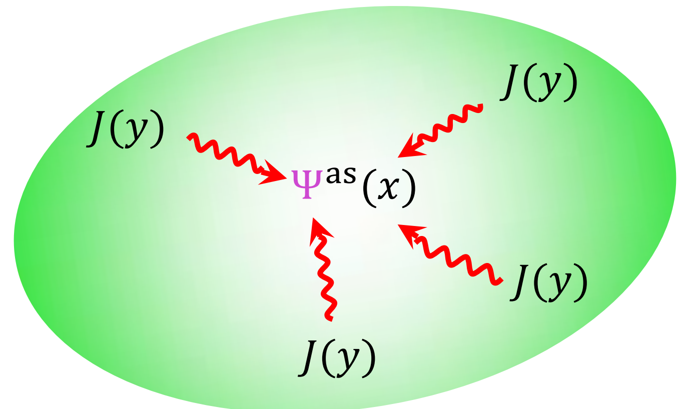
$a_{\mathbf{k}}^{\mathrm{out}}$ $a_{\mathbf{k}}^{\mathrm{in}}$



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An example with a scalar field

- Ψ^{as} is free particle, so we can expand with plane waves as

$$\Psi^{\text{as}}(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(\Psi_k^{\text{as}}(x^0) a_{\mathbf{k}}^{\text{as}} + \Psi_k^{\text{as}*}(x^0) b_{-\mathbf{k}}^{\text{as}\dagger} \right)$$

plane wave

(time dependent) wave func.

$$0 = \ddot{\Psi}_k^{\text{as}} + (\mathbf{k}^2 + M^2)\Psi_k^{\text{as}}$$

creation/annihilation op.

- inner product relation : $\dot{\Psi}_k^{\text{as}*}\Psi_k^{\text{as}} - \Psi_k^{\text{as}*}\dot{\Psi}_k^{\text{as}} = i/Z$

which comes from conditions

$$\left[a_{\mathbf{k}}^{\text{as}},\ a_{\mathbf{k}'}^{\text{as}\dagger}\right] = (2\pi)^3\delta^3(\mathbf{k}-\mathbf{k}')\ ,\ Z\left[\Psi^{\text{as}}(\mathbf{x}), \dot{\Psi}^{\text{as}}(\mathbf{y})\right]_{t\to t^{\text{as}}} = i\hbar\delta^3(\mathbf{x}-\mathbf{y})$$

$$a_{\mathbf{k}}^{\text{as}} = -iZ\int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}}\left(\dot{\Psi}_k^{\text{as}*}\Psi^{\text{as}} - \Psi_k^{\text{as}*}\dot{\Psi}^{\text{as}}\right)$$

An example with a scalar field

Relation between $a_{\mathbf{k}}^{\text{in}}$ and $a_{\mathbf{k}}^{\text{out}}$

$$a_{\mathbf{k}}^{\text{out}} = -iZ \int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}} \left(\dot{\Psi}_k^{\text{out}*} \Psi^{\text{out}} - \Psi_k^{\text{out}*} \Psi^{\text{out}} \right)$$

$$\Psi^{\text{out}}(x^{\text{out}}) = \Psi^{\text{in}}(x^{\text{out}}) - i\sqrt{Z} \int d^4y \left[\Psi^{\text{in}}(x^{\text{out}}), \Psi^{\text{in},*}(y) \right] J(y)$$

$$a_{\mathbf{k}}^{\text{out}} = \alpha_k a_{\mathbf{k}}^{\text{in}} + \beta_k a_{-\mathbf{k}}^{\text{in}\dagger} - i\sqrt{Z} \int d^4x\, e^{-i\mathbf{k}\cdot\mathbf{x}} \left(\alpha_k \Psi_k^{\text{in}*} - \beta_k \Psi_k^{\text{in}} \right) J(y)$$

(usual) Bogoliubov tf low

Interaction effects

$$\alpha_k \equiv -iZ \left(\dot{\Psi}_k^{\text{out}*} \Psi_k^{\text{in}} - \Psi_k^{\text{out}*} \dot{\Psi}_k^{\text{in}} \right)$$

$$\beta_k \equiv -iZ \left(\dot{\Psi}_k^{\text{out}*} \Psi_k^{\text{in}*} - \Psi_k^{\text{out}*} \dot{\Psi}_k^{\text{in}*} \right)$$

$$\Psi_k^{\text{in}} = \alpha_k \Psi_k^{\text{out}} + \beta_k \Psi_k^{\text{out}*}$$

$$\Psi_k^{\text{out}} = \alpha_k^* \Psi_k^{\text{in}} - \beta_k^* \Psi_k^{\text{in}*}$$

$$|\alpha_k|^2 - |\beta_k|^2 = 1$$

An example with a scalar field

Produced (occupation) number :

$$n_k = \langle 0^{\rm in} | a_{\mathbf{k}}^{\rm out\dagger} a_{\mathbf{k}}^{\rm out} | 0^{\rm in} \rangle$$

$$= \left| \left(\beta_k a_{-\mathbf{k}}^{\rm in\dagger} - i\sqrt{Z} \int d^4x \, e^{-i\mathbf{k}\cdot\mathbf{x}} \left(\alpha_k \Psi_k^{\rm in*} - \beta_k \Psi_k^{\rm in} \right) J \right) | 0^{\rm in} \rangle \right|^2$$

$$= \begin{cases} V \cdot |\beta_k|^2 + \cdots & (\beta_k \neq 0) \\ 0 \quad + Z \left| \int d^4x \, e^{-i\mathbf{k}\cdot\mathbf{x}} \, \Psi_k^{\rm in*} \, J \, | 0^{\rm in} \rangle \right|^2 + \cdots & (\beta_k = 0) \end{cases}$$

→ Particles can be produced even if $\beta_k = 0$!

3. Calculation of particle number

Equation of Motion (again) :

$$\phi \;:\quad 0 = (\partial^2 + \cancel{g^2|\chi|^2})\phi + \cancel{\tfrac{1}{2} g \psi_\chi^\dagger \psi_\chi^\dagger}$$

$$\chi \;:\quad 0 = \left(\partial^2 + g^2|\phi|^2 + \cancel{\tfrac{1}{2} g^2 |\chi|^2}\right)\chi + \cancel{g \psi_\phi^\dagger \psi_\chi^\dagger}$$

$$\psi_\phi \;:\quad 0 = \bar{\sigma}^\mu \partial_\mu \psi_\phi + \cancel{i g \chi^* \psi_\chi^\dagger}$$

$$\psi_\chi \;:\quad 0 = \bar{\sigma}^\mu \partial_\mu \psi_\chi + i g \phi^* \psi_\chi^\dagger + \cancel{i g \chi^* \psi_\phi^\dagger}$$

ϕ : macroscopic

χ , ψ_ϕ, ψ_χ : microscopic

EOM for asymptotic fields (as = in, out):

$$\phi^{\mathrm{as}} \;:\; 0 = \partial^2 \phi^{\mathrm{as}}$$

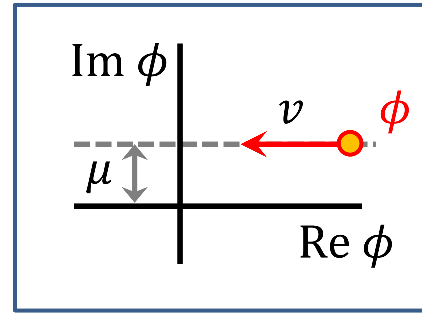
$$\chi^{\mathrm{as}} \;:\; 0 = (\partial^2 + g^2 |\langle\phi\rangle|^2)\chi^{\mathrm{as}}$$

$$\psi_\phi^{\mathrm{as}} \;:\quad 0 = \bar{\sigma}^\mu \partial_\mu \psi_\phi^{\mathrm{as}}$$

$$\psi_\chi^{\mathrm{as}} \;:\quad 0 = \bar{\sigma}^\mu \partial_\mu \psi_\chi^{\mathrm{as}} + i g \langle\phi^*\rangle \psi_\chi^{\mathrm{as}\dagger}$$

Solutions for Asymptotic fields

Assuming $\langle\phi\rangle = \langle\phi\rangle(t)$ for simplicity, then



- $$\phi^{\mathrm{as}} = \langle\phi^{\mathrm{as}}\rangle + \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(\phi_k^{\mathrm{as}} a_{\phi\mathbf{k}}^{\mathrm{as}} + \phi_k^{\mathrm{as}*} b_{\phi-\mathbf{k}}^{\mathrm{as}\dagger} \right)$$

$0 = \partial^2 \phi^{\mathrm{as}}$

$$\sqrt{Z_\phi}\langle\phi^{\mathrm{in}}\rangle = vt + i\mu \ , \quad \sqrt{Z_\phi}\phi_k^{\mathrm{in}} = \sqrt{Z_\phi}\phi_k^{\mathrm{out}} = \frac{1}{\sqrt{2|\mathbf{k}|}} e^{-i|\mathbf{k}|t}$$

→ $\beta_{\phi k} = 0$

$$\phi_k^{\mathrm{out}} = \alpha_k^* \phi_k^{\mathrm{in}} - \beta_k^* \phi_k^{\mathrm{in}*}$$

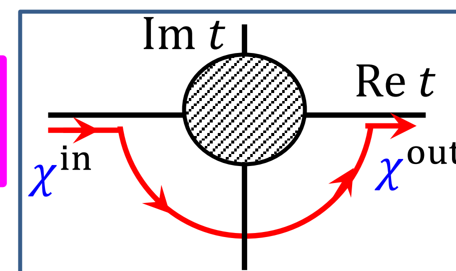
- $$\chi^{\mathrm{as}} = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(\chi_k^{\mathrm{as}} a_{\chi\mathbf{k}}^{\mathrm{as}} + \chi_k^{\mathrm{as}*} b_{\chi-\mathbf{k}}^{\mathrm{as}\dagger} \right)$$

$$\sqrt{Z_\chi}\chi_k^{\mathrm{in}} \sim \frac{1}{\sqrt{2\omega_k}} \exp\left[-i\int^t dt' \omega_k(t')\right]$$ (valid for $|t| \gtrsim 1/\sqrt{gv}$)

$= (\partial^2 + g^2|\langle\phi\rangle|^2)\chi^{\mathrm{as}}$

$$\left(\omega_k \equiv \sqrt{\mathbf{k}^2 + Z_\phi g^2 |\langle\phi\rangle|^2} \right)$$

(analytic continuation) → $\beta_{\chi k} \sim -i \exp\left[-\pi \frac{k^2 + g^2\mu^2}{2gv}\right]$



Solutions for Asymptotic fields

eigen spinor for helicity op. :
$k^i \bar{\sigma}^i e_{\mathbf{k}}^{\pm} = \pm|\mathbf{k}|\bar{\sigma}^0 e_{\mathbf{k}}^{\pm}$

- $$\psi_{\phi}^{\rm as} = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \left(e_{\mathbf{k}}^{+} \psi_{\phi k}^{+,{\rm as}} a_{\psi_\phi \mathbf{k}}^{+{\rm as}} + e_{\mathbf{k}}^{-} \psi_{\phi k}^{-,{\rm as}*} a_{\psi_\phi -\mathbf{k}}^{-{\rm as}\dagger} \right)$$

$$\sqrt{Z_\phi}\psi_{\phi k}^{\pm,{\rm in}} = \sqrt{Z_\phi}\psi_{\phi k}^{\pm,{\rm out}} = e^{-i|\mathbf{k}|t}$$

$0 = \bar{\sigma}^\mu \partial_\mu \psi_\phi^{\rm as}$

$\rightarrow$ $\beta_{\psi_\phi k}^{\pm} = 0$

- $$\psi_{\chi}^{\rm as} = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} \sum_{s=\pm} \left(e_{\mathbf{k}}^{s} \psi_{\chi k}^{(+)s,{\rm as}} a_{\psi_\chi \mathbf{k}}^{s} - \sigma^0 e_{-\mathbf{k}}^{s\dagger} \psi_{\chi k}^{(-)s,{\rm as}*} a_{\psi_\chi -\mathbf{k}}^{s\dagger} \right)$$

$$\sqrt{Z_\chi}\psi_{\chi k}^{(\pm)s,{\rm in}} \sim \frac{1}{2}\left(\sqrt{1-\frac{gvt}{\omega_k}} \pm \sqrt{1+\frac{gvt}{\omega_k}} e^{i\theta_k^s} \right) \exp\left[-i \int^t dt' \omega_k(t') \right]$$

$0 = \bar{\sigma}^\mu \partial_\mu \psi_\chi^{\rm as} + ig\langle \phi^* \rangle \psi_\chi^{{\rm as}\dagger}$

(valid for $|t| \gtrsim 1/\sqrt{gv}$)

$$\left(\omega_k \equiv \sqrt{\mathbf{k}^2 + Z_\phi g^2 |\langle \phi \rangle|^2}, \quad e^{i\theta_k^s} \equiv \frac{s|\mathbf{k}| - ig\mu}{\sqrt{\mathbf{k}^2 + g^2\mu^2}} \right)$$

(analytic continuation) $\rightarrow$ $\beta_{\psi_\chi k}^{s} \sim -s \cdot \exp\left[-\pi \frac{k^2 + g^2\mu^2}{2gv} \right]$

Produced Particle number

- $n_{\chi k} = \left|\beta_{\chi k}\right|^2 + \cdots = \exp\left[-\pi \frac{k^2+g^2\mu^2}{gv}\right] + \cdots$
- $n^s_{\psi_\chi k} = \left|\beta^s_{\psi_\chi k}\right|^2 + \cdots = \exp\left[-\pi \frac{k^2+g^2\mu^2}{gv}\right] + \cdots$

→ leading term is obtained

- $n_{\phi k} = \left|\beta_{\phi k}\right|^2 + \cdots = 0 + \cdots$
- $n^s_{\psi_\phi k} = \left|\beta^s_{\psi_\phi k}\right|^2 + \cdots = 0 + \cdots$

Focus on!

→ need to calculate next to leading order

Calculation of $n_{\psi_\phi k}$

$$n_{\psi_\phi k} = \left\langle 0^{\text{in}} \middle| a^{\text{s,out}\dagger}_{\psi_\phi \mathbf{k}} a^{\text{s,out}}_{\psi_\phi \mathbf{k}} \middle| 0^{\text{in}} \right\rangle$$

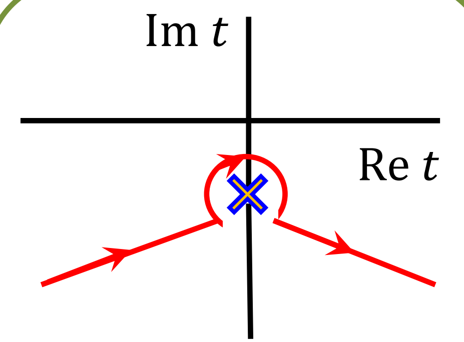
$$= g^2 Z_\phi Z_\chi^2 \left| \int d^4x \, e^{-i\mathbf{k}\cdot\mathbf{x}} \psi^{\text{in}*}_{\phi k} \cdot \chi^{\text{in}*} \cdot e^{s\dagger}_{\mathbf{k}} \psi^{\text{in}\dagger}_{\chi} \, |0^{\text{in}}\rangle \right|^2 + \cdots$$

$$\sim V \cdot g^2 Z_\phi Z_\chi^2 \int \frac{d^3p}{(2\pi)^3} \sum_{r=\pm} \frac{1}{2} \left(1 - sr \frac{\mathbf{p}\cdot\mathbf{k}}{pk}\right) \left| \int dt \, \psi^{\text{in}*}_{\phi k} \, \chi^{\text{in}}_{|\mathbf{k}+\mathbf{p}|} \, \psi^{(+)r,\text{in}}_{\chi p} \right|^2$$

$$\sqrt{Z_\phi} \psi^{\text{in}}_{\phi k} = e^{-i|\mathbf{k}|t}$$

$$\sqrt{Z_\chi} \chi^{\text{in}}_k \sim \frac{1}{\sqrt{2\omega_k}} e^{-i\int^t dt' \omega_k(t')}$$

$$\sqrt{Z_\chi} \psi^{(+)s,\text{in}}_{\chi k} \sim \frac{1}{2} \left(\sqrt{1 - \frac{gvt}{\omega_k}} \pm \sqrt{1 + \frac{gvt}{\omega_k}} e^{i\theta^s_k} \right) e^{-i\int^t dt' \omega_k(t')}$$



- This integral is too complicated to perform

→ We estimated in special case $\mathbf{k} = 0$ with ***steepest decent method***

Analytical Result

$$n^{s}_{\psi_\phi k=0}/V = C \cdot \frac{g^2}{4\pi} \cdot \exp\left[-\pi \frac{g^2\mu^2}{gv}\right]$$

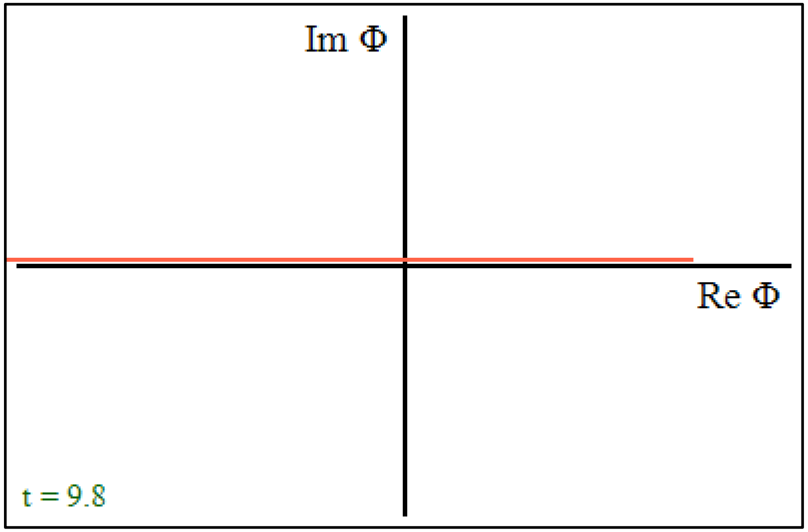
$$\left(C = \left(\frac{3}{2}\right)^{17/6} \frac{\Gamma(4/3)^2}{(\pi^2 e)^{2/3}} \sim 0.28 \right)$$

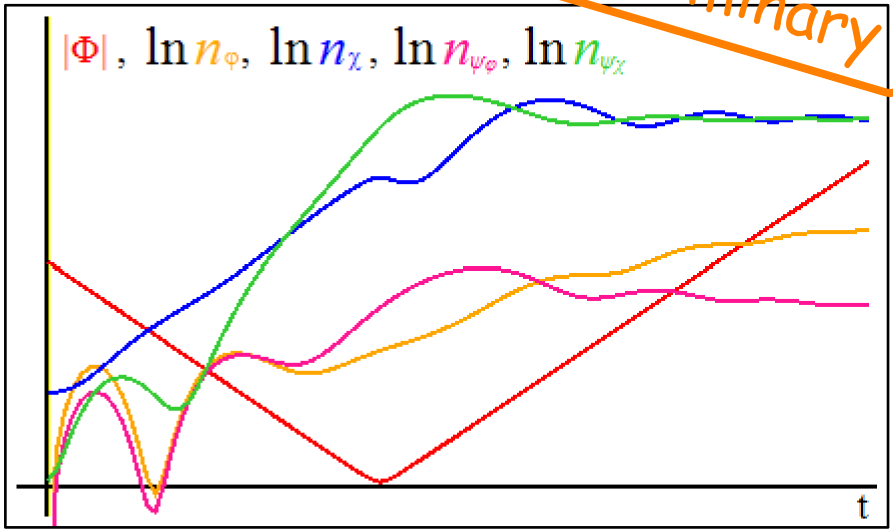
(c.f.) $n_{\chi k}/V \sim n^{s}_{\psi_\chi k}/V \sim \exp\left[-\pi \frac{k^2+g^2\mu^2}{gv}\right]$

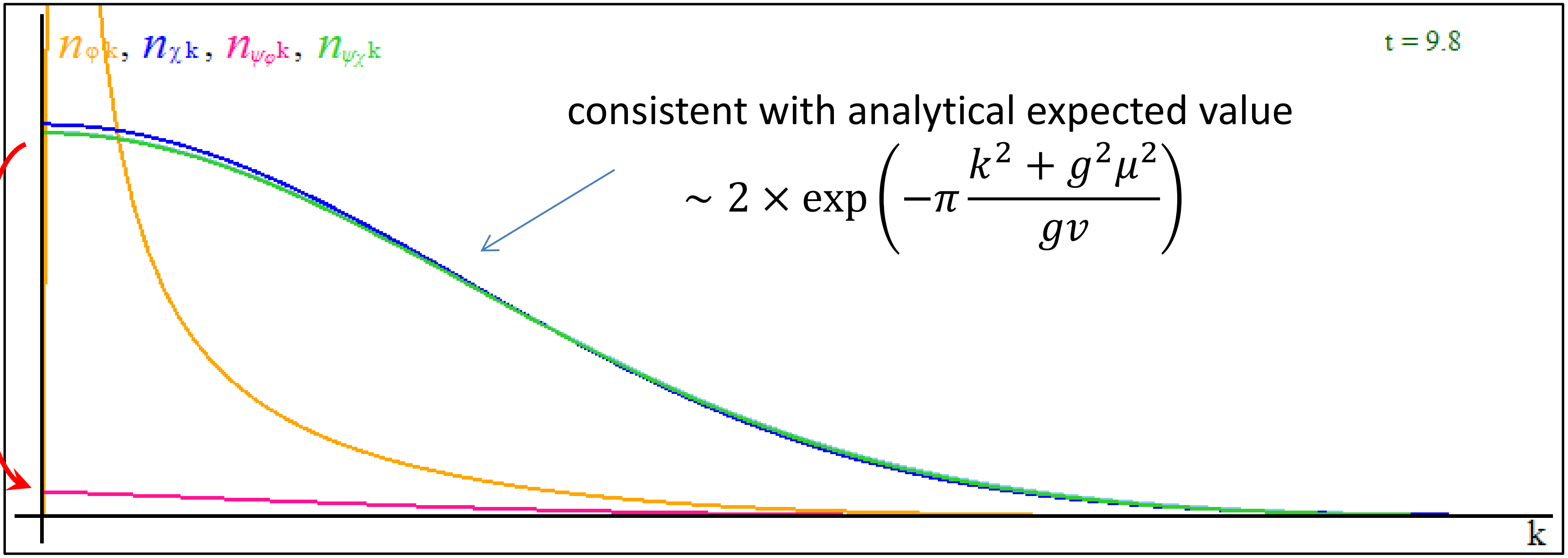
- Produced number of ψ_ϕ is suppressed by factor g^2 comparing with χ or ψ_χ
  - This results is consistent with perturbativity

Numerical Results

Preliminary







consistent with analytical expected value

$$\sim 2 \times \exp\left(-\pi \frac{k^2 + g^2\mu^2}{gv}\right)$$

$$0.28 \times \frac{g^2}{4\pi} \times$$

4. Summary

1. We constructed the Bogoliubov transformation taking into account interaction effects
2. We calculated produced particle's (occupation) number

- $n_{\chi k}/V \sim n^s_{\psi_\chi k}/V \sim \exp\left[-\pi \frac{k^2+g^2\mu^2}{gv}\right]$
- $n^s_{\psi_\phi k=0}/V \sim 0.28 \cdot \frac{g^2}{4\pi} \cdot \exp\left[-\pi \frac{g^2\mu^2}{gv}\right] \neq 0$
  - Massless particle can be produced, however the production is suppressed by the coupling
- $n_{\phi k=0}/V$ … now in progress
  - The numerical result shows massless bosonic statistics.
  - It would be interesting to see modification induced by supersymmetry breaking terms

    - this is issue under investigation