Soft supersymmetry breaking terms and unification of Yukawa matrices

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Unification - SU(5) model: matter & Higgs sector

$$U(1) \times SU(2) \times SU(3) \subset SU(5)$$

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$$\underbrace{(\mathbf{3}, \mathbf{2}, \tfrac{1}{6})}_{q} \oplus \underbrace{(\mathbf{3}^*, \mathbf{1}, -\tfrac{2}{3})}_{u*} \oplus \underbrace{(\mathbf{1}, \mathbf{1}, 1)}_{e} = \mathbf{10}$$

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The minimal (SUSY) model includes following Higgs fields:

$$\underbrace{\mathbf{5}_H,\ (\mathbf{5}^*{}_H)}_{\text{for electroweak SSB}} \qquad \underbrace{\mathbf{24}_H}_{\text{to break SU(5) to U(1)xSU(2)xSU(3)}}$$

Problem's anatomy in SU(5)

In SM and MSSM the fermion masses are independent parameters and are given by 3 Yukawa matrices:

$$Y^u \to m_u,\ m_c,\ m_t$$
$$Y^d \to m_d,\ m_s,\ m_b$$
$$Y^e \to m_e,\ m_\mu,\ m_\tau$$

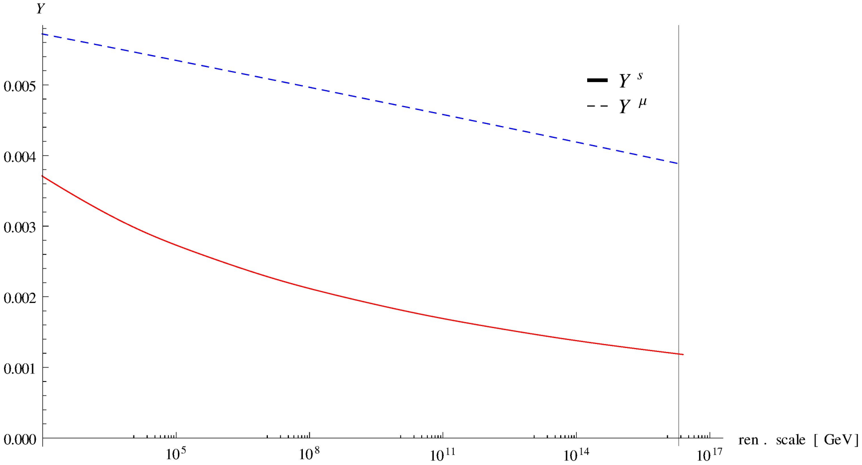
In the minimal SU(5) Grand Unified Theory the symmetry requires:

$$Y_d = Y_e,\ Y_s = Y_\mu,\ Y_b = Y_\tau$$

flavour mixing (CKM matrix can be included in) $\mathbf{Y_u}$

Yukawa unification

Unsuccesful unification of strange and mu Yukawa couplings:
$\tan\beta = 10$, $M_{1/2} = m_0 = 600\,GeV$, $A^{de} = A^u = 0$



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Yukawa unification - Solution 1 - modify GUT structure

Change the boundary condition at the high scale

- additional Higgs fields, e.g.
  $\mathbf{45}_H \to m_b = m_\tau,\ m_s = \frac{1}{3} m_\mu,\ m_d = 3 m_e$ Georgi, Jarlskog 1979
- correction O(1) from higher-dim. operators
  - original idea accompanying GUTs in '70s
  - many modern treatments: 0903.2793, 1009.6000, 1101.5423, 1109.3396, 1211.0516 [arXiv]
  - also with other mechanisms: 1211.6529, 1202.4012
- many combinations with various masses' ratios: *S. Antusch, S.F. King, M. Spinrath 1311.0877*

Yukawa unification - Solution 2

Manipulate the boundary condition between SM and MSSM - play with threshold corrections

'06 Diaz-Cruz, Murayama, Pierce, arXiv: hep-ph/0012275

'09 Ts. Enkhbat, arXiv:0909.5597

Our analysis:

- full 1-loop chirality changing threshold corrections in MSSM (implemented as modification to Softsusy 3.3.8 Allanach, hep-ph/0104145)
- simpler ansatz - flavour violation in $\mathbf{Y_u}$
- quantitatively testing flavour observables

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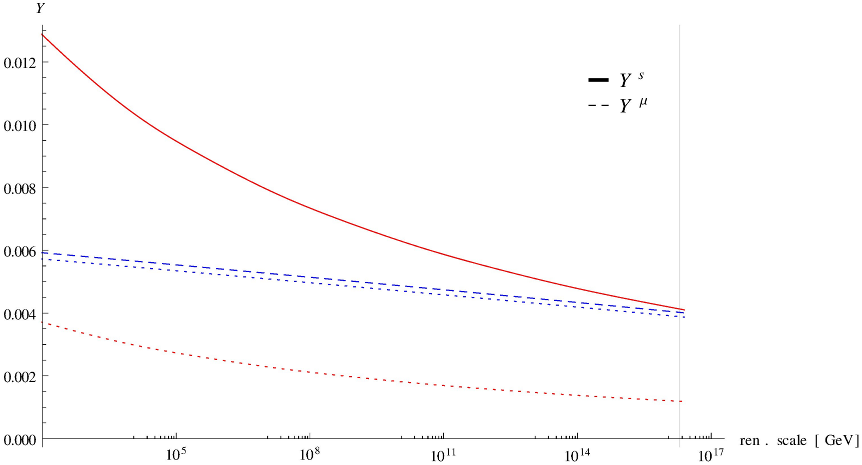

(dotted - running for unadjusted lower boundary condition)

Soft parameters

Soft-supersymmetry breaking A terms in MSSM:

$$\mathcal{L}_{soft} \ni \tilde{q}\mathbf{A}^u\tilde{u}h_u + \tilde{q}\mathbf{A}^d\tilde{d}h_d + \tilde{l}\mathbf{A}^e\tilde{e}h_d$$

Yukawa couplings can be unified within MSSM

with big flavour-diagonal A terms

making the MSSM vacuum metastable

Chosen parameters - working assumptions

1. $\tan\beta$
2. GUT scale parameters:
   - $M_{1/2}$ common gaugino mass
   - m_0 common sfermion mass
   - A^u_{33}, $\mathbf{A}^{de}$ matrix diagonal in the SCKM basis
3. EWSB scale parameters:
   - μ, m_{A^0} Higgs sector parameterization
4. SM parameters fixed by measurements

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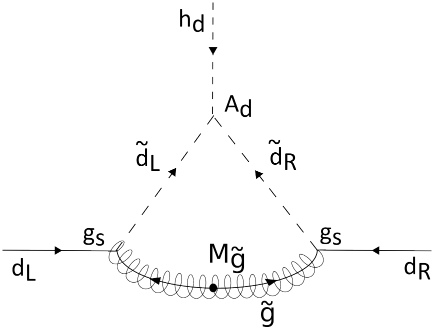
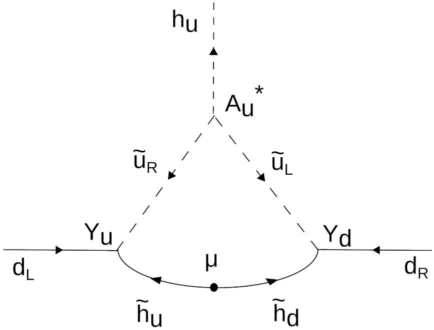
Some of the diagrams involved in threshold corrections





Dominant corrections in complete form

i - generation
Σ, ϵ - self-energies

$$m_{q_i} = v_q Y^{q_i} + \Sigma_{ii}^{q,LR}(Y^q)$$

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$$m_{q_i} = v_q Y^{q_i} + \Sigma_{ii}^{q,LR}(Y^q)$$

$$m_{d_i} = v_d Y^{d_i} + \Sigma_{ii}^{\not{Y}} + v_u Y^{d_i} \epsilon_i^d + O(\frac{v^2}{M_{SUSY^2}})$$

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$$Y_{ii}^d = \frac{m_i^d - \Sigma_{\not{Y}}^{d_L R}}{v_d[1 + \tan\beta \cdot \epsilon^d]}$$

Dependence on MSSM parameters

$\mathbf{A}^d_{ii}$ can be used to adjust the magnitude of threshold correction to achieve unification for given values of other parameters

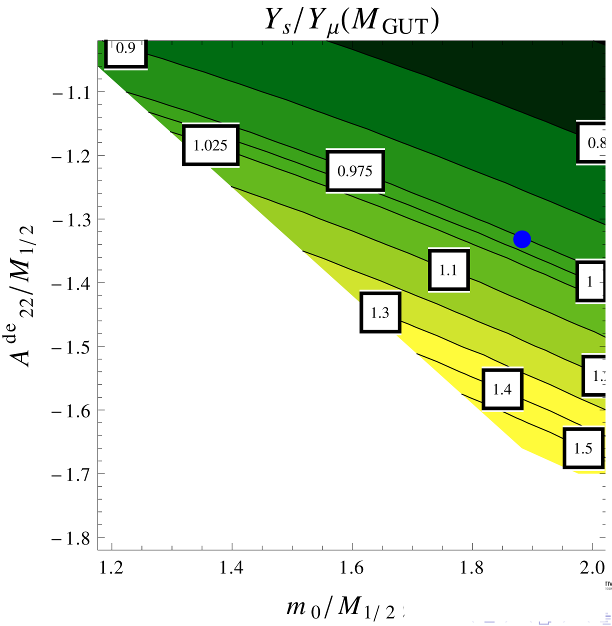
$$\mathbf{Y}^d_{ii} = \frac{m_i^{d,SM} - \Sigma^{d,LR}_{\not{Y}}(\alpha_s M_{\tilde{g}} A^d_{ii}, m_{\tilde{q}_i}, m_{\tilde{d}_i})}{v_d[1 + \tan\beta \cdot \epsilon^d(\mu, M_{\tilde{B}}, M_{\tilde{W}}, M_{\tilde{g}}, m_{\tilde{q}_i}, m_{\tilde{d}_i})]}$$

A. Crivellin, L. Hofer, J. Rosiek, JHEP 1107 (2011) 017 [arXiv:1103.4272]

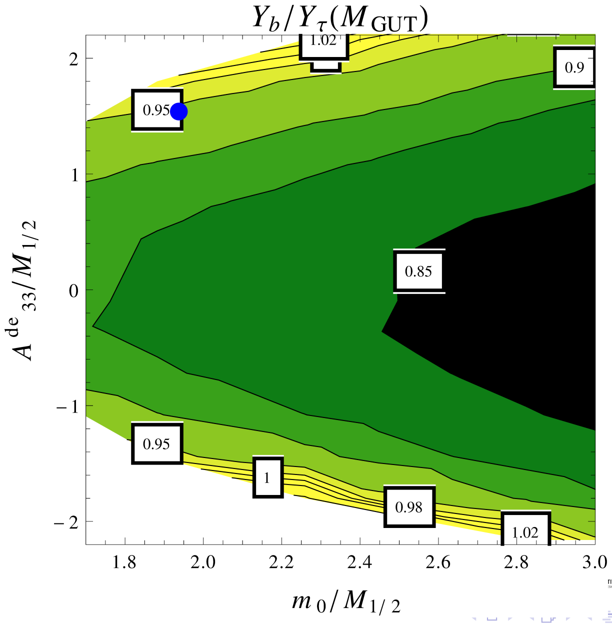
Plots that follow were obtained by variation of one of the model parameters around the point:

$\tan\beta$	$M_{1/2}$	μ	m_0	$\frac{A^{de}_{11}}{M_{1/2}}$	$\frac{A^{de}_{22}}{M_{1/2}}$	$\frac{A^{de}_{33}}{M_{1/2}}$	$\frac{A^{u}_{33}}{M_{1/2}}$	m_{A^0}
28.12	920.73	-1235.4	1733.5	0.0327	-1.458	1.519	-1.0910	4042

Strange quark and muon



Bottom - tau unification



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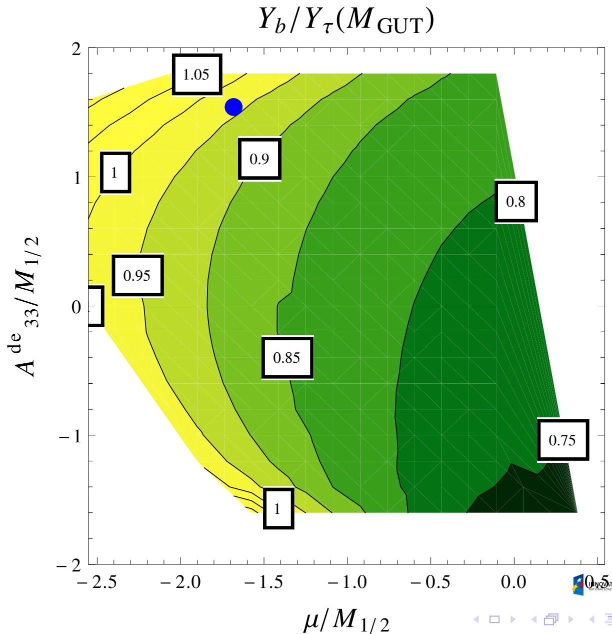

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Examples of points with Yukawa unification

$\tan\beta$	$\frac{M_{1/2}}{\mathrm{GeV}}$	$\frac{m_0}{M_{1/2}}$	$\frac{\mu}{\mathrm{GeV}}$	$\frac{A^{de}_{11}}{M_{1/2}}$	$\frac{\mathbf{A}^{de}_{22}}{M_{1/2}}$	$\frac{\mathbf{A}^{de}_{33}}{M_{1/2}}$	$\frac{\mathbf{A}^{u}_{33}}{M_{1/2}}$	$\frac{m_{A0}}{\mathrm{GeV}}$
8	983.9	2.01	-1096.	0.0083	-0.68	1.55	-1.52	2764.
20.7	956.6	1.99	-1450	0.023	-1.16	1.88	-1.20	2500
40.8	979.8	1.94	-1050.	0.054	-1.66	0.09	-1.40	4700

δY_d	δY_s	δY_b	$\frac{A_d}{\tilde{m}_1}$	$\frac{A_s}{\tilde{m}_2}$	$\frac{A_b}{\tilde{m}_3}$
-0.63	2.8	0.14	0.00826	-0.715	1.5
-0.64	2.5	0.12	0.0232	-1.37	1.5
-0.65	2.5	0.044	0.0355	-1.27	-0.3

$$\delta Y_x = \frac{v_d Y_x - M_x}{M_x}(M_Z),\ \tilde{m} = \sqrt{\frac{m^2_{\tilde{q}} + m^2_{\tilde{d}} + m^2_{H_d}}{3}}$$

Analysis setup

Only 3 variables are sensitive to nonzero A terms that are necessary for Yukawa unification in this approach:

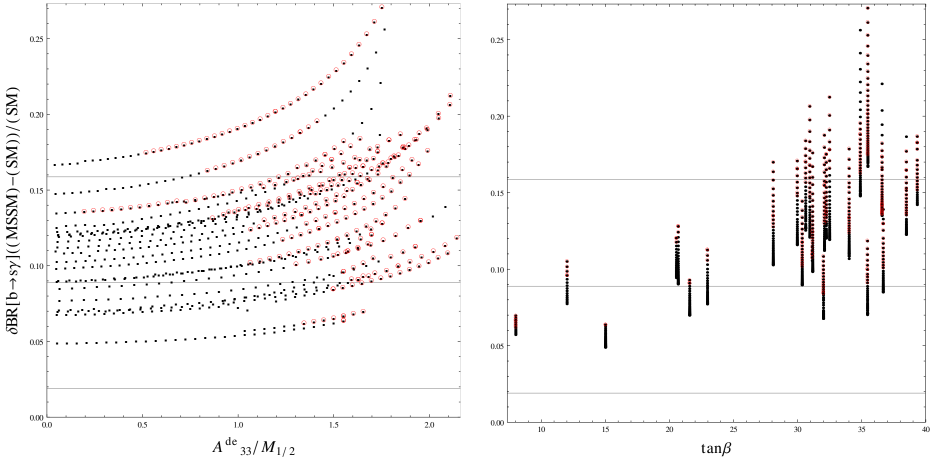
- $BR[B_s \to \mu^+\mu^-]$
- $BR[B_d \to \mu^+\mu^-]$
- $BR[B^0 \to X_s\gamma]$

(others don't vary significantly w.r.t. A terms or their magnitude is tiny).

Flavour observables calculated with `SUSY_FLAVOR v2.10`

Plots present 26 points (incl. examples) for which an appropriate A term was varied between 0 and 150% of the value necessary for unification.

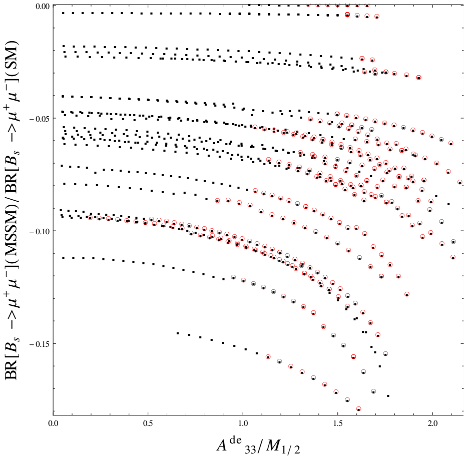
$\delta\mathcal{B}_\gamma \equiv (\mathcal{B}_\gamma^{\text{MSSM}} - \mathcal{B}_\gamma^{\text{SM}})/\mathcal{B}_\gamma^{\text{SM}}$ vs A_{33}^{de} and $\tan\beta$

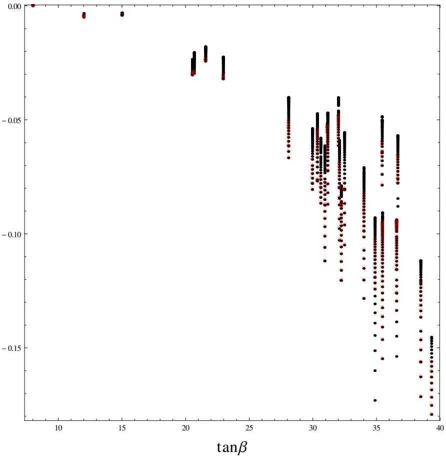


Points fulfilling $\mathbf{Y}_d(M_{GUT}) = \mathbf{Y}_e(M_{GUT})$ in red circles

$\delta\mathcal{B}_{s\mu} \equiv (\mathcal{B}^{\rm MSSM}_{s\mu} - \mathcal{B}^{\rm SM}_{s\mu})/\mathcal{B}^{\rm SM}_{s\mu}$ vs A^{de}_{33} and $\tan\beta$

Experimental values for δ: mean -0.19*SM, sigma 0.2*SM

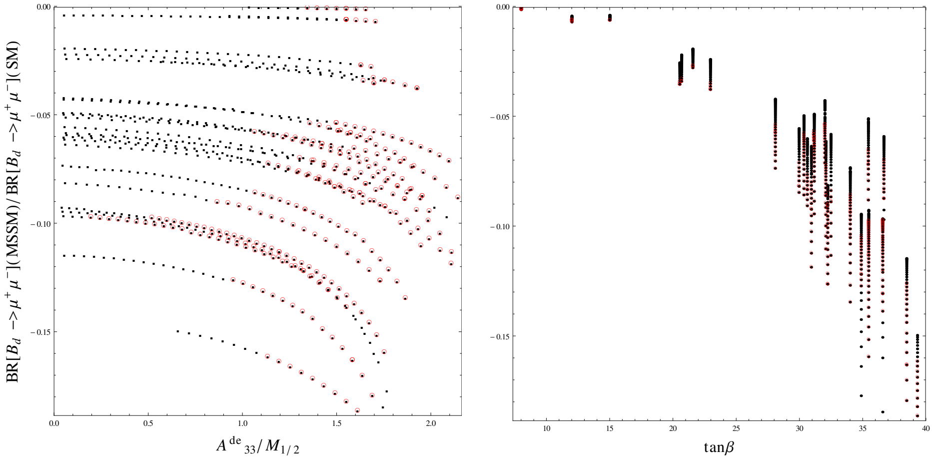




Points fulfilling $\mathbf{Y}_d(M_{GUT}) = \mathbf{Y}_e(M_{GUT})$ in red circles.

$\delta\mathcal{B}_{d\mu} \equiv (\mathcal{B}^{\rm MSSM}_{d\mu} - \mathcal{B}^{\rm SM}_{d\mu})/\mathcal{B}^{\rm SM}_{d\mu}$ vs A^{de}_{33} and $\tan\beta$

Exp: $\left(3.6^{+1.6}_{-1.4}\right) \times 10^{-10}$ SM: $(1.06 \pm 0.09) \times 10^{-10}$



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Relevant stability condition

Along the direction in space of scalar fields of MSSM where

$$|H_1| = |\tilde{s}_L| = |\tilde{s}_R|$$

a deeper, charge and color breaking minimum develops if A^d_{22} is of the order considered here. The stability conditions
given by Casas, Lleyda, Munoz, arXiv: hep-ph/9507294

$$\frac{A_{ii}}{Y_{ii}\tilde{m}} < O(1)$$

are violated:

$$\rightarrow \frac{A_{22}}{Y_{22}\tilde{m}_2}(Q_{EWSB}) \approx 2 * 10^2$$

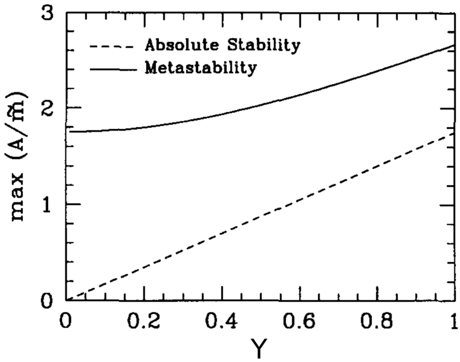
Metastable but durable

The lifetime of the correct MSSM vacuum is longer than the age of the Universe if

$$\frac{A_{22}}{\tilde{m}} < 1.75 \qquad \text{where} \qquad \tilde{m} = \sqrt{\frac{m_{\tilde{q}}^2 + m_{\tilde{d}}^2 + m_{H_d}^2}{3}}$$

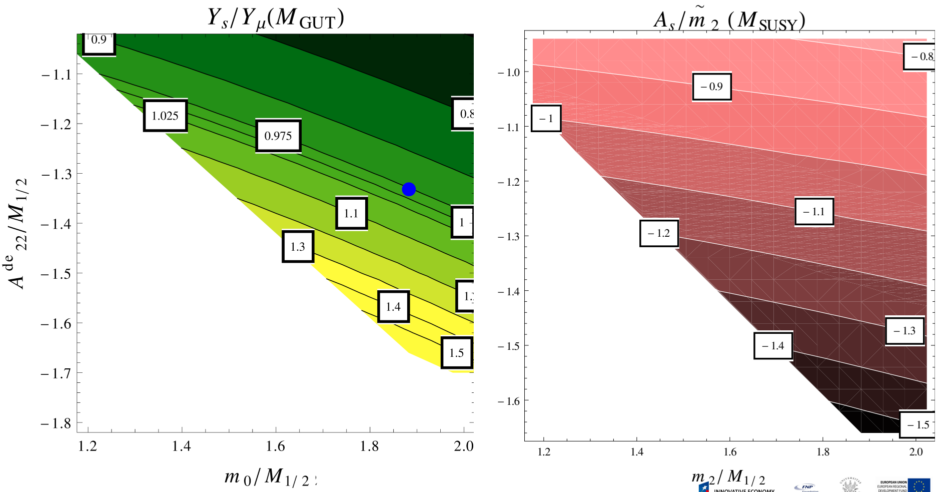
Borzumati, Farrar, Polonsky, Thomas, Nuclear Physics B 555 (1999) 53-115:

F. Borzumati et al. / Nuclear Physics B 555 (1999) 53–115



Unavoidable?

Have big does $A^{de}/\tilde{m}$ have to be?



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Conclusion

Yukawa couplings can be unified within MSSM

with big diagonal A terms

making MSSM vacuum metastable

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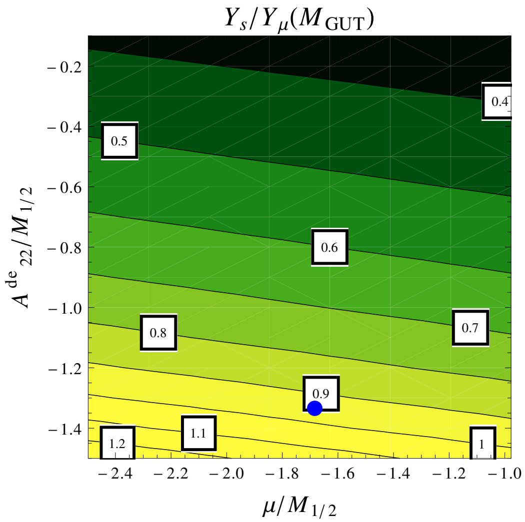
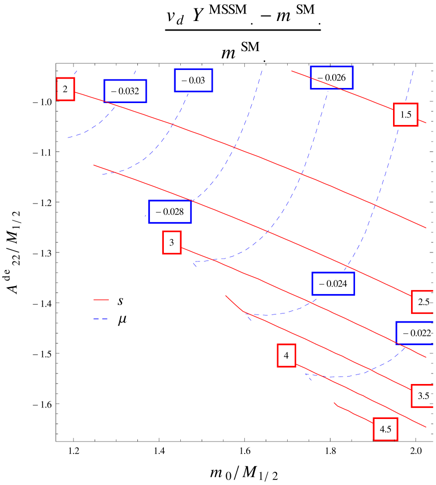
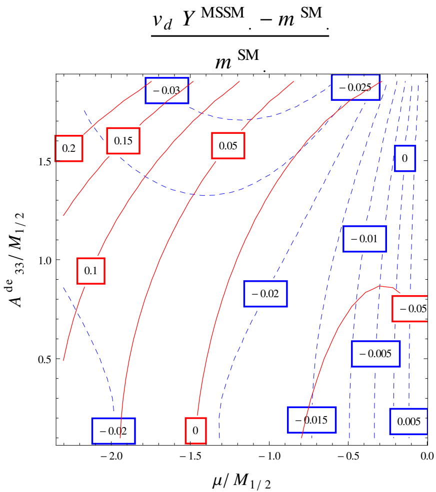

$Y_s/Y_\mu(M_{GUT})$
$A^{de}_{22}/M_{1/2}$
$\mu/M_{1/2}$
−0.2
−0.4
−0.6
−0.8
−1.0
−1.2
−1.4
−2.4
−2.2
−2.0
−1.8
−1.6
−1.4
−1.2
−1.0
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2

Δm_s dependence on A^{de}_{22} and m_0

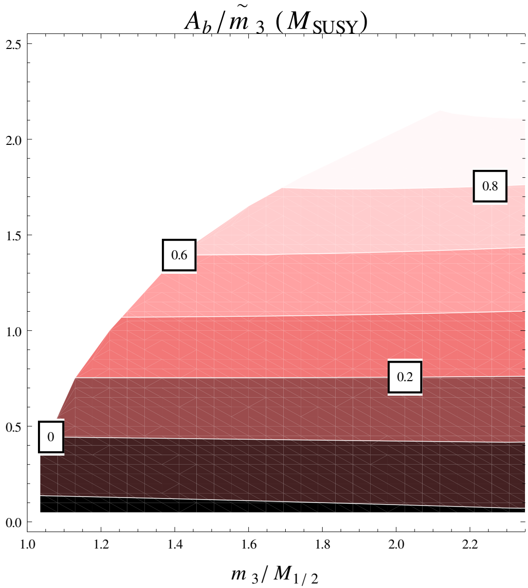


Threshold corrections to Y_s (red, solid) and Y_μ (blue, dashed)

Δm_b dependence on A_{33}^{de} and μ



Threshold corrections to Y_b (red, solid) and Y_τ (blue, dashed)



$A_b/\tilde{m}_3\ (M_{SUSY})$
2.5
2.0
1.5
1.0
0.5
0.0
1.0
1.2
1.4
1.6
1.8
2.0
2.2
0.8
0.6
0.2
0
$m_3/M_{1/2}$

Minimal Supersymmetric Standard Model (MSSM)

SM field	g_μ	W_μ	B_μ	q	u	d	l	e	$\varphi \to h_d$	$\varphi^\dagger \to h_u$
superpartner	$\tilde{g}$	$\tilde{W}$	$\tilde{B}$	$\tilde{q}$	$\tilde{u}$	$\tilde{d}$	$\tilde{l}$	$\tilde{e}$	$\tilde{h}_d$	$\tilde{h}_u$

$$\mathcal{L}_{MSSM} = \mathcal{L}_{kin} + \mathcal{L}_{SUSY} + \mathcal{L}_{soft}$$

The MSSM superpotential defines supersymmetric interactions between superfields (capital lett.) from $\mathcal{L}_{SUSY}$:

$$W_{MSSM} = Q\mathbf{Y}^u U H_u + Q\mathbf{Y}^d D H_d + L\mathbf{Y}^e E H_d + \mu H_d H_u$$

Softly broken supersymmetry is parameterized by $\mathcal{L}_{soft}$: masses of superpartners:

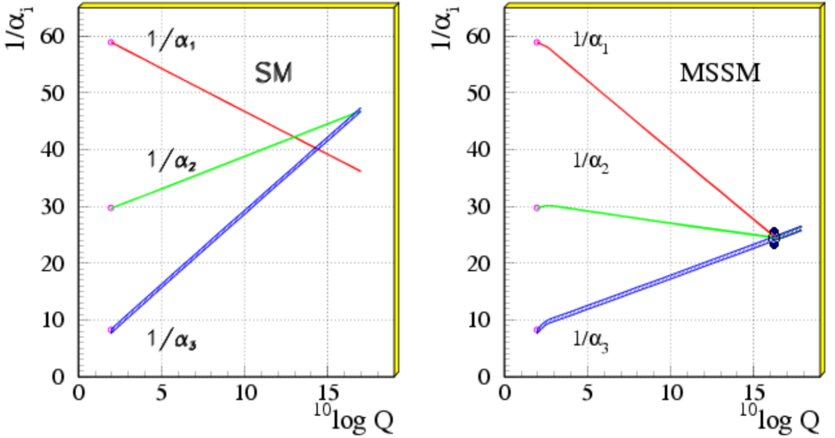
$$\mathcal{L}_{soft} \ni M_{gaugino},\ m_{\tilde{f}},\ m_{h_d},\ m_{h_u}$$

bi- and trilinear Higgs-sfermion-sfermion interaction:

$$\mathcal{L}_{soft} \ni \tilde{q}\mathbf{A}^u \tilde{u} h_u + \tilde{q}\mathbf{A}^d \tilde{d} h_d + \tilde{l}\mathbf{A}^e \tilde{e} h_d + B\mu h_d h_u$$

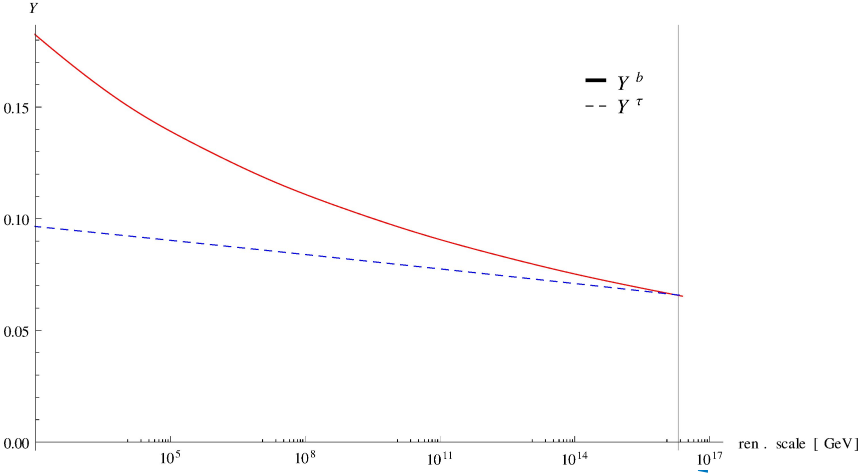
Gauge couplings unification

Gauge coupling constants as measured by LEP (1991 - from PDG 2012)

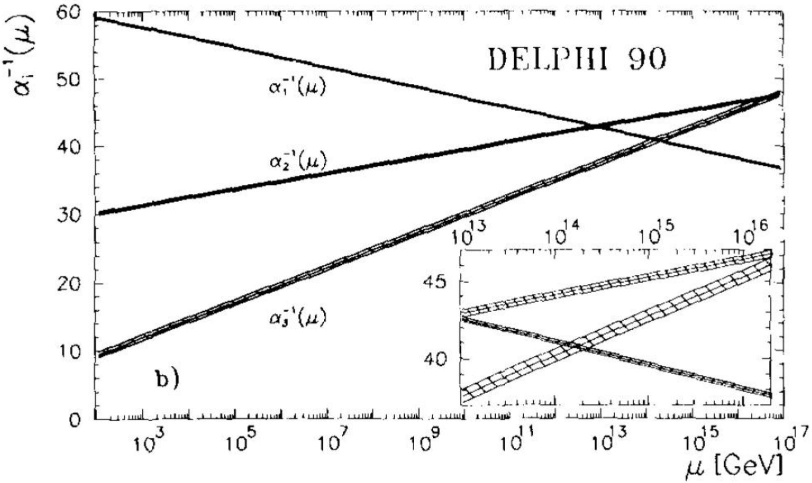


Bottom - tau Yukawa unification

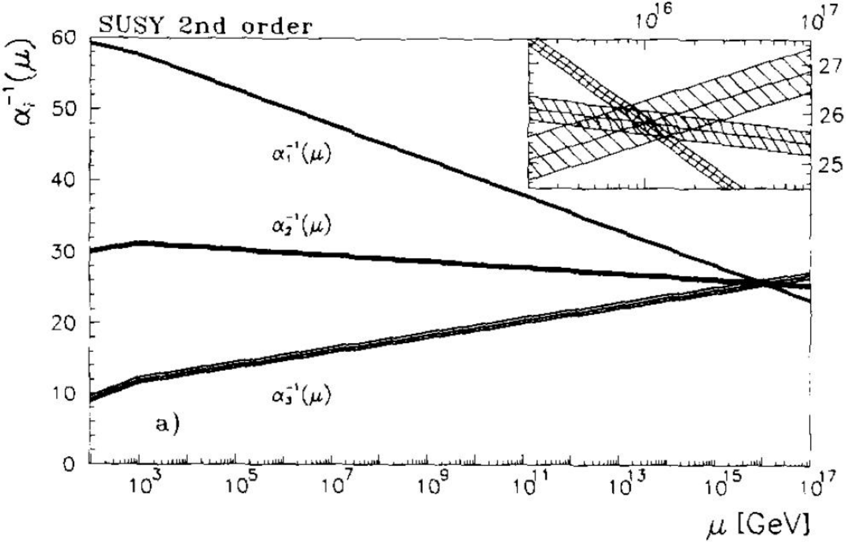
- initial pheno success of non-SUSY GUTs
- most studies - MSSM threshold corrections just for 3rd family



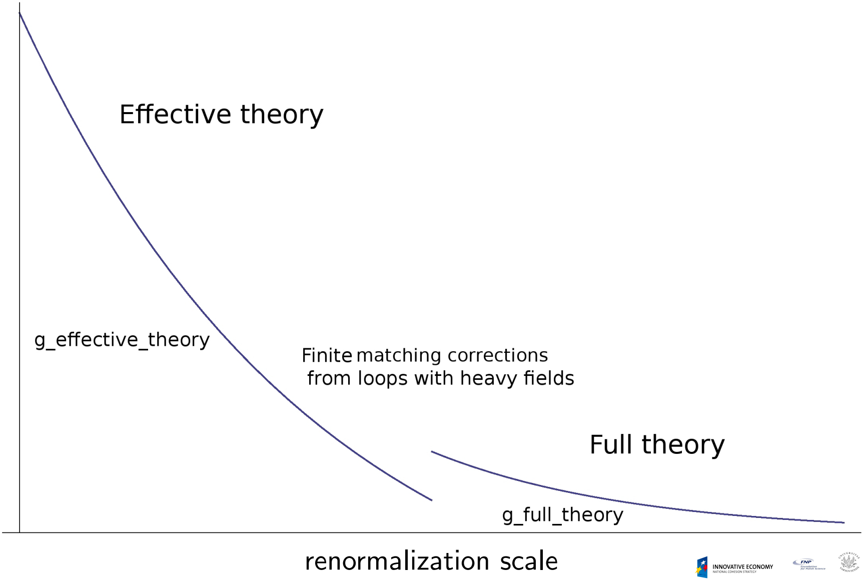
Gauge couplings unification



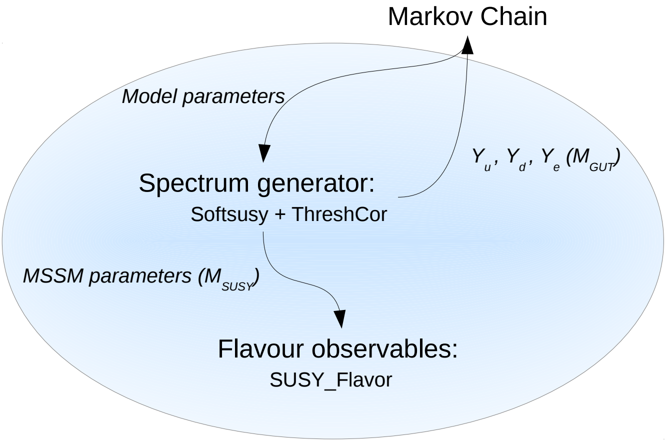
What are threshold corrections?



Threshold corrections = finite matching corrections

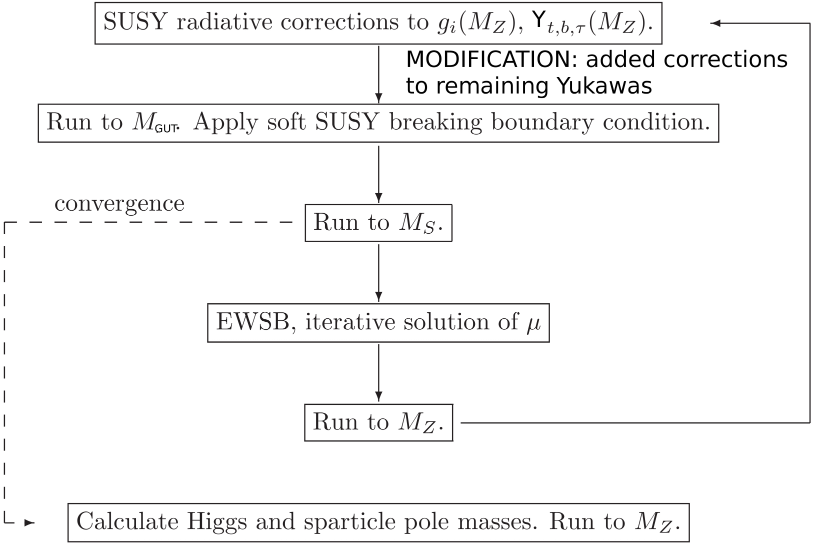


Calculation procedure

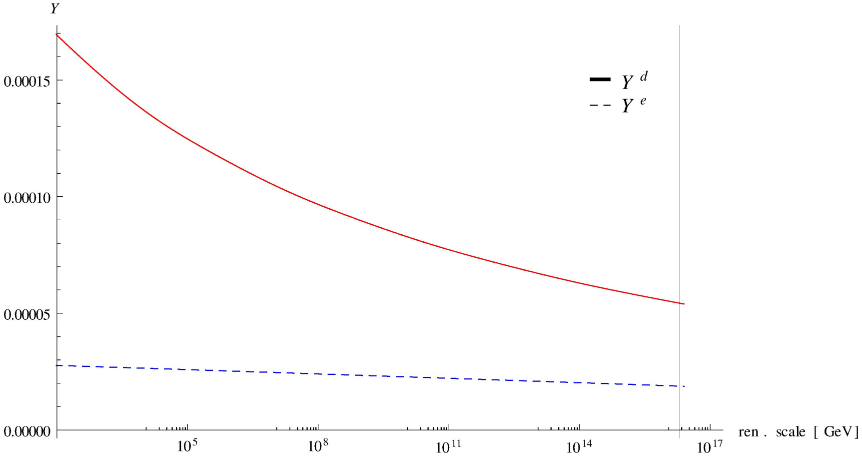


Calculation procedure - modified Softsusy

B. C. Allanach, Softsusy 3.3.8



Down quark and electron 1



Down quark and electron 2