Momentum-Dependent Two-Loop Corrections to the Neutral Higgs-Boson Masses in the MSSM

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Based on 1404.7074 (accepted for publication in EPJC)
and SecDec 2 (C.P.C. 184 2552-2561, C.P.C. 184 396-408)

SUSY 2014, Manchester, July 22th

http://www.feynhiggs.de/ *http://secdec.hepforge.org/*

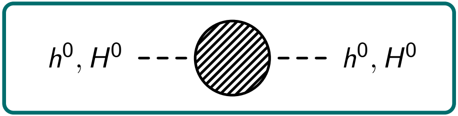
The neutral $\mathcal{CP}$-even Higgs-boson masses in the MSSM

- Feature in the MSSM: Light Higgs-boson mass can be predicted!
- The tree-level neutral $\mathcal{CP}$-even Higgs-boson masses

$$M^{2,\text{tree}}_{\text{Higgs}} = \begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta & -(M_A^2 + M_Z^2)\sin\beta\cos\beta \\ -(M_A^2 + M_Z^2)\sin\beta\cos\beta & M_A^2\cos^2\beta + M_Z^2\sin^2\beta \end{pmatrix}$$

  are limited to $m_h \leq \min(M_Z, M_A)\,|\cos(2\beta)|$

- Higher order corrections shift the Higgs-boson masses considerably



- These lead to maximal values for $m_{h_{max}} \approx 135$ GeV

Status: Radiative corrections in the real MSSM

Higher-order corrections to the Higgs mass sector in the rMSSM:

1-loop 2-loop 3-loop RGE approach

- Ellis, Ridolfi, Zwirner '91; Okada, Yamaguchi, Yanagida '91; Haber & Hempfling '91; Brignole '92; Chankowski, Pokorski, Rosiek '92 '94; Dabelstein '95
- Hempfling & Hoang '94; Carena et al. '95 '96; Espinosa et al. '95 '00 '01; Heinemeyer, Hollik, Weiglein et al. '98 '99 '99 '00; Zhang '99; Degrassi, Slavich et al. '01 '03; Brignole, Degrassi, Slavich, Zwirner '02; Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '05 '13; S. P. Martin '02 '03 '04 '05
- S.P. Martin '07; Harlander, Kant, Mihaila, Steinhauser '08 '10
- Binger '04; Giudice, Strumia '11; Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '13; Draper, Lee, Wagner '13; Bagnaschi, Giudice, Slavich, Strumia '14

Many more publications...

Public codes implementing the rMSSM corrections

FeynHiggs Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '00 '03 '07 '14

SoftSusy Allanach '02 SPheno Porod '03; Porod, Staub '11

CPsuperH Carena, Choi, Drees, Ellis, Lee, Pilaftsis, Wagner '04 '09 '12

Suspect Djouadi, Kneur, Moultaka '07

H3m Kant, Harlander, Mihaila, Steinhauser '10

Summary of the implemented rMSSM corrections before Apr '14:

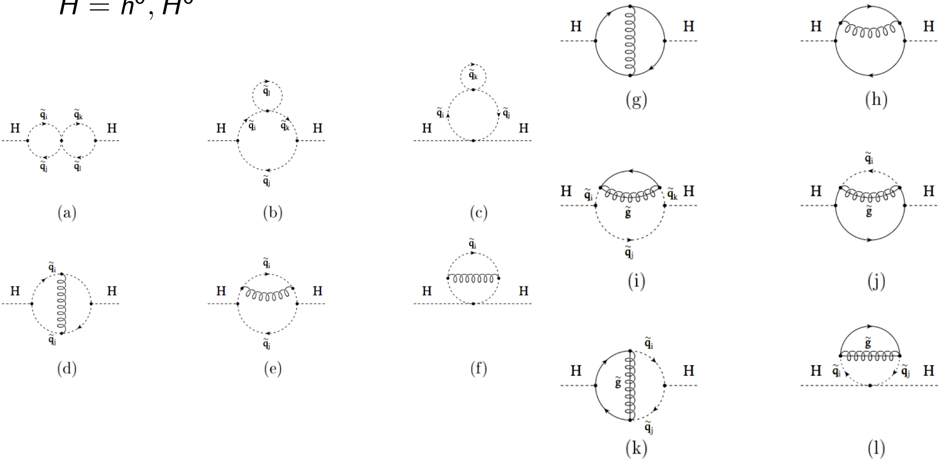
1-loop complete
2-loop $\mathcal{O}(\alpha_s\alpha_t)$, $\mathcal{O}(\alpha_t^2)$, $\mathcal{O}(\alpha_s\alpha_b)$, $\mathcal{O}(\alpha_t\alpha_b)$, $\mathcal{O}(\alpha_b^2)$ at $p^2 = 0$
3-loop $\mathcal{O}(\alpha_s^2\alpha_t)$ at $p^2 = 0$

dominant correction @ 2-loop: $\mathcal{O}(\alpha_s\alpha_t)$ $(p^2 = 0)$

$\longrightarrow$ **next improvement:** $\mathcal{O}(\alpha_s\alpha_t)$ for $p^2 \neq 0$

Higgs-boson self-energy diagrams for $\mathcal{O}(\alpha_s\alpha_t)$

$H = h^0, H^0$

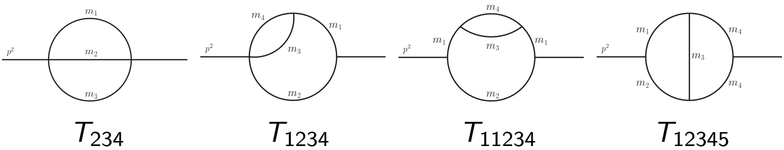


Technical complication: momentum dependence

Treatment of loop integrals

Tensor reduction to only scalar integrals with TwoCalc & FormCalc possible G. Weiglein et al. '93, Hahn et al. '99 '08

- Many of the resulting integrals (mainly 1-loop) are known analytically
- Full analytic results unknown for 4 different two-loop topologies



- These integrals are treated numerically with SecDec

The program SecDec 2.1

Idea and method of sector decomposition pioneered by
Hepp '66, Denner & Roth '96, Binoth & Heinrich '00

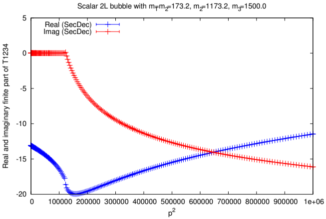
SecDec is a public tool to numerically compute various sorts of integrals contributing to higher-order computations.

It can tackle:

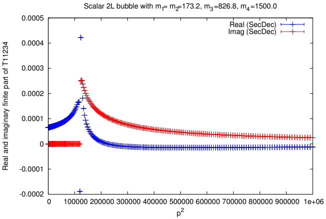
- General Feynman loop integrals, contracted tensor integrals to in principle arbitrary rank
- More general user-defined polynomial / parametric functions
- Since SecDec 2.0: Extension to general kinematics including mass thresholds! SB, Carter & Heinrich '12
- Fast evaluation of diverse integral topologies
- Widely used for numerical calculations/checks

Numerical evaluation of momentum dependent integrals

- 34 mass configurations are run with SecDec, e.g.



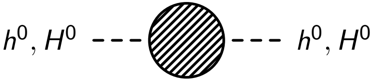
T_{1234}, finite part



T_{11234}, finite part

- differences of kinematic invariants of up to 14 orders of magnitude
- rel. accuracy better than 10^{-5}, timings range from $0.01 - 100$secs

Two-loop renormalization for neutral $\mathcal{CP}$-even Higgs-boson self-energies



- Renormalization procedure consistent with other higher-order corrections in `FeynHiggs` hep-ph/0202166
- Mass renormalization in the OS scheme:

$$\delta M_A^{2(2)},\ \delta t_1^{(2)},\ \delta t_2^{(2)},\ \delta m_{\tilde{t}_1}^{(1)},\ \delta m_{\tilde{t}_2}^{(1)},\ \delta m_t^{(1)},\ \delta A_t^{(1)}$$

- Field renormalization in a "hybrid OS/$\overline{DR}$" scheme:

$$\delta Z_{H_1}^{(2)},\ \delta Z_{H_2}^{(2)},\ \delta\tan\beta^{(2)} = \frac{1}{2}\left(\delta Z_{H_2}^{(2)} - \delta Z_{H_1}^{(2)}\right)$$

- Resulting input parameters: $m_t, \mu, X_t, M_{SUSY}, m_{\tilde{g}}, \tan\beta, m_A$

  $X_t = A_t - \mu\cot\beta$ and A_t the soft SUSY breaking parameters

Effect from inclusion of $p^2 \neq 0$ terms

- Inclusion of field renormalization necessary
- Renormalization condition for A-boson mass must be altered
- Two-loop A-boson mass counter term reads

$$\delta M_A^{2(2)} = \mathrm{Re}\, \Sigma_{AA}^{(2)}(p^2 = M_A^2)$$

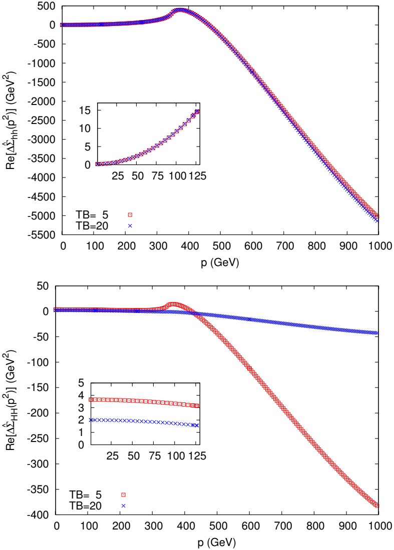
- Consequence: additional constant shift which is physical!

Analysis in m_h^{max} benchmark scenario

Carena, Heinemeyer, Stal, Wagner, Weiglein '13

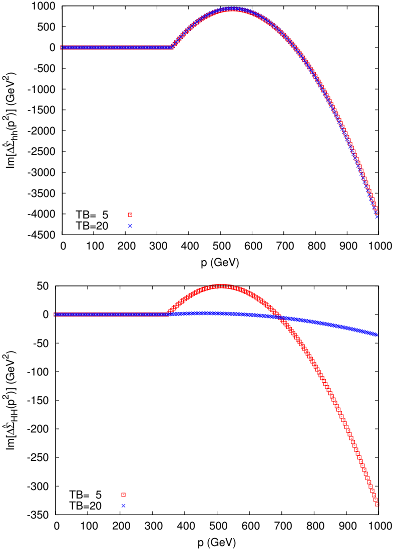
- $m_t = 173.2$ GeV,
- $\mu = 200$ GeV,
- $X_t = 2\, M_{SUSY}$,
- $M_{SUSY} = 1$ TeV,
- $m_{\tilde{g}} = 1.5$ TeV,
- $m_A = 250$ GeV,
- $\tan\beta = 5, 20$

Momentum dependent 2L self-energies - h-H basis



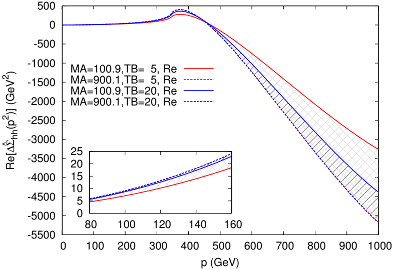
Real part

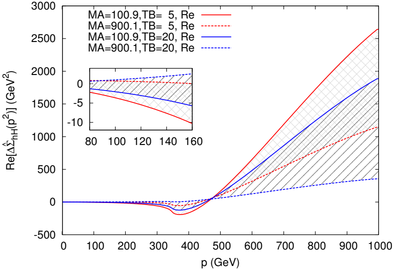
Momentum dependent 2L self-energies - *h*-*H* basis

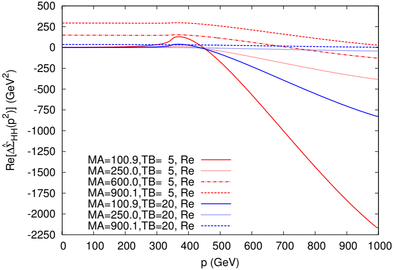


Imaginary part

Dependence on SUSY parameters







m_h^{max} benchmark scenario

$M_A = 100.9 - 900.1$ GeV,
$\tan\beta = 5,\ 20$

Computation of the neutral $\mathcal{CP}$-even MSSM Higgs-boson masses

- Self-energy corrections are included in the inverse Higgs-boson propagator matrix

$$\Gamma \equiv \Delta_{\text{Higgs}}^{-1} = \begin{pmatrix} p^2 - m^2_{H,\text{tree}} + \hat{\Sigma}_H(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m^2_{h,\text{tree}} + \hat{\Sigma}_h(p^2) \end{pmatrix}$$

  with renormalized self-energies $\hat{\Sigma}$ up to the two-loop level
- The masses are the real parts of the poles of the propagator matrix Δ_{Higgs}
- Strategy: Find complex solutions to $\text{Det}(\Gamma) = 0$

New corrections to the neutral $\mathcal{CP}$-even MSSM Higgs-boson masses

$$\left[p^2 - m^2_{h,\text{tree}} + \hat{\Sigma}_{hh}(p^2)\right]\left[p^2 - m^2_{H,\text{tree}} + \hat{\Sigma}_{HH}(p^2)\right] - \left[\hat{\Sigma}_{hH}(p^2)\right]^2 = 0$$

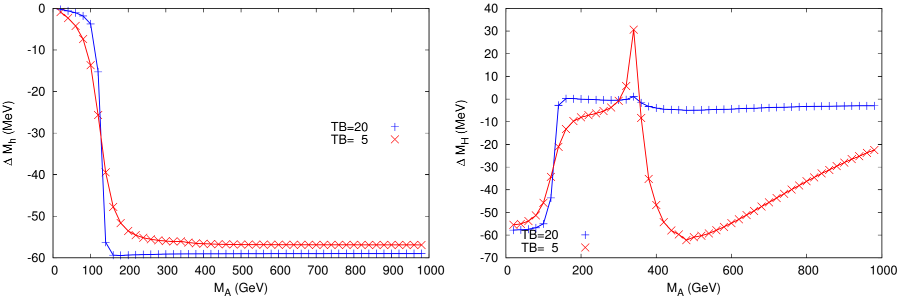
Three steps:

1. Compute M_h and M_H from the self-energies previously implemented in `FeynHiggs`
2. Compute momentum dependent renormalized $\mathcal{O}(\alpha_s\alpha_t)$ self-energies for $p^2 = M_h^2$ and $p^2 = M_H^2$
3. Include new self-energy contributions as constant shifts into `FeynHiggs` and find poles M_h^{new} and M_H^{new}

- corrections available in `FeynHiggs` 2.10.1

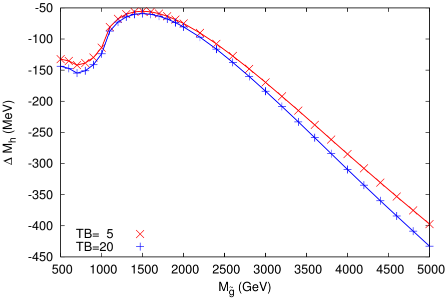
The mass shifts are $\Delta M_{\{h,H\}} = M^{\text{new}}_{\{h,H\}} - M_{\{h,H\}}$

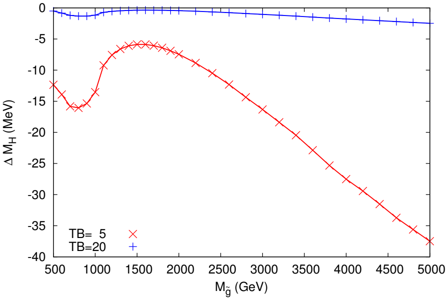
Variation of mass shifts with M_A



- Light Higgs-boson mass: additional shifts at most of the order of the expected linear collider precision ($\sim$ 0.05 GeV)
- Heavy Higgs-boson mass: additional corrections suppressed with tan β

Variation of mass shifts with $M_{\tilde{g}}$





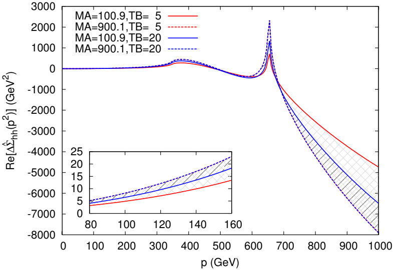
- Light Higgs-boson mass: dependence on $M_{\tilde{g}}$ larger than on M_A, additional shifts are around current exp. accuracy for high $M_{\tilde{g}}$

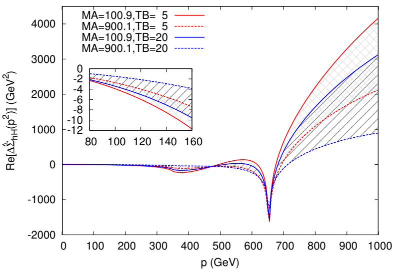
Analysis in light stop benchmark scenario

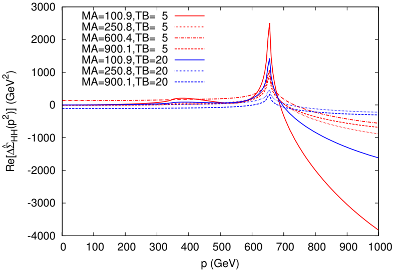
Carena, Heinemeyer, Stal, Wagner, Weiglein '13

- $m_t = 173.2$ GeV,
- $\mu = 200$ GeV,
- $X_t = 2\, M_{SUSY}$,
- $M_{SUSY} = 0.5$ TeV
  $\rightarrow$ light stops: $m_{\tilde{t}_1} = 326.8$ GeV, $m_{\tilde{t}_2} = 673.2$ GeV,
- $m_{\tilde{g}} = 1.6$ TeV,
- $m_A = 250$ GeV,
- $\tan\beta = 5, 20$

Momentum dependent 2L self-energies





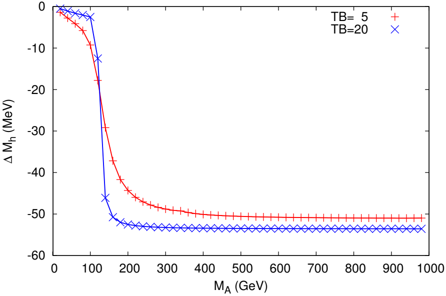


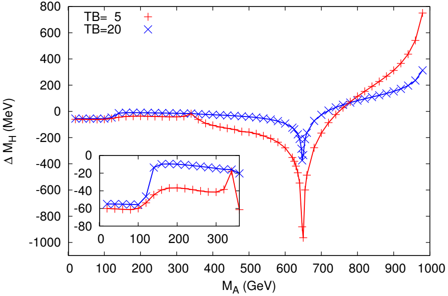
light stop benchmark scenario

$M_A = 100.9 - 900.1$ GeV,
$\tan\beta = 5,\ 20$

large threshold at $p = 2m_{\tilde{t}_1}$

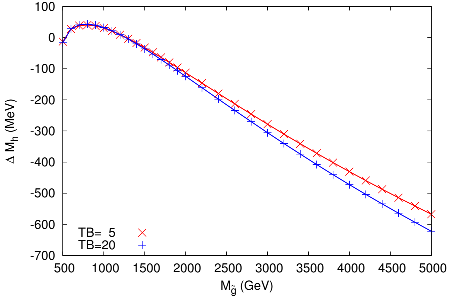
Variation of mass shifts with M_A





- Light Higgs-boson mass: additional shifts at most of the order of the expected linear collider precision
- Heavy Higgs-boson mass: peak corrections around thresholds

Variation of mass shifts with $M_{\tilde{g}}$



Light Higgs-boson mass:

- strong dependence on $M_{\tilde{g}}$
- additional shifts around current exp. accuracy for high $M_{\tilde{g}}$
- shifts larger than in m_h^{max} scenario

Summary & Outlook

Summary

- We computed the momentum dependent 2-loop corrections to the $\mathcal{CP}$-even MSSM Higgs-boson masses adopting a renormalization scheme consistent with `FeynHiggs`
  $\rightarrow$ Reduction of theoretical uncertainty on Higgs-boson masses
- Numerical computation of integrals containing thresholds with the program `SecDec 2.1`
- The additional shifts to the light Higgs-boson mass reaches up to -0.6 GeV for $M_{\tilde{g}}$ around 5 TeV
- Results are implemented in the program `FeynHiggs` version 2.10.1

Outlook

- Improve on timings
- Go beyond the $\mathcal{O}(\alpha_s \alpha_t)$ approximation
- Further phenomenological applications to other massive two-loop calculations using `SecDec`

Backup

Renormalized two-loop self-energies

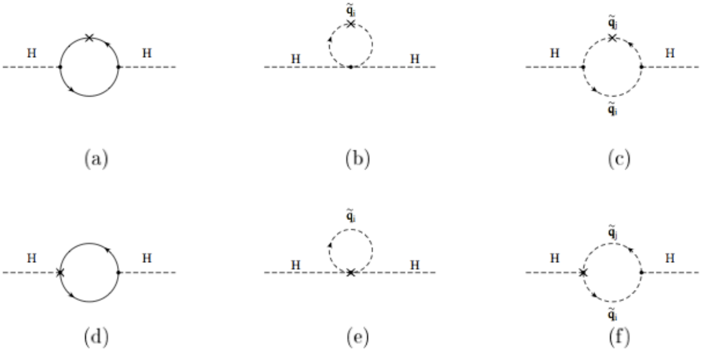
Renormalized self-energies in unphysical ϕ_1-ϕ_2 basis

$$\begin{aligned}
\hat{\Sigma}^{(2)}_{\phi_1^0\phi_1^0}(p^2) &= \Sigma^{(2)}_{\phi_1^0\phi_1^0}(p^2) + p^2\delta Z^{(2)}_{\phi_1} - \delta V^{(2)}_{\phi_1^0\phi_1^0} \\
\hat{\Sigma}^{(2)}_{\phi_2^0\phi_2^0}(p^2) &= \Sigma^{(2)}_{\phi_2^0\phi_2^0}(p^2) + p^2\delta Z^{(2)}_{\phi_2} - \delta V^{(2)}_{\phi_2^0\phi_2^0} \\
\hat{\Sigma}^{(2)}_{\phi_1^0\phi_2^0}(p^2) &= \Sigma^{(2)}_{\phi_1^0\phi_2^0}(p^2) - \delta V^{(2)}_{\phi_1^0\phi_2^0} \ .
\end{aligned}$$

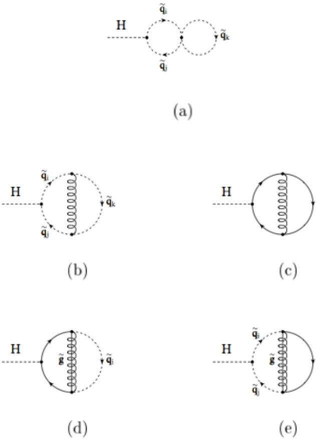
Rotation of the unphysical $\hat{\Sigma}^{(2)}_{\phi_i\phi_j}$ self-energies into the physical $h - H$ basis

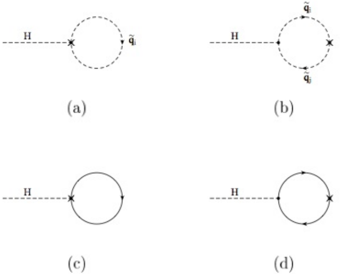
$$\begin{aligned}
\hat{\Sigma}^{(2)}_{H} &= \cos^2\alpha\,\hat{\Sigma}^{(2)}_{\phi_1\phi_1} + \sin^2\alpha\,\hat{\Sigma}^{(2)}_{\phi_2\phi_2} + 2\sin\alpha\cos\alpha\hat{\Sigma}^{(2)}_{\phi_1\phi_2} \\
\hat{\Sigma}^{(2)}_{h} &= \sin^2\alpha\,\hat{\Sigma}^{(2)}_{\phi_1\phi_1} + \cos^2\alpha\,\hat{\Sigma}^{(2)}_{\phi_2\phi_2} - 2\sin\alpha\cos\alpha\hat{\Sigma}^{(2)}_{\phi_1\phi_2} \\
\hat{\Sigma}^{(2)}_{hH} &= -\sin\alpha\cos\alpha\,(\hat{\Sigma}^{(2)}_{\phi_1\phi_1} - \hat{\Sigma}^{(2)}_{\phi_2\phi_2}) + (\cos^2\alpha - \sin^2\alpha)\,\hat{\Sigma}^{(2)}_{\phi_1\phi_2}
\end{aligned}$$

Diagrams for sub-loop renormalization

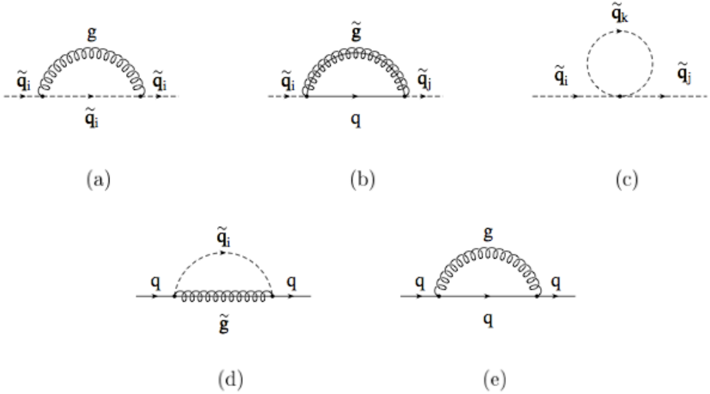


Tadpole diagrams needed in the renormalization





Counter term insertions for sub-loop renormalization



Install SecDec 2.1

- **Download**:
  http://secdec.hepforge.org

- **Install**:
  tar xzvf SecDec.tar.gz
  cd SecDec-2.1
  ./install

- **Prerequisites**:
  Mathematica (version 6 or above), Perl, Fortran and/or C++ compiler

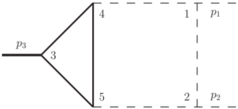
User Input I

- param.input: parameters for integrand specification and numerical integration

```
################ input parameters for sector decomposition ################
#------------------------------------------------------------------------------------------------
# subdirectory for the mathematica output files (will be created if non-existent) :
# if not specified, a directory with the name of the graph given below will be created by default
subdir=2loop
#------------------------------------------------------------------------------------------------
# if outputdir is not specified: default directory for
# the output will have integral name (given below) appended to directory above,
# otherwise specify full path for Mathematica output files here
outputdir=
#------------------------------------------------------------------------------------------------
# graphname (can contain underscores, numbers, but should not contain commas)
graph=P126
#------------------------------------------------------------------------------------------------
# number of propagators:
propagators=6
#------------------------------------------------------------------------------------------------
# number of external legs:
legs=3
#------------------------------------------------------------------------------------------------
# number of loops:
loops=2
#------------------------------------------------------------------------------------------------
# construct integrand (F and U) via topological cuts (only possible for scalar integrals)
# default is 0 (no cut construction used)
cutconstruct=1
########################################################
# parameters for subtractions and epsilon expansion
########################################################
```

User Input II

- template.m: definition of the integrand (Mathematica syntax)




```
(******* USER INPUT for construction of integrand ***************************)

(******* Use with cutconstruct=1 ********************************************)

proplist={{ms[1],{3,4}},{ms[1],{4,5}},{ms[1],{5,3}},
          {0,{1,2}},{0,{1,4}},{0,{2,5}}};

(******* Use with cutconstruct=0 ********************************************)
(*
momlist={k1,k2};
proplist={k1^2-ms[1],(k1+p3)^2-ms[1],(k1-k2)^2-ms[1],
         (k2+p3)^2,(k2+p1+p3)^2,k2^2};
numerator={1};
*)

(******* Propagator powers (optional) ***************************************)

powerlist=Table[1,{i,Length[proplist]}];

(******* On-shell conditions (optional) *************************************)
onshell={ssp[1]->0,ssp[2]->0,ssp[3]->sp[1,2],sp[1,3]->0,sp[2,3]->0};

(******* Set Dimension ******************************************************)

Dim=4-2*eps;
(**************************************************************************)
```

Program Test Run

- ./launch -p param.input -t template.m

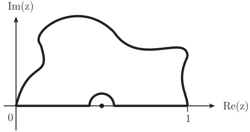
```
********** This is SecDec version 2.0 **********
Authors: Sophia Borowka, Jonathon Carter, Gudrun Heinrich
************************************************
graph = P126
primary sectors 1,2,3,4,5,6, will be calculated
calculating F and U . . .
done
written to /home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126/FUN.m

results of the decomposition will be written to
/home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126
doing sector decomposition . . .
done

working on pole structure: 2 logarithmic poles, 0 linear poles, 0 higher poles
C++ functions created for pole structure 2l0h0
compiling 2l0h0/epstothe0 ...
doing numerical integrations in P126/2l0h0/epstothe0
compiling 2l0h0/epstothe-1 ...
doing numerical integrations in P126/2l0h0/epstothe-1
compiling 2l0h0/epstothe-2 ...
doing numerical integrations in P126/2l0h0/epstothe-2
working on pole structure: 1 logarithmic pole, 0 linear poles, 0 higher poles
C++ functions created for pole structure 1l0h0
compiling 1l0h0/epstothe0 ...
doing numerical integrations in P126/1l0h0/epstothe0
compiling 1l0h0/epstothe-1 ...
doing numerical integrations in P126/1l0h0/epstothe-1
working on pole structure: 0 logarithmic poles, 0 linear poles, 0 higher poles
C++ functions created for pole structure 0l0h0
compiling 1l0h0/epstothe0 ...
doing numerical integrations in P126/0l0h0/epstothe0
Output written to /home/pcl335a/sborowka/Work/SecDecBeta/loop/2loop/P126/P126_pfull.res

To remove intermediate files, execute the command /home/pcl335a/sborowka/Work/SecDecBeta/loop/launchcleanP126
```

Get the Result

- resultfile P126_full.res

```
**********************************************
***OUTPUT: P126 p5 ***************
 point: 7.0
 ext. legs: 0.0 0.0 7.0
 prop. mass: 1.0 0. 0. 0. 0. 0.
Prefactor=-Exp[-2EulerGamma*eps]
**********************************************
****** eps^-2 coeff ******

result       =0.07563683
             +0.1003924148 I
error        =0.000493522517701388
             + 0.00139691015080074 I
CPUtime (all eps^-2 subfunctions)  =0.04
CPUtime (longest eps^-2 subfunction)  =0.01
.
.
.

****** eps^0 coeff ******

result       =0.906978296750816
             -0.908781551612644 I
error        =0.00754504726896407
             + 0.0442867373250588 I
CPUtime (all eps^0 subfunctions)  =2.44
CPUtime (longest eps^0 subfunction)  =0.51

**********************************************

Time taken for decomposition = 2.005725

Total time for subtraction and eps expansion = 41.5057 secs
Time taken for longest subtraction and eps expansion = 17.8613 secs
```

Deformation of the integration contour to integrate mass thresholds



- Integrand is analytically continued into the complex plane

$$\mathcal{F}(\vec{t}) \to \mathcal{F}(\vec{t} + \mathrm{i}\vec{y}(\vec{t})) = \mathcal{F}(\vec{t}) + \mathrm{i}\sum_j y_j(\vec{t})\frac{\partial \mathcal{F}(\vec{t})}{\partial t_j} + \mathcal{O}(y(\vec{t})^2)$$

- The integration contour is deformed by

$$\vec{t} \to \vec{z} = \vec{t} + \mathrm{i}\vec{y} \ ,$$

$$y_j(\vec{t}) = -\lambda t_j(1 - t_j)\frac{\partial \mathcal{F}(\vec{t})}{\partial t_j}$$

Soper '99

Soper, Nagy, Binoth; Kurihara et al., Anastasiou/Beerli et al., Freitas et al., Weinzierl et al.