On the determination of QCD parameters from τ hadronic spectral moment fits including duality violations

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ongoing *work done in collaboration with:*
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i. introduction

ii. main features of our analysis

iii. preliminary results

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Fundamental quantity

$$R_\tau^{S=0} = \frac{\Gamma[\tau \to \text{hadrons}^{S=0}\,\nu_\tau]}{\Gamma[\tau \to e^-\bar{\nu}_e\,\nu_\tau]} = R_\tau^V + R_\tau^A$$

- Related to the correlators

$$\Pi_{\mu\nu}^{V,A}(q) = i\int d^4x\, e^{iqx}\langle 0|T\{J_\mu^{V,A}(x)J_\nu^{V,A}(0)^\dagger\}|\rangle \qquad J_\mu^{V/(A)} = \bar{u}(x)\gamma_\mu(\gamma_5)d(x)$$

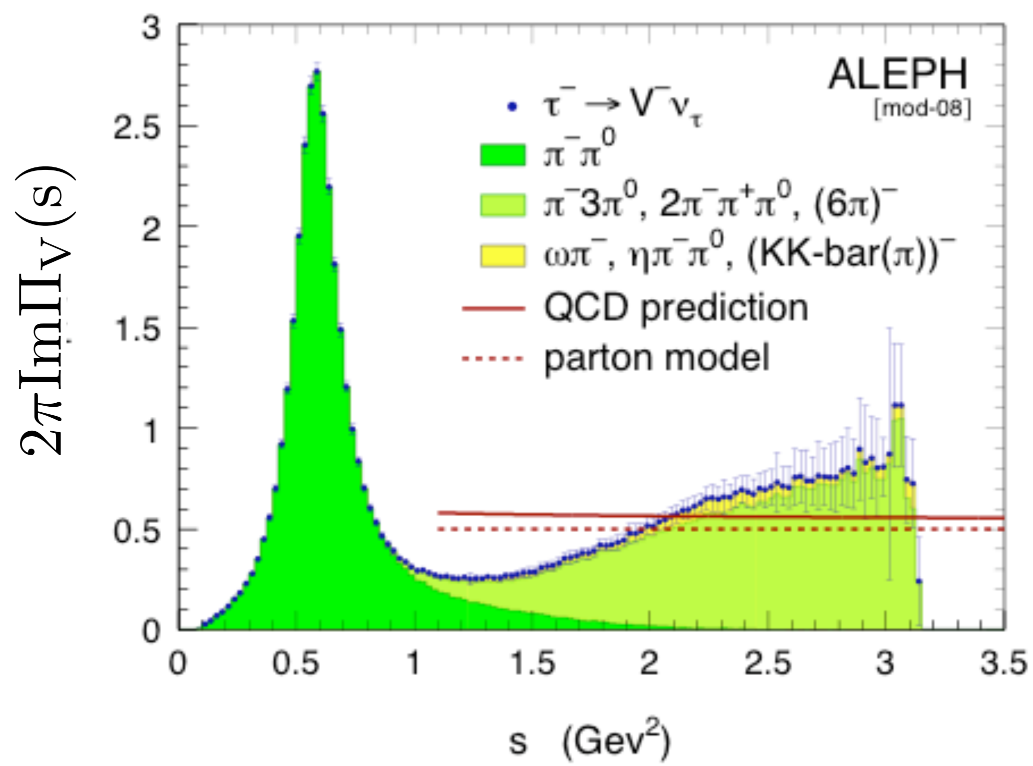
via

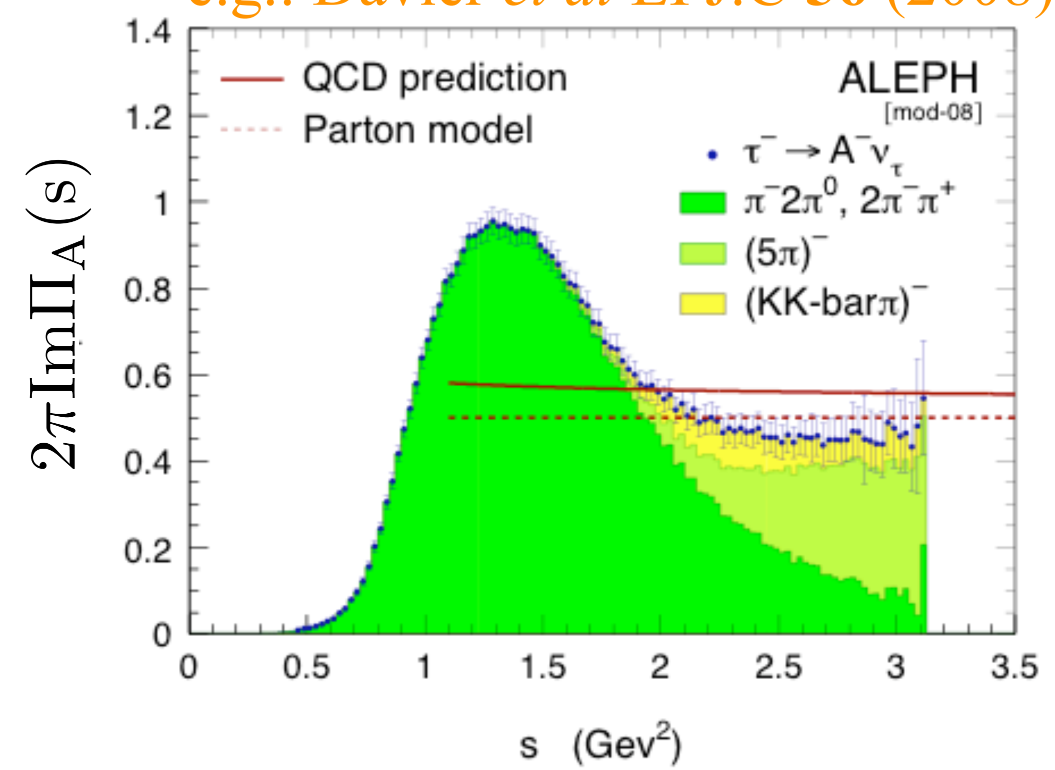
$$R_\tau^{V/A} = 12\pi\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\Pi^{(1)}_{\mathrm{V/A}} + \mathrm{Im}\Pi^{(0)}_{\mathrm{V/A}}\right]$$

$\mathrm{Im}\Pi_{V/A}$
can then be determined experimentally

$\mathrm{Im}\Pi^{\mathrm{exp}}_{V/A}$

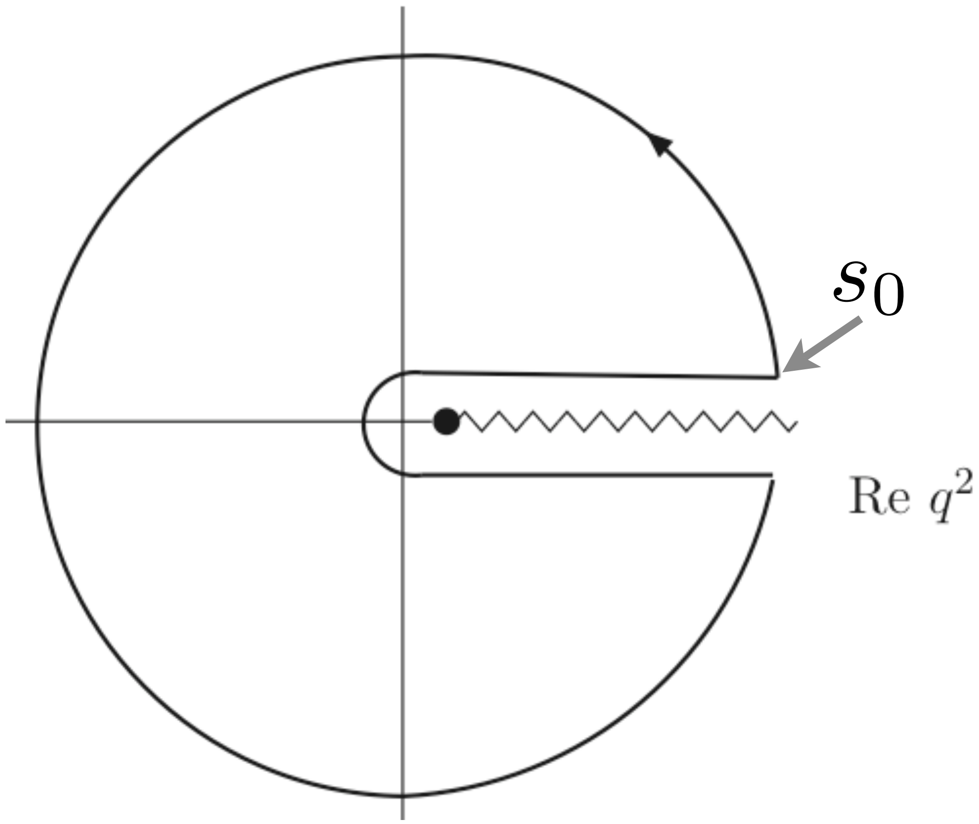
e.g.: Davier *et al* EPJ.C **56** (2008)





Cauchy's theorem

Theoretical strategy to deal with integrals of the correlators using analytical structure of $\Pi(s)$



$$\int_0^{s_0} ds\, w(s)\frac{1}{\pi}\underbrace{\mathrm{Im}\Pi(s)}_{\text{experiment}} = -\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\overbrace{\Pi(z)}^{\text{theory}}$$

Braaten, Narison, and Pich (1992)

R_τ: special case ⟶ $s_0 = m_\tau^2$ and $w(s) = w_\tau(s)$

- Different functions *w(s)* emphasize different aspects of $\Pi(s)$.
- On the contour the common lore is to use the OPE expansion of $\Pi(s)$.

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main features of previous analyses

- Fits to moments of $\Pi(s)$ in terms of QCD parameters (α_s, condensates, ...).

exp ⟷ theory

$$\int_0^{m_\tau^2} ds\, w(s) \frac{1}{\pi} \mathrm{Im}\Pi(s) = -\frac{1}{2\pi i} \oint_{|z|=m_\tau^2} dz\, w(z) \left[\Pi^{(0)}_{\mathrm{OPE}}(z) + \Pi^{(2)}_{\mathrm{OPE}} + \Pi^{(4)}_{\mathrm{OPE}} + \cdots\right]$$

α_s $\quad m_q^2 \quad m_q\langle q\bar{q}\rangle, \langle aG^2\rangle$

- Fits with pinched weights, using the physical tau mass, main results from $V+A$
  Le Diberder and Pich (1992), ALEPH Collaboration, OPAL Collaboration, Davier et al.

- Fits using several different "tau-masses" (s_0).
  Maltman and Yavin (2008)

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Why revisit the determination of QCD parameters from tau decay data?

Results for α_s from different works are barely compatible: tiny uncertainties.

Gluon condensate value inconsistent between *V* and *A* analyses.

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exp ⟷ theory

$$\int_0^{s_0} ds\, w(s) \frac{1}{\pi} \mathrm{Im}\Pi(s) = -\frac{1}{2\pi i} \oint_{|z|=s_0} dz\, w(z) \left[\Pi^{(0)}_{\mathrm{OPE}}(z) + \Pi^{(2)}_{\mathrm{OPE}} + \Pi^{(4)}_{\mathrm{OPE}} + \cdots + \Delta_{V/A}(z) \right]$$

α_s m_q^2 $m_q\langle q\bar{q}\rangle$ $\langle aG^2\rangle$

- Duality Violations $\Delta_{V/A}$ (DVs)

  Blok et al. (1998), Bigi et al. (1999), Catà, Golterman, and Peris (2005)

- Perturbative component of $\Pi(s)$: RG resummation FO vs CI vs ... Not completely resolved. → **see talks by Caprini and Jamin**

  Pivovarov (1992) , Le Diberder and Pich (1992), Beneke and Jamin (2008), Caprini and Fischer (2009)

- What is the best strategy to extract information from $\mathrm{Im}\Pi^{\mathrm{exp}}_{V/A}$? Fits varying $s_0, w(s)$, ...

  see e.g. Maltman and Yavin (2008), Dominguez et al. (2004)

- Direct fit to $\mathrm{Im}\Pi_{V/A}$

  CGP (2009)

Fits with DVs, combining the spectral function, different moments and energies.

- Theory side of the sum rule

$$-\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\overbrace{\Pi_{V/A}}^{\text{theory}}(z) = -\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\ \Pi^{\rm OPE}_{V/A}(z)$$

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duality violations

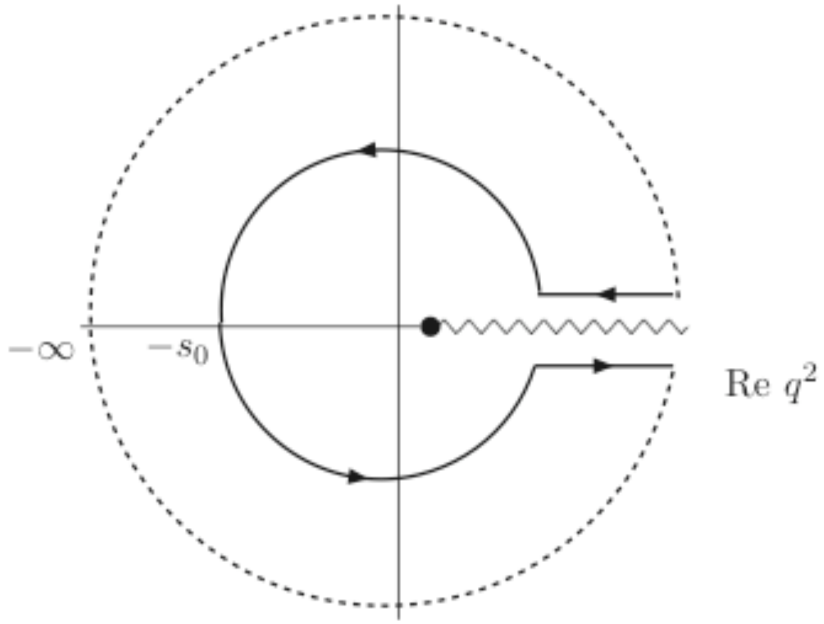
- Theory side of the sum rule

$$-\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\overbrace{\Pi_{V/A}(z)}^{\text{theory}} = -\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\left[\Pi^{\rm OPE}_{V/A}(z) + \Delta_{V/A}(z)\right]$$

Duality violations (DVs) $\longrightarrow$ $\Delta_{V/A}(s) = \Pi_{V/A}(s) - \Pi^{\rm OPE}_{V/A}(s)$

- Very little is known about DVs
  - $\Delta(z) \to 0$ as $|z| \to \infty$
  - only substantial near the Minkowski axis

Duality violation correction to the sum rule



$$\mathcal{D}_{V/A}(s_0) = -\int_{s_0}^{\infty} ds\, w(s)\frac{1}{\pi}{\rm Im}\Delta_{V/A}(s)$$

CGP (2005)

requires information beyond m_τ^2

- Reasonable *Ansatz* for the duality violations

$$\frac{1}{\pi}\mathrm{Im}\Delta_{V/A}(s) \underset{\text{large } s}{\longrightarrow} \kappa_{V/A}\, e^{-\gamma_{V/A} s} \sin\left(\alpha_{V/A} + \beta_{V/A} s\right)$$

  - Asymptotic regime of a large-N_c inspired model with a tower of resonances with Regge behavior.
  - Exponentially damped oscillatory behavior from increasing widths of resonances. Blok et al, (1998); Bigi et al. (1999), CGP (2005)
  - Existing standard analyses $\kappa_{V/A} = 0$

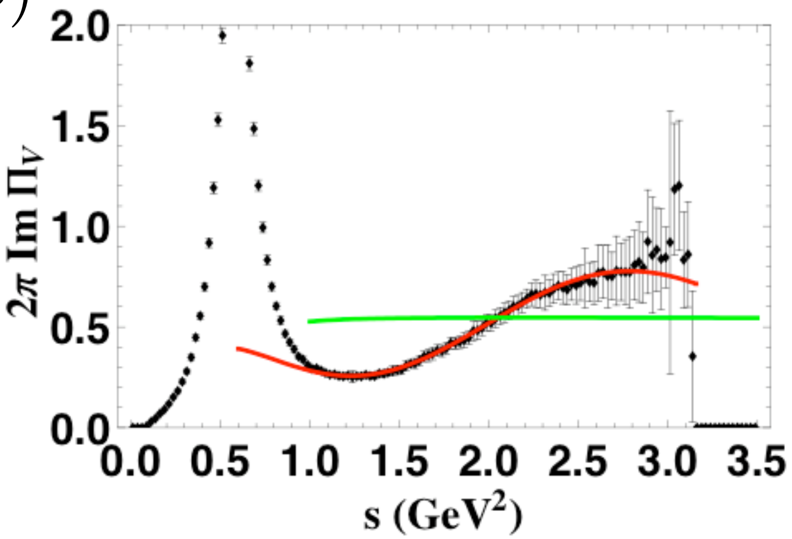
- Four parameters to be determined for *V* and *A* ***separately*** (8 in total).
- Onset of asymptoticity tested in the spectral function: successful fits.

$$\frac{1}{\pi}\mathrm{Im}\Pi_{V/A}(s) \underset{\text{large } s}{\longrightarrow} \frac{N_c}{12\pi^2}\left[1+\hat{\rho}(s)\right] + \kappa_{V/A}\, e^{-\gamma_{V/A} s} \sin\left(\alpha_{V/A} + \beta_{V/A} s\right)$$

Pert. QCD result

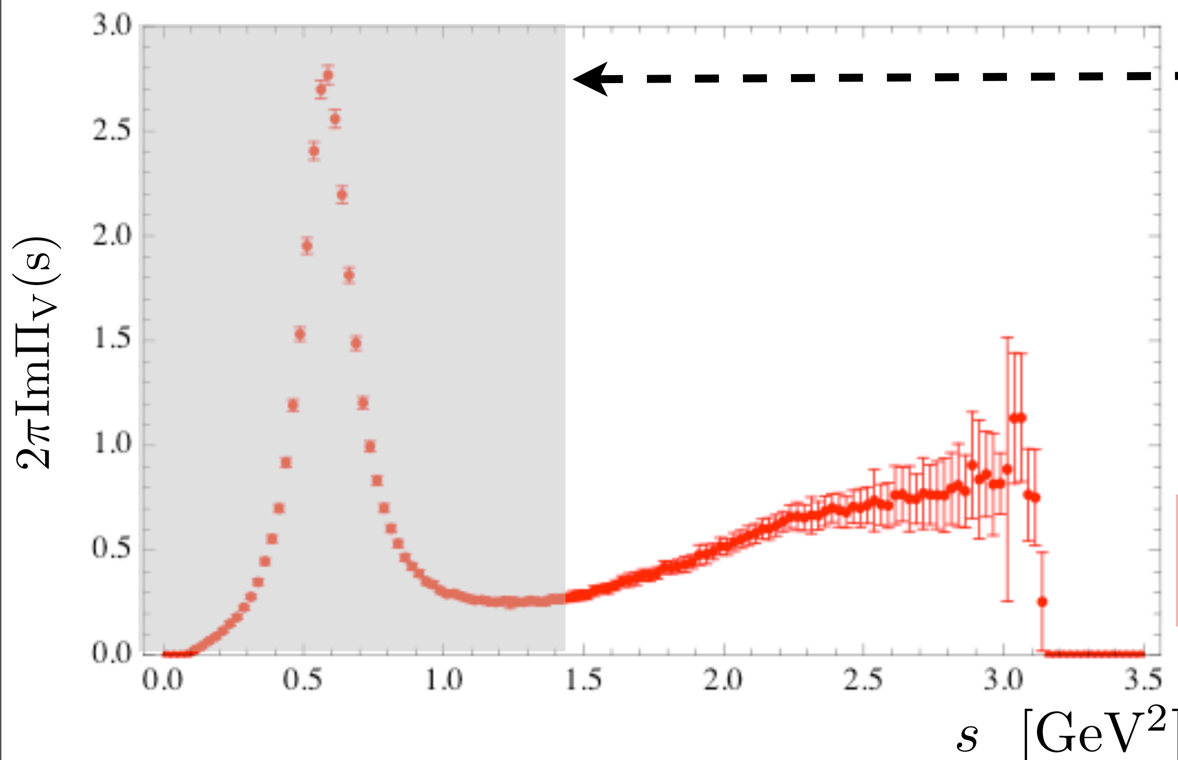
$\mathcal{O}(\alpha_s^4)$

CGP PRD **79** (2009).



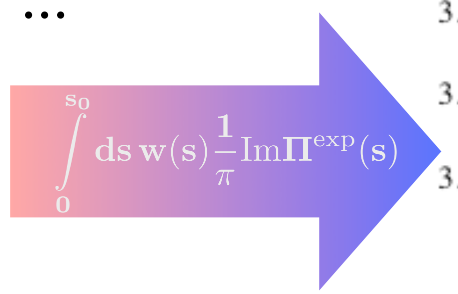
fit strategy

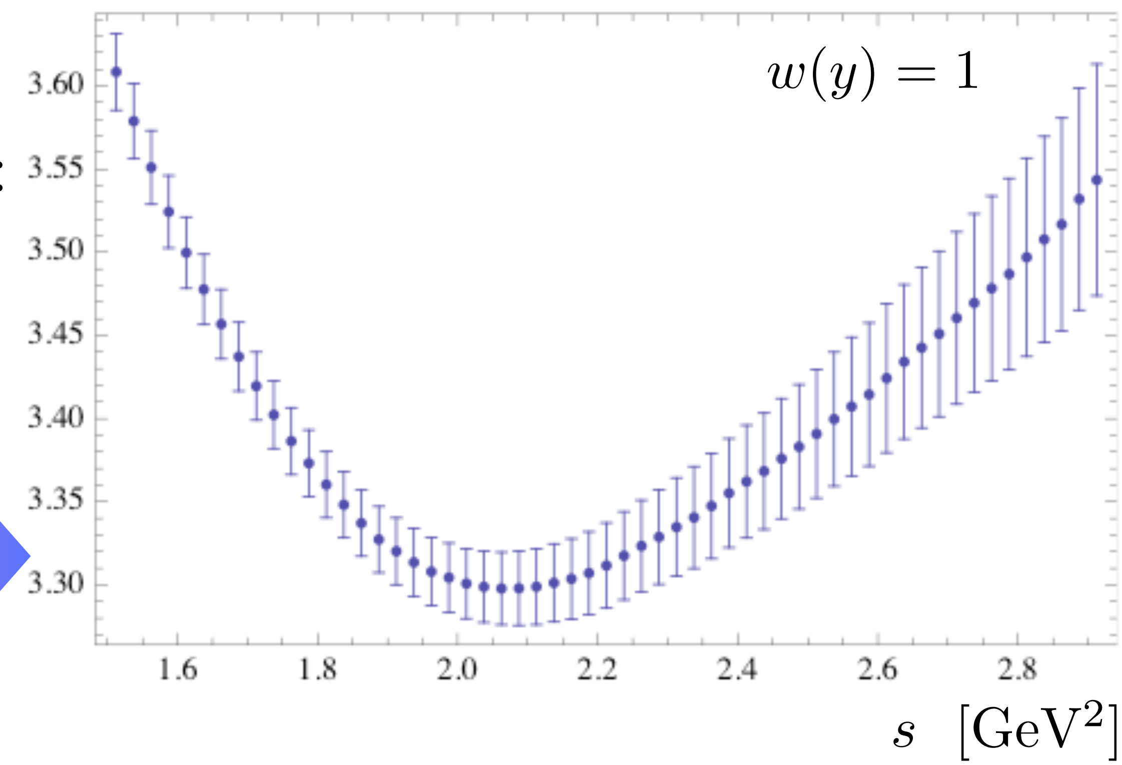
- Not restricted to pinched weights. Possible fit to the spectral function. ⟷ **DVs are included.**



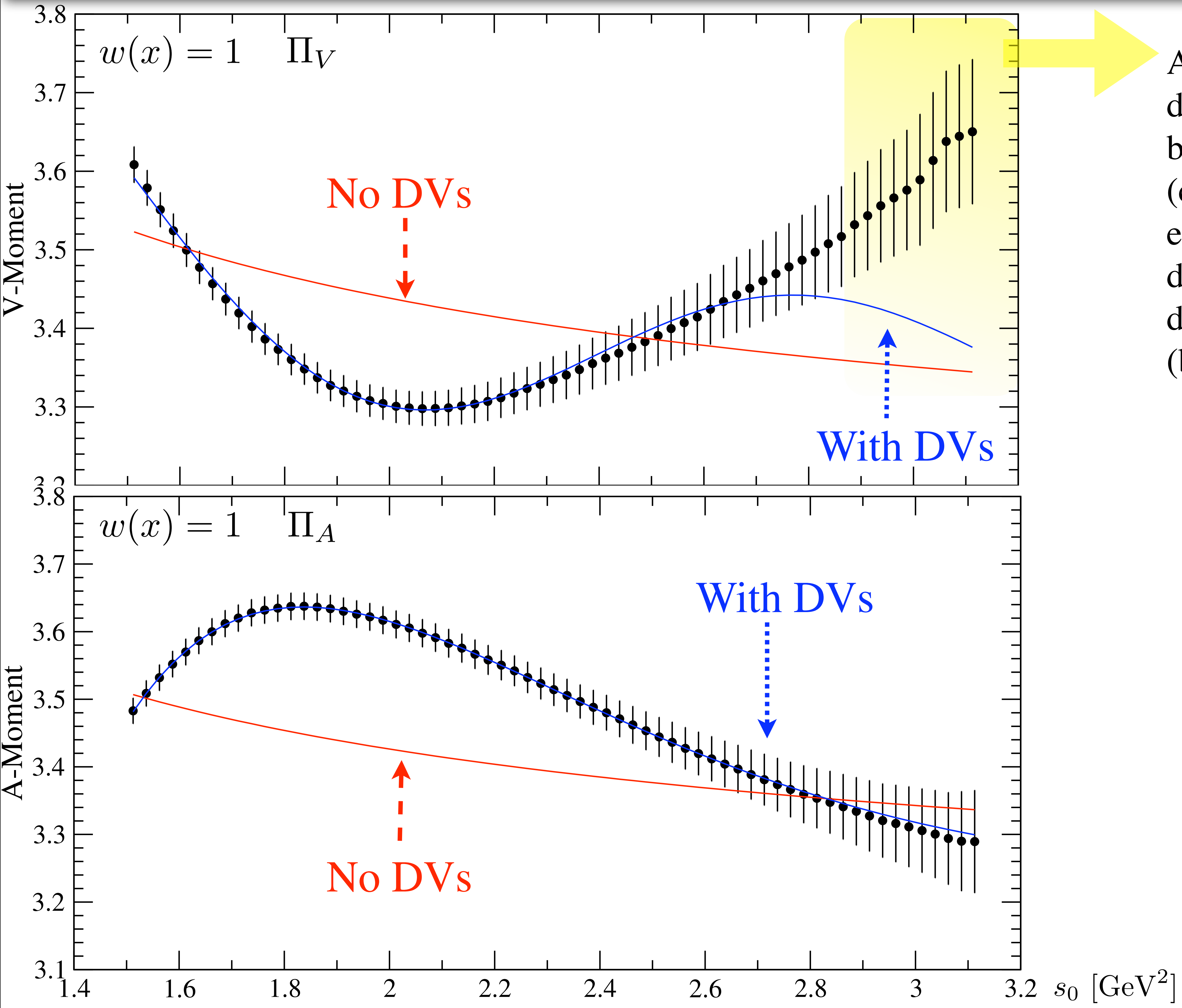
$s_{\min}$

Determined by: behavior of PT series, OPE, χ^2 ...



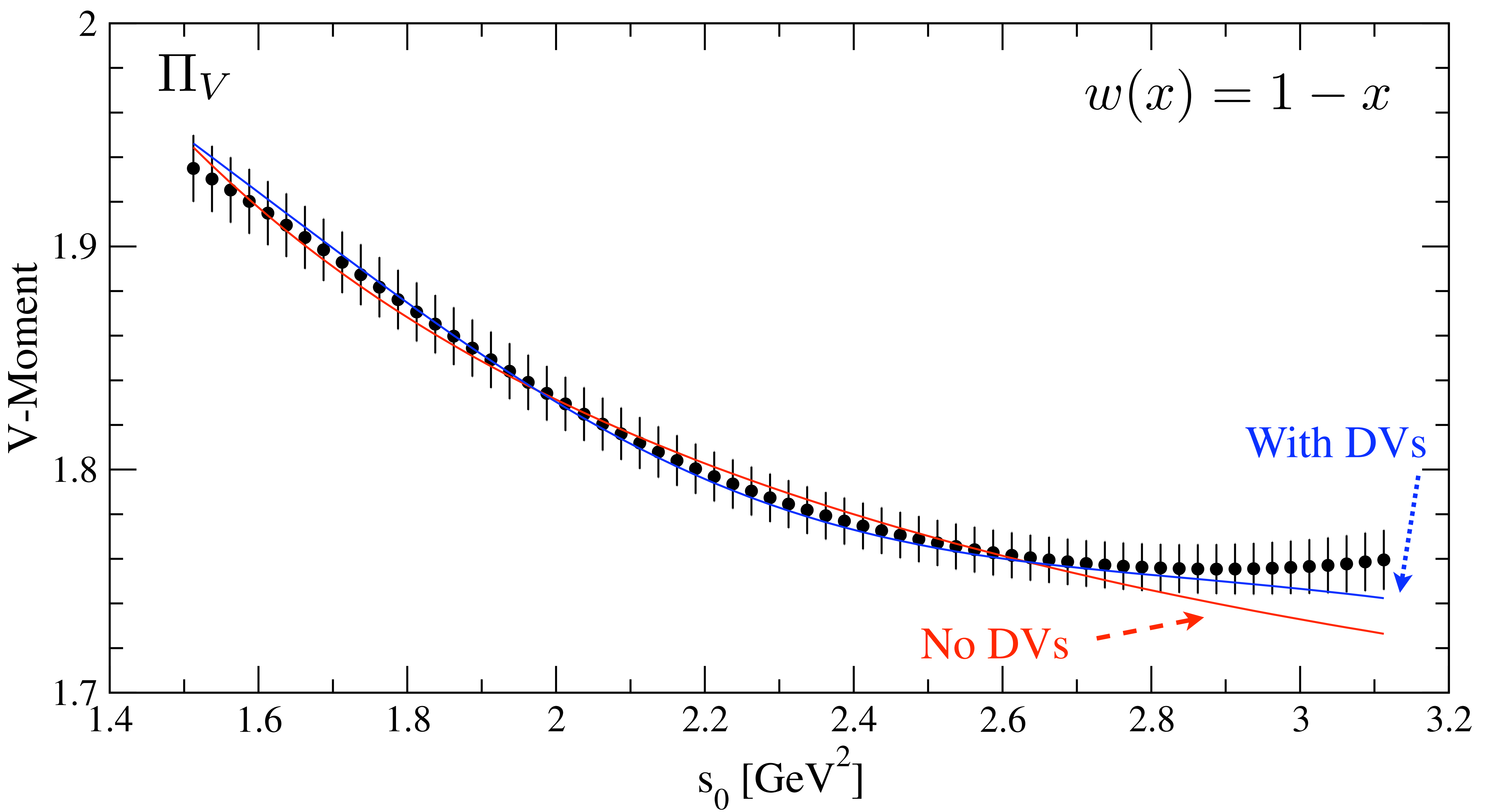


- Possible joint analysis of moments and of the spectral function.
- Fits employing an interval of energy (s_0): needed to fix DVs parameters.
- Independent treatment of DVs in *V* and *A* analyses.
- Weinberg sum rules: consistency check [within errors].
- Moments highly correlated. Role of correlations?



ALEPH 4π distribution could be too large (conflict with electroproduction data). With OPAL data no problem (but larger errors).

Data: ALEPH 2005



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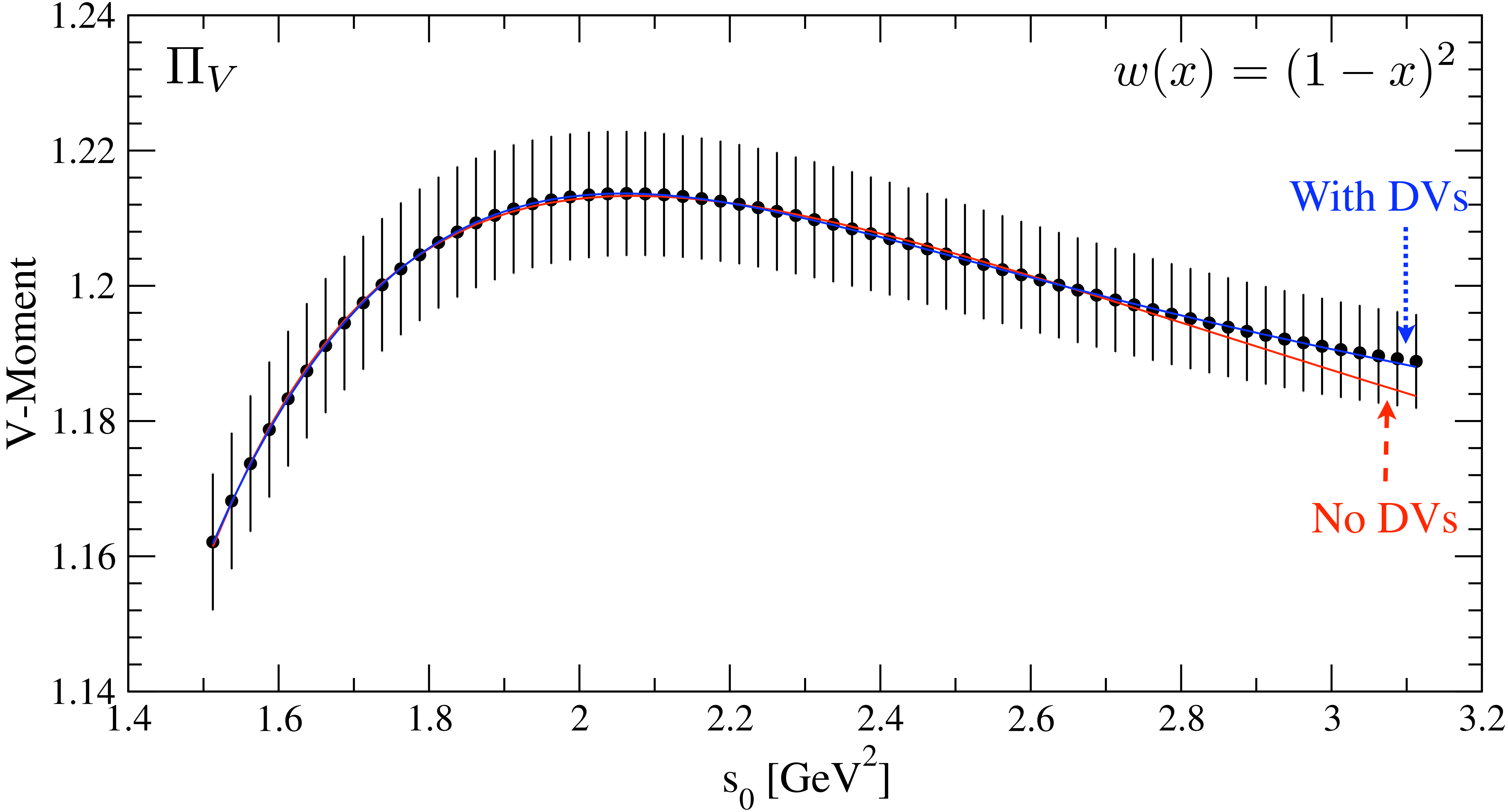

Doubly pinched weight: effect is still visible.

Data: ALEPH 2005

If one wants to achieve

- Self-consistent determination of α_s and condensates.
- Weinberg sum rules satisfied.
- Consistency between V and A.
- Stability with respect to s_0 .

Duality Violations are indispensable for desired precision.

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Improvements in the spectral function using BaBar and Belle data are highly desirable!