$g-2$ of the muon and $\Delta\alpha$ re-evaluated

UNIVERSITY OF LIVERPOOL

Thomas Teubner

Kaoru Hagiwara, Ruofan Liao, Alan Martin and Daisuke Nomura

I. Introduction: SM contributions to $g-2$

II. Recent developments in $(g-2)_\mu$: Hadronic Vacuum Polarisation contributions

- 2π: KLOE 2009 and 2010, BaBar 2009 analyses
- Inclusive vs. sum of exclusive data below 2 GeV
- New HLMNT10 compilation; comparison SM vs. BNL

III. $\Delta\alpha(q^2)$: Running QED coupling in the space- and time-like region. $\alpha(M_Z^2)$

IV. Conclusions and Outlook

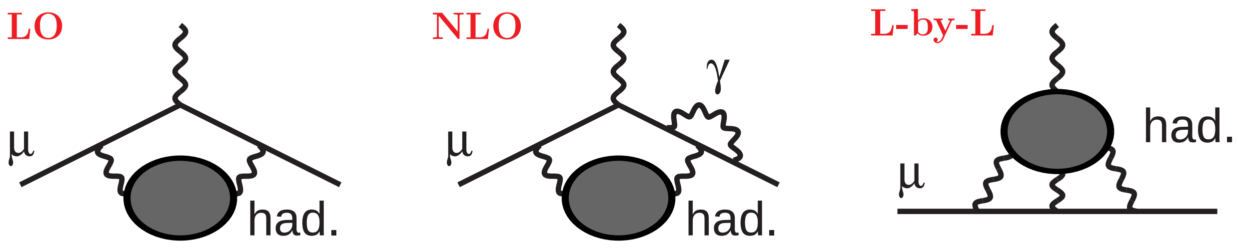
I. Introduction: SM contributions to $g-2$

- $a_\mu = (g-2)_\mu/2 = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had} + a_\mu^{\rm New\ Physics?}$
- QED: Predictions consolidated, further work (numerical five-loop) ongoing, big surprises very unprobable, error formidably small: $a_\mu^{\rm QED} = 116584718.08(15) \cdot 10^{-11}$ ✓

  Kinoshita et al.
- EW: reliable two-loop predictions, accuracy fully sufficient: $a_\mu^{\rm EW} = (154 \pm 2) \cdot 10^{-11}$ ✓

  Czarnecki et al., Knecht et al.
- Hadronic contributions: uncertainties completely dominate $\Delta a_\mu^{\rm SM}$!

$$a_\mu^{\rm had} = a_\mu^{\rm had,VP\ LO} + a_\mu^{\rm had,VP\ NLO} + a_\mu^{\rm had,Light-by-Light}$$



▶ Hadronic contributions from low γ virtualities not calculable with perturbative QCD

— Lattice simulations difficult; promising first steps, but accuracy not (yet?) sufficient

- ▶ Vacuum Polarisation contributions from exp. $\boldsymbol{\sigma(e^+e^- \to \gamma^* \to hadrons)}$ data

  [or from $\tau \to \nu_\tau + hadrons$ spectral functions, but problem of isospin corrections]

  via *dispersion integral* (based on analyticity and unitarity):

$$a_\mu^{\rm had,VP\ LO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^\infty {\rm d}s\, \sigma^0_{\rm had}(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.63\ldots 1)$$

  $\longrightarrow$ Weighting with kernel K towards smallest energies

  $\longrightarrow$ Similar approach with different kernel functions for NLO VP contributions $a_\mu^{\rm had,VP\ NLO}$

- ▶ Light-by-Light:

– No dispersion relation for L-by-L. *First Principles* calculations from lattice QCD are underway by two groups: QCDSF and T Blum et al. Both approaches promising but at an early stage and no results yet.

  [First results based on Dyson-Schwinger eqs. reported by C Fischer et al. at QCHS9.]

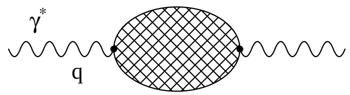
$\longrightarrow$ Convergence of different recent model calculations. Below we will use the recent compilation from J Prades, E de Rafael, A Vainshtein: $a_\mu^{\rm L-by-L} = (10.5 \pm 2.6)\cdot 10^{-10}$

– Similar recent result from F Jegerlehner, A Nyffeler: $a_\mu^{\rm L-by-L} = (11.6 \pm 4.0)\cdot 10^{-10}$

II. Recent developments in $(g-2)_\mu$: Hadronic VP contributions

- Compilation of $\sigma^0_{\rm had}(s)$

- For low energies, need to sum ~ 24 exclusive channels. $[2\pi,\ 3\pi,\ KK,\ 4\pi,\ \ldots]$
- $1.43-2$ GeV: sum exclusive channels or use (old) inclusive data?
- above 2 GeV: inclusive data *and/or* use of perturbative QCD.
- In each channel: Data combination from many experiments, non-trivial w.r.t. error analysis/correlations/different energy ranges.
  [HLMNT use adaptive binning and non-linear $\chi^2_{\rm min}$ fit with full cov.-matrices.]
- Note: $\sigma^0(s)$ must be the *undressed* hadronic cross section (i.e. photon VP *subtracted* $[\sigma^0(s)=\sigma(s)\cdot(\alpha/\alpha(s))^2]$, otherwise double-counting with $a_\mu^{\rm had,VP\ NLO}$)



- but must *include final state photon radiation*.

⇝ Uncertainty in treatment of radiative corrections, especially for older data sets!
Assign additional error. HLMNT: $\delta a_\mu^{\rm had,VP+FSR} \simeq 2\times10^{-10}$ $[\sim 10\cdot\Delta a_\mu^{\rm EW}]$

▶ Most important channels with changes in input data since ∼2006

The main exps. for 'low' energy hadronic cross sections in e^+e^-; channels

– CMD-2, [VEPP-2M], Novosibirsk (K^+K^-, $2\pi^+2\pi^-\pi^0$, $2\pi^+2\pi^-2\pi^0$)

– SND, [VEPP-2M], Novosibirsk (K^+K^-, $K^0_S K^0_L$)

– KLOE, [DAΦNE], Frascati ($\pi^+\pi^-(\gamma)$, $\omega\pi^0$)

– BaBar, [PEP-II], SLAC, Stanford ($\pi^+\pi^-(\gamma)$, $K^+K^-\pi^0$, $K^0_S\pi K$, $2\pi^+2\pi^-\pi^0$, $K^+K^-\pi^+\pi^-\pi^0$, $2\pi^+2\pi^-\eta$, $2\pi^+2\pi^-2\pi^0$)

– BELLE, [KEKB], KEK, Tsukuba

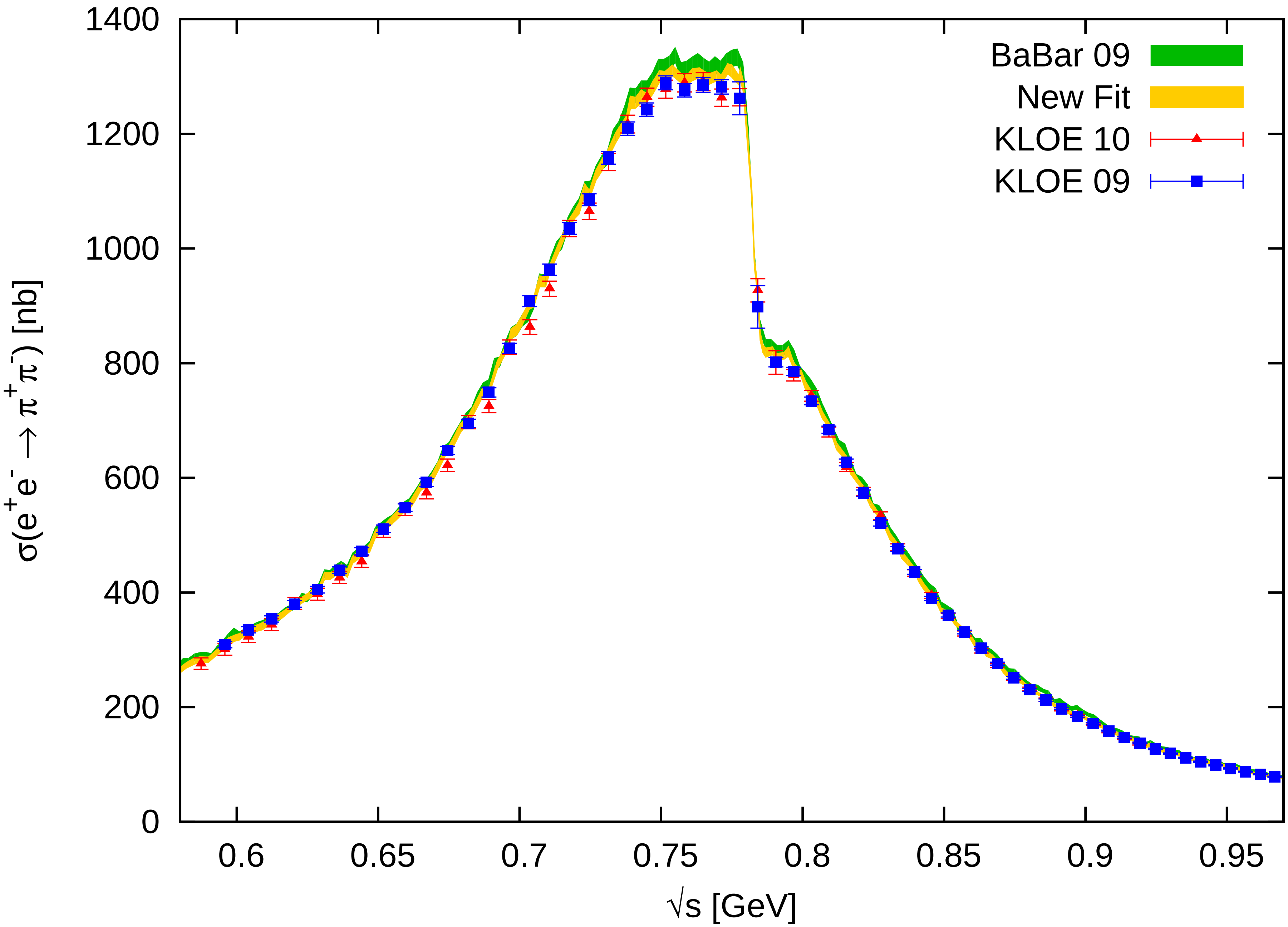
– BES, [BEPC], Beijing (inclusive $R = \sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$ data)

– CLEO, [CESR], Cornell (inclusive R)

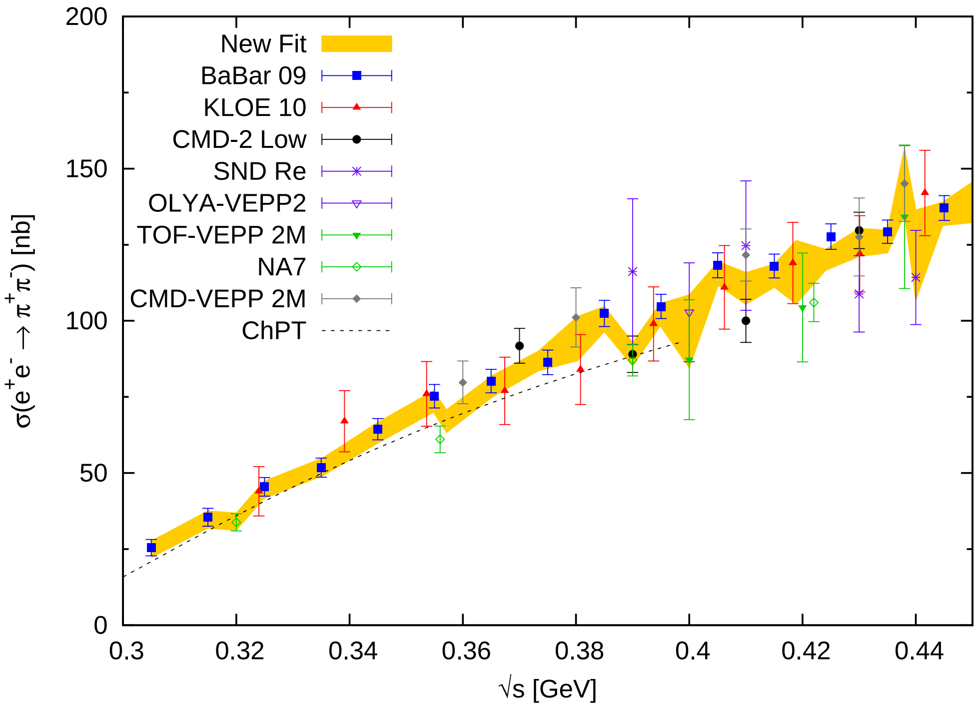
- In principle inclusion of new data in updated analysis straightforward..

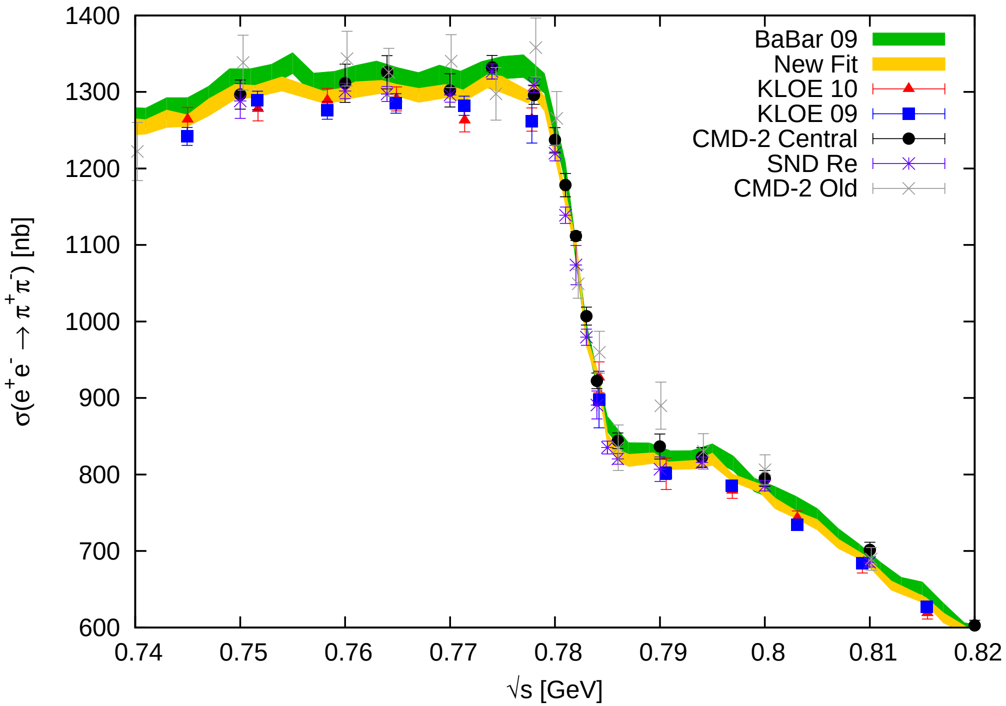
  Concentrate on two cases where not: most important $\boldsymbol{2\pi}$ and the $\mathbf{1.43 - 2}$ GeV region.

▶ The most important 2π channel ($> 70\%$) 879 data points, overall picture fine



Zoom in low energy (2π threshold) and ρ-peak / ρ-ω interference region



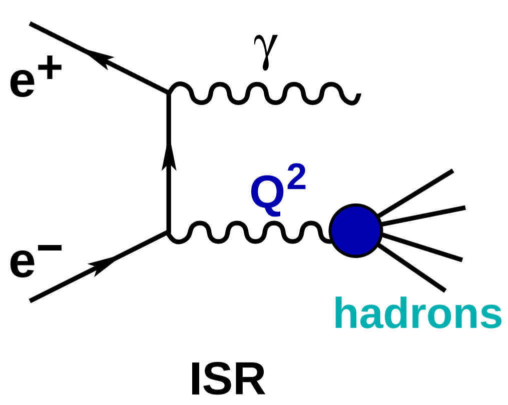


- 'Direct Scan': Very good agreement between data from CMD-2 and SND, fully consistent with earlier data.
- Low energy points crucial for recent improvements of $a_\mu^{\pi\pi}$.
- 'Radiative Return': *KLOE* and *BaBar* show slight tension with the Direct Scan data, and with each other;

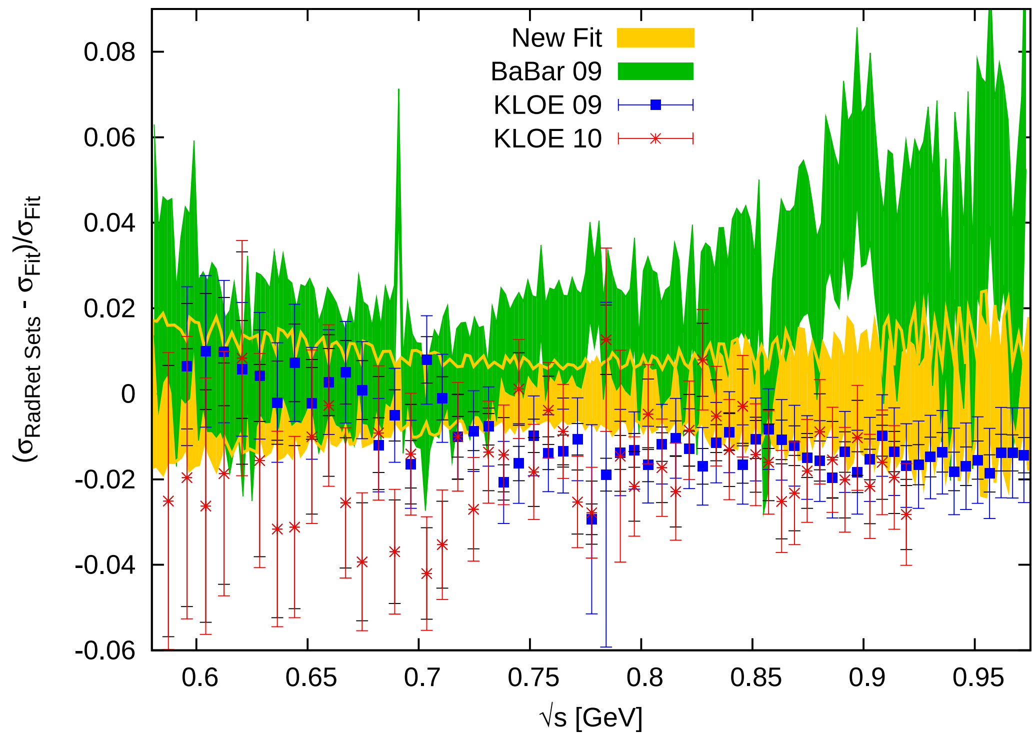
$\longrightarrow$ Differences in shape and BaBar high at medium and higher energies:

KLOE 09/10 and BaBar 09 $\pi\pi(\gamma)$ Radiative Return data compared to combination of all

Radiative Return (at fixed e^+e^- energy) has recently developed (TH + EXP) into a powerful method with great potential, complementary to direct energy scan!



Normalised difference of cross sections:



- New method used by 'meson factories', where high statistics compensates α/π suppression of γ radiation.
- Results for 2π channel slightly different in shape, but completely different method, Monte Carlos etc.

⇝ HLMNT 10: Combination of all data, including the latest KLOE 10 set, on the same footing, i.e. before integration (yellow band). [Still good $\chi^2_{\rm min}/{\rm d.o.f.} \sim 1.5$ of the overall 2π combination fit.]

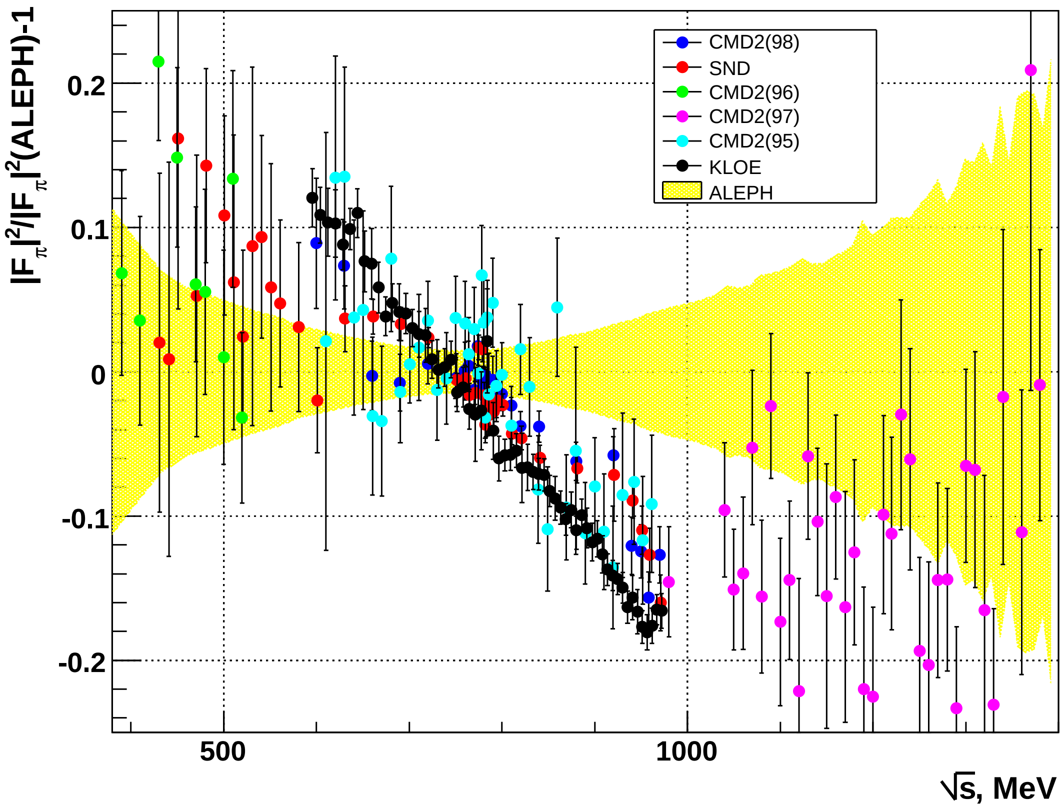
HLMNT 10 (prel.): $a_\mu^{2\pi}(0.32\text{–}2\,\text{GeV}) = (504.23 \pm 2.97)\cdot 10^{-10}$ [RadRet. data pull a_μ up by ~ 5.5 units!]

- ## What about the τ data?
  - CVC hypothesis (isospin-symm.) connects $\tau^- \to \pi^-\pi^0\nu_\tau$ to $e^+e^- \to \rho, \omega \to \pi^+\pi^-$
  - Sizeable isospin-symmetry violations [from radiative corrections, mass differences ($m_{\pi^-} \neq m_{\pi^0}$), $\rho - \omega$ interf.] ($\to$ Cirigliano+Ecker+Neufeld)
  - Role of possible $\rho^0 - \rho^\pm$ mass difference?
  - Width difference $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$?

    Large effects possible!

    How reliable are the model calculations?

S Eidelman (ICHEP06): τ compared to e^+e^- data



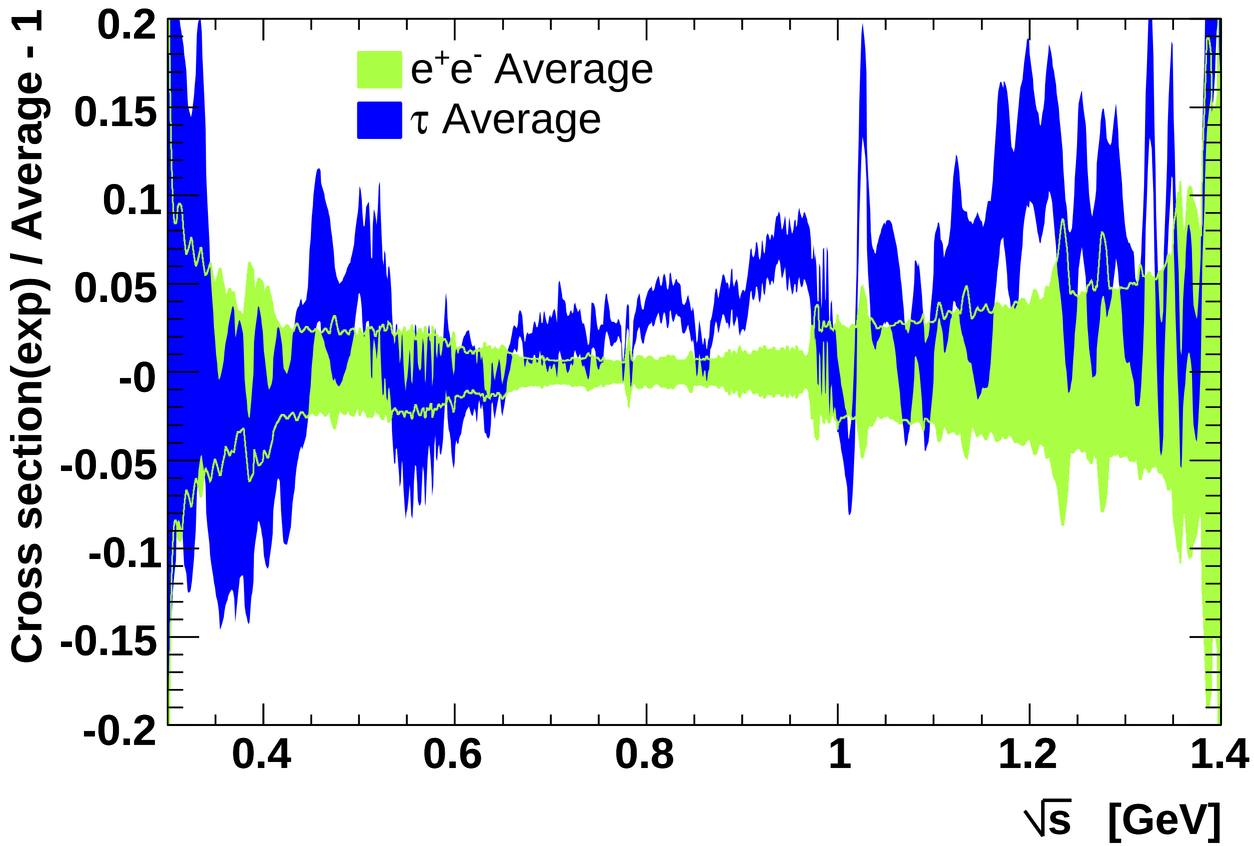
$\to$ Disagreement between τ and e^+e^- data already for $[B_\tau - B_{CVC}]_{\pi\pi^0}$: up to 4.5 σ?!

$\hookrightarrow$ Is everything under control *at the % level*? Is something wrong with data? H^-?

- KLOE Rad. Ret. agrees much better with e^+e^- scan experiments, BaBar somewhat;

$\to$ Recent work of Davier et al. gives better agreement: $\to$

Fig. from Davier et al., EPJC66 (2010) 1



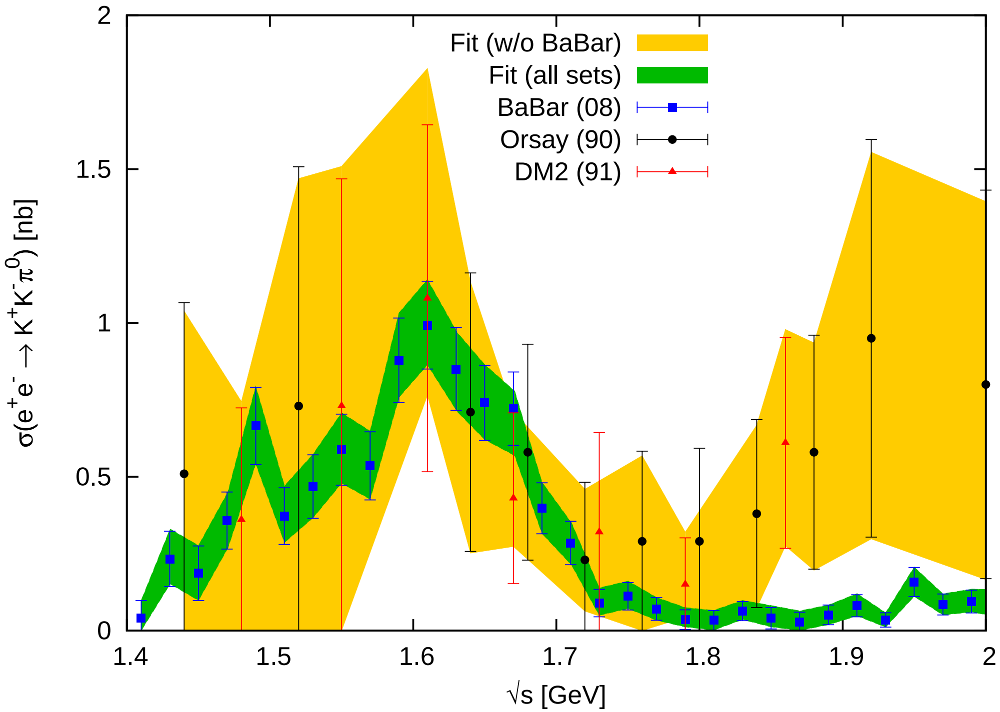
→ Disagreement between τ and e^+e^- data less severe than previously but still not solved.

→ Work from Benayoun et al. [EPJC55 (2008) 199; C65 (2010) 211, C68 (2010) 355]:
mixing + isospin breaking effects in model based on *Hidden Local Symmetry*
⇝ τ compatible with and confirm e^+e^- ?!

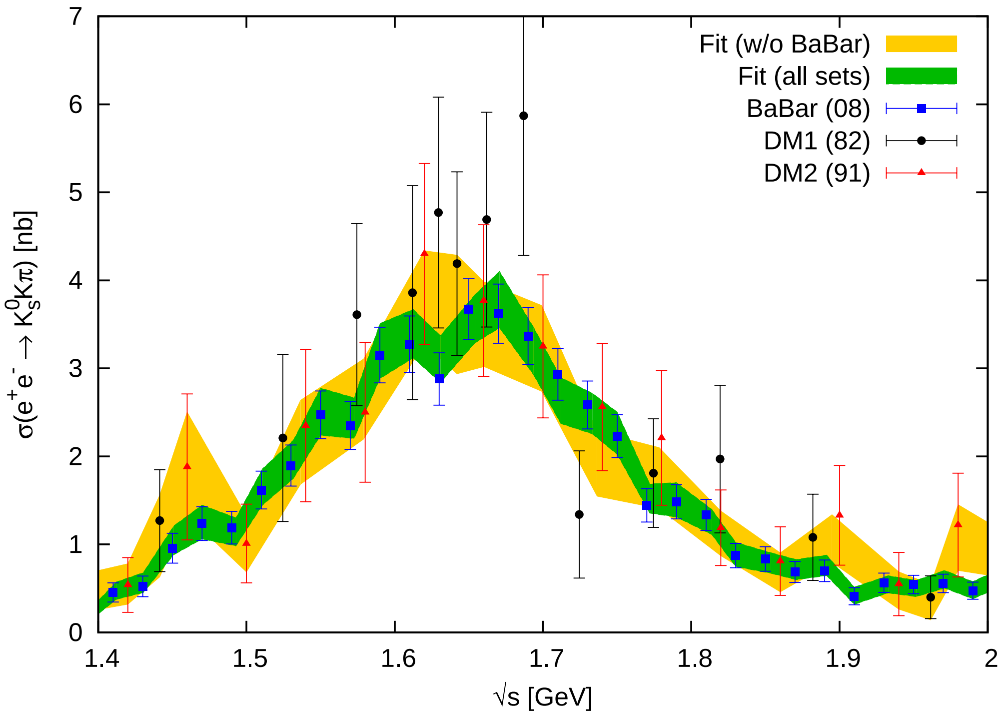
▶ :-(Not only) our choice: better not use τ data for $g-2$ predictions.

Region below 2 GeV: influence of recent BaBar Radiative Return analyses

$K^+K^-\pi^0$ channel



$K_S^0K\pi$ channel

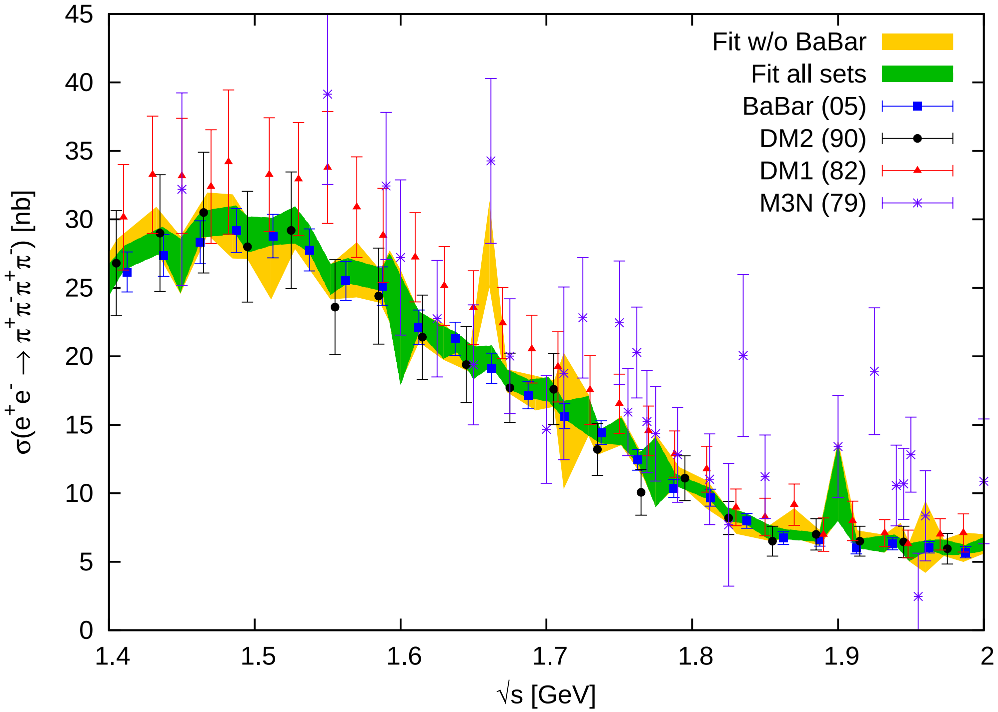


$\longrightarrow$ Big improvements over earlier data compilations in many channels. [$\rightarrow$ Malaescu]

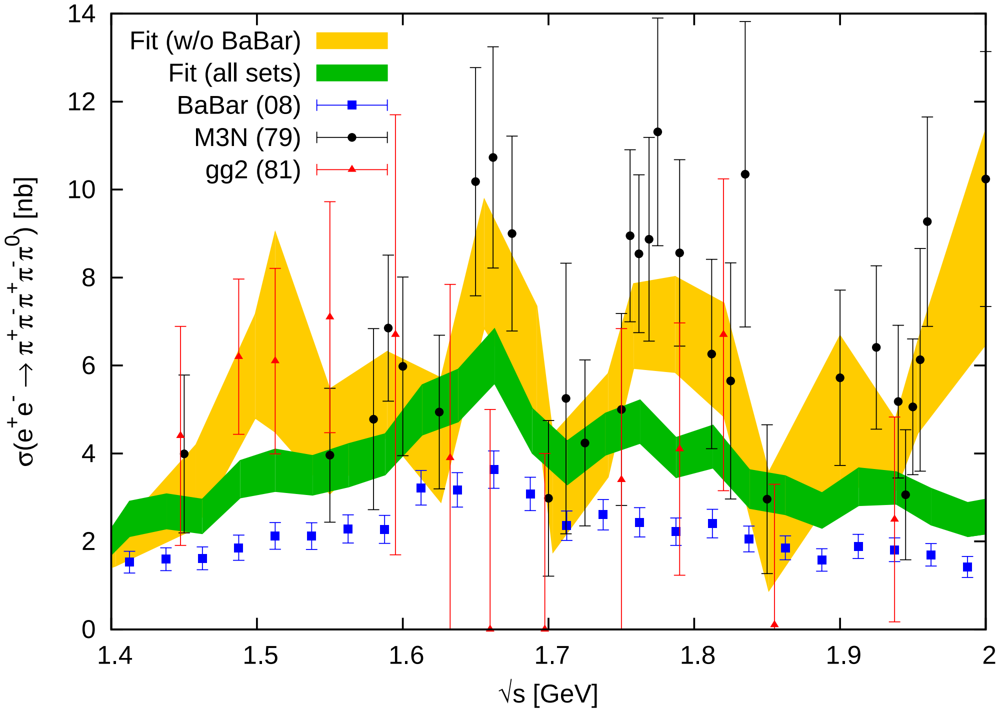
BaBar Radiative Return data lower than less precise older data in most channels.

▶ Region below 2 GeV: influence of recent BaBar Radiative Return analyses (contd)

$2\pi^-2\pi^-$ channel



$2\pi^+2\pi^-\pi^0$ channel

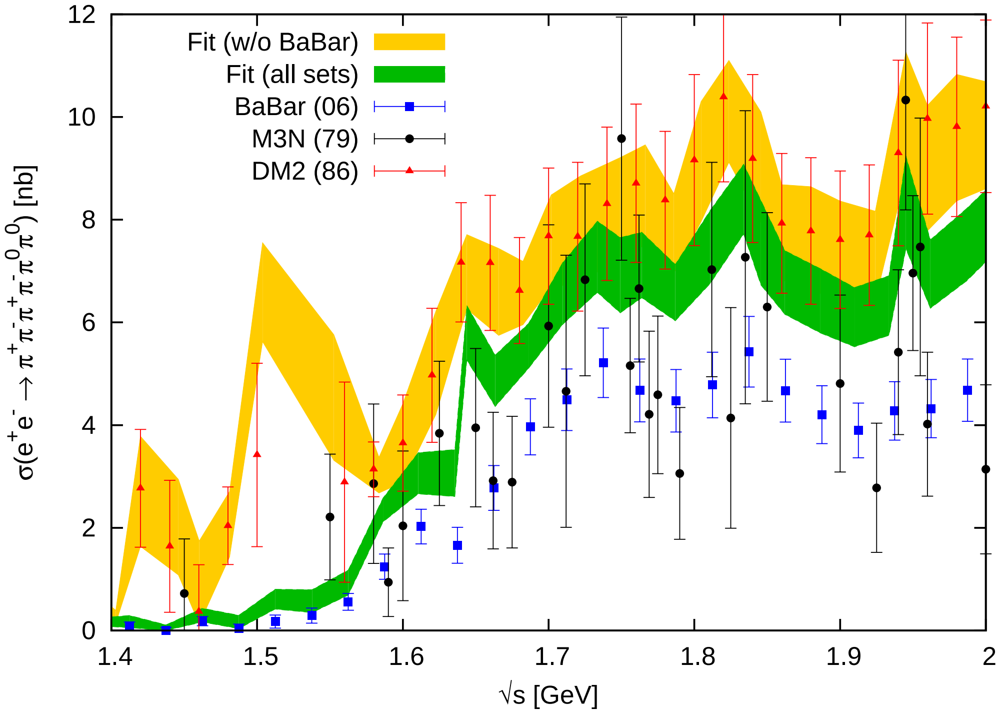


$\longrightarrow$ BaBar lower in $2\pi^+2\pi^-\pi^0$ channel, fit responds by bad $\chi^2_{\rm min}$

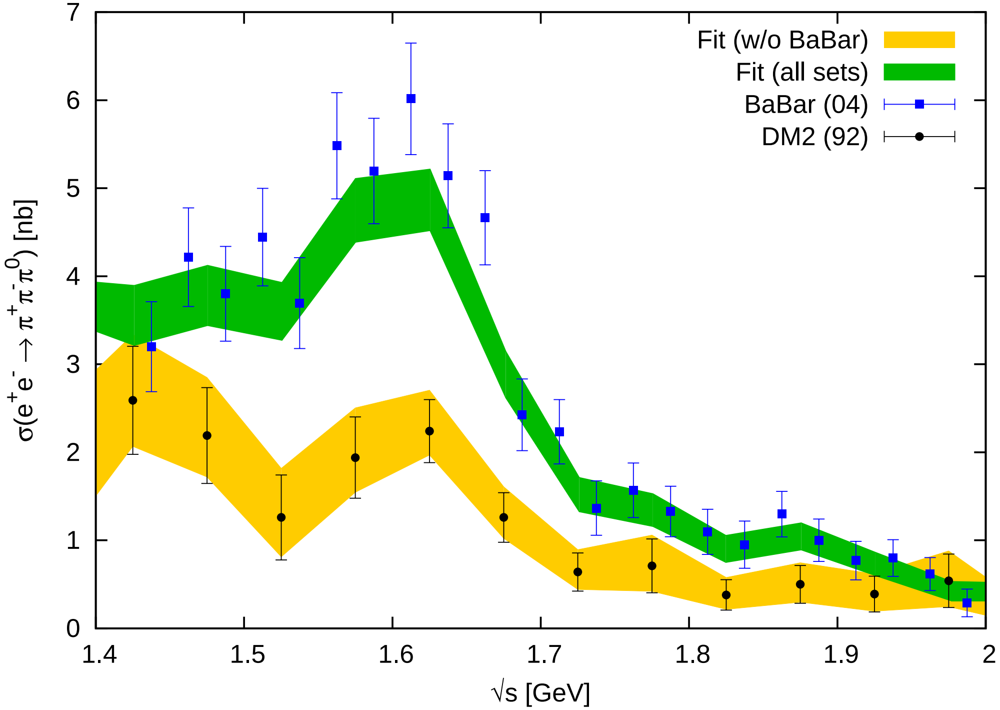
$\rightsquigarrow$ Errors for $g-2$ 'inflated' by $\sqrt{\chi^2_{\rm min}/{\rm d.o.f.}}$ [scaling up by 1.29 here.]

► Region below 2 GeV: influence of recent BaBar Radiative Return analyses (contd 2)

$2\pi^-2\pi^-2\pi^0$ channel



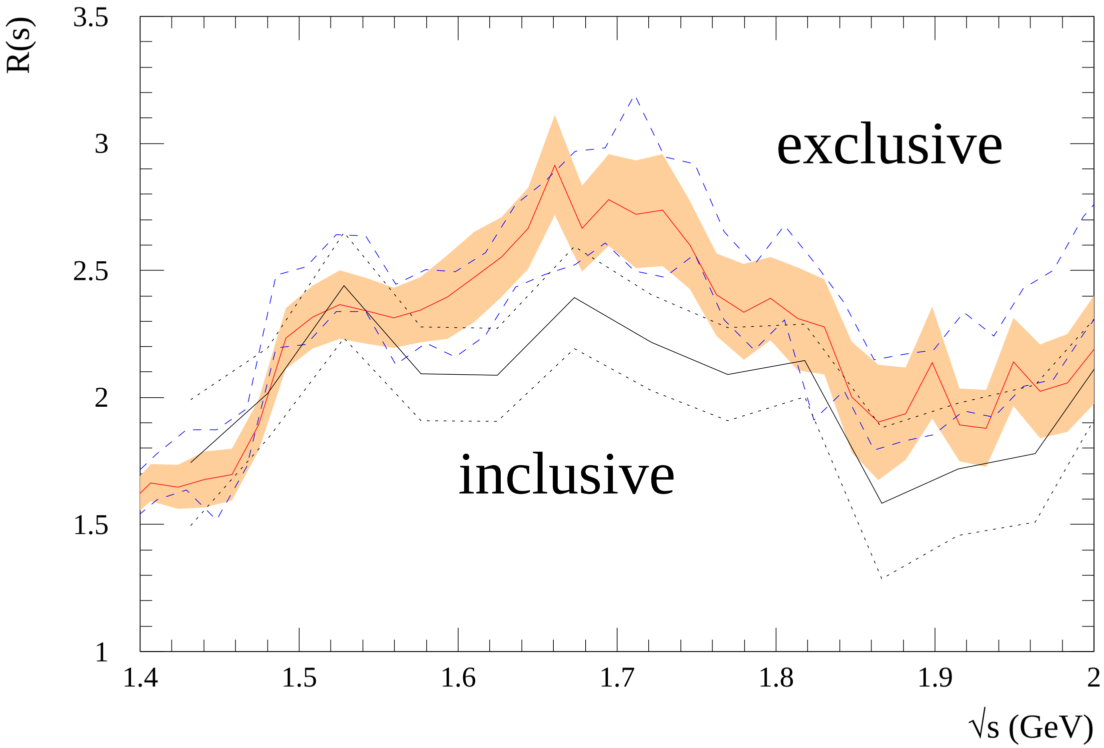
$\pi^+\pi^-\pi^0$ channel



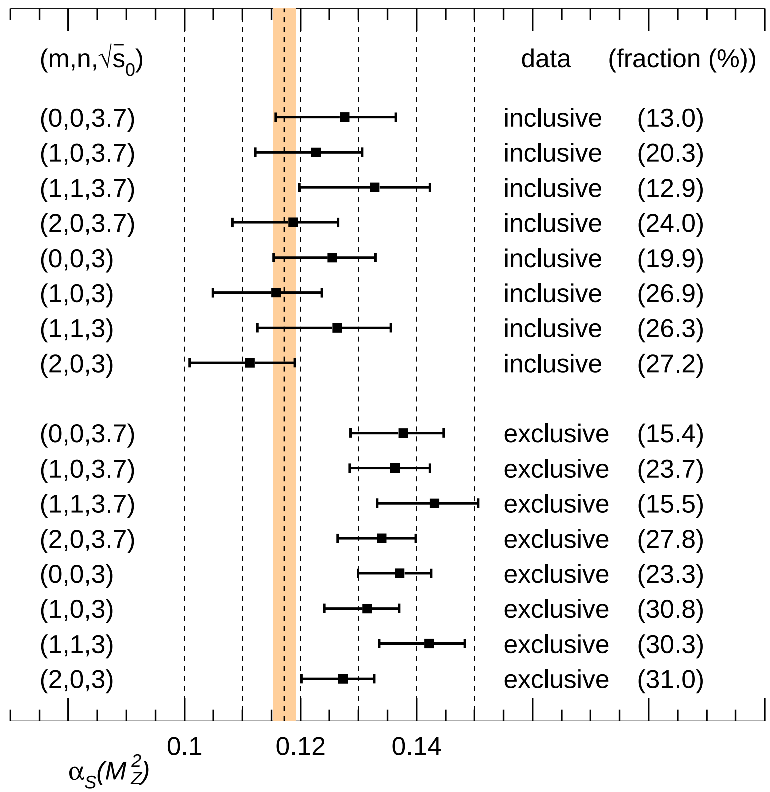
$\longrightarrow$ Again 'bad' $\chi^2_{\rm min}/{\rm d.o.f.}$ of 2.7 and 2.9. Data not really compatible, inflate error.

▶ Region below 2 GeV: *inclusive* vs. *sum over exclusive*

Data blue: old excl. analysis, red/orange: new



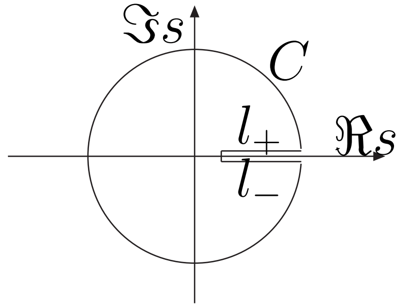
Sum-rules 'determining' α_S (old):



- Shape similar, but normalisation different
- Question of completeness/quality of sum of exclusive data vs. reliability/systematics of old inclusive data ($\gamma\gamma 2$, MEA, M3N, BBbar)
- HMNT previously (2003/06) have used incl. data, in line with sum-rule analysis

Check against perturbative QCD: QCD $\sum$-rule analysis

- Evaluate QCD $\sum$-rules of the form:

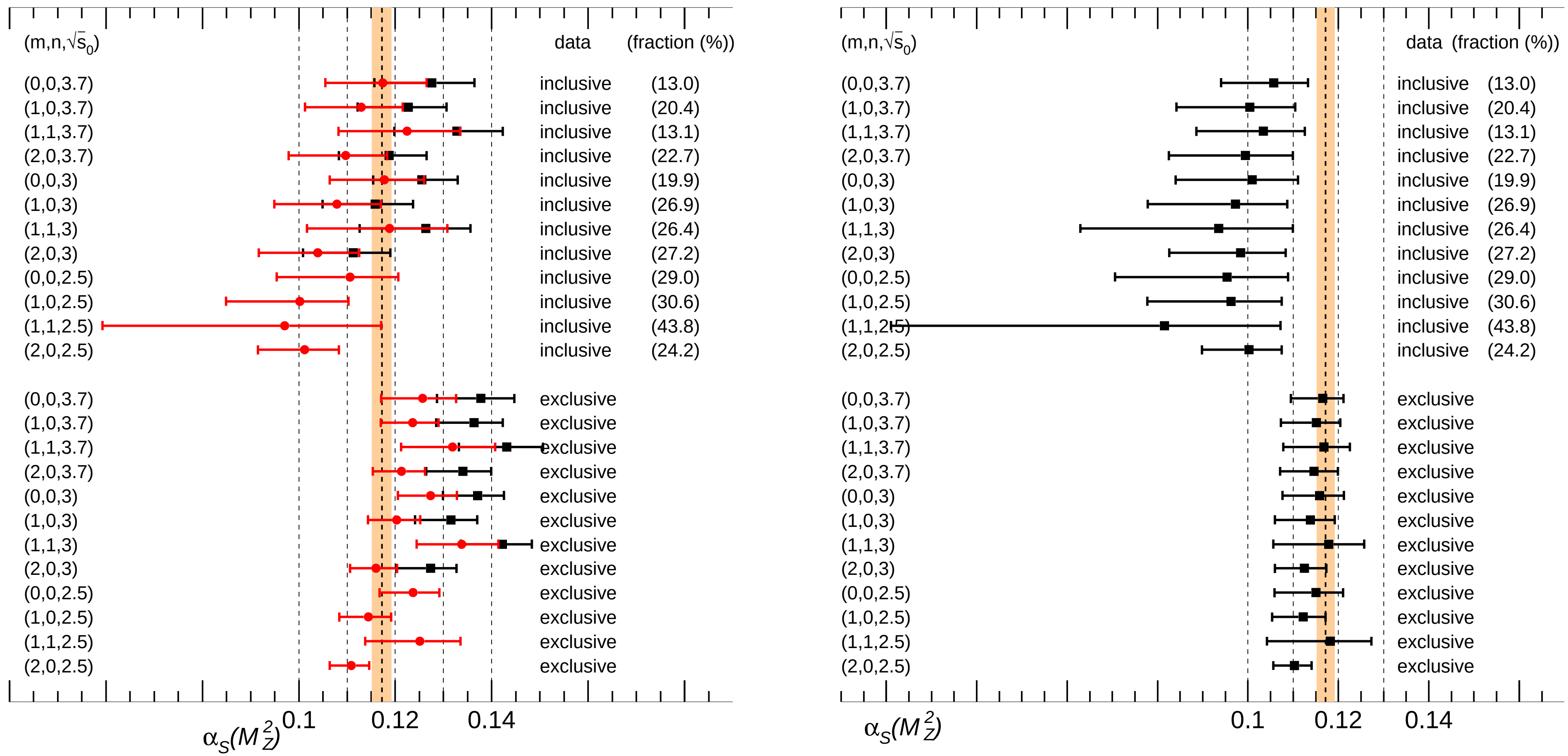


$$\int_{s_{\rm th}}^{s_0} {\rm d}s\, R(s) f(s) = \int_C {\rm d}s\, D(s) g(s)\,, \qquad \text{with} \quad D(s) \equiv -12\pi^2 s \frac{\rm d}{{\rm d}s}\left(\frac{\Pi(s)}{s}\right)$$

- The Adler D function is calculable in pQCD: $D(s) = D_0(s) + D_{\rm m}(s) + D_{\rm np}(s)\,.$
- Take $f(s) = (1 - s/s_0)^m (s/s_0)^n$ to maximise sensitivity to the required region, $g(s)$ follows.
- Choose s_0 below the open charm threshold ($n_f = 3$ for pQCD).
- For $m = 1$, $n = 0$ one gets e.g.

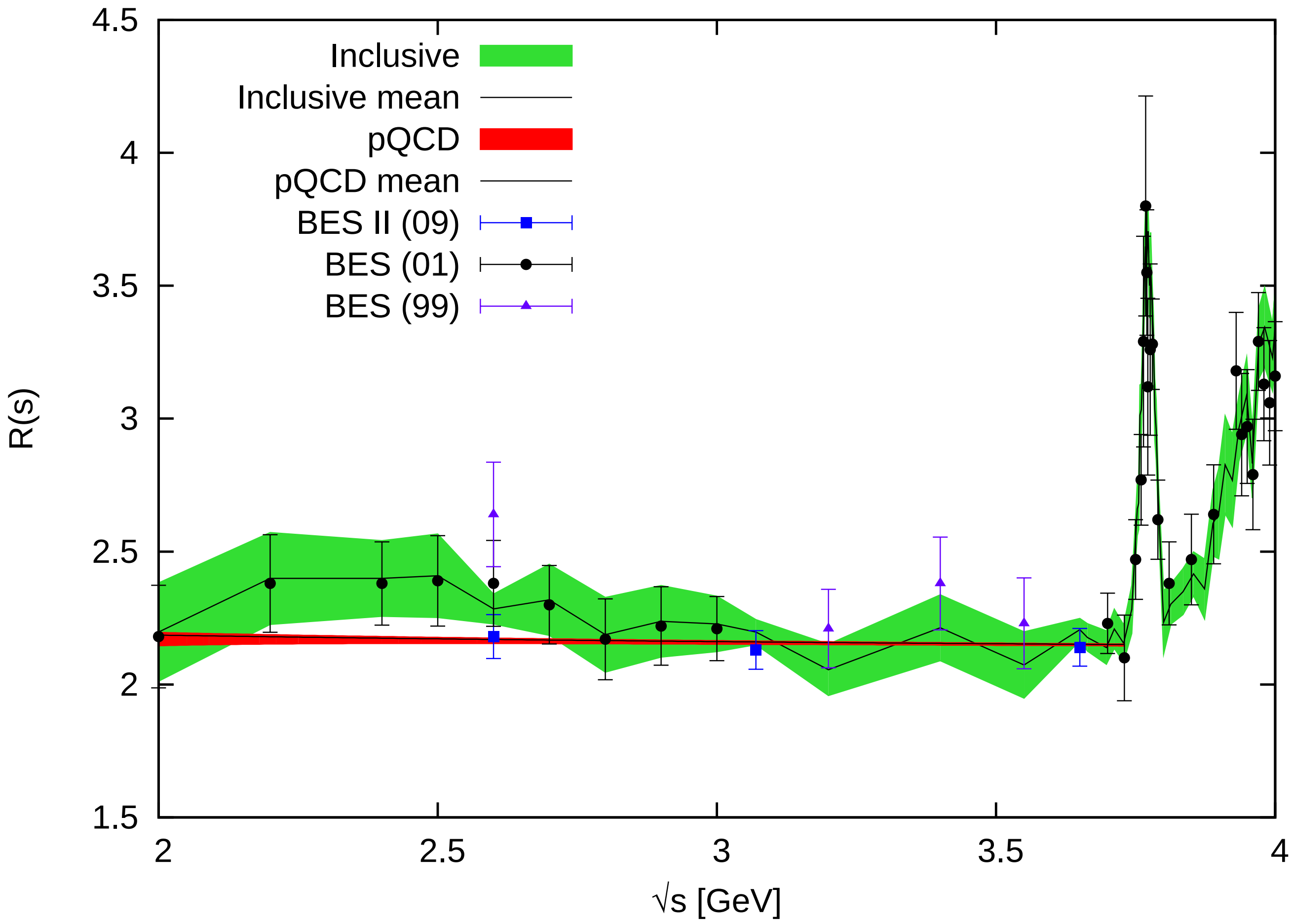
$$\int_{s_{\rm th}}^{s_0} {\rm d}s\, R(s) \left(1 - \frac{s}{s_0}\right) = \frac{i}{2\pi} \int_C {\rm d}s \left(-\frac{s}{2s_0} + 1 - \frac{s_0}{2s}\right) D(s)\,.$$

▶ New sum-rule analysis R: data only   If pQCD for 2 GeV $< \sqrt{s} < \sqrt{s_0}$: $\longrightarrow$



- Changes in data have changed the picture $\longrightarrow$ *sum over exclusive* in line with QCD.
- Still rely on isospin relations for missing channels. [Sizeable error from $K\bar{K}\pi\pi$!]
- For HLMNT 10: Use of more precise *sum over exclusive* ($\hookrightarrow$ shift up by $\sim +3 \cdot 10^{-10}$).

Perturbative QCD vs. inclusive data above 2 GeV (below charm threshold)



- R_{uds} from pQCD mostly below data fit in region above 2 GeV
- Latest BES data agree very well with pQCD
- For $2 < \sqrt{s} < 3.7$ GeV we now use pQCD but with (larger) BES errors
  $\hookrightarrow$ small shift downwards for $g-2$ ($\sim -1.4 \cdot 10^{-10}$) and $\Delta\alpha$

The different SM contributions numerically

HLMNT 10 (prel.)

Source	contr. to $a_\mu \times 10^{11}$	remarks
QED	$116\,584\,718.08 \pm 0.15$	up to 5-loop (Kinoshita+Nio, Passera)
	(was $116\,584\,719.35 \pm 1.43$)	▶ incl. recent updates of α
EW	154 ± 2	2-loop, Czarnecki+Marciano+Vainshtein
		(agrees very well with Knecht+Peris+Perrottet+deRafael)
LO hadr.	$7053 \pm 39 \pm 7 \pm 7 \pm 19$	Davier *et al.* '09 (τ)
	$6955 \pm 40 \pm 7$	Davier *et al.* '09 (e^+e^-)
	$\mathbf{6894 \pm 42 \pm 18}$	Hagiwara+Martin+Nomura+T '06
new:	$\mathbf{6951 \pm 40 \pm 21}$	HLMNT 10 (prel.), incl. BaBar 09 and KLOE 09/10 2π
NLO hadr.	$\mathbf{-98.2 \pm 0.7 \pm 0.4}$	HLMNT, in agreem. with Krause '97, Alemany+D+H '98
L-by-L	105 ± 26	▶ Prades+deRafael+Vainshtein
agrees with	< 159 (95% CL)	upper bound from Erler+Toledo Sánchez from PHD
< Nov. 2001:	(-85 ± 25)	the 'famous' sign error, $2.6\,\sigma \to 1.6\,\sigma$
$\sum$	$\mathbf{116591830 \pm 48}$	with HLMNT 10 (prel.)

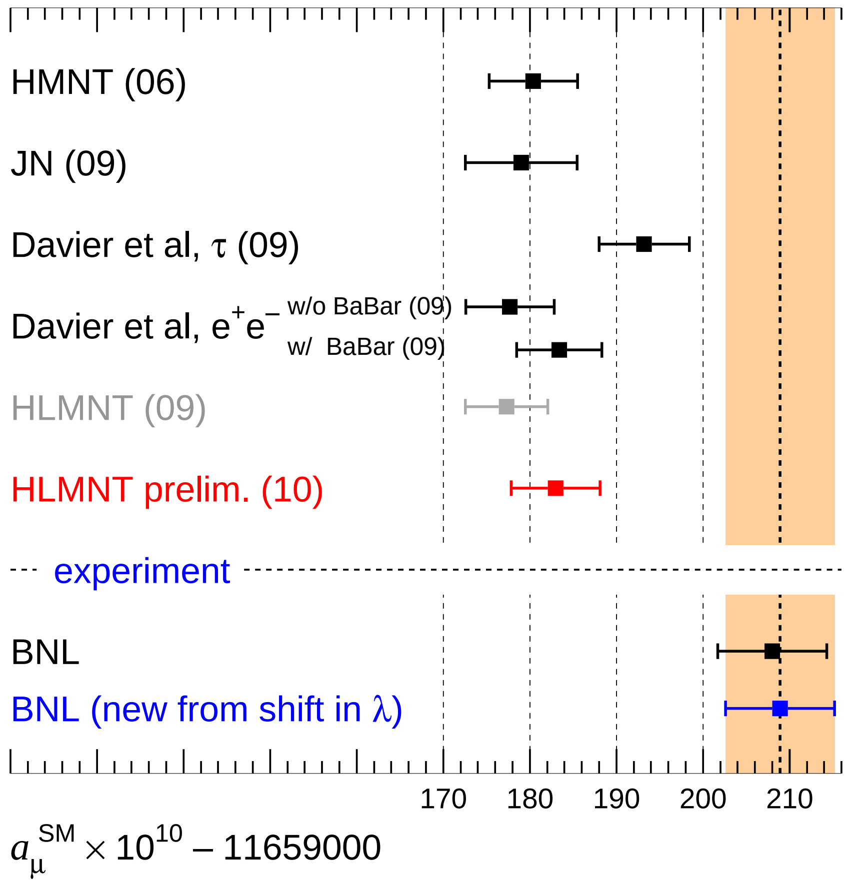
Now the theory prediction of $g-2$ is more precise than its measurement from BNL

SM vs BNL: A sign for New Physics?

Covered storage ring (Pic. from the g–2 Collab.)

Various choices w.r.t. data, way to compile, τ (?!), L-by-L: $a_\mu^{\rm SM}$ always stays $< a_\mu^{\rm EXP}$

$a_\mu^{\rm SM}$ compared to BNL world av.



Davier et al.: $1.9/3.9/3.2\,\sigma$

JN 09: $3.2\,\sigma$ [179.0 ± 6.5]

HLMNT 09: $4.0\,\sigma$ [w/out BaBar 09 2π]

Recent changes

TH: Improved LO hadronic (from e^+e^-)

[Many new data from CMD-2, SND, KLOE, BaBar, CLEO, BES. Now use sum of excl. (BaBar RadRet!) data below 2 GeV.]

$(6894 \pm 46) \cdot 10^{-11} \longrightarrow (6951 \pm 45) \cdot 10^{-11}$ (prel.)

TH: Use of recent L-by-L compilation [PdeRV]

$a_\mu^{\rm L-by-L} = (10.5 \pm 2.6) \cdot 10^{-10}$

EXP: Small shift of BNL's value due to CODATA's shift of muon to proton magn. moment ratio:

Was $a_\mu = 116\;592\;080(63) \times 10^{-11}$

$\rightarrow a_\mu = 116\;592\;089(63) \times 10^{-11}$ $(0.5ppm)$

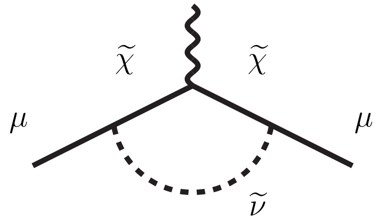
▶ With this input HLMNT get

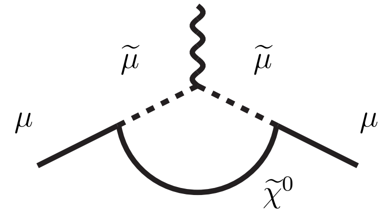
$a_\mu^{\rm EXP} - a_\mu^{\rm TH} = (25.9 \pm 8.1) \cdot 10^{-10}$, $\sim \mathbf{3.2\sigma}$

SUSY contributions in a_μ ?

$$a_\mu^{\mathrm{SUSY,1-loop}} \simeq \frac{\alpha}{8\pi \sin^2\theta_W} \tan\beta \; \mathrm{sign}(\mu) \, \frac{m_\mu^2}{M_{\mathrm{SUSY}}^2}$$

They mainly come from:

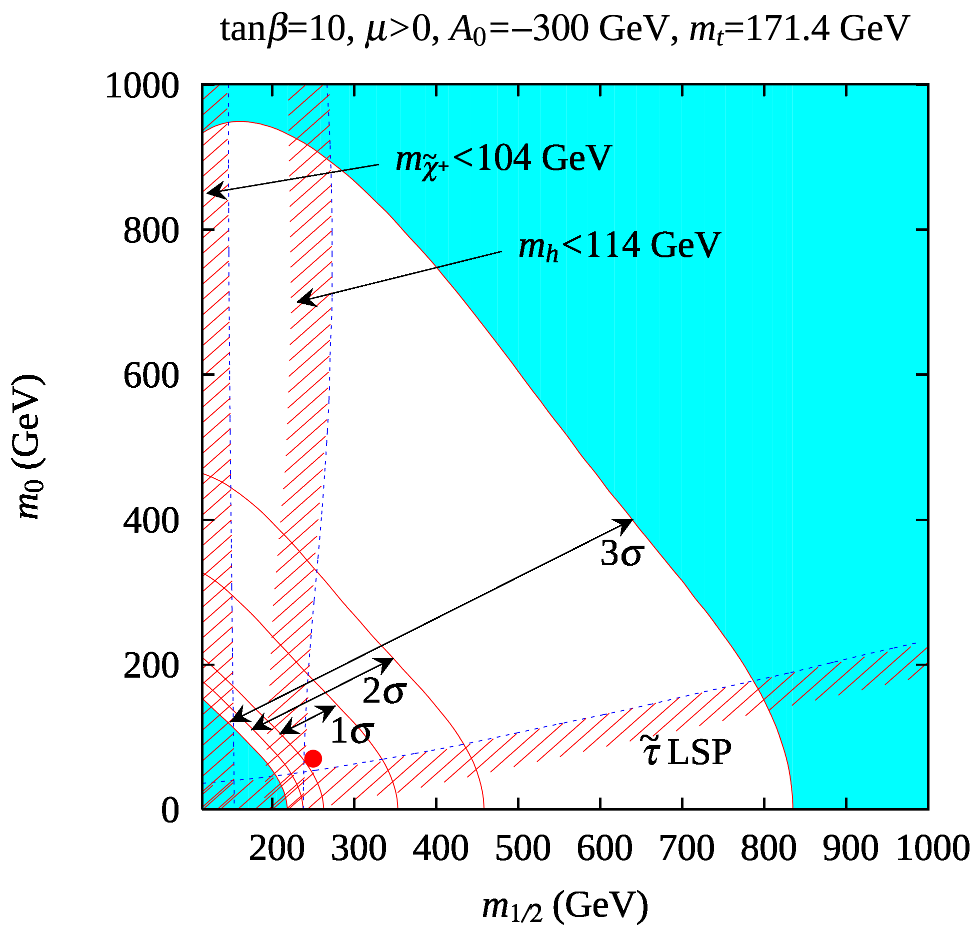




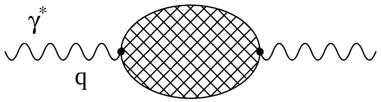
$\longrightarrow$ SUSY is a good candidate to explain $\Delta a_\mu = a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}}$, but

- no chargino at LEP
- so far no light Higgs
- limits on lightest charged SUSY part.
- $+$ limits from direct searches
- SPS 1a′ in 1σ band from $g-2$

$\longrightarrow$ Many other BSM scenarios, like e.g. Universal Extra Dimensions, seem a less natural solution.



III. The 'running coupling' $\alpha_{\rm QED}(q^2)$ and the Higgs mass



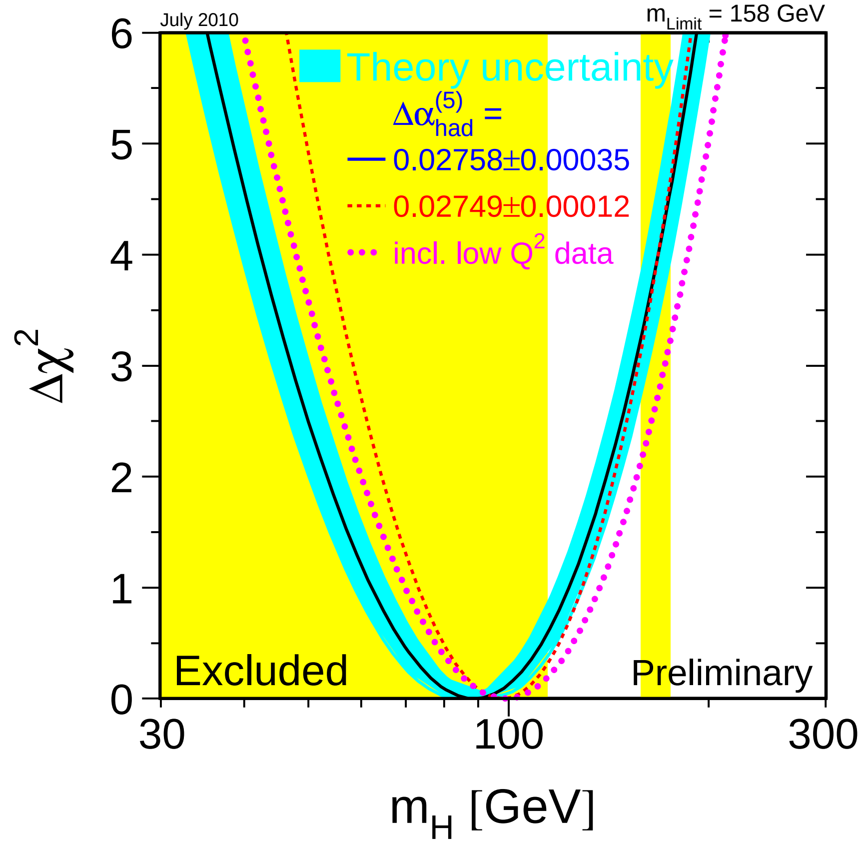
- Vacuum polarisation leads to the 'running' of α from $\alpha(q^2=0) = 1/137.035999084(51)$ to $\alpha(q^2=M_Z^2) \sim 1/129$
- $\alpha(s) = \alpha / \left(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha_{\rm had}(s)\right)$
- Again use of a dispersion relation: $\Delta\alpha^{(5)}_{\rm had} = -\frac{\alpha s}{3\pi} P \int_{s_{\rm th}}^{\infty} \frac{R_{\rm had}(s')\, ds'}{s'(s'-s)}$

▶ HLMNT-routine for $\alpha(q^2)$ and $R_{\rm had}$ available

- Hadronic uncertainties ⇝ α is the least well known Electro-Weak SM parameter of $[G_\mu, M_Z$ and $\alpha(M_Z^2)]$!
- We find: $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02759 \pm 0.00015$

i.e. $\alpha(M_Z^2)^{-1} = 128.953 \pm 0.020$ (HLMNT 10 prel.)

Fit of the SM Higgs mass: LEP EWWG



- $M_H = 89^{+35}_{-26}$ GeV ($m_t = (173.3 \pm 1.1)$ GeV)

($M_H < 158$ GeV (95% CL), < 185 GeV incl. direct limit $M_H < 114$ GeV.)

- M_H moves further down with new $\Delta\alpha$.

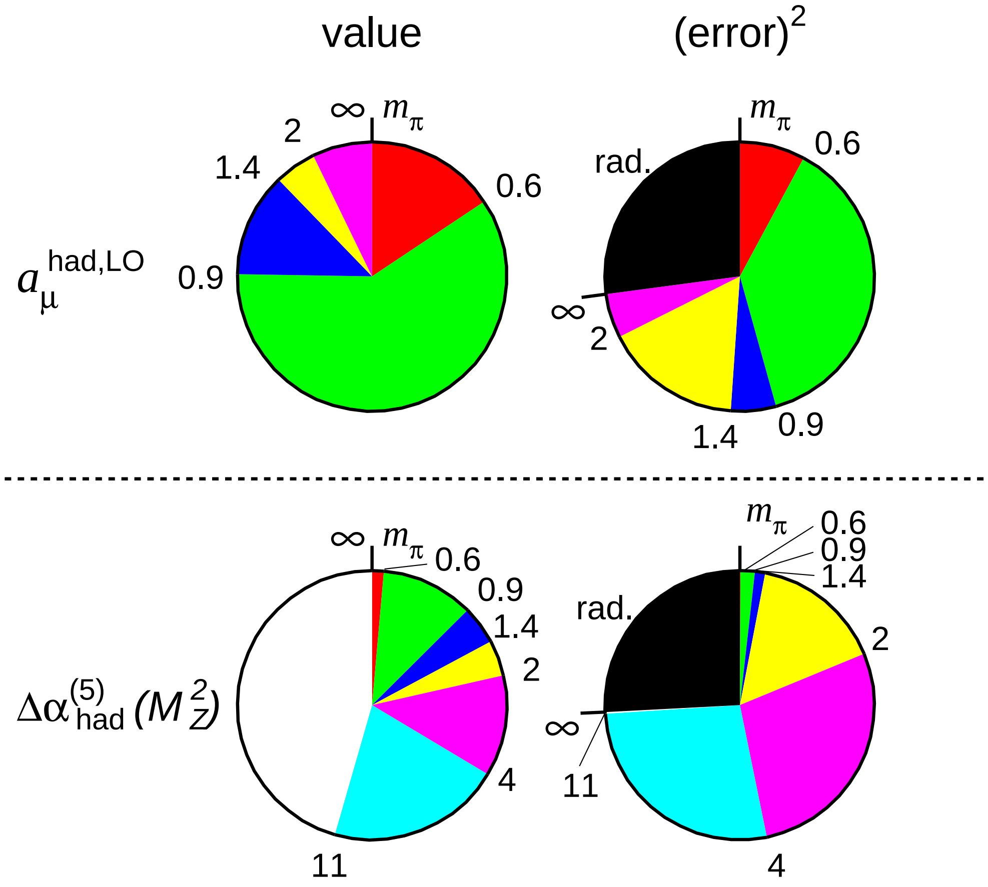
IV. Outlook / Conclusions

Where is improvement needed most urgently? Hadronic VP still the biggest error in $a_\mu^{\rm SM}$

Pie diagrams of contributions to a_μ and $\alpha(M_Z)$ and their errors^2: enjoy!

Prospects for squeezing the error!

- More 'Radiative Return' in progress at KLOE. [$\rightarrow$ Venanzoni]
- Further prospects with DAΦNE-2.
- Big improvement envisaged with CMD-3 and SND at VEPP2000. [$\rightarrow$ Shwartz]
- At higher energies, BES-III at BEPCII in Beijing is on; opportunities for BELLE.



- $(g-2)_\mu$ strongly tests *all sectors* of the SM and constrains possible physics beyond.
- Recently new data from Novosibirsk (CMD-2 and SND), Beijing (BES), Cornell (CLEO), and Frascati (KLOE) and SLAC (BaBar) using the new method of *Radiative Return*, have led to improvements and consolidation of $a_\mu^{\rm SM}$.
- With the same data compilation as for $g-2$, also the hadronic contributions to $\Delta\alpha(q^2)$ have been determined; in turn $\boldsymbol{\alpha(M_Z^2)}$ has been improved considerably. M_H !?
- Interaction of TH + MC + EXP is most important to achieve even higher precision $\rightsquigarrow$ join the WG Radio Montecar Low. $\to$ Satellite meeting this Sat.+Sun. in Liverpool

▶ Discrepancy betw. the SM pred. of $\boldsymbol{g-2}$ and the BNL measurement persists at $\boldsymbol{> 3\,\sigma}$.

- SUSY could, quite naturally, explain the discrepancy;
  SUSY parameter space already strongly constrained by $g-2$.

▶ New $g-2$ experiments planned at Fermilab and J-PARC. [$\longrightarrow$ Roberts and Mibe]

▶ Will $a_\mu^{\rm SM}$ match the planned accuracy? $\rightsquigarrow$ Light-by-Light may become limiting factor!

`The coming years will be exciting, and not only for the LHC`

Extras:

$\Delta\alpha(q^2)$: Vacuum Polarisation in the space- and time-like

- Why Vacuum Polarisation / running α corrections ?

Precise knowledge of VP / $\alpha(q^2)$ needed for:

– Corrections for data used as input for $g-2$: 'undressed' $\sigma^0_{\rm had}$

$$a_\mu^{\rm had,LO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^\infty {\rm d}s\, \sigma^0_{\rm had}(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.63\ldots 1)$$

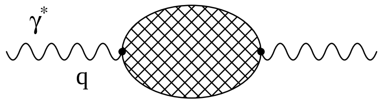
– Determination of α_s and quark masses from total hadronic cross section $R_{\rm had}$ at low energies and of resonance parameters.

– Part of higher order corrections in Bhabha scattering important for precise Luminosity determination.

– $\alpha(M_Z^2)$ a fundamental parameter at the Z scale (the least well known of $\{G_\mu, M_Z, \alpha(M_Z^2)\}$), needed to test the SM via precision fits/constrain new physics.

$\longrightarrow$ Ingredient in MC generators for many processes.

- Dyson summation of Real part of one-particle irreducible blobs $\mathbf{\Pi}$ into the effective, real running coupling $\boldsymbol{\alpha}_{\text{QED}}$:

$\Pi \;=$ 

Full photon propagator $\sim\; 1 \;+\; \Pi \;+\; \Pi\cdot\Pi \;+\; \Pi\cdot\Pi\cdot\Pi \;+\; \ldots$

$$\rightsquigarrow \qquad \alpha(q^2) \;=\; \frac{\alpha}{1-\mathrm{Re}\Pi(q^2)} \;=\; \alpha\,/\,\left(1-\Delta\alpha_{\mathrm{lep}}(q^2)-\Delta\alpha_{\mathrm{had}}(q^2)\right)$$

- The Real part of the VP, $\mathrm{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\mathrm{Im}\Pi \sim \sigma(e^+e^-\to hadrons)$:

$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2) \;=\; -\frac{q^2}{4\pi^2\alpha}\,\mathrm{P}\int_{m_\pi^2}^{\infty}\frac{\sigma^0_{\mathrm{had}}(s)\,\mathrm{d}s}{s-q^2}\;, \quad \sigma_{\mathrm{had}}(s) = \frac{\sigma^0_{\mathrm{had}}(s)}{|1-\Pi|^2}$$

[$\to$ σ^0 requires ‘undressing’, e.g. via $\cdot(\alpha/\alpha(s))^2$ $\rightsquigarrow$ iteration needed]

- Observable cross sections σ_{had} contain the |full photon propagator|^2, i.e. |infinite sum|^2.
  $\longrightarrow$ To include the subleading Imaginary part, use dressing factor $\frac{1}{|1-\Pi|^2}$.

Comparison of different compilations

- Timelike $\alpha(s)$ from Fred Jegerlehner's (2003 routine as available from his web-page)

$$\alpha(s=E^2) \;=\; \alpha \,/\, \left(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha^{(5)}_{\rm had}(s) - \Delta\alpha^{\rm top}(s)\right)$$

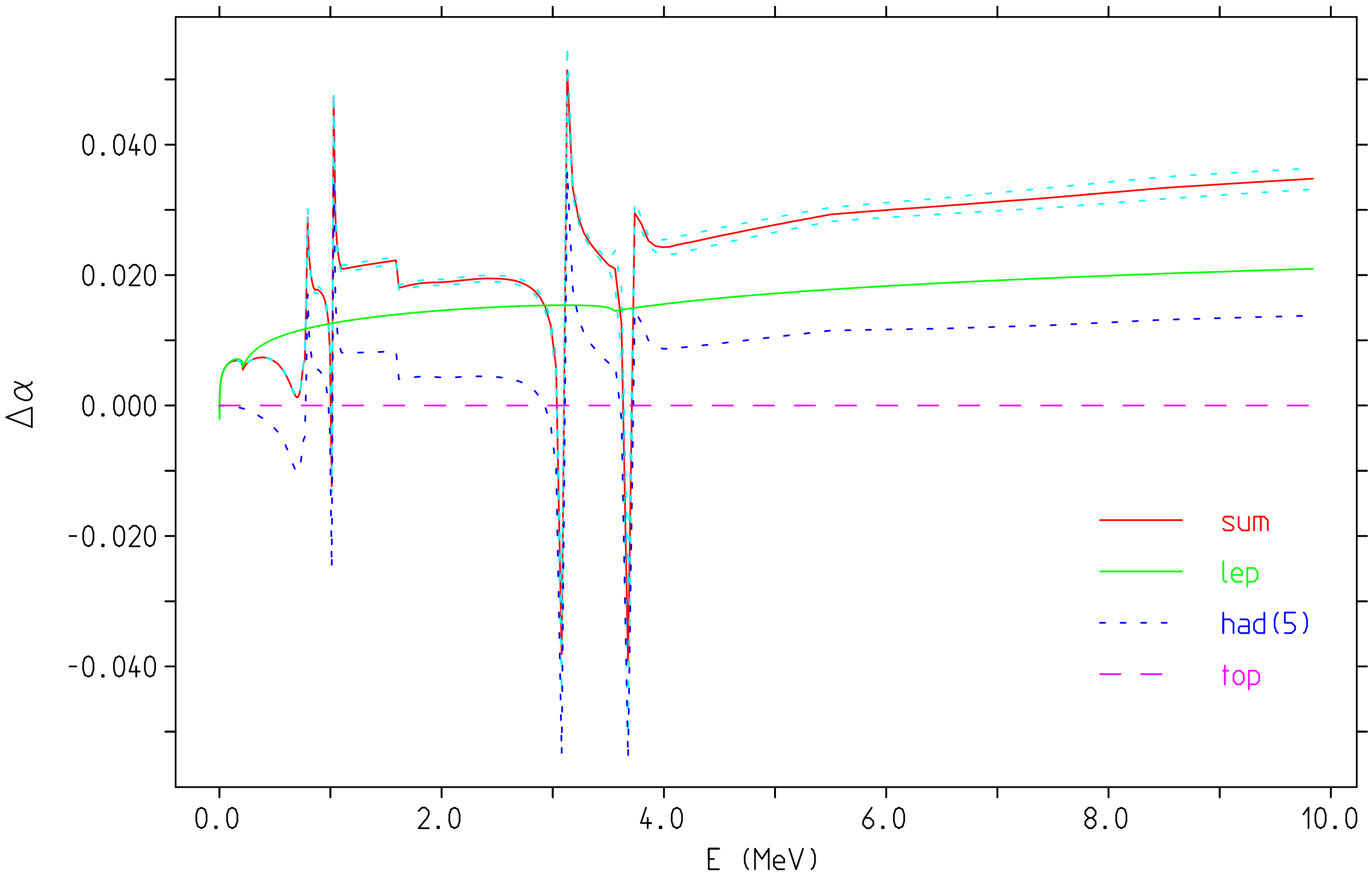
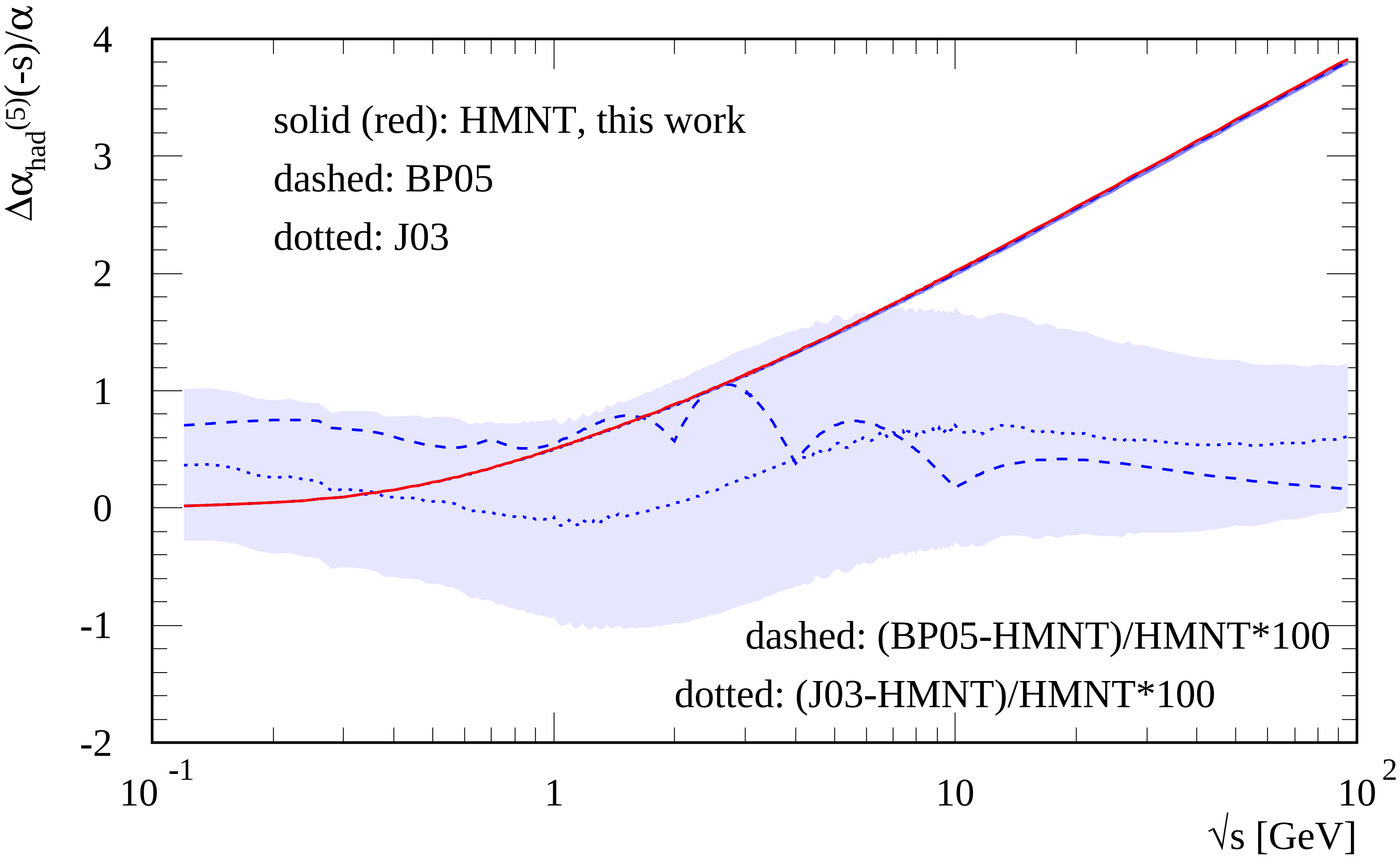


Figure from Fred Jegerlehner

- HMNT's evaluation of $\alpha_{\mathrm{QED}}(q^2)$ compared to other parametrisations:

Spacelike $\Delta\alpha^{(5)}_{\mathrm{had}}(-s)/\alpha$ (smooth $\alpha(q^2 < 0)$)

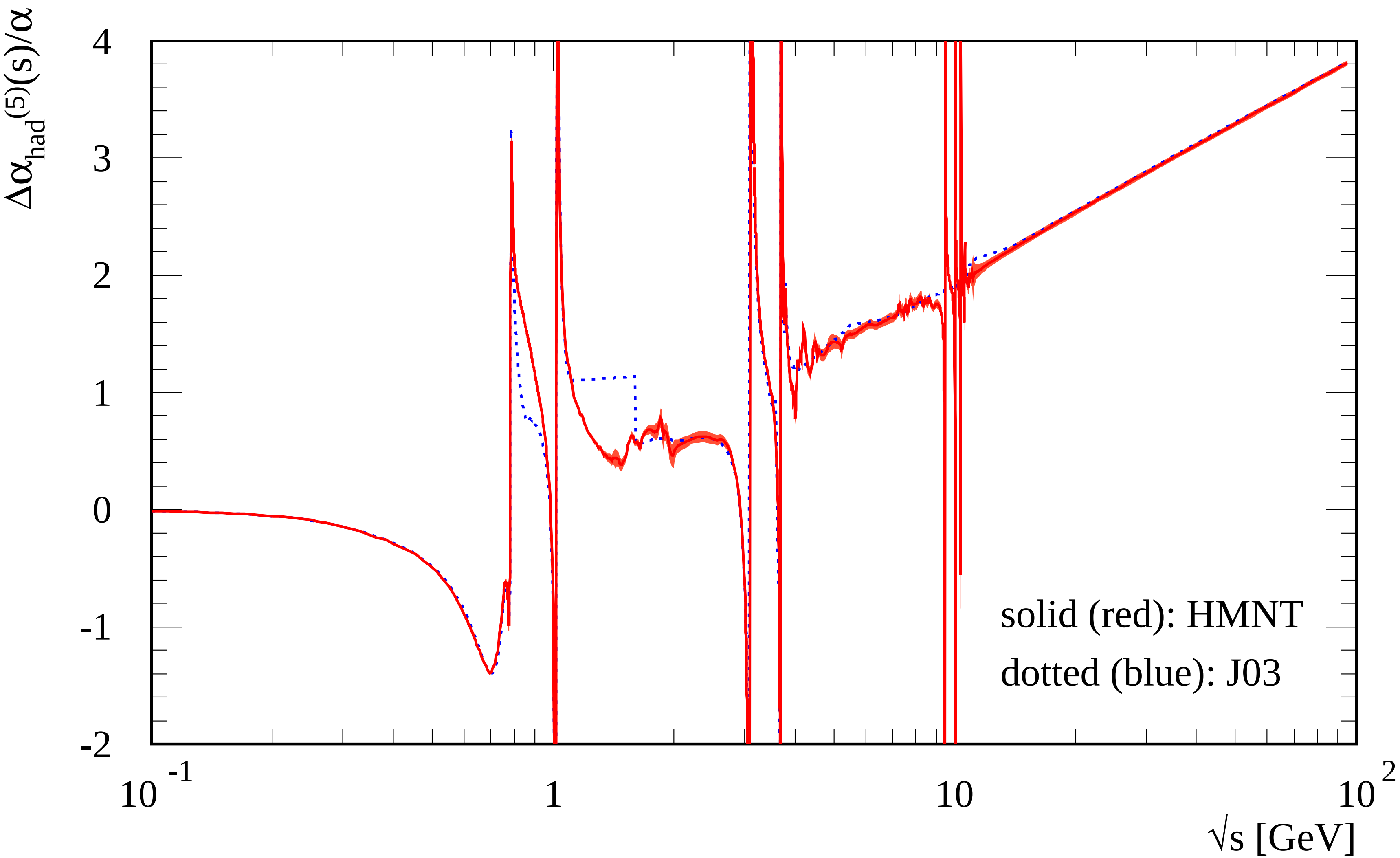


— Differences between parametrisations clearly visible but within error band (of HMNT)

— Few-parameter formula from Burkhardt+Pietrzyk slightly 'bumpy' but still o.k.

— What is in your MC?

Timelike $\alpha(s = q^2 > 0)$ follows resonance structure:



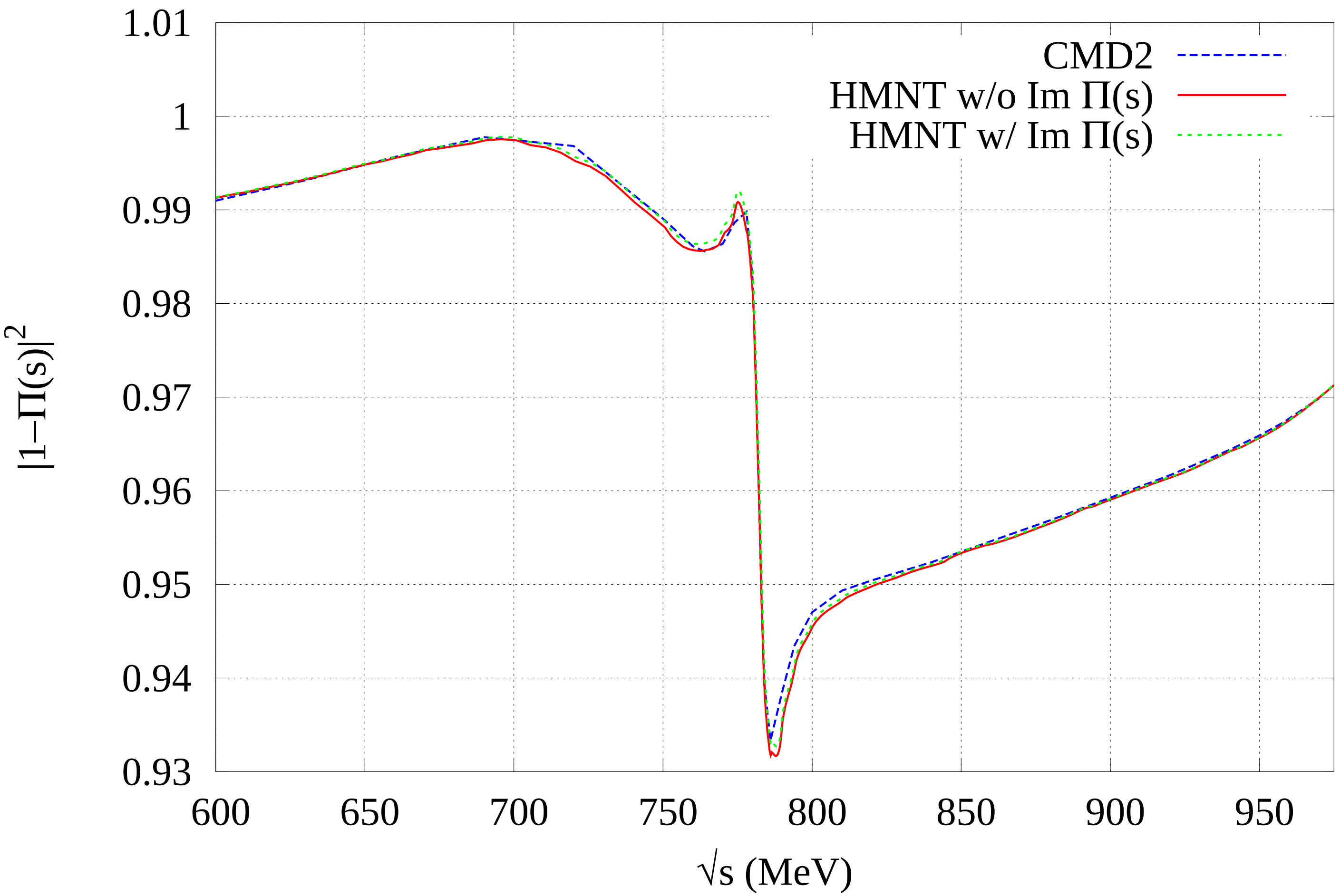
— Step below just a feature of unfortunate grid.

— Difference below 1 GeV not expected from data.

[Comparisons with other parametrisations confirm HMNT.]

- HMNT compared to Novosibirsk's new parametrisation

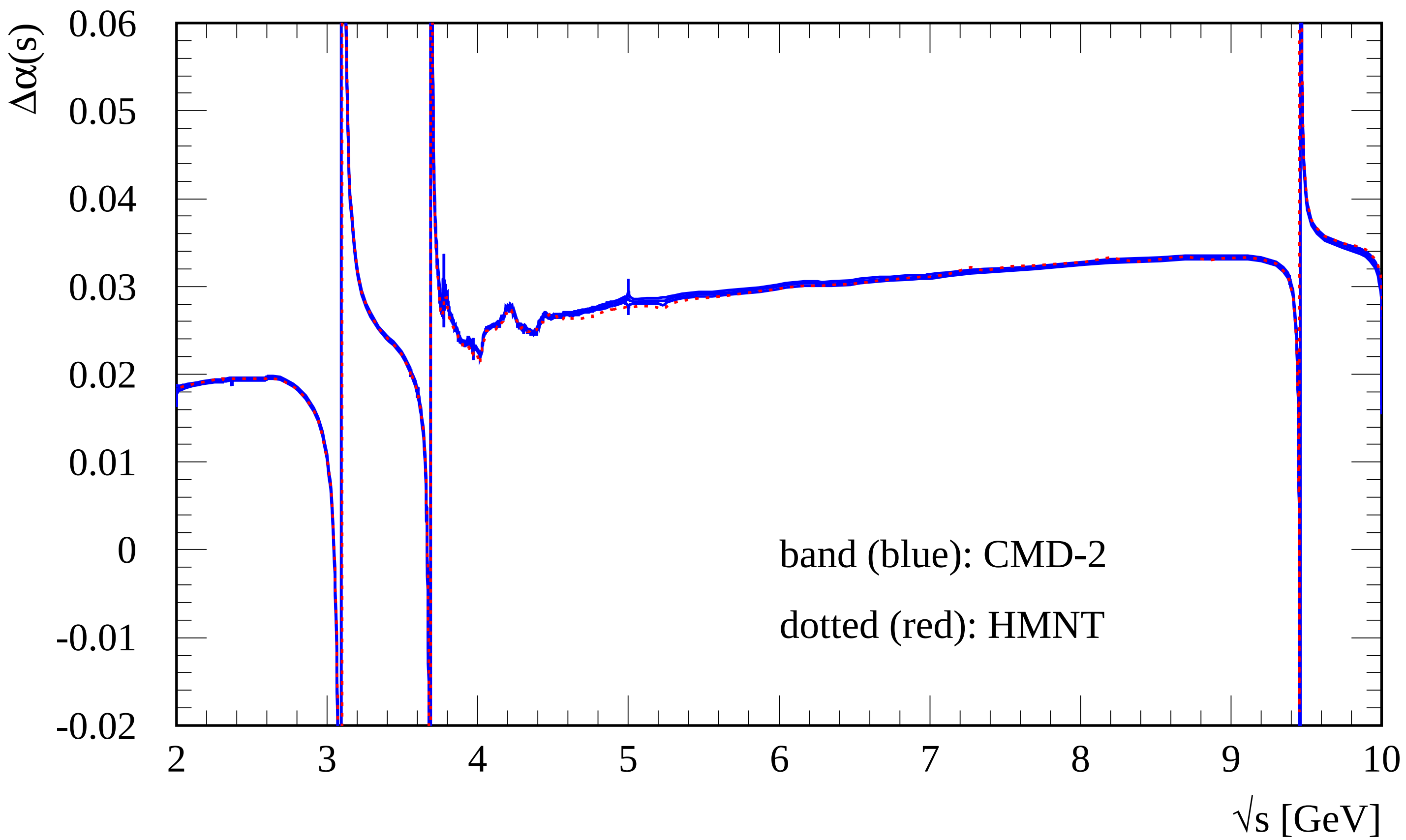
Timelike $|1 - \Pi(s)|^2 \;\sim\; (\alpha(s)/\alpha)^2$ in ρ central energy region: A relevant correction!



(Different sign and prefactor, $-e^2$, used for Π by HMNT.)

$\longrightarrow$ Small but visible differences, as expected from independent compilations.

- HMNT compared to Novosibirsk   Timelike, $\Delta\alpha(q^2)$



$\longrightarrow$ Differences of about one per-mille in the 'undressing' factor, up to -3/+5 per-mille in the $\rho - \omega$ interference regime, but likely to cancel at least partly in applications.

$\longrightarrow$ As expected small contributions from $\mathrm{Im}\Pi$.

- What about $\Delta\alpha(M_Z^2)$?

$\longrightarrow$ With the same data compilation of $\sigma^0_{\rm had}$ as for $g-2$ HLMNT find:

$$\boldsymbol{\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \;=\; 0.02760 \pm 0.00015}$$ (HLMNT 09 prelim.)

i.e. $\alpha(M_Z^2)^{-1} \;=\; 128.947 \pm 0.020$ [HMNT '06: $\alpha(M_Z^2)^{-1} \;=\; 128.937 \pm 0.030$]

Earlier compilations:

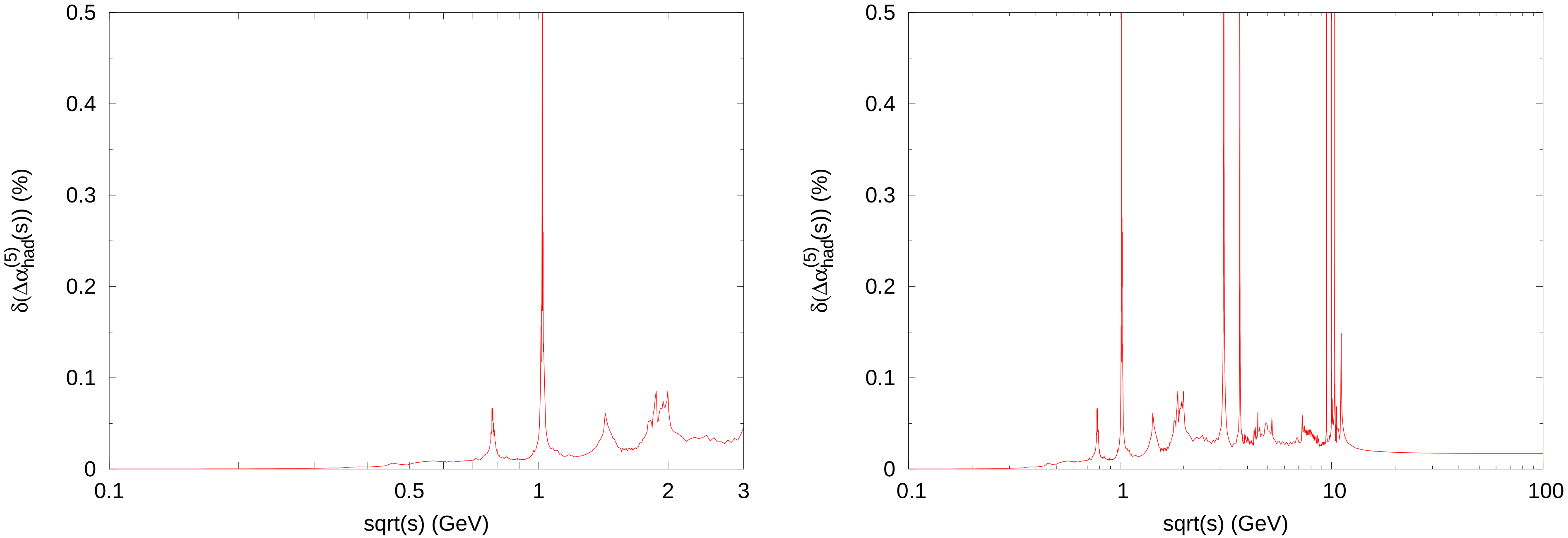
Group	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$	remarks
Burkhardt+Pietrzyk '05	0.02758 ± 0.00035	data driven
Troconiz+Yndurain '05	0.02749 ± 0.00012	pQCD
Kühn+Steinhauser '98	0.02775 ± 0.00017	pQCD
Jegerlehner '08	0.027594 ± 0.000219	data driven/pQCD
($M_0 = 2.5$ GeV)	0.027515 ± 0.000149	Adler fct, pQCD
HMNT '06	0.02768 ± 0.00022	data driven

Adler function: $$D(-s) \;=\; \frac{3\pi}{\alpha} s \frac{\rm d}{{\rm d}s}\Delta\alpha(s) \;=\; -(12\pi^2) s \frac{{\rm d}\Pi(s)}{{\rm d}s}$$

allows use of pQCD and minimizes dependence on data.

$\delta\left(\Delta\alpha_{\rm had}^{(5)}(s)\right)$ of HMNT compilation

Error of VP in the timelike regime at low and higher energies:



$\longrightarrow$ Below one per-mille (and typically $\sim 5 \cdot 10^{-4}$), apart from Narrow Resonances where the bubble summation is not well justified.