Constraints on Kπ form factors from K_{l3} and τ decays

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Outline

1. Introduction and Motivations
2. Dispersive representation of the form factors
3. Fits to the $\tau \rightarrow K\pi\nu_\tau$ and K_{l3} decays
4. Conclusion and outlook

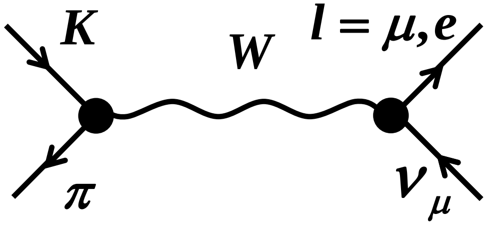
1. Introduction and Motivations

Motivations

- Studying τ and K_{l3} decays ⇨ indirect searches of new physics, several possible high-precision tests:

K_{l3} $\left(K \to \pi \underset{\uparrow}{l} \nu_l\right)$ decays :

$(l = e, \mu)$



Hadronic matrix element :

$$\boxed{\langle \pi(p_\pi)|\, \bar{\mathrm{s}}\gamma_\mu \mathrm{u} \,|\mathrm{K}(\mathrm{p_K})\rangle = \left[(p_K + p_\pi)_\mu - \frac{\Delta_{K\pi}}{t}(p_K - p_\pi)_\mu\right] \underset{\text{vector}}{\underset{\uparrow}{f_+(t)}} + \frac{\Delta_{K\pi}}{t}(p_K - p_\pi)_\mu \, \underset{\text{scalar}}{\underset{\uparrow}{f_0(t)}}}$$

→ $t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2$

→ Normalization $\bar{f}_{0,+}(t) = \dfrac{f_{0,+}(t)}{f_+(0)}$

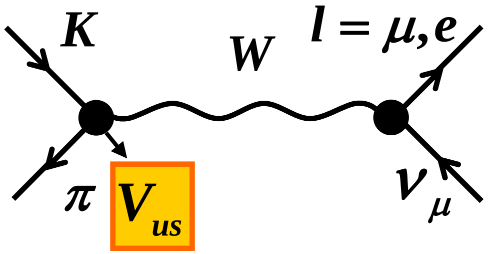
Motivations

- Studying τ and K_{l3} decays ⇒ indirect searches of new physics, several possible high-precision tests :

  1) Extraction of V_{us}

From K_{l3} $\left(K \to \pi l \nu_l\right)$ decays

$\left(l = e, \mu\right)$



$$\boxed{\Gamma_{K^{+/0}l3} = N\left|f_+(0)V_{us}\right|^2 I^l_{K^{+/0}}}$$

with

$$\boxed{I^l_{K^{+/0}} = \int dt \frac{1}{m^8_{K^{+/0}}}\lambda^{3/2}F\left(t, \bar{f}_+(t), \bar{f}_0(t)\right)}$$

From Lattice QCD

$f_+(0)$

$\boxed{f_+(0)\left|V_{us}\right|} \longrightarrow \boxed{\left|V_{us}\right|}$ ⇒ Test of unitarity $\boxed{\left|V_{ud}\right|^2 + \left|V_{us}\right|^2 + \left|V_{ub}\right|^2 \overset{?}{=} 1}$

$0^+ \to 0^+$ β decays

K_{l3} decays

Negligible (B decays)

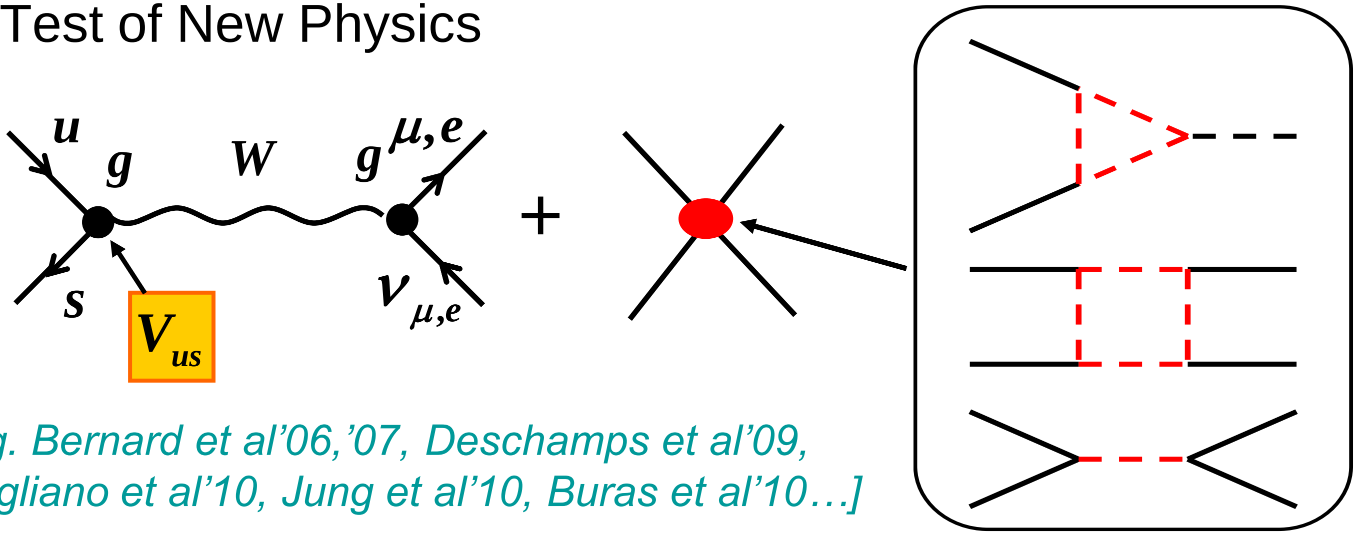
2) Callan-Treiman (CT) theorem

$$C = \bar{f}_0(\underset{\overset{\nearrow}{m_K^2 - m_\pi^2}}{\Delta_{K\pi}}) = \frac{F_K}{F_\pi f_+(0)} + \Delta_{CT} = \underbrace{\frac{F_K\left|V^{us}\right|}{F_\pi\left|V^{ud}\right|}\,\frac{1}{f_+(0)\left|V^{us}\right|}\,\left|V^{ud}\right|}\, r + \Delta_{CT}$$

Very precisely known
from Br(K_{l2}/π_{l2}), $\Gamma(K_{e3})$ and $\left|\mathcal{V}_{ud}\right|$

- In the Standard Model $\boxed{r = 1}$ $\left(\ln C_{SM} = 0.2141(73)\right)$
- In presence of new physics, new couplings : $\boxed{r \neq 1}$

- Test of New Physics



SUSY loops
Z', Charged Higgs,
Right-Handed
Currents,….

*[E.g. Bernard et al'06,'07, Deschamps et al'09,
Cirigliano et al'10, Jung et al'10, Buras et al'10…]*

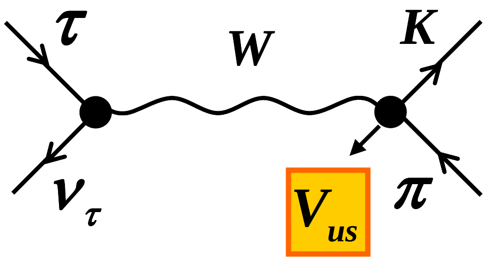
- $\bar{f}_+(t)$ accessible in K_{e3} and $K_{\mu3}$ decays
- $\bar{f}_0(t)$ only accessible in $K_{\mu3}$ (suppressed by m_l^2/M_K^2) + correlations

  ⇨ difficult to measure.

- New data from *Belle* and *BaBar* on $\tau \to K\pi\nu_\tau$ decays

  ⇨ Use them to constrain the form factors and especially $\bar{f}_0$

- $\tau \to K\pi\nu_\tau$ decays



Hadronic matrix element: Crossed channel

$$\boxed{\langle \mathbf{K}\pi |\ \bar{\mathbf{s}}\gamma_\mu \mathbf{u}\ |\mathbf{0}\rangle = \left[(p_K - p_\pi)_\mu + \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu \right] f_+(s) - \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu\ f_0(s)}$$

vector

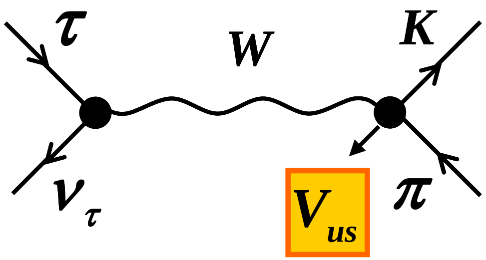
scalar

with $s = q^2 = (p_K + p_\pi)^2$

- New data from *Belle* and *BaBar* on $\tau \to K\pi\nu_\tau$ decays

  ⇨ Use them to constrain the form factors and especially $\bar{f}_0$

- $\tau \to K\pi\nu_\tau$ decays



Extraction of V_{us} from this decay:

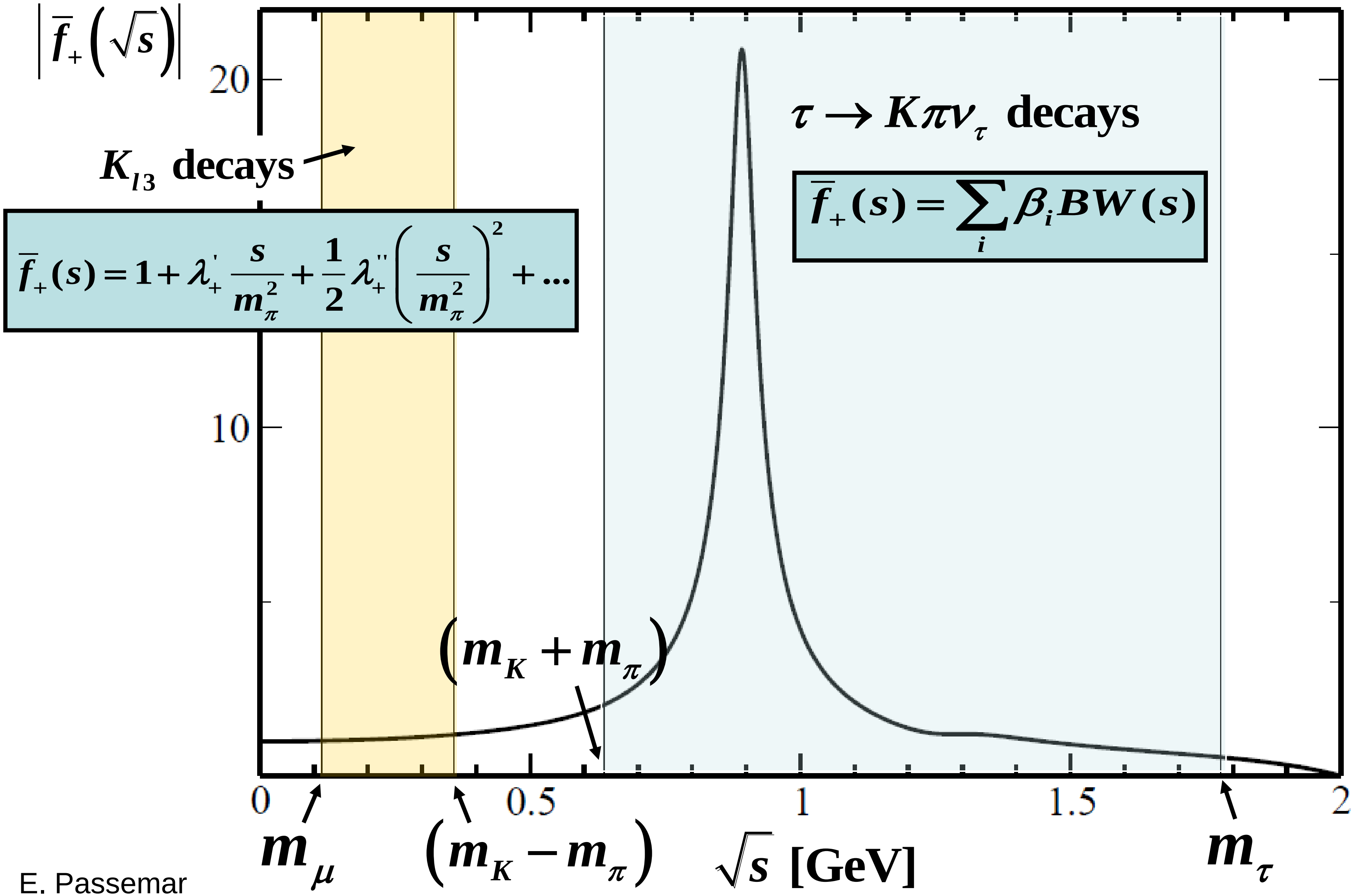
$$\boxed{\Gamma_{\tau\to K\pi\nu_\tau} \equiv \Gamma_{K\pi} = \frac{G_F^2 m_\tau^3}{32\pi^3} C_K S_{EW} \left| f_+(0)\mathrm{V}_{us} \right|^2 I_K^\tau \left(1+\delta_{EM}\right)^2}$$

$$\boxed{I_K^\tau = \int_{(m_K+m_\pi)^2}^{m_\tau^2} ds\,\lambda^{3/2} F\left(s, \bar{f}_+(s), \bar{f}_0(s)\right)}$$

~0.1% level expected
Neglected at this stage
Work in progress by F. Flores

Determination of the form factors: Vector

- Dominance of K*(892) resonance

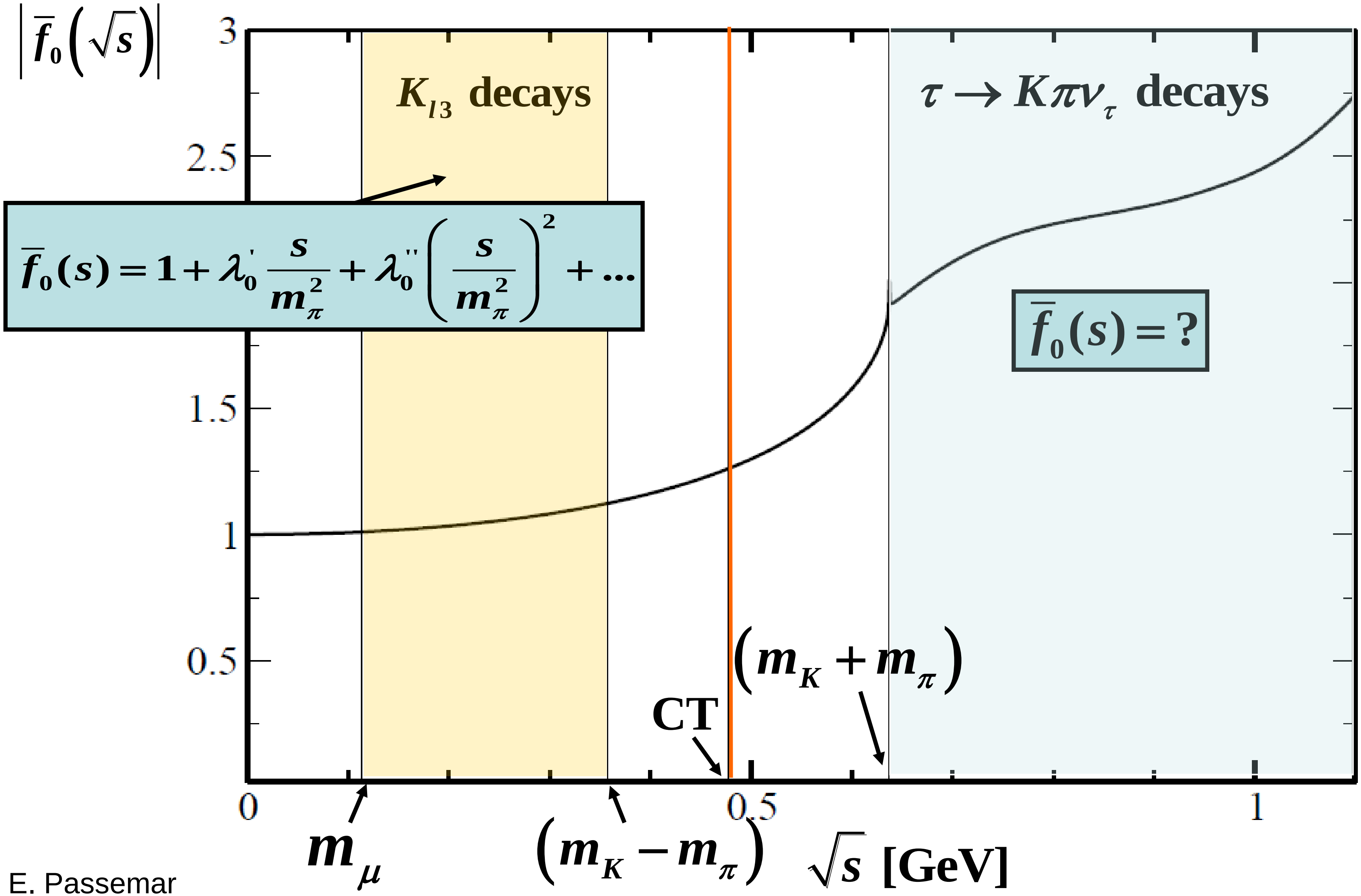


$$\bar f_+(s) = 1 + \lambda_+^{'} \frac{s}{m_\pi^2} + \frac{1}{2}\lambda_+^{''}\left(\frac{s}{m_\pi^2}\right)^2 + \ldots$$

$$\bar f_+(s) = \sum_i \beta_i BW(s)$$

Determination of the form factors: Scalar

In this case no obvious dominance of a resonance



$$\bar{f}_0(s) = 1 + \lambda_0^{'}\frac{s}{m_\pi^2} + \lambda_0^{''}\left(\frac{s}{m_\pi^2}\right)^2 + \ldots$$

$$\boxed{\bar{f}_0(s) = ?}$$

E. Passemar

2. Dispersive representation of the form factors

Strategy

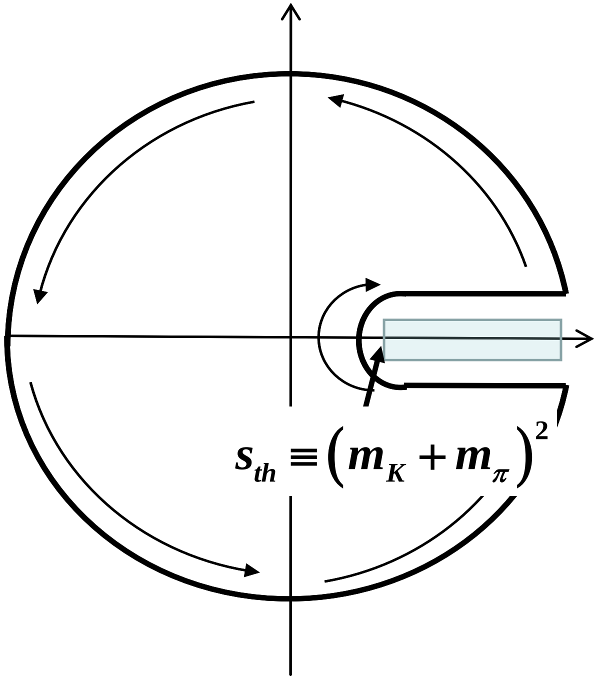
- K_{l3} decays: dispersive representation introduced to improve on the form factors extraction *[Bernard, Oertel, E.P., Stern'06,'10]*

  Also coupled channel analysis *[Jamin, Oller & Pich'02,'06]*

  ⇨ Adapt the dispersive representation to analyse the $\tau \to K\pi\nu_\tau$ decays

- Omnès representation: ⇨

$$\boxed{\bar{f}_{+,0}(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$



$\phi_{+,0}$ (s): phase of the form factor

- $s < s_{in}: \ \phi_{+,0}(s) = \delta_{K\pi}(s)$ *[Watson theorem]*

  $K\pi$ scattering phase

- $s \geq s_{in}: \ \phi_{+,0}(s)$ unknown

⇨ $\phi_{+,0}(s) = \phi_{+,0as}(s) = \pi \pm \pi \ \left(\bar{f}_{+,0}(s) \to 1/s\right)$

[Brodsky&Lepage]

- Subtract dispersion relation to weaken the high energy contribution of the phase. Improve the convergence but sum rules to be satisfied !

Strategy

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  ⇨ Adapt the dispersive representation to analyse the $\tau \to K\pi\nu_\tau$ decays

- Omnès representation: ⇨

$$\boxed{\bar{f}_{+,0}(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$

- Dispersion relation with n subtractions in $\bar{s}$:

$$\boxed{\bar{f}_{+,0}(s) = \exp\left[P_{n-1}(s) + \frac{(s-\bar{s})^n}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{(s'-\bar{s})^n}\frac{\phi_{+,0}(s')}{s'-s-i\varepsilon}\right]}$$

Dispersive Representation: Scalar

- For $\bar{f}_0(s)$ ⇒ Dispersion relation with 3 subtractions: 2 in s=0 and 1 in $s=\Delta_{K\pi}$

[Callan-Treiman]

$$\bar{f}_0(s) = \exp\left[\frac{s}{\Delta_{K\pi}}\left(\ln C + \left(s-\Delta_{K\pi}\right)\left(\frac{\ln C}{\Delta_{K\pi}} - \frac{\lambda_0^{'}}{m_\pi^2}\right) + \frac{\Delta_{K\pi}\, s\left(s-\Delta_{K\pi}\right)}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^2}\,\frac{\phi_0(s')}{\left(s'-\Delta_{K\pi}\right)\left(s'-s-i\varepsilon\right)}\right)\right]$$

For $s<s_{in}$: $K\pi$ scattering phase extracted from the data

[Buettiker, Descotes-Genon, & Moussallam ’02]

⇒ 2 parameters to fit to the data $\boxed{\ln C = \ln \bar{f}(\Delta_{K\pi})}$ and $\boxed{\lambda_0^{'}}$

NB: One more subtraction than in the K_{l3} case ⇒ τ decays at higher energy

Improve the convergence !

Dispersive Representation: Vector

- For $\bar{f}_+(s)$ ⇒ Dispersion relation with 3 subtractions in s=0

$$\bar{f}_+(s) = \exp\left[\lambda'_+ \frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda''_+ - \lambda'^2_+\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^3}\frac{\phi_+(s')}{\left(s'-s-i\varepsilon\right)}\right]$$

[Boito, Escribano & Jamin’09,’10]

- For ϕ_+ (s):In this case instead of the data, use of a parametrization including 2 resonances K*(892) and K*'(1414) :

[Jamin, Pich & Portolés '08]

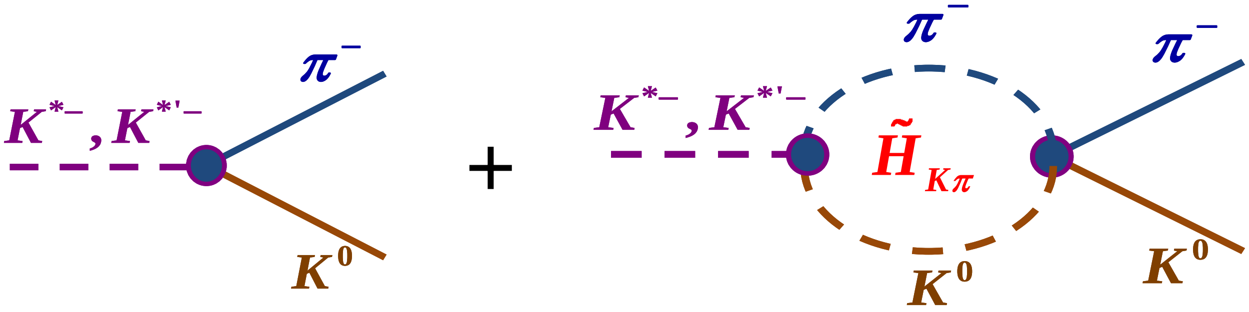
$$\boxed{\bar{f}_+(s) = \left[\frac{m_{K^*}^2 - \kappa_{K^*}\left(\mathrm{Re}\,\tilde{H}_{K\pi}(0) + \mathrm{Re}\,\tilde{H}_{K\eta}(0)\right) + \beta s}{D\left(m_{K^*},\Gamma_{K^*}\right)} - \frac{\beta s}{D\left(m_{K^{*\prime}},\Gamma_{K^{*\prime}}\right)} \right]}$$

⇒

$$\boxed{\tan\delta_{K\pi}^{P,1/2} = \frac{\mathrm{Im}\,\bar{f}_+(s)}{\mathrm{Re}\,\bar{f}_+(s)}}$$

with $\boxed{D\left(m_n,\Gamma_n\right) = m_n^2 - s - \kappa_n \sum \mathrm{Re}\,\tilde{H} - i m_n \Gamma_n(s)}$

- Parametrization that takes into account the loop effects :



- Future improvements: ⇒ take also into account the inelastic channels

Dispersive Representation: Vector

- For $\bar{f}_+(s)$ ⇨ Dispersion relation with 3 subtractions in s=0

$$\boxed{\bar{f}_+(s) = \exp\left[\lambda_+^{'}\frac{s}{m_\pi^2} + \frac{1}{2}\left(\lambda_+^{''} - \lambda_+^{'2}\right)\left(\frac{s}{m_\pi^2}\right)^2 + \frac{s^3}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'^3}\frac{\phi_+(s')}{(s'-s-i\varepsilon)}\right]}$$

- 7 parameters to fit to the data:
  - $\boxed{\lambda_+^{'}}$ and $\boxed{\lambda_+^{''}}$ ⇨ can be combined with K_{l3} fits
  - Resonance parameters: $\boxed{m_{K^*}, \Gamma_{K^*}, m_{K^{*'}}, \Gamma_{K^{*'}}, \beta}$

Mixing parameter

3. Fits to the $\tau \to K\pi\nu_\tau$ and K_{l3} decays

Presentation

- Fit to the $\tau \to K\pi\nu_\tau$ decay data from Belle [Epifanov et al'08]

$$\boxed{N_{events} \propto N_{tot}\; b_w \;\frac{1}{\Gamma_{K\pi}}\frac{d\Gamma_{K\pi}}{d\sqrt{s}}} \;\Rightarrow\; \boxed{\chi^2_\tau = \sum_{bins}\left(\frac{N_{events}-N_\tau}{\sigma_{N_\tau}}\right)^2}$$

with

Number of events/bin

bin width

$$\boxed{\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 m_\tau^3}{32\pi^3 s} C_K^2 S_{EW} \left|f_+(0)V_{us}\right|^2 \left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+\frac{2s}{m_\tau^2}\right) q^3_{K\pi}(s) \left|\bar{f}_+(s)\right|^2 + \frac{3\Delta^2_{K\pi}}{4s} q_{K\pi}(s) \left|\bar{f}_0(s)\right|^2\right]}$$

Normalization disappears by taking the ratio $\frac{1}{\Gamma_{K\pi}}\frac{d\Gamma_{K\pi}}{d\sqrt{s}}$ ⇒ fit independant

of V_{us}, to be determined from the Br $\boxed{B_{K\pi} = \frac{\Gamma_{K\pi}}{\Gamma_\tau}}$ measurement

- Possible combination with K_{l3} decay data fits

$$\boxed{\chi^2 = \chi_\tau^2 + \left(\lambda_+ - \lambda_+^{K_{l3}}\right)^T V^{-1}\left(\lambda_+ - \lambda_+^{K_{l3}}\right) + \left(\frac{\ln C - \ln C^{K_{l3}}}{\sigma_{\ln C}}\right)} \quad \text{with} \quad \lambda_+ = \begin{pmatrix} \lambda_+^{'} \\ \lambda_+^{''} \end{pmatrix}$$

⇨ should be redone with the data to take into account the correlations between the λ's and lnC !

K_{l3} data: average from the *Flavianet Kaon WG'10*

- Results expressed in terms of the physical poles

Solutions of $\boxed{D\left(m_n^{fit}, \Gamma_n^{fit}\right) = 0}$ in the complex plane

⇨ $$\boxed{\sqrt{s_{pol}} = m_n^{phys} - \frac{i}{2}\Gamma_n^{phys}}$$

4. Conclusion and Outlook

- Possibility to get interesting constraints on the $K\pi$ form factors from $\tau \to K\pi\nu_\tau$ decays
  - New data available from Belle and BaBar
  - Build a parametrization using dispersion relations to have a precise parametrization and theoretically well motivated to fit the data

- Dispersive parametrization ⇒ possible combination with K_{l3} data to have constraints from different energy regions

- Possibility to test the Standard Model: ⇒ $\boxed{V_{us}}$ and $\boxed{\ln C}$

- Work still in progress !

- Results not competitive yet but improvements underways

- Use the representation to measure the asymmetries ⇒ disentangle scalar and vector form factors.