Study of the hadronic three-prong decays of the tau lepton in the Belle experiment

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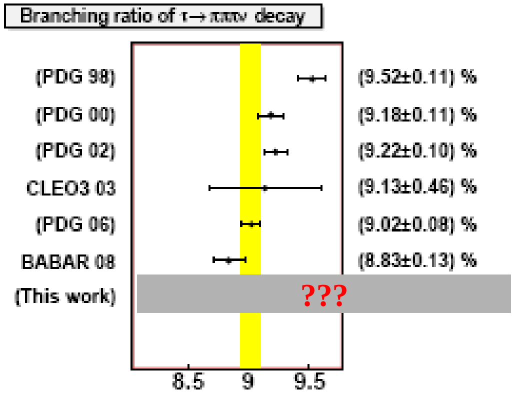
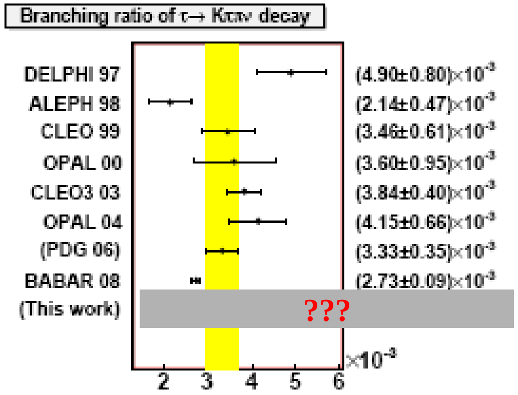
Three-prong decays of tau lepton

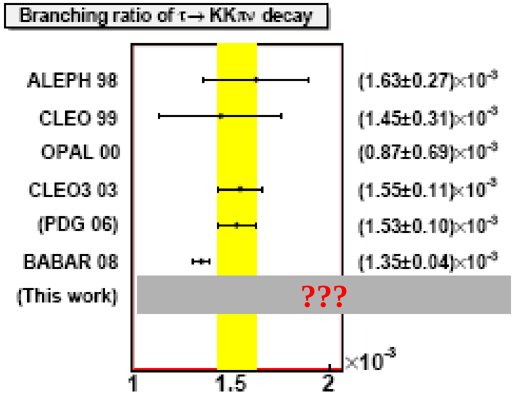
- In this study, we consider the three-prong decays with no neutral particles.
  - $\tau\to\pi\pi\pi\nu_\tau$ ($a_1\to\rho\pi$), $\tau\to K\pi\pi\nu_\tau$ ($K_1\to K^*\pi$, $K_1\to K\rho$), $\tau\to KK\pi\nu_\tau$, $\tau\to KKK\nu_\tau$

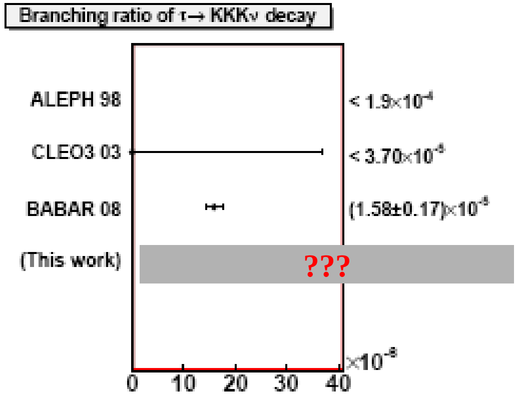
- BF have been measured with large error
- Precise measurements showed large scatter of BF
- Unfolded mass spectra have not been measured

- A preliminary result had been shown at TAU08, now we update BF result with considering unfolded mass spectra.









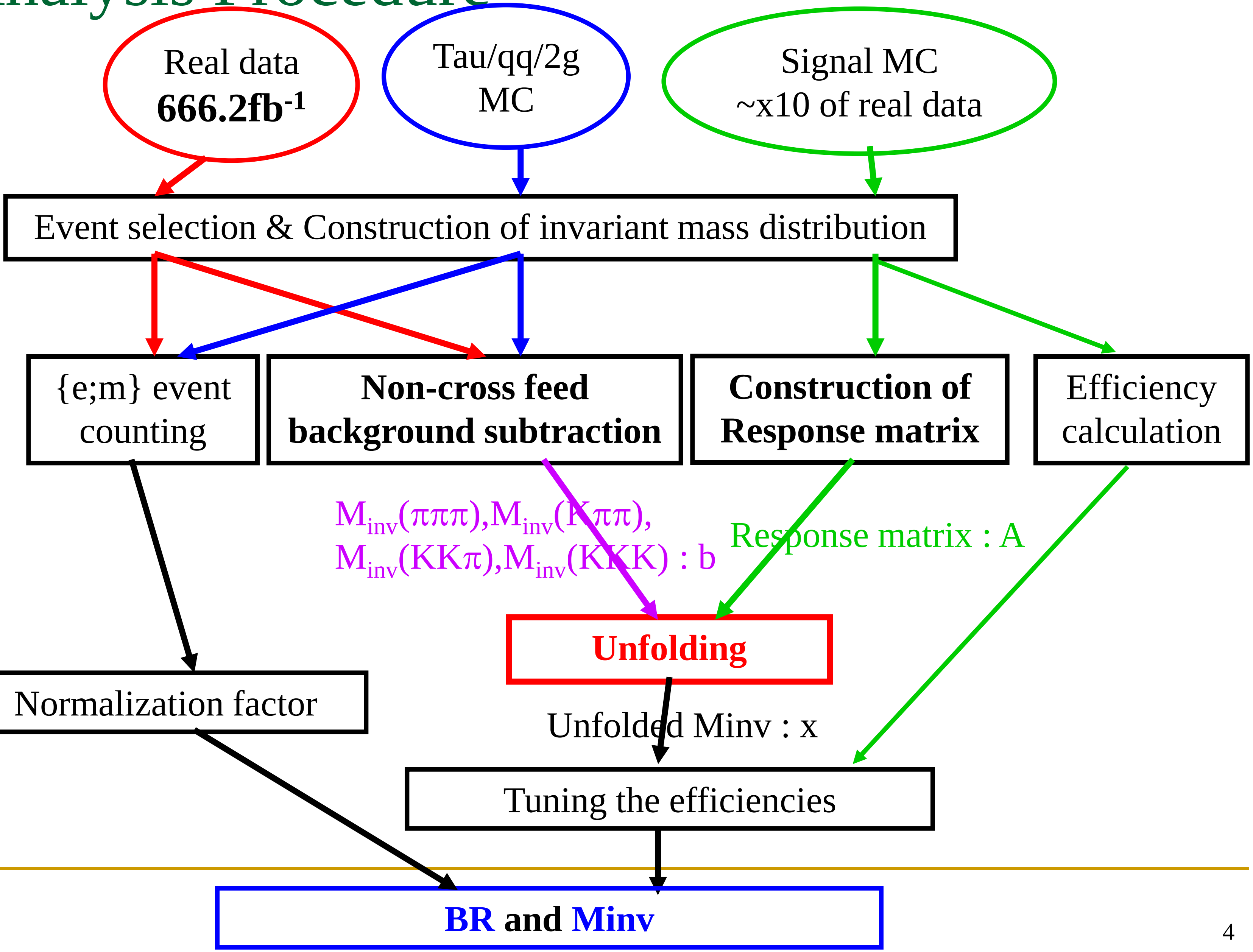
Hadronic three-prong decays

Importance : To measure the spectral function, Vus, Leptonic CP violation, Wess-Zumino anomaly...

- Status before this work
  - Precise measurement (BaBar,2008) showed large deviation from previous world averages.
  - No study on the mass spectrum
- Difficulty : Estimation of cross-feeding background
  - Main Background of $\tau \to K\pi\pi\nu$ is $\tau \to \pi\pi^+\pi^-\nu_\tau$
  - **4 modes are correlated and should be studied simultaneously.**
  - **Kaon identification** is the most important in this analysis
  - We use **unfolding** to estimate cross-feed BG.

Main backgrounds	Discrimination	Background estimation
Bhabha, dimuon ($e^+e^- \to e^+e^-, \mu^+\mu^-$)	Reconstructed total energy, momentum, multiplicity	Estimated from MC
Continuum ($e^+e^- \to qq$, q=uds)	Event topology, thrust, invariant mass, multiplicity	
Two-photon	Small transverse momentum, Missing mass/momentum reconstruction	
Beam background	Quality of decay vertex	
Other tau decays	**Ks, pi0, gamma veto**	
Cross-feed		**Unfolding**

Analysis Procedure



Real data
666.2fb^{-1}

Tau/qq/2g
MC

Signal MC
~x10 of real data

Event selection & Construction of invariant mass distribution

{e;m} event counting

Non-cross feed background subtraction

Construction of Response matrix

Efficiency calculation

$M_{inv}(\pi\pi\pi)$,$M_{inv}(K\pi\pi)$,
$M_{inv}(KK\pi)$,$M_{inv}(KKK)$: b

Response matrix : A

Unfolding

Normalization factor

Unfolded Minv : x

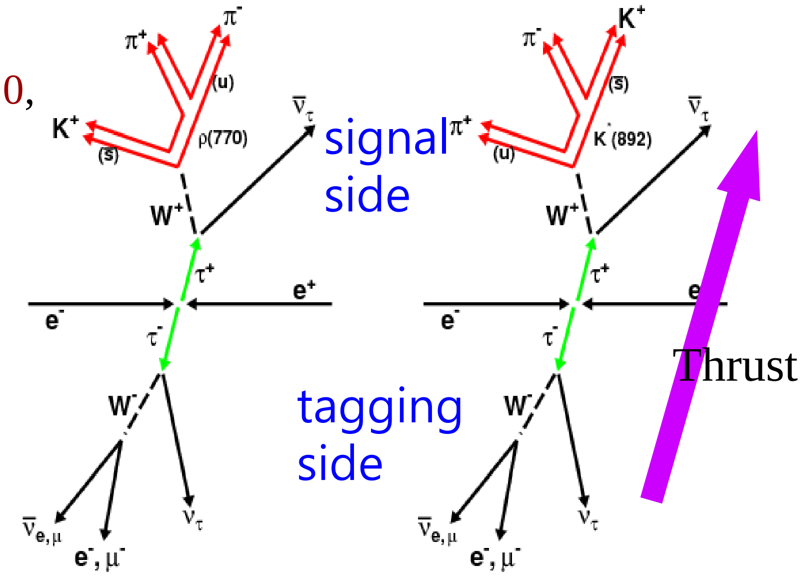
Tuning the efficiencies

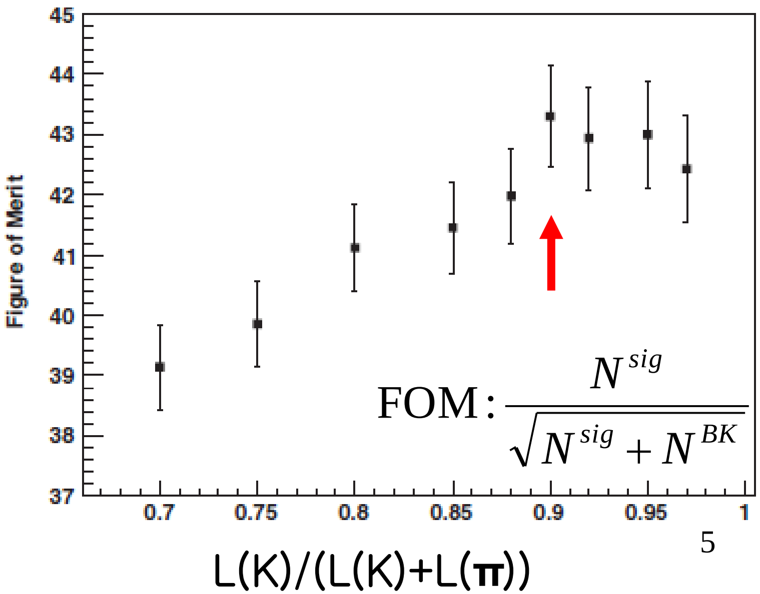
BR and Minv

Event selection for $\tau^- \rightarrow h^- h^+ h^- \nu$

1. Tau candidate selection
   - Number of charged track = 4 and sum of charge = 0, with proper Calorimeter energy, vertex, and transverse momentum requirement
2. Background suppression
   - Require missing momentum should be properly reconstructed, to suppress 2-γ and q-qbar background
   - Tight likelihood ratio cut for Kaon ID
   - Select a event with one e or μ, and three K or π
   - Event shape : thrust, 3-1 prong topology
   - Invariant mass cut : require tau mass range
3. Final event selection
   - No Ks, π^0, energetic gamma candidate

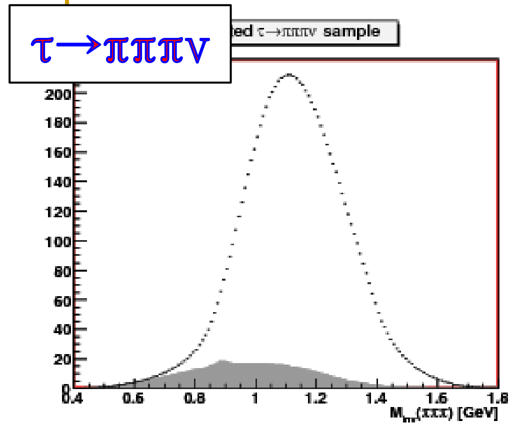
KID is the most important : FOM study result a tight cut for kaon (L(K)/(L(K)+L(π))>0.9)

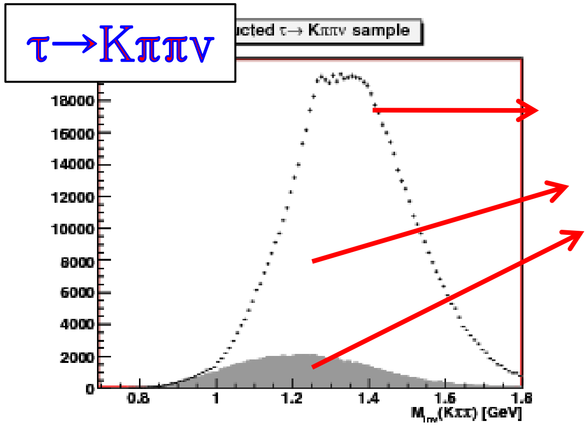


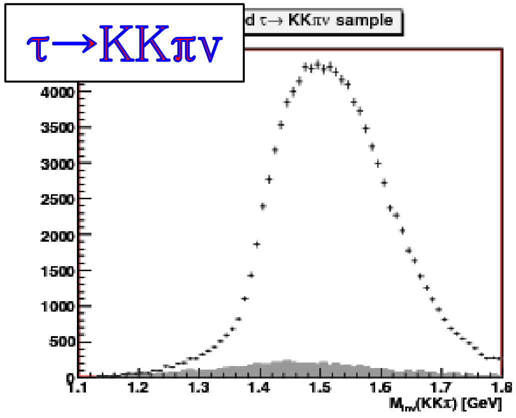


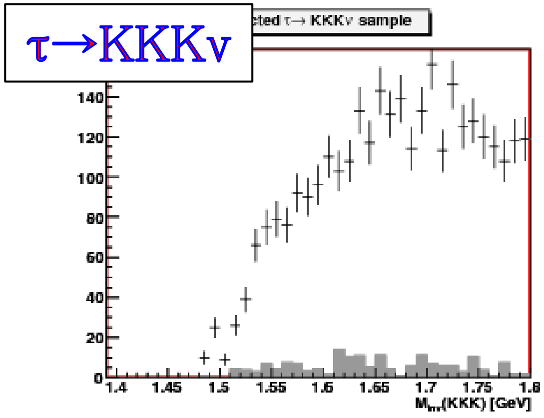
$$\text{FOM}: \frac{N^{sig}}{\sqrt{N^{sig} + N^{BK}}}$$

Result of event selection, other BG estimation









Data : N(rec)

Signal + Cross Feed BG : τ→hhhν

Other BG : N(others)

τ→K_Sπν, τ→ππππ^0ν , q-qbar,… etc

Mode	**Rec. Eff. (as τ→hhhν)**
τ→hhhν	24 ~ 28 %
τ→K_Sπν	1.8 %
τ→ππππ^0ν	6.0 %
Continuum	0.004 %
Two photon	0.0002%

Mode	**N(rec)**	**N(others)**	**Main component in N(others)**
τ→πππν	8.86×10^6	9.35×10^5(10.6%)	τ→ππππ^0ν (64.2%)
τ→Kππν	7.94×10^5	9.60×10^4(12.1%)	τ→ππππ^0ν (34.3%)
τ→KKπν	1.08×10^5	7.16×10^3(6.66%)	ee→qq (30.3%)
τ→KKKν	3.16×10^3	1.71×10^2(5.41%)	ee→qq (53.0%)

Branching ratio calculation

$$N_i^{true} = \varepsilon^{-1}{}_{ij}(N_j^{rec} - N_j^{Others})$$

N_i^{true} : Number of true signal events for *i*-th mode
N_i^{rec} : Number of reconstructed events for *i*-th mode
N_i^{Others} : Number of estimated background for *i*-th mode from non-three-prong decay

ε : efficiency migration matrix

The efficiency is tuned by using the unfolded mass spectra :

1. **Estimate the efficiency with MC**
2. **Scale the efficiency distribution over invariant mass using the unfolded spectra**
3. **By summing normalizing, re-estimate the efficiency matrix**

Efficiency migration matrix ε & the differences after tuning

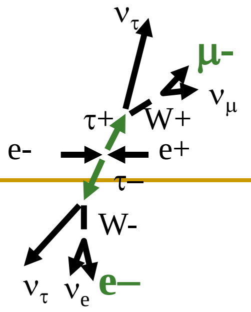
rec true	τ→πππν	τ→Kππν	τ→KKπν	τ→KKKν
τ→πππν	0.22 (-0.1%)	0.079 (-0.1%)	0.002 (-1.1%)	6.4×10^{-2} (+2.6%)
τ→Kππν	0.012 (-0.2%)	0.18 (-0.5%)	0.047 (+1.4%)	0.019 (-3.1%)
τ→KKπν	3.9×10^{-4} (+0.3%)	4.7×10^{-3} (-4.4%)	0.12 (-1.0%)	0.051 (-0.4%)
τ→KKKν	5.0×10^{-6} (+4.5%)	1.3×10^{-4} (-9.1%)	2.3×10^{-3} (-14.0%)	0.081 (+1.2%)

- Effect of mass dependence of efficiency and the discrepancies btw unfolded and MC distribution
- Efficiency : 10 ~ 20%, Fake rate from πππ to Kππ is sufficiently small

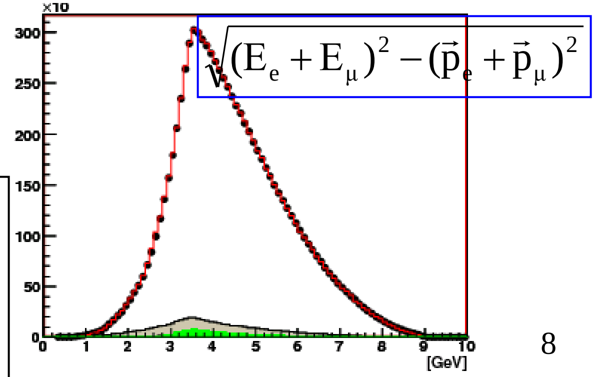
Systematic uncertainties and Normalization

Source	ππππ	Kππ	KKπ	KKK	Method
Tracking eff.	**+2.2/-2.0**	**+2.1/-2.0**	**+2.1/-2.0**	**+2.1/-1.9**	$D^*\to\pi D^0$, $D^0\to\pi\pi K_S$, $K_S\to\pi^+\pi^-$
Particle ID	**±1.9**	**+4.0/-4.1**	**±2.3**	**+5.4/-5.6**	$\gamma\gamma\to ee/\mu\mu$, $D^{*+}\to D^0\pi_s^+(D^0\to K^-\pi^+)$ Apply the eff. change and obtain BF
Mass spectrum	±0.1	±0.1	±0.1	+0.5/-0.8	Apply 1σ variations of unfolded mass spectrum to obtain new eff. Matrix
Trigger	±0.5	±0.5	±0.6	±0.6	Trigger simulation
γ veto	±0.9	±1.5	±1.7	±0.5	Different cut for γ energy
K_S veto	±0.2	±0.2	±0.1	±0.3	Remove K_S veto condition
BG estimation	±0.3	±1.3	±0.3	±0.4	Varying BFs of other τ decay BG modes
Leptonic τ decay	±0.2	±0.2	±0.2	±0.2	Err. of BF($\tau\to e/\mu\nu\nu$)
Tot. [%]	+3.1/-3.0	±5.0	±3.6	+5.9/-6.0	

$$Br_{\tau\to K\pi\pi\nu} = \frac{N_{sig,K\pi\pi}\cdot\varepsilon_{e\mu}}{N_{sig,e\mu}\cdot\varepsilon_{K\pi\pi}}\cdot\frac{Br_{\tau\to e\nu\nu}\cdot Br_{\tau\to\mu\nu\nu}}{Br_{\tau\to e\nu\nu}+Br_{\tau\to\mu\nu\nu}}$$



Normalization by {τ→eνν;τ→μνν}



$$\sqrt{(E_e+E_\mu)^2-(\vec{p}_e+\vec{p}_\mu)^2}$$

Data, MC sum ((e;μ) signal MC + other tau decay + 2 photon)
Other tau decay, 2 photon

Result on branching ratio

	Branching ratio
$\tau^- \rightarrow \pi^-\pi^+\pi^-\nu_\tau$	$\mathbf{(8.42 \pm 0.00(st.)^{+0.26}_{-0.25}(sy.)) \times 10^{-2}}$
$\tau^- \rightarrow K^-\pi^+\pi^-\nu_\tau$	$\mathbf{(3.30 \pm 0.01(st.)^{+0.16}_{-0.17}(sy.)) \times 10^{-3}}$
$\tau^- \rightarrow K^-K^+\pi^-\nu_\tau$	$\mathbf{(1.55 \pm 0.01(st.)^{+0.06}_{-0.05}(sy.)) \times 10^{-3}}$
$\tau^- \rightarrow K^-K^+K^-\nu_\tau$	$\mathbf{(3.29 \pm 0.17(st.)^{+0.19}_{-0.20}(sy.)) \times 10^{-5}}$

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Correlation coefficient

	$\tau^- \rightarrow K^-\pi^+\pi^-\nu_\tau$	$\tau^- \rightarrow K^-K^+\pi^-\nu_\tau$	$\tau^- \rightarrow K^-K^+K^-\nu_\tau$
$\tau^- \rightarrow \pi^-\pi^+\pi^-\nu_\tau$	**+0.175**	**+0.049**	**-0.053**
$\tau^- \rightarrow K^-\pi^+\pi^-\nu_\tau$		**+0.080**	**+0.035**
$\tau^- \rightarrow K^-K^+\pi^-\nu_\tau$			**-0.008**

Strong correlation between
$\tau^- \rightarrow \pi^-\pi^+\pi^-\nu_\tau$ and $\tau^- \rightarrow K^-\pi^+\pi^-\nu_\tau$

Unfolding to measure the mass spectrum

- In general, unfolding is a "inverse problem" :

$$\hat{A}x = b$$

  - x : real (unfolded) distribution (so, cross-feed background subtracted)
  - b : distribution before unfolding (including (only) cross-feed background. Other backgrounds are estimated/subtracted using MC)
  - A : detector response matrix (expansion of efficiency matrix(4x4))

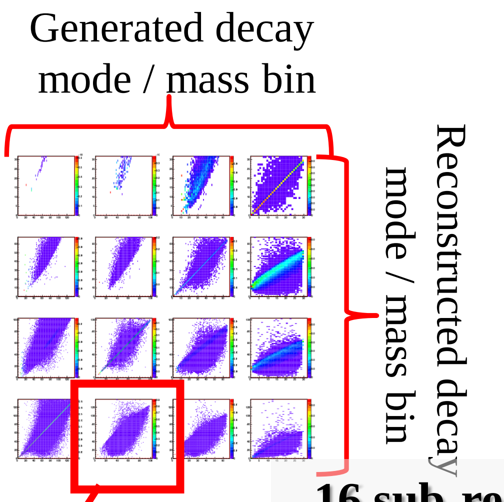
$$\hat{A} = \frac{\text{Number of event reconstructed at i-th bin and generated at j-th bin}}{\text{Number of event generated at j-th bin}}$$

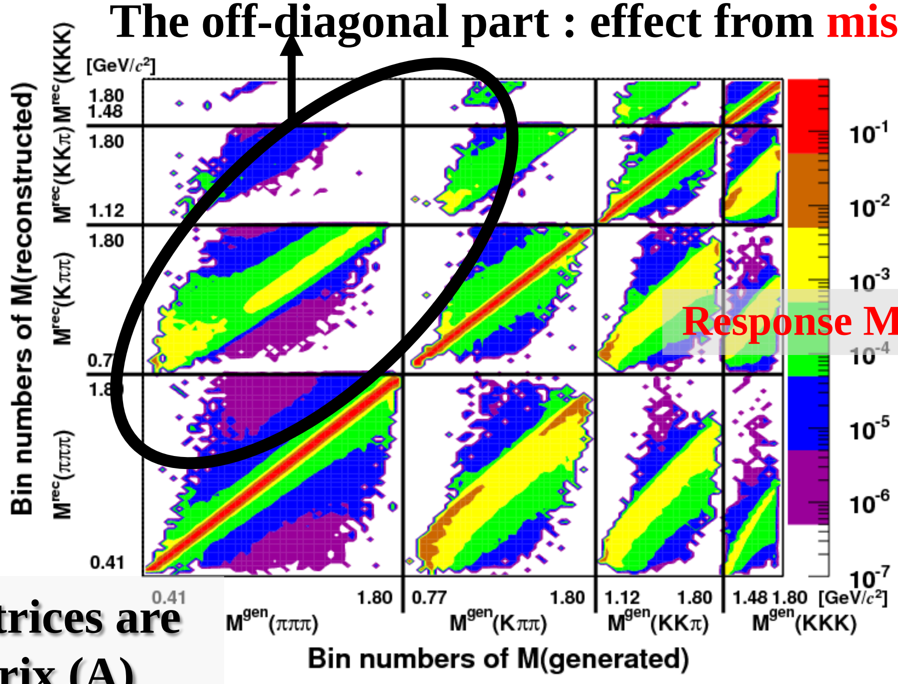
$$\text{(Bin : decay mode and binning on invariant mass)}$$

- 4 modes are combined and the response matrix includes the mode-migration effects

  → **Cross-feeding BG from mis-PID can be subtracted** after unfolding
- Algorithm : Developed in ALEPH
  - Singular Value Decomposition, Tikhonov-type regularization

Response matrix (A)/mass spectrum before unfolding (b)



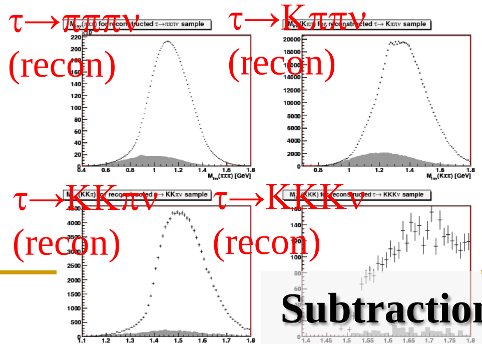


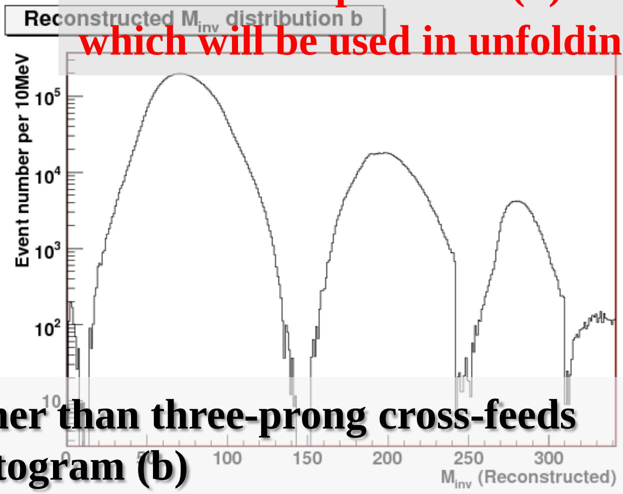
The off-diagonal part : effect from mis-PID

Response Matrix (A)

16 sub-respose matrices are merged to a matrix (A)

Ex) Response matrix (probability) that real $\tau \to K\pi\pi\nu$ events are reconstructed as $\tau \to \pi\pi\pi\nu$ mode





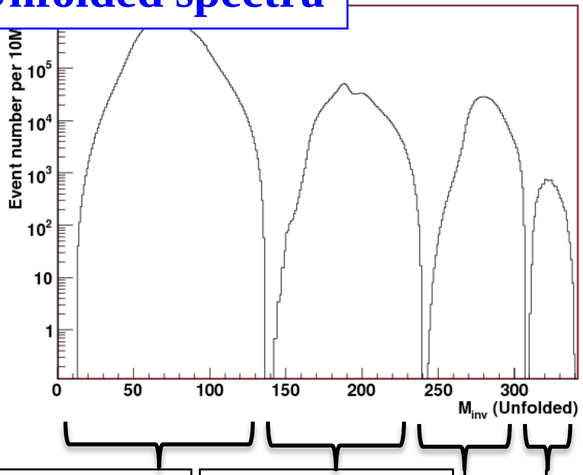
Recon. Mass spectrum (b) which will be used in unfolding

Subtraction of BG other than three-prong cross-feeds

Merged to a single histogram (b)

Result of Unfolding (x) and check

Unfolded spectra



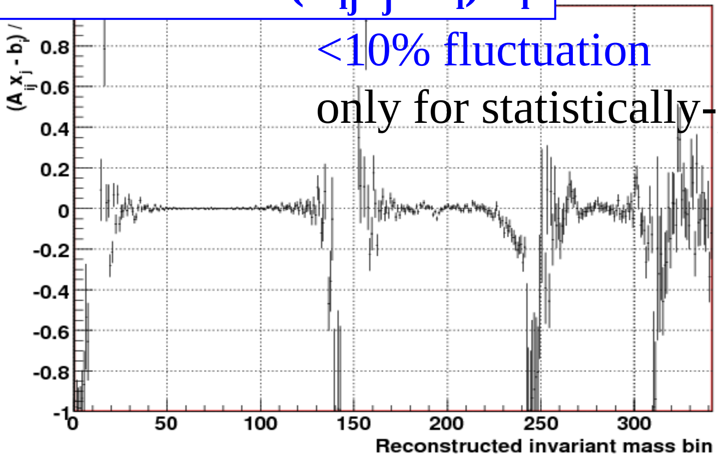
0.41~1.8
M(πππ) [GeV]

0.77~1.8
M(Kππ) [GeV]

1.12 ~ 1.8
M(KKπ) [GeV]

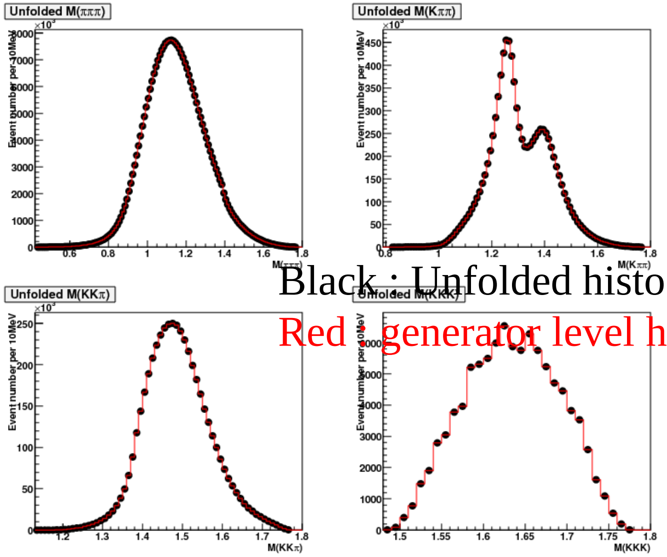
1.48~1.8
M(KKK) [GeV]

Residual Norm : $(A_{ij}x_j - b_i)/b_i$



<10% fluctuation

only for statistically-poor bins



Black : Unfolded histogram

Red : generator level histogram

Check on unfolding algorithm :

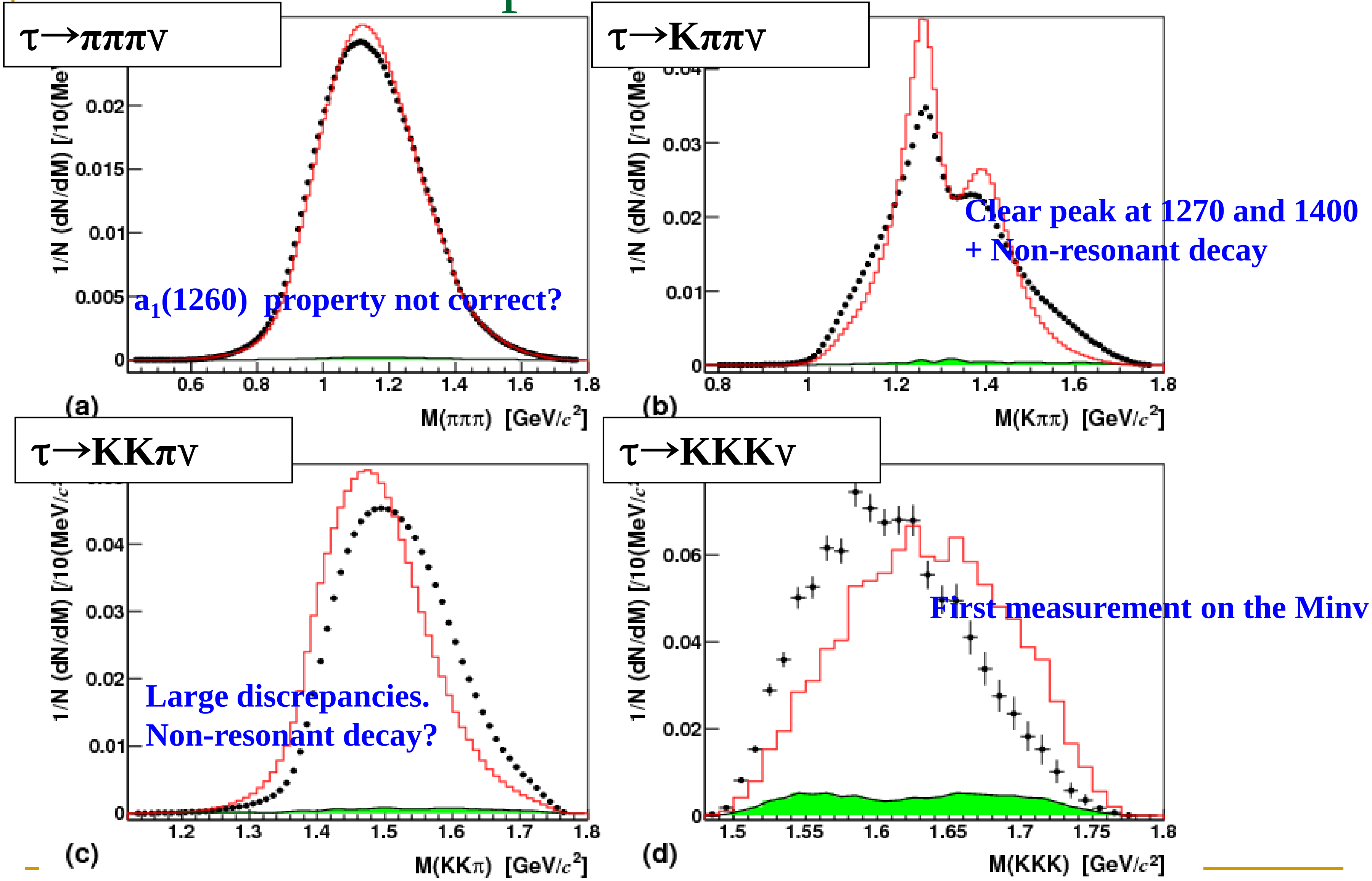
Use independent signal MC sample,

unfold and compare with generator level mass spectra

Less than 0.5% diff. for all mass range → UNF1 err.

- Many other tests on the unfolding algorithm with different samples
  : Max. ~1% discrepancies between the generated and unfolded distributions.

Unfolded mass spectrum



$\tau\to\pi\pi\pi\nu$

$a_1(1260)$ property not correct?

$\tau\to K\pi\pi\nu$

Clear peak at 1270 and 1400
+ Non-resonant decay

$\tau\to KK\pi\nu$

Large discrepancies.
Non-resonant decay?

$\tau\to KKK\nu$

First measurement on the Minv

Red:TAUOLA MC

Black:unfolded mass spectrum

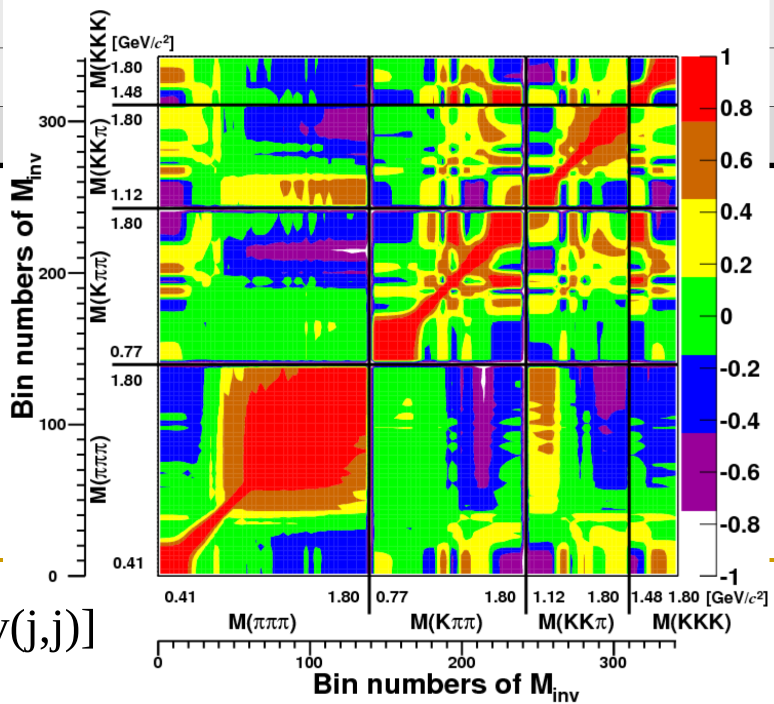
Error bar : Statistical error

Green band : Systematic error

Systematic errors of unfolded mass spectra

Source	**πππ**	**Kππ**	**KKπ**	**KKK**	**Method**
Unfolding (1)	0.5	0.1	0.4	0.6	Unfolding an independent signal MC sample
Unfolding (2)	0.1	1.5	0.9	4.0	Varying the effective rank of unfolding
Kaon ID	0.4	0.8	1.1	0.9	KID eff. variation from D* sample, Varying Response matrix.
BG estimation	0.2	0.5	0.3	1.1	Varying BF of other BG component
γ veto	0.4	0.4	0.9	3.2	Different cut for γ energy
Momentum scale	0.0	0.2	0.3	3.1	$\varphi \to K^+K^-$,$D^+ \to K^-\pi^+\pi$+ recon. for mom. scale Assign 0.01% variation of recon. mass spectrum
Tot. sys.	0.7	1.9	1.7	6.2	
Stat.	0.2	0.8	0.8	4.6	
Tot. [%]	0.8	2.0	1.9	7.7	

(*) Error at the peak of mass spectrum

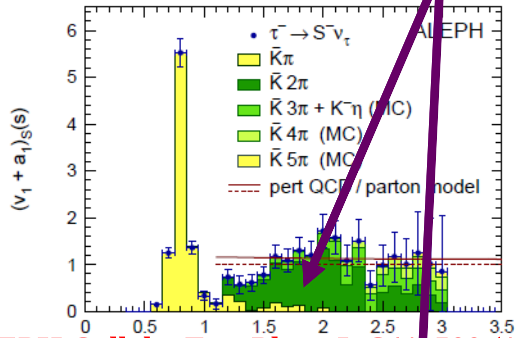


Correlation
= cov(i,j)/√[cov(i,i) cov(j,j)]

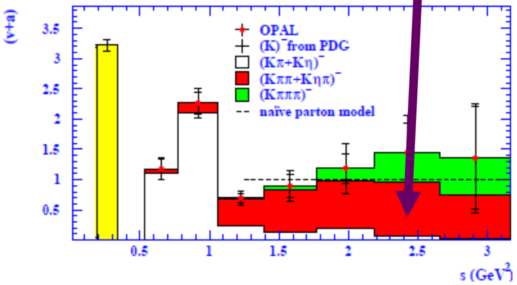
Result : Contributions to spectral function

$$v_1^S(s)/a_1^S(s) = \frac{m_\tau^2}{6|V_{us}|^2 S_{ew}} \left(1-\frac{s}{m_\tau^2}\right)^{-2} \left(1+\frac{2s}{m_\tau^2}\right)^{-1} \frac{B(\tau \to (V/A)^{(S=-1,J=1)}\nu_\tau)}{B(\tau \to e^-\bar{\nu}_e\nu_\tau)} \frac{1}{N_{V/A}} \frac{dN_{V/A}}{ds}$$

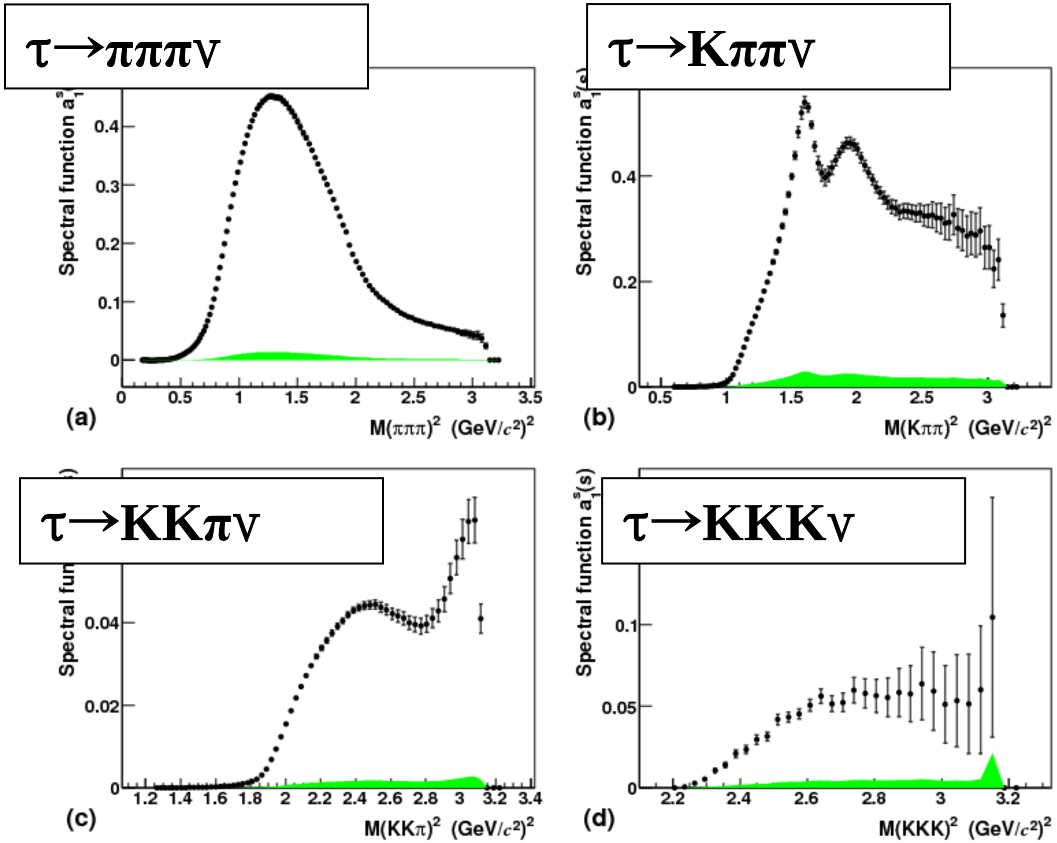
Large contribution from τ→Kππν



ALEPH Collab., Eur. Phys. J. C11, 599 (1999).



OPAL Collab., Eur. Phys. J. C35, 437 (2004).



τ→πππν

τ→Kππν

τ→KKπν

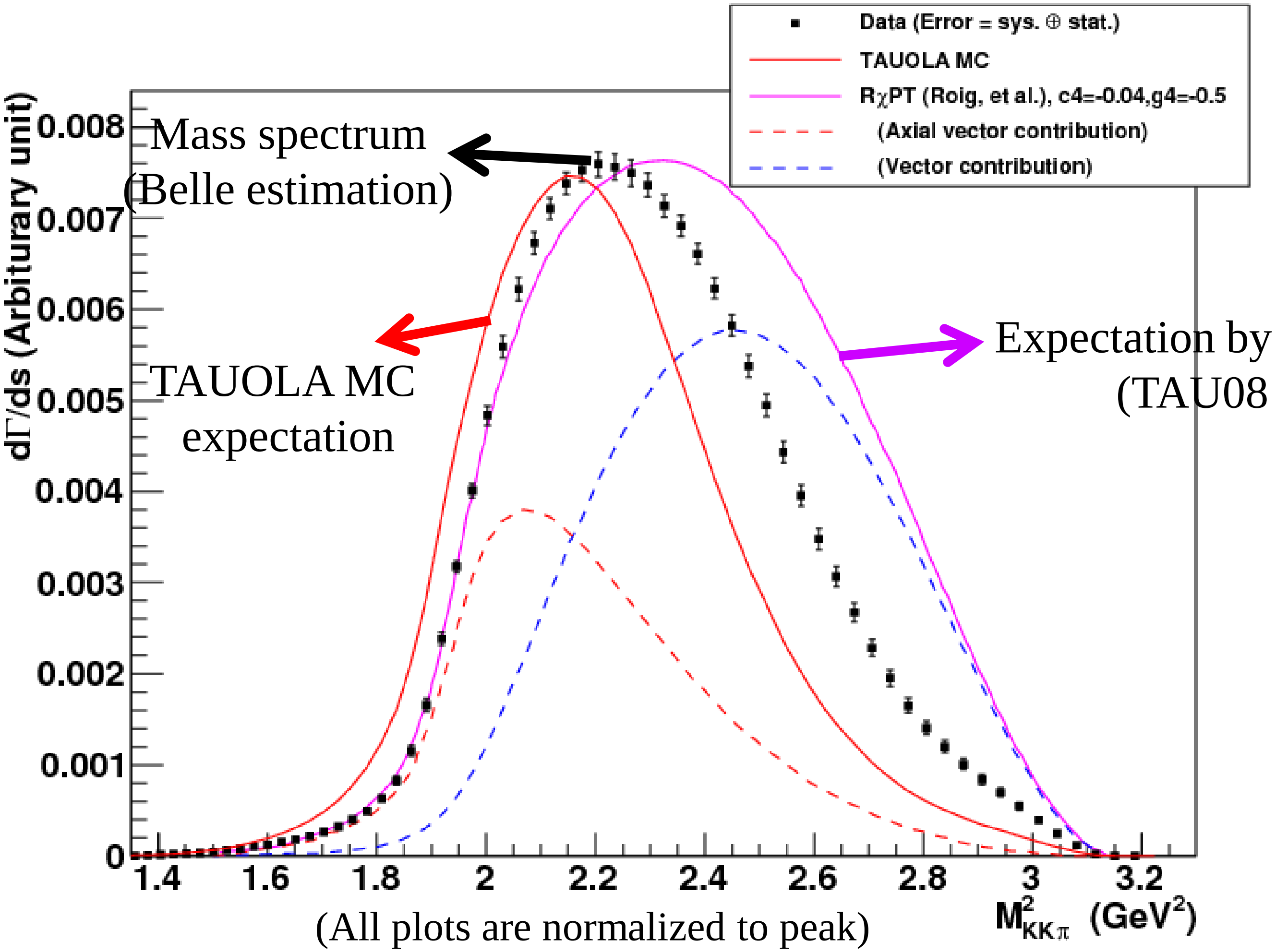
τ→KKKν

Black : spectral function, Err. from mass spectrum

Green : Error from other sys. source

The spectral function can be improved significantly.

Result : Mass spectrum of $\tau \to KK\pi\nu$ decay



Mass spectrum
(Belle estimation)

TAUOLA MC
expectation

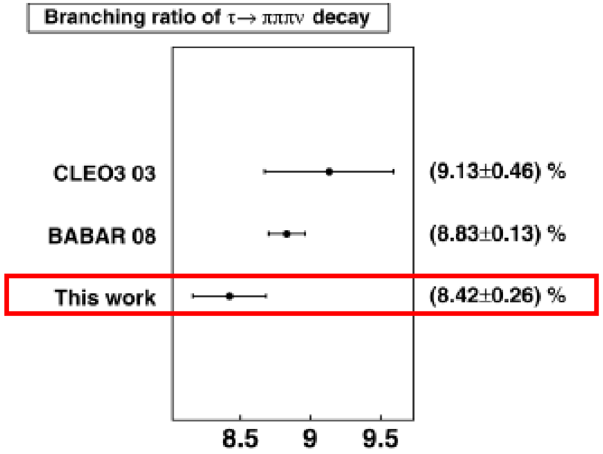
Expectation by Roig et.al,
(TAU08 talk)

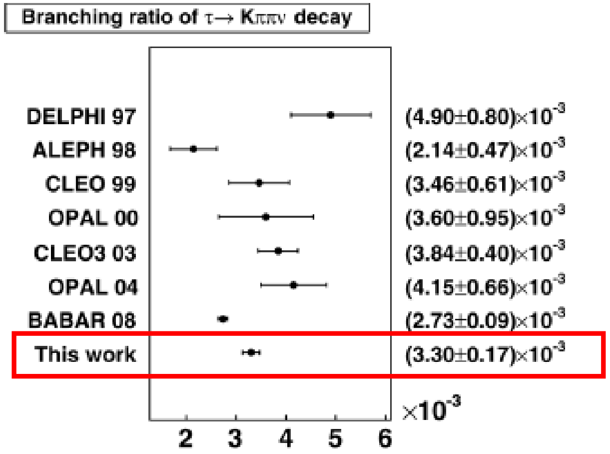
(All plots are normalized to peak)

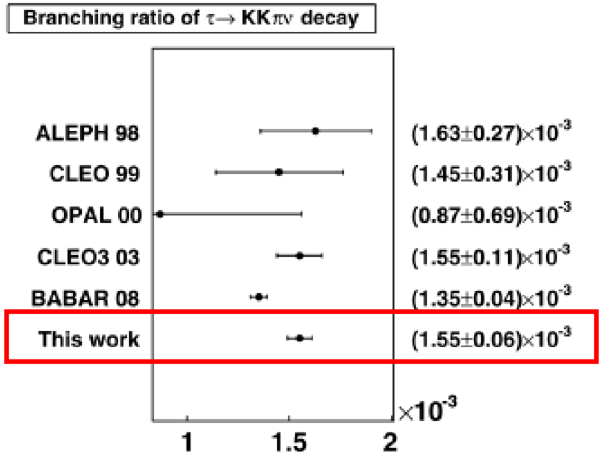
Theoretical work on the decay dynamics is still necessary.

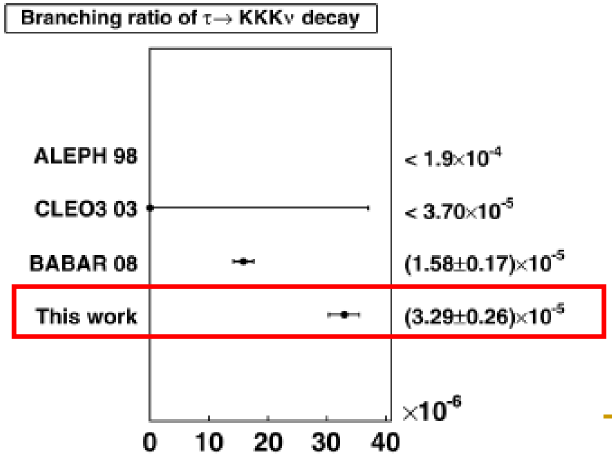
Summary and Conclusion

- We measured the branching ratio and mass spectra of τ→πππν, τ→Kππν, τ→KKπν, τ→KKKν decays, using 666.2fb^{-1} data collected at **Belle** experiment.

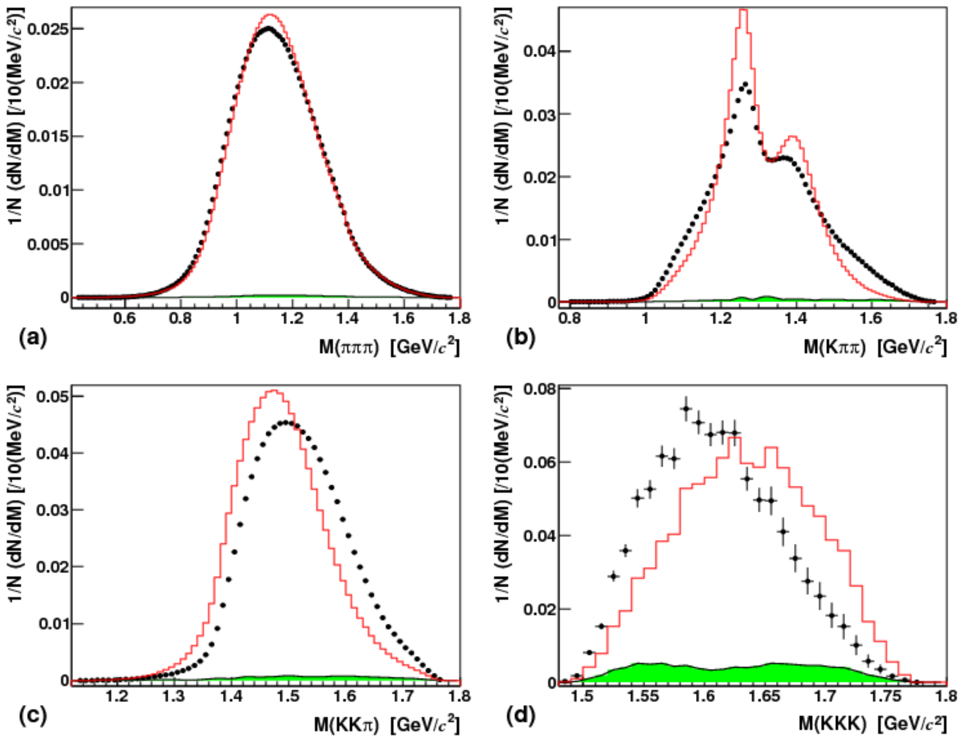

Branching ratio of τ→ πππν decay


Branching ratio of τ→ Kππν decay


Branching ratio of τ→ KKπν decay


Branching ratio of τ→ KKKν decay

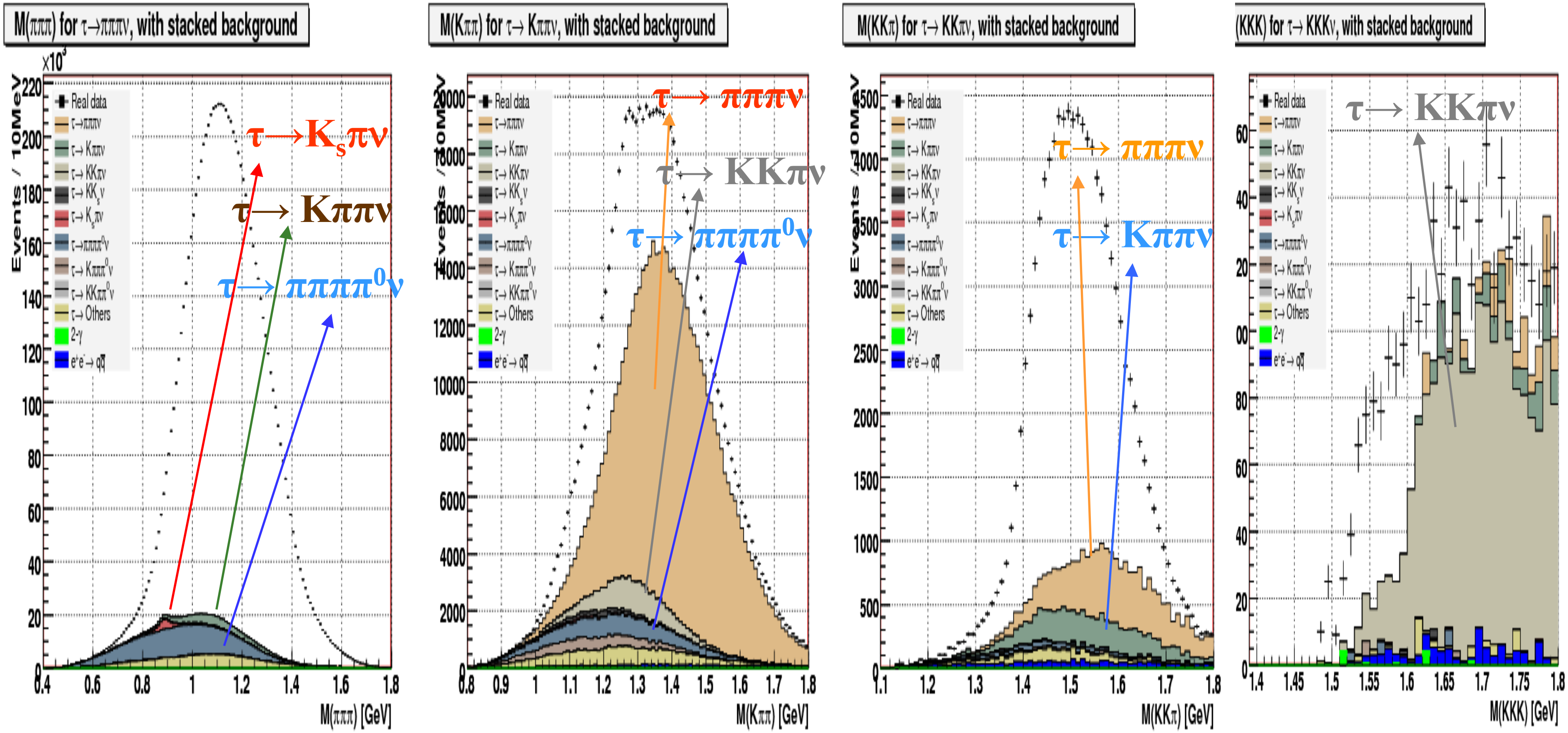
- We also measured the mass spectra of three-prong decays of τ.



- A special unfolding techniques to remove any dependence to the MC model and to subtract the dominant cross-feed backgrounds.
- Comparable accuracies with large statistics BF measurements
- Large differences with MC (current theoretical) predictions on the mass spectra
- First measurements on the mass spectra with large statistics.

Backup slides

Comparison of M_{INV} distribution of $\tau^- \rightarrow h^- h^+ h^- \nu_\tau$ for data and MC background



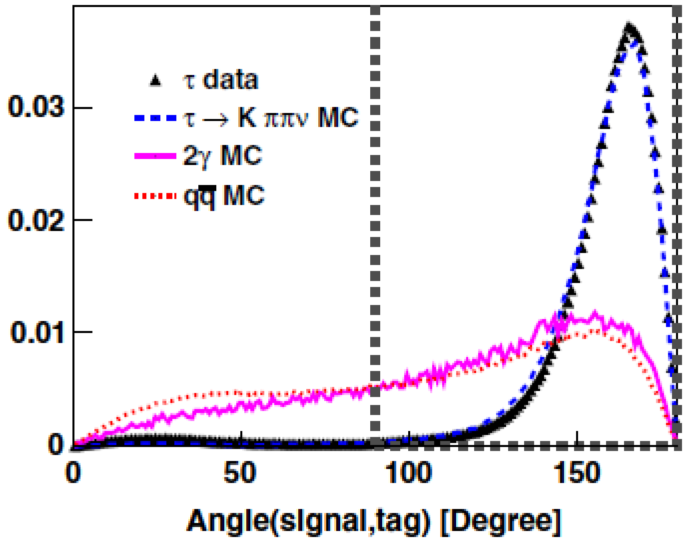
Black point : real data

Colored (stacked) histogram

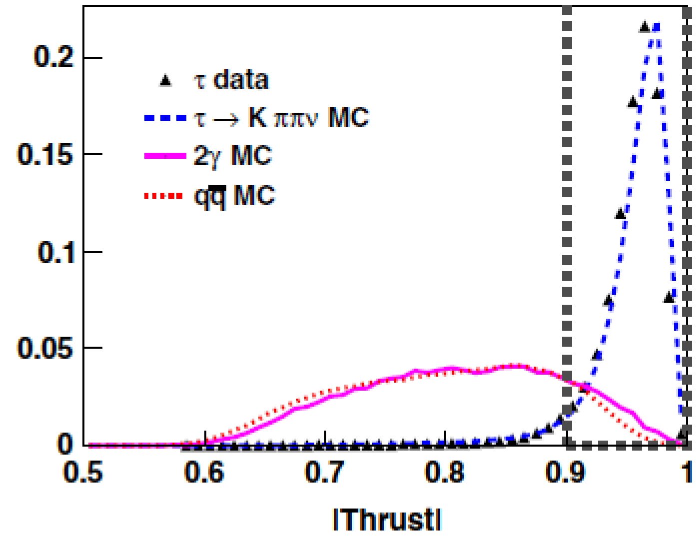
: background estimated from MC

Data : 666.2 fb^{-1} (613M tau pair sample)

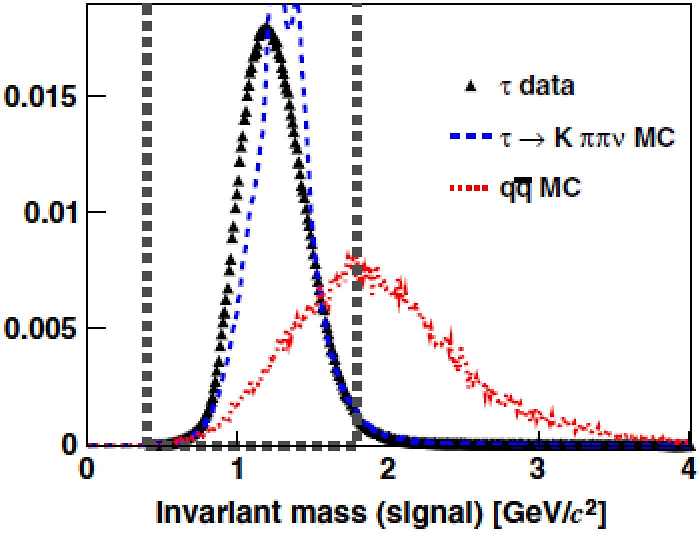
on/off resonance


τ data
τ → K ππν MC
2γ MC
$q\bar{q}$ MC
Arbitrary units / degree
0.03
0.02
0.01
0
0
50
100
150
Angle(signal,tag) [Degree]

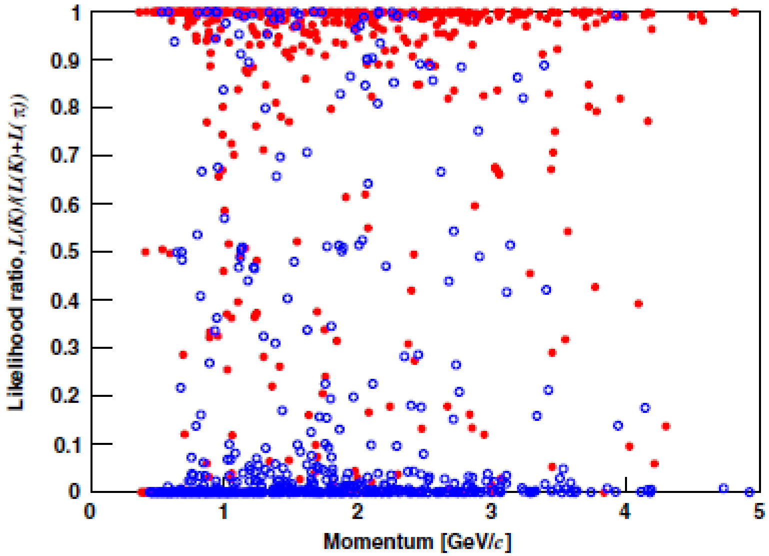
(a)

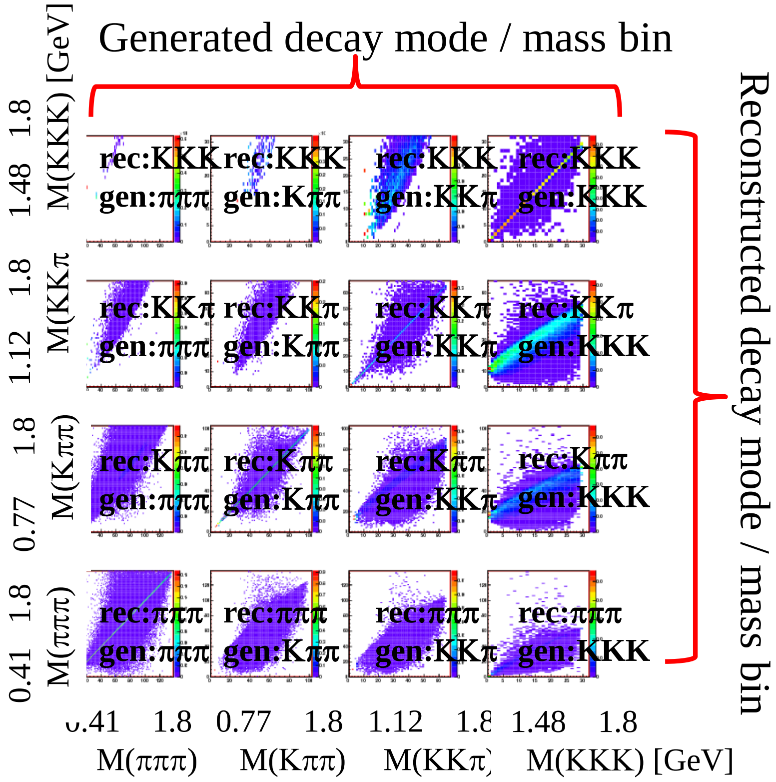

τ data
τ → K ππν MC
2γ MC
$q\bar{q}$ MC
Arbitrary units
0.2
0.15
0.1
0.05
0
0.5
0.6
0.7
0.8
0.9
1
|Thrust|

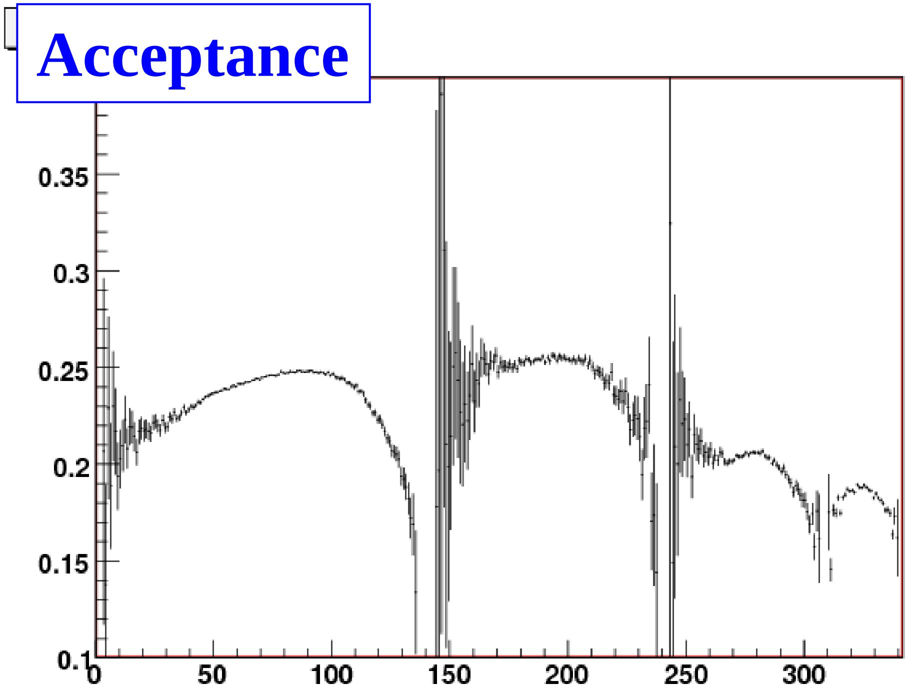
(b)


τ data
τ → K ππν MC
$q\bar{q}$ MC
Arbitrary units / (10MeV/c^2)
0.015
0.01
0.005
0
0
1
2
3
4
Invariant mass (signal) [GeV/c^2]

(c)


Likelihood ratio, $L(K)/(L(K)+L(\pi))$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5
Momentum [GeV/c]


Generated decay mode / mass bin
Reconstructed decay mode / mass bin
rec:KKK gen:πππ
rec:KKK gen:Kππ
rec:KKK gen:KKπ
rec:KKK gen:KKK
rec:KKπ gen:πππ
rec:KKπ gen:Kππ
rec:KKπ gen:KKπ
rec:KKπ gen:KKK
rec:Kππ gen:πππ
rec:Kππ gen:Kππ
rec:Kππ gen:KKπ
rec:Kππ gen:KKK
rec:πππ gen:πππ
rec:πππ gen:Kππ
rec:πππ gen:KKπ
rec:πππ gen:KKK
1.48 1.8 M(KKK) [GeV]
1.12 1.8 M(KKπ)
0.77 1.8 M(Kππ)
0.41 1.8 M(πππ)
0.41 1.8 M(πππ)
0.77 1.8 M(Kππ)
1.12 1.8 M(KKπ)
1.48 1.8 M(KKK) [GeV]


Acceptance
0.35
0.3
0.25
0.2
0.15
0.1
0
50
100
150
200
250
300

Test of unfolding

- Testing of unfolding is important since we used some new techniques in constructing the b and A.

- Tikhonov regularization check with ρ(770) MC sample
- Tikhonov regularization check with $\tau \rightarrow hhh\nu$ MC sample
- Tikhonov regularization check with L-curve analysis
- Truncated SVD (TSVD) method check
- Conjugated Gradient Least Square (CGLS) method
- Unfolding of ToyMC samples to measure the correction factor

- All test results maximum ~1% discrepancies between the generated distribution and unfolded distribution.

Regularization

- Direct inversion of matrix A (although Singular value decomposition is used) is know to generate fast-fluctuating solution. So we need special regularization techniques.
- Most well known regularization method is "Tikhonov regularization"

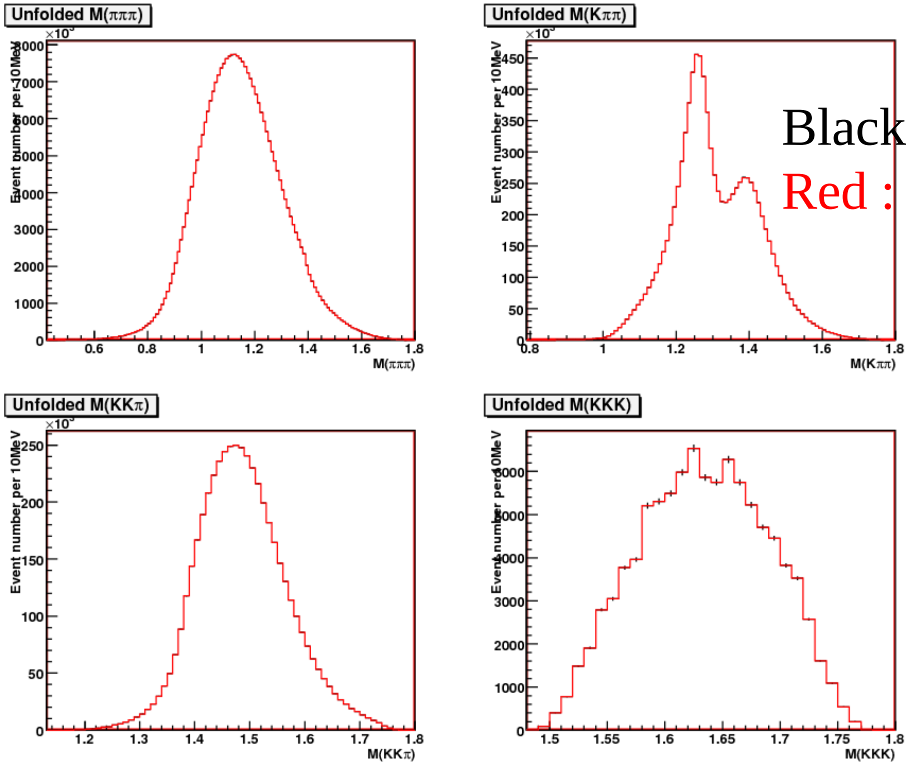
$$min\{||Ax-b||_2^2+\lambda^2||Lx||_2^2\}$$

(A.Hoecker and V.Karvelishvili, NIMA 372 (1996))

- Very popular method in HEP
- λ : regularization parameter
- L : a-priori information on the curvature of the solution
- Also, the initial distribution (x^{init}) is required to rescale the histogram. Signal MC samples are used.
- This method also involves the inversion of any matrix, and Singular value decomposition (SVD) is used.

Tikhonov regularization test with τ→hhhν MC sample

- Deferent (independent) MC sample which is enriched by the signal modes(τ→hhhν) are generated as a test data.
- This test data were unfolded using the same algorithm.
  - The same process with unfolding the real data, but input was MC
  - Used statistically independent MC set to construct response matrix



Black : Unfolded histogram

Red : generator level histogram

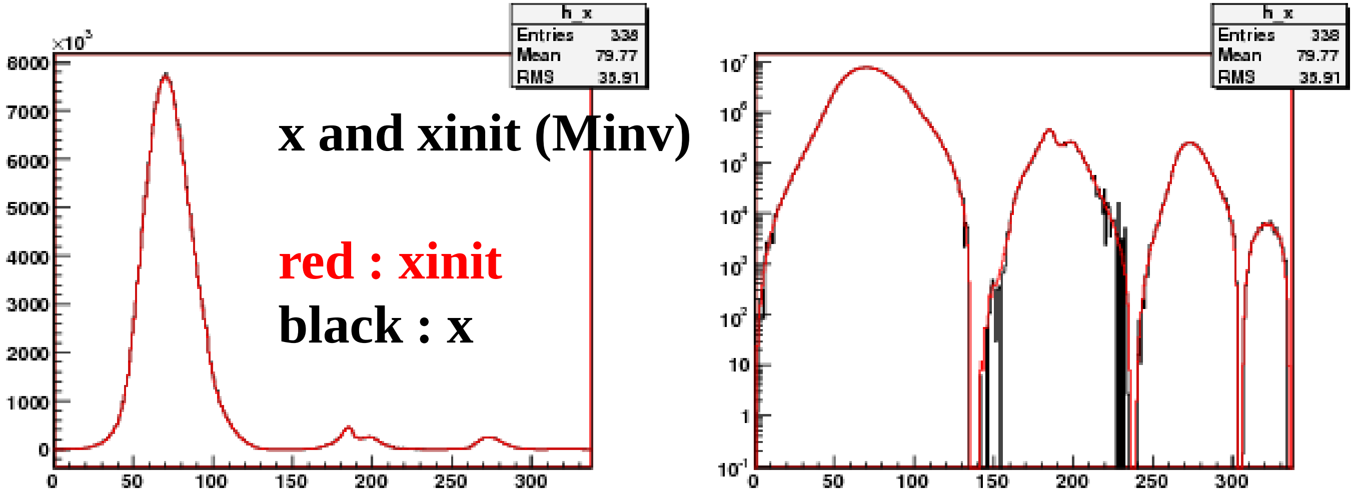
Truncated SVD method

- When solving reverse problem Ax=b, response matrix A can be inverted using SVD (Singular value decomposition)

$$A = U\Sigma V^T \qquad x_i = A^{-1}_{ij} b_j = \sum_j \sum_k^{\boxed{rank(A)}} \frac{V_{ik}U^T_{kj}}{\sigma_k} b_j$$

- TSVD : rank(A) can be truncated to "effective rank (A) = λ", to suppress fluctuating solution

$$x_i = A^{-1}_{ij} b_j = \sum_j \sum_k^{\boxed{\lambda}} \frac{V_{ik}U^T_{kj}}{\sigma_k} b_j$$



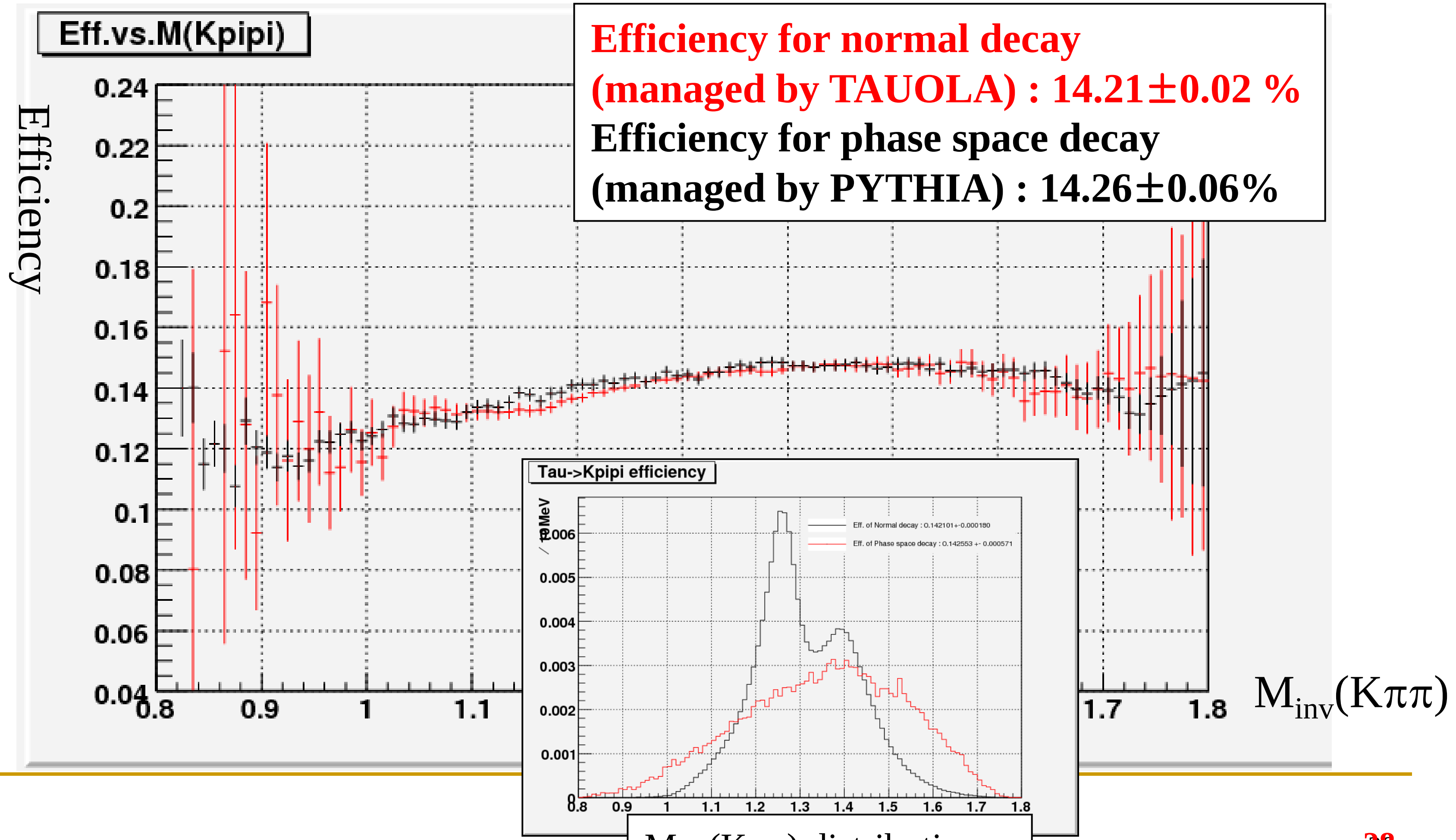
x and xinit (Minv)

red : xinit

black : x

Efficiency dependence on $M(K\pi\pi)$

- To check any dependence of efficiency to decay model, phase space decay of signal mode is simulated.



Efficiency for normal decay (managed by TAUOLA) : 14.21±0.02 %

Efficiency for phase space decay (managed by PYTHIA) : 14.26±0.06%

$M_{inv}(K\pi\pi)$ distribution