Recent progress in hadronic τ decays

- Perturbative contribution to R_τ.
- Description of $\tau \to \nu_\tau K\pi$.

Hadronic τ decay rate

Consider the physical quantity R_τ: (Braaten, Narison, Pich 1992)

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \text{hadrons}\,\nu_\tau(\gamma))}{\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau(\gamma))} = 3.640 \pm 0.010\,.$$

R_τ is related to the QCD correlators $\Pi^{T,L}(x)$: $(x \equiv s/M_\tau^2)$

$$R_\tau = 12\pi \int_0^1 dx\,(1-x)^2 \Big[(1+2x)\,\mathrm{Im}\Pi^T(x) + \mathrm{Im}\Pi^L(x)\Big]\,,$$

with the appropriate combinations

$$\Pi^J(x) = |V_{ud}|^2 \Big[\Pi_{ud}^{V,J} + \Pi_{ud}^{A,J}\Big] + |V_{us}|^2 \Big[\Pi_{us}^{V,J} + \Pi_{us}^{A,J}\Big]\,.$$

Hadronic τ decay moments

Additional information can be inferred from the moments

$$R_\tau^{kl} \equiv \int_0^1 dx\,(1-x)^k x^l\,\frac{dR_\tau}{dx} = R_{\tau,V}^{kl} + R_{\tau,A}^{kl} + R_{\tau,S}^{kl}\,.$$

Theoretically, R_τ^{kl} can be expressed as:

$$R_\tau^{kl} = N_c\,S_{\rm EW}\Big\{(|V_{ud}|^2+|V_{us}|^2)\Big[1+\delta^{kl(0)}\Big] + \sum_{D\geq 2}\Big[|V_{ud}|^2\delta_{ud}^{kl(D)}+|V_{us}|^2\delta_{us}^{kl(D)}\Big]\Big\}\,.$$

$\delta_{ud}^{kl(D)}$ and $\delta_{us}^{kl(D)}$ are corrections in the Operator Product Expansion, the most important ones being $\sim m_s^2$ and $m_s\langle\bar{q}q\rangle$.

Adler function

The perturbative part $\delta^{(0)}$ is related to the Adler function $D(s)$:

$$D(s) \equiv -s\,\frac{\mathrm{d}}{\mathrm{d}s}\,\Pi_V(s) = \frac{N_c}{12\pi^2}\sum_{n=0}^{\infty} a_\mu^n \sum_{k=1}^{n+1} k\, c_{n,k}\,\ln^{k-1}\Big(\frac{-s}{\mu^2}\Big)$$

where $a_\mu \equiv \alpha_s(\mu)/\pi$.

Resumming the Log's with the scale choice $\mu^2 = -s \equiv Q^2$:

$$D(Q^2) = \frac{N_c}{12\pi^2}\sum_{n=0}^{\infty} c_{n,1}\, a^n(Q^2)$$

As a consequence, only the coefficients $c_{n,1}$ are independent:

$$c_{0,1} = c_{11} = 1\,, \quad c_{2,1} = 1.640\,, \quad c_{3,1} = 6.371\,,$$

$$c_{4,1} = 49.076\,!!$$

(Baikov, Chetyrkin, Kühn 2008)

RG-improvement

Fixed order perturbation theory amounts to choose $\mu^2 = M_\tau^2$:

$$\delta_{\rm FO}^{(0)} = \sum_{n=1}^{\infty} a^n(M_\tau^2) \sum_{k=1}^{n+1} k\, c_{n,k} J_{k-1} = \sum_{n=1}^{\infty} [c_{n,1}+g_n] a^n(M_\tau^2)$$

A given perturbative order n depends on all coefficients $c_{m,1}$ with $m \leq n$, and on the coefficients of the QCD β-function.

Contour improved perturbation theory employs $\mu^2 = -M_\tau^2 x$:

(Pivovarov; Le Diberder, Pich 1992)

$$\delta_{\rm CI}^{(0)} = \sum_{n=1}^{\infty} c_{n,1} J_n^a(M_\tau^2) \qquad \text{with}$$

$$J_n^a(M_\tau^2) = \frac{1}{2\pi i} \oint\limits_{|x|=1} \frac{dx}{x} (1-x)^3 (1+x) a^n(-M_\tau^2 x)$$

Employing $\alpha_s(M_\tau) = 0.34$, the numerical analysis results in:

$$
\begin{array}{lccccc}
 & a^1 & a^2 & a^3 & a^4 & a^5 \\
\delta^{(0)}_{\rm FO} = & 0.108 & +0.061 & +0.033 & +0.017 & (+0.009) = 0.220\,(0.229) \\
\delta^{(0)}_{\rm CI} = & 0.148 & +0.030 & +0.012 & +0.009 & (+0.004) = 0.198\,(0.202)
\end{array}
$$

Contour improved PT appears to be better convergent.

The difference between both approaches amounts to 0.022 !

From the uniform convergence of $\delta^{(0)}_{\rm FO}$, and the assumption that the series is not yet asymptotic, one may also infer

$$c_{5,1} \approx 283\,,$$

leading to a difference of $\delta^{(0)}_{\rm FO} - \delta^{(0)}_{\rm CI} = 0.027$.

Borel transform

To further investigate the difference between CI and FOPT, let us consider the Borel-transformed Adler function.

$$4\pi^2 D(s) \equiv 1+\widehat{D}(s) \equiv 1+\sum_{n=0}^{\infty} r_n \alpha_s(s)^{n+1},$$

where $r_n = c_{n+1,1}/\pi^{n+1}$. The Borel-transform reads:

$$\widehat{D}(\alpha_s) = \int_0^\infty \mathrm{d}t\, \mathrm{e}^{-t/\alpha_s} B[\widehat{D}](t)\,; \quad B[\widehat{D}](t) = \sum_{n=0}^{\infty} r_n \frac{t^n}{n!}\,.$$

Generally, the Borel-transform $B[\widehat{D}]$ developes poles and cuts at integer values p of $u \equiv \beta_1 t/(2\pi)$. (Except at $u=1$.)

The poles at negative p are called UV renormalon poles and the ones at positive p IR renormalons.

Borel model

To proceed, realistic model $B[\widehat{D}](u)$: (Beneke, MJ 2008)

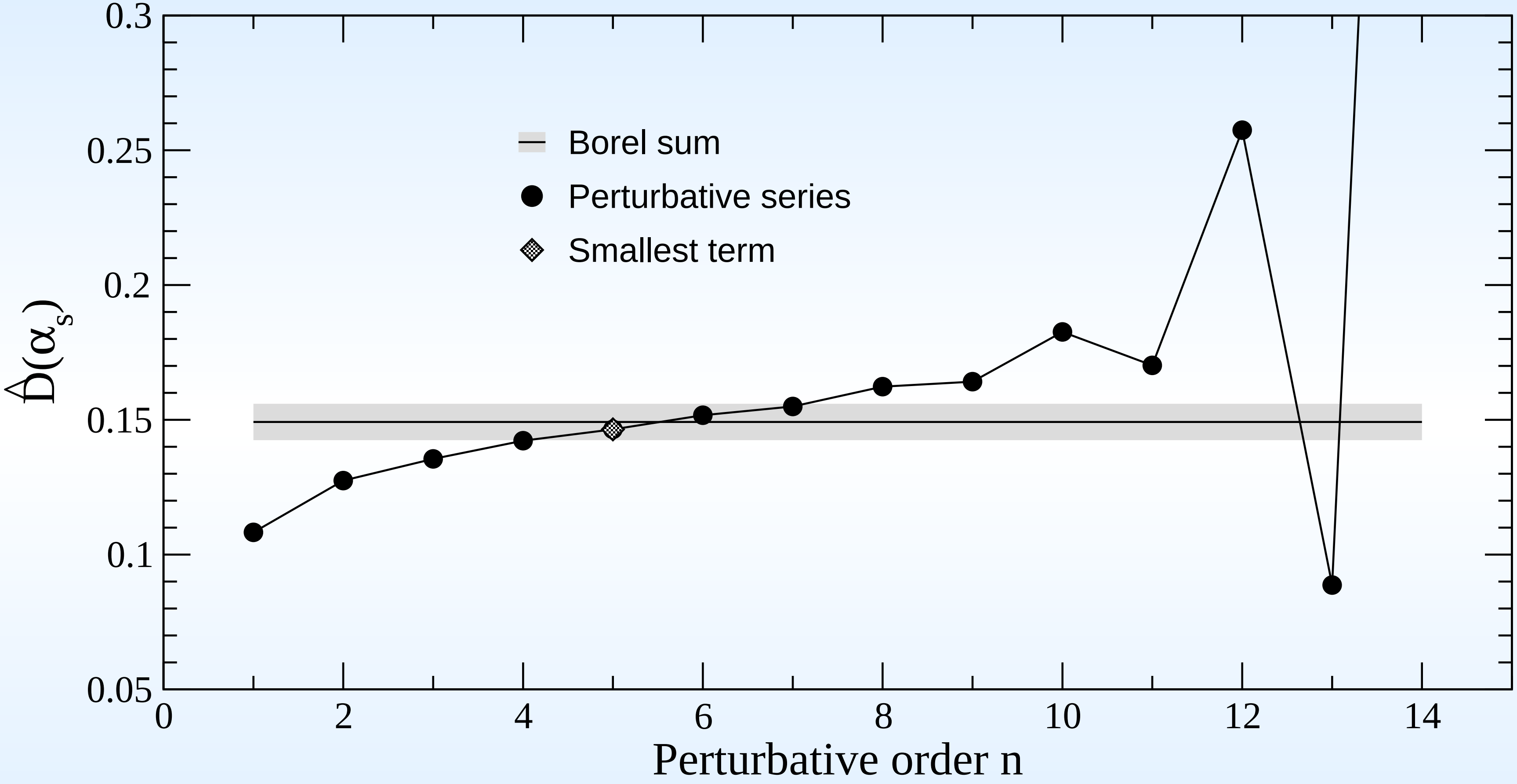
$$B[\widehat{D}](u) \;=\; B[\widehat{D}_1^{\rm UV}](u) + B[\widehat{D}_2^{\rm IR}](u) + B[\widehat{D}_3^{\rm IR}](u) + d_0^{\rm PO} + d_1^{\rm PO}\, u\,,$$

where

$$B[\widehat{D}_p](u) = \frac{d_p}{(p\pm u)^{1+\gamma}}\,\Big[\,1 + b_1(p\pm u) + b_2(p\pm u)^2\,\Big]\,.$$

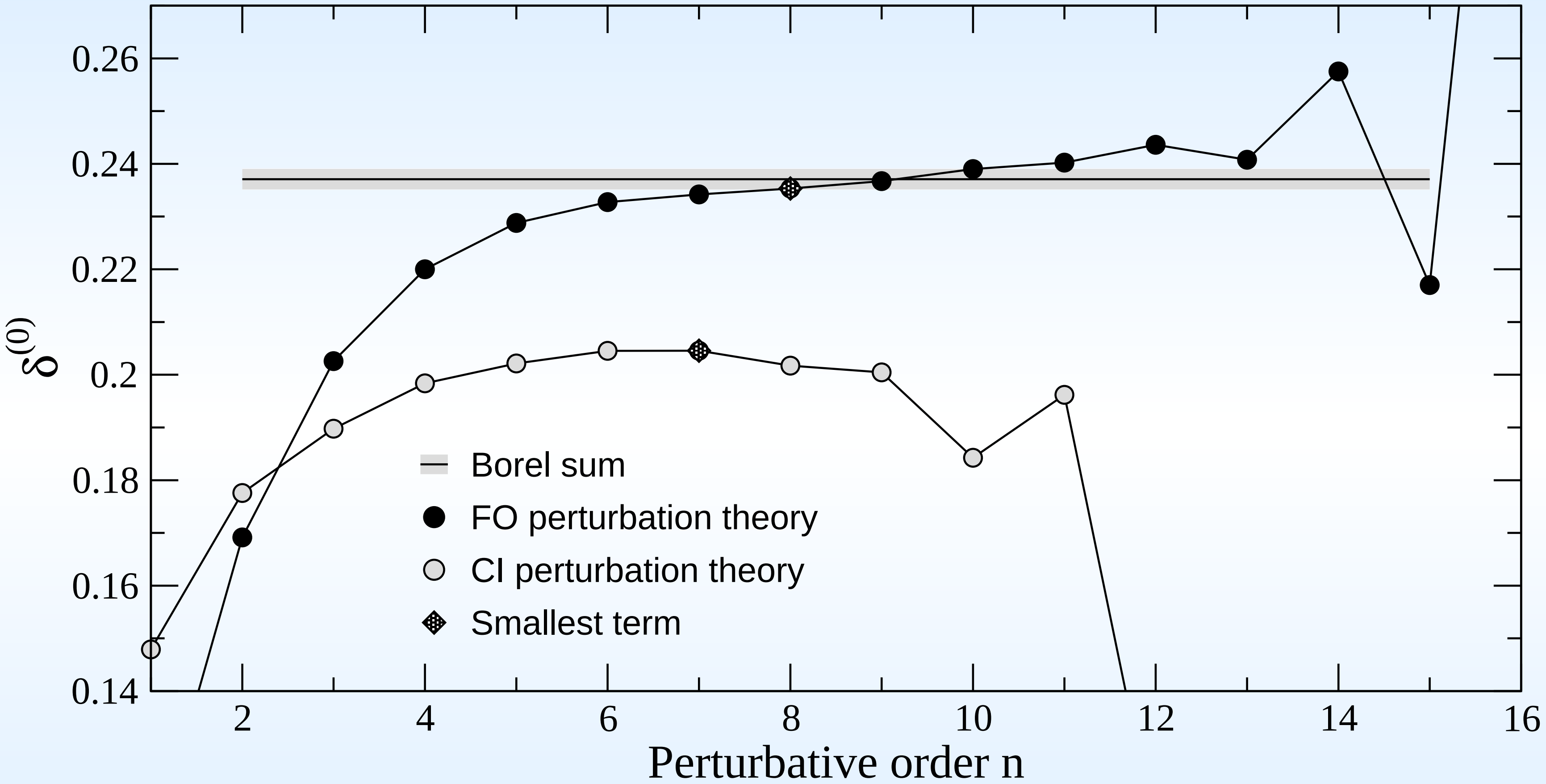
- Our main model incoporates the leading UV pole ($u=-1$), as well as the two leading IR renormalons ($u=2{,}3$).
- It should reproduce the exactly known $c_{n,1},\;\; n\le 4$.
- For both UV and IR, the residues d_p are free while γ, $b_{1,2}$ depend on anomalous dimensions and β-coefficients.

Adler function



$$c_{5,1} = 283\,, \quad \alpha_s(M_\tau) = 0.34\,.$$

Central Borel model



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Borel model

$c_{n,1}$ composition in central Borel model:

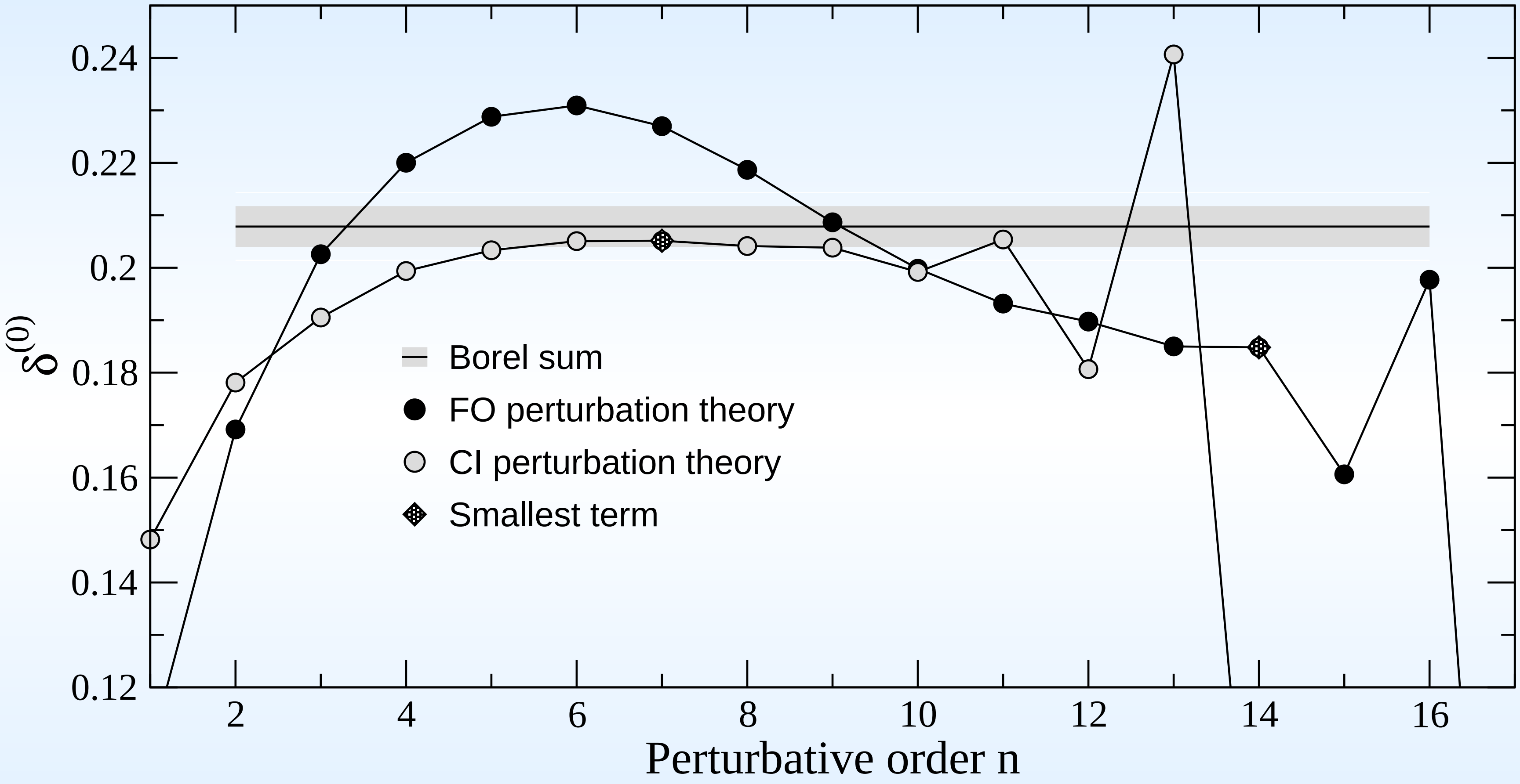
	$c_{2,1}$	$c_{3,1}$	$c_{4,1}$	$c_{5,1}$	$c_{6,1}$
IR_2	-77.8	82.4	100.4	135.9	97.5
IR_3	152.0	28.7	-10.0	-20.2	-13.3
UV_1	22.5	-11.2	9.7	-15.6	15.8

$c_{n,1}$ composition in Borel model with $d_2^{\mathrm{IR}} = 0$:

	$c_{2,1}$	$c_{3,1}$	$c_{4,1}$	$c_{5,1}$	$c_{6,1}$
IR_3	-743.3	-140.5	49.1	98.9	99.1
IR_4	662.8	244.2	47.7	6.3	-7.2
UV_1	7.5	-3.7	3.2	-5.2	8.1

Large cancellations occur. Appears unnatural.

$d_2^{\mathrm{IR}} = 0$ Borel model



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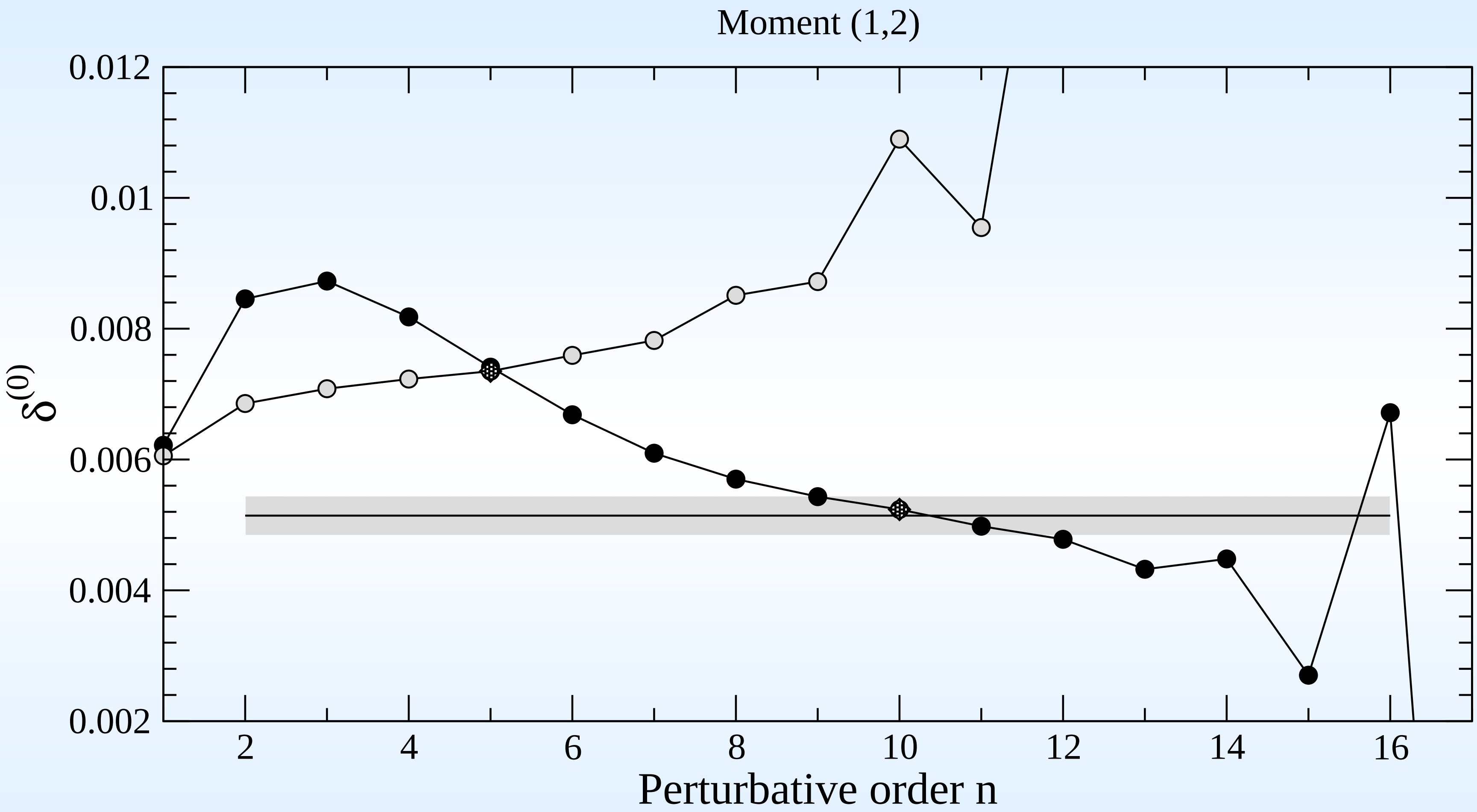
$(1,2)$ moment



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Employing the hadronic decay rate into light quarks

$$R_{\tau,V+A} = N_c\,|V_{ud}|^2\,S_{\rm EW}\Big[\,1+\delta^{(0)}+\delta^{\rm NP}_{V+A}\Big]$$

with $\delta^{\rm NP}_{V+A} = (-\,7.1\pm 3.1)\cdot 10^{-3}$, one finds

$$\delta^{(0)} = \frac{R_{\tau,V+A}}{3|V_{ud}|^2 S_{\rm EW}} - 1 - \delta^{\rm NP}_{V+A} = 0.2042(38)(33)$$

The first uncertainty is due to $R_{\tau,V+A}$, while the remaining error is dominated by $\delta^{\rm NP}_{V+A}$.

Adjusting α_s such as to reproduce $\delta^{(0)}$: (Beneke, MJ 2008)

$$\alpha_s(M_\tau) = 0.3156(30)(51) \;\Rightarrow\; \alpha_s(M_Z) = 0.1180(8)$$

Duality violations neglected! See talk by Diogo Boito.

Exclusive decay spectra

Viable information can be obtained from the decay spectra for exclusive τ-decay channels.

A first step in this direction is a reliable description of the $\tau \to \nu_\tau K\pi$ decay spectrum:

(MJ, Pich, Portolés 2006/08)
(Boito, Escribano, MJ 2008/10)
(talk by Emilie Passemar)

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 |V_{us}|^2 M_\tau^3}{32\pi^3 s} \left(1 - \frac{s}{M_\tau^2}\right)^2 \times$$

$$\left[\left(1 + 2\,\frac{s}{M_\tau^2}\right) q_{K\pi}^3 \, |F_+^{K\pi}(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s}\, q_{K\pi} |F_0^{K\pi}(s)|^2 \right].$$

To this end the $K\pi$ vector and scalar form factors $F_+^{K\pi}(s)$ and $F_0^{K\pi}(s)$ are required as an input.

$K\pi$ form factors

A description of the $K\pi$ vector form factor can be obtained within chiral perturbation theory with resonances (RχPT):

$$F_+^{K\pi}(s) = \frac{m_{K^*}^2}{m_{K^*}^2 - s - \kappa\, \mathrm{Re}\widetilde{H}_{K\pi}(s) - i m_{K^*}\gamma_{K^*}(s)} \,.$$

The parameters of this model, namely m_{K^*} and γ_{K^*}, can be fitted from experimental data for p-wave $K\pi$ scattering, or from the τ data.

The physical parameters M_{K^*} and Γ_{K^*} can be inferred from the pole of $F_+^{K\pi}(s)$ in the complex s-plane.

Also a second resonance contribution can easily be included.

$K\pi$ form factors

The scalar form factor $F_0^{K\pi}(s)$ has been obtained from a dispersion relation analysis of S-wave $K\pi$ scattering data.

(MJ, Oller, Pich 2000/02)

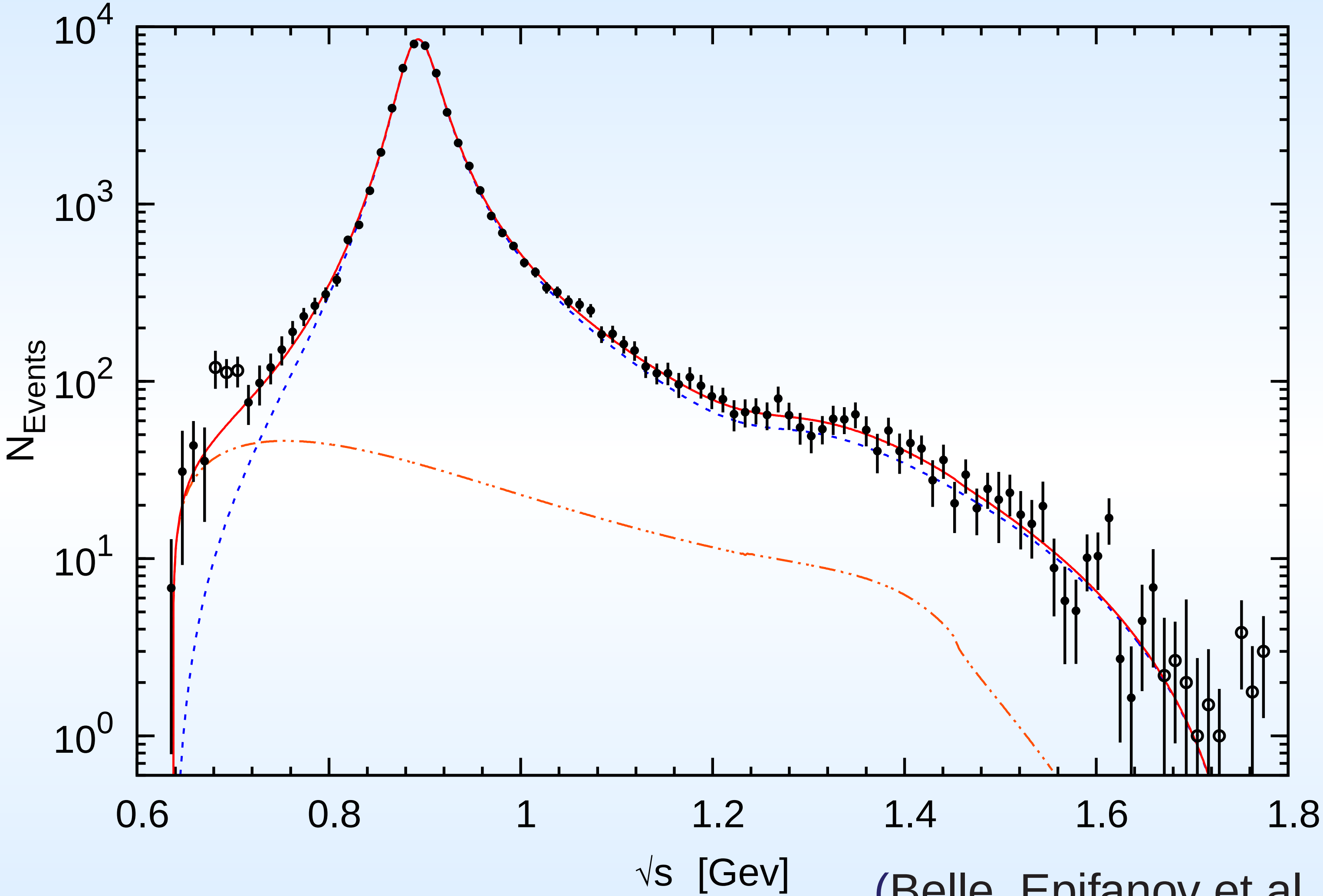
As a prediction of the model, we obtain the slope and curvature of the vector form factor $F_+^{K\pi}(s)$: (Boito, Escribano, MJ 2010)

$$\lambda_+^{\prime} = (25.49 \pm 0.31) \cdot 10^{-3}\,, \quad \lambda_+^{\prime\prime} = (12.22 \pm 0.14) \cdot 10^{-4}\,.$$

Results on slope and curvature from K_{l3} decays have been included as a constraint in the fit.

Allows for an improved determination of the phase-space integrals needed in $|V_{us}|$ analysis from K_{l3}.

Belle $K\pi$ spectrum



(Belle, Epifanov et al. 2007)

$$M_{K^*} = 892.0 \pm 0.5 \text{ MeV}, \qquad \Gamma_{K^*} = 46.5 \pm 1.1 \text{ MeV}$$

Conclusions

- FOPT appears to provide the more reliable approach to the perturbative series for $\delta^{(0)}$ while CIPT misses cancellations.

$$\Rightarrow \quad \alpha_s(M_Z) = 0.1180 \pm 0.0008$$

- Good description of the exclusive decay $\tau \to \nu_\tau K\pi$ allows to extract resonance as well as form factor parameters.

- Better data on exclusive and inclusive τ decay spectra would be helpful to resolve theoretical issues.

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- ☞ Good description of the exclusive decay $\tau \to \nu_\tau K\pi$ allows to extract resonance as well as form factor parameters.

- ☞ Better data on exclusive and inclusive τ decay spectra would be very helpful to resolve theoretical issues.

Thank You for Your attention !