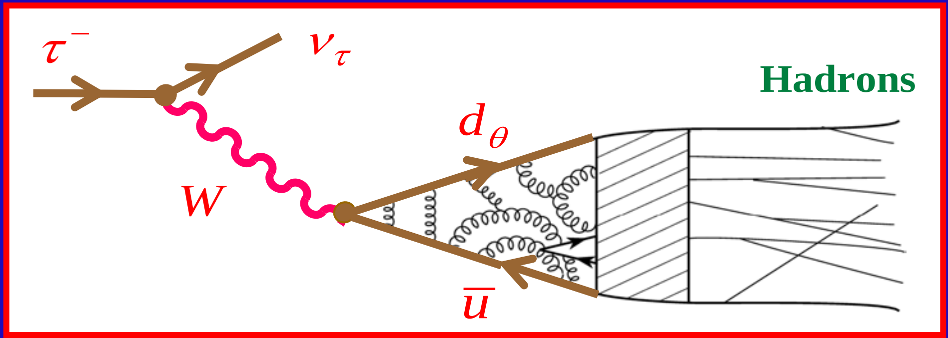
Hadronic

τ Decays

A. Pich

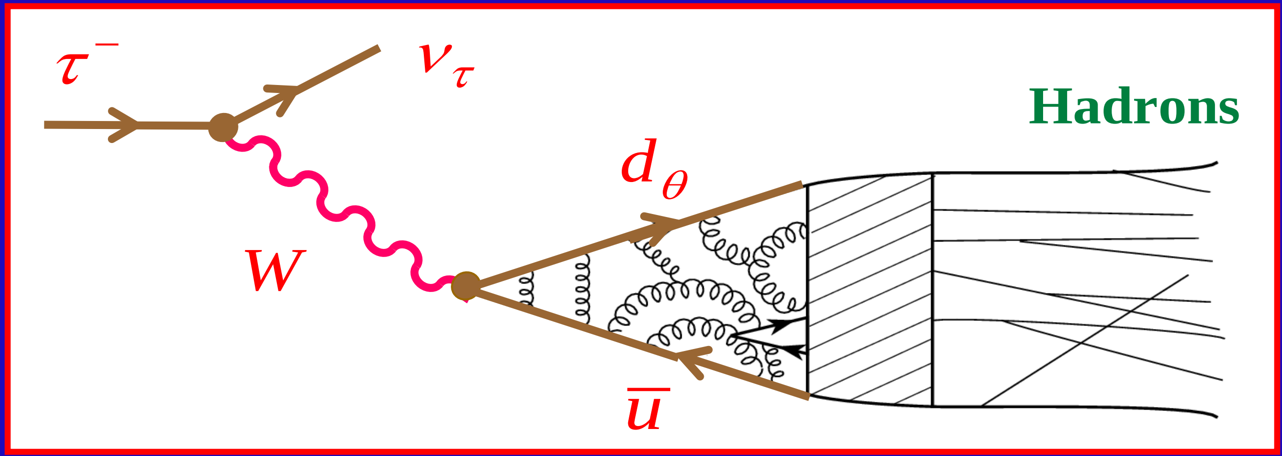
IFIC, Valencia



The 11th International Workshop on Tau Lepton Physics
Manchester, UK, 13-17 September 2010

To the memory
of our friend
Ximo Prades

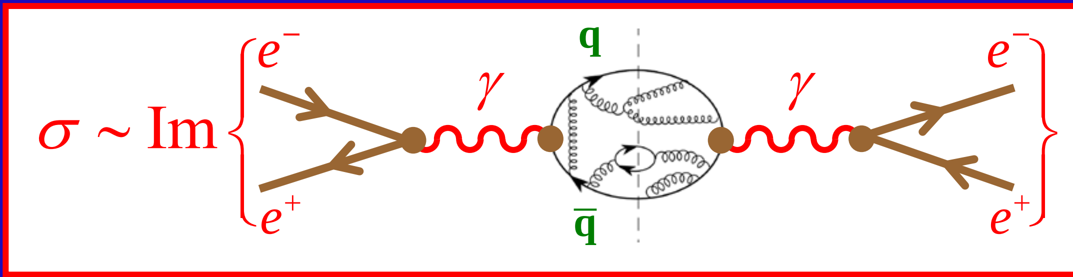
HADRONIC TAU DECAY



$$d_\theta = V_{ud}\; d + V_{us}\; s$$

Only lepton massive enough to decay into hadrons

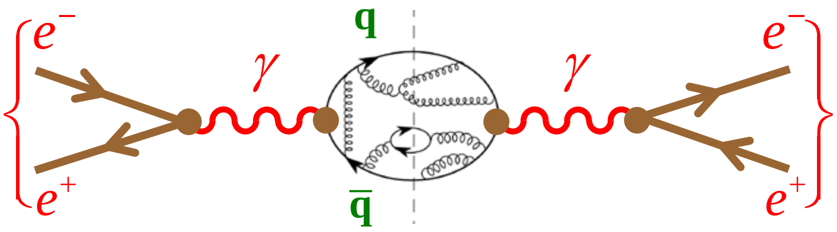
$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{Hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau\, e^-\, \overline{\nu}_e\right)} \approx N_C \qquad ; \qquad R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.640 \pm 0.010$$



$$\frac{\sigma(e^+e^-\to\text{had})}{\sigma(e^+e^-\to\mu^+\mu^-)} = 12\pi\,\text{Im}\,\Pi_{\text{em}}(s)$$

$$\Pi^{\mu\nu}_{\text{em}}(q) \equiv i\int d^4x\; e^{iqx}\,\langle 0|T[J^{\mu}_{\text{em}}(x)\,J^{\nu}_{\text{em}}(0)]|0\rangle \;=\; \left(-g^{\mu\nu}q^2 + q^\mu q^\nu\right)\Pi_{\text{em}}(q^2)$$

$$\sigma \sim \mathrm{Im}$$



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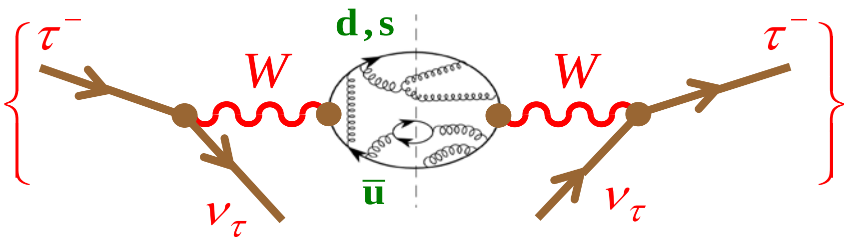
$$\Gamma_{\tau \to \nu_\tau + \mathrm{had}} \sim \mathrm{Im}$$

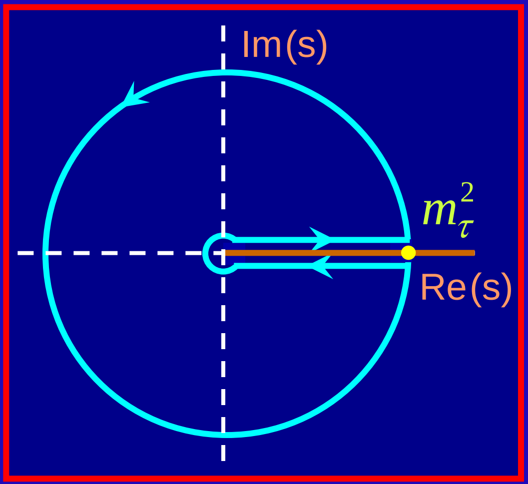


$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \mathrm{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^{m_\tau^2} dx \left(1 - \frac{s}{m_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{m_\tau^2}\right) \mathrm{Im}\,\Pi^{(1)}(s) + \mathrm{Im}\,\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2 \left[\Pi^{(J)}_{ud,V}(s) + \Pi^{(J)}_{ud,A}(s)\right] + |V_{us}|^2 \left[\Pi^{(J)}_{us,V}(s) + \Pi^{(J)}_{us,A}(s)\right]$$

$$\Pi^{\mu\nu}_{ij,J}(q) \equiv i \int d^4x \; e^{iqx} \, \langle 0 | T[J^{\mu}_{ij}(x)\, J^{\nu}_{ij}(0)^\dagger] | 0 \rangle = \left(-g^{\mu\nu} q^2 + q^\mu q^\nu\right) \Pi^{(1)}_{ij,J}(q^2) + q^\mu q^\nu \, \Pi^{(0)}_{ij,J}(q^2)$$

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{had})}{\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu}_e)} = 12\pi \int_0^1 dx\, (1-x)^2 \left[(1+2x)\, \text{Im}\,\Pi^{(1)}(x m_\tau^2) + \text{Im}\,\Pi^{(0)}(x m_\tau^2)\right]$$



$$R_\tau = 6\pi i \oint_{|x|=1} dx\, (1-x)^2 \left[(1+2x)\, \Pi^{(0+1)}(x m_\tau^2) - 2x\, \Pi^{(0)}(x m_\tau^2)\right]$$

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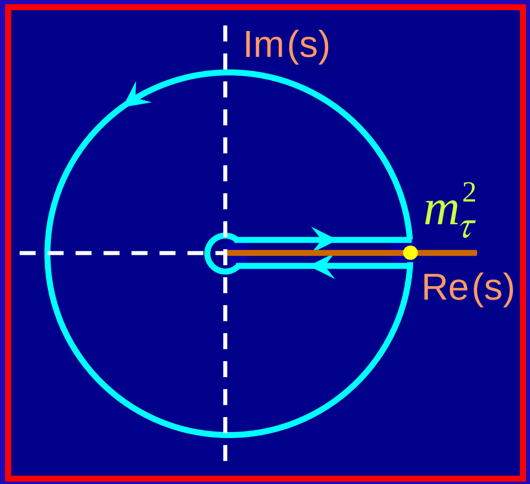

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$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle}{(-s)^{D/2}}$$

OPE

Braaten-Narison-Pich

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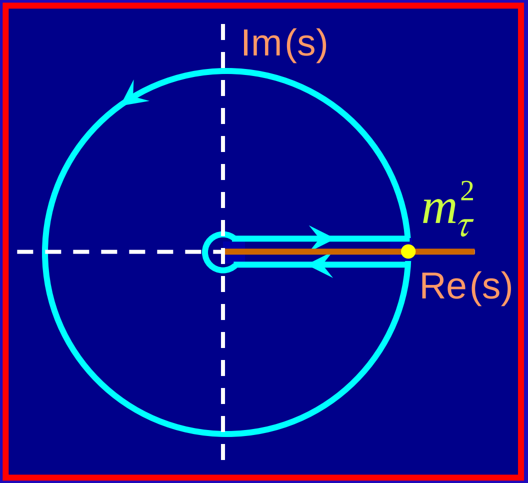

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OPE

$$R_\tau = N_C\, S_{\mathrm{EW}} \left(1 + \delta_{\mathrm{P}} + \delta_{\mathrm{NP}}\right) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$S_{\mathrm{EW}} = 1.0201\,(3)$; $\delta_{\mathrm{NP}} = -0.0059 \pm 0.0014$

Marciano-Sirlin, Braaten-Li, Erler

Fitted from data (Davier et al)

$\delta_{\mathrm{P}} = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + \ldots \approx 20\%$; $a_\tau \equiv \alpha_s(m_\tau)/\pi$

Perturbative: $(m_q=0)$

$K_4 = 49.07570$ (Baikov-Chetyrkin-Kühn '08)

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n \quad ; \quad K_0 = K_1 = 1 \;,\; K_2 = 1.63982 \;,\; K_3 = 6.37101$$

$$\delta_{\rm P} = \sum_{n=1} K_n\, A^{(n)}(\alpha_s) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \cdots$$

Le Diberder- Pich '92

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

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$$\Rightarrow \quad \delta_{\mathrm{P}} = \sum_{n=1} K_n\, A^{(n)}(\alpha_s) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \cdots$$

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Power Corrections:

Braaten-Narison-Pich '92

$$\Pi^{(0+1)}_{\mathrm{OPE}}(s) \approx \frac{1}{4\pi^2}\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-s)^n}$$

$$C_4\langle O_4\rangle \approx \frac{2\pi}{3}\langle 0|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

$$\delta_{\mathrm{NP}} \approx \frac{-1}{2\pi i}\oint_{|x|=1} dx\;(1-3x^2+2x^3)\sum_{n\geq 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-x m_\tau^2)^n} = -3\,\frac{C_6\langle O_6\rangle}{m_\tau^6} - 2\,\frac{C_8\langle O_8\rangle}{m_\tau^8}$$

Suppressed by m_τ^6 [additional chiral suppression in $C_6\langle O_6\rangle^{V+A}$]

Similar predictions for $R_{\tau,V}$, $R_{\tau,A}$, $R_{\tau,S}$ and the moments

$$R_\tau^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}$$

Different sensitivity to power corrections through k , l

The non-perturbative contribution to R_τ can be obtained from the invariant-mass distribution of the final hadrons:

$$\delta_{\mathrm{NP}} = -0.0059 \pm 0.0014$$

Davier et al. (ALEPH data)

Recent $\alpha_s(m_\tau)$ Analyses

Reference	Method	δ_P	$\alpha_s(m_\tau)$	$\alpha_s(m_Z)$
Baikov et al	CIPT, FOPT	0.1998 (43)	**0.332 (16)**	**0.1202 (19)**
Davier et al	CIPT	0.2066 (70)	**0.344 (09)**	**0.1212 (11)**
Beneke-Jamin	BSR + FOPT	0.2042 (50)	**0.316 (06)**	**0.1180 (08)**
Maltman-Yavin	PWM + CIPT		**0.321 (13)**	**0.1187 (16)**
Menke	CIPT, FOPT	0.2042 (50)	**0.342 (11)**	**0.1213 (12)**
Narison	CIPT, FOPT		**0.324 (08)**	**0.1192 (10)**
Caprini-Fischer	BSR + CIPTm	0.2042 (50)	**0.321 (10)**	
Cvetič et al	βexp + CIPT	0.2040 (40)	**0.341 (08)**	**0.1211 (10)**
Pich	CIPT	0.2038 (40)	**0.342 (12)**	**0.1213 (14)**

CIPT: Contour-improved perturbation theory
FOPT: Fixed-order perturbation theory
BSR: Borel summation of renormalon series
CIPTm: Modified CIPT (conformal mapping)
βexp: Expansion in derivatives of the coupling (β function)
PWM: Pinched-weight moments

Perturbative Uncertainty on $\alpha_s(m_\tau)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n\; a(-s)^n$$

$$\delta_P = \underbrace{\sum_{n=1} K_n\, A^{(n)}(\alpha_s)}_{\text{CIPT}} = \underbrace{\sum_{n=0} r_n\; a_\tau^n}_{\text{FOPT}}$$

$$r_n = K_n + g_n$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\, a_\tau(-x m_\tau^2)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

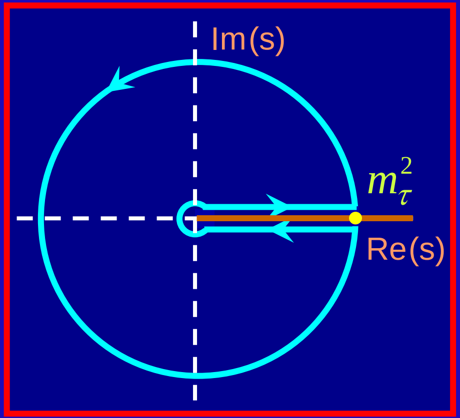
n	1	2	3	4	5
K_n	1	1.6398	6.3710	49.0757	
g_n	0	3.5625	19.9949	78.0029	307.78
r_n	1	5.2023	26.3659	127.079	

The dominant corrections come from the contour integration

Le Diberder- Pich 1992

Large running of α_s along the circle $s = m_\tau^2\, e^{i\varphi}$, $\varphi \in [0, 2\pi]$

$$A^{(n)}(a_\tau) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}(1-2x+2x^3-x^4)\, a_\tau(-x m_\tau^2)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$



$$A^{(1)}(a_\tau) = a_\tau - \frac{19}{24}\beta_1 a_\tau^2 + \left[\beta_1^2\left(\frac{265}{288}-\frac{\pi^2}{12}\right)-\frac{19}{24}\beta_2\right] a_\tau^3 + \cdots$$

$$a(-s) \simeq \frac{a_\tau}{1-\frac{\beta_1}{2}a_\tau \log\left(-s/m_\tau^2\right)} = \frac{a_\tau}{1-i\frac{\beta_1}{2}a_\tau\phi} = a_\tau\sum_n\left(i\frac{\beta_1}{2}a_\tau\phi\right)^n \qquad ; \qquad \phi\in[0,2\pi]$$

FOPT expansion only convergent if $a_\tau < 0.13$ (0.11) [at 1 (3) loops]

Experimentally $a_\tau \approx 0.11$ ➡ **FOPT should not be used** (divergent series)

The difference between FOPT and CIPT grows at higher orders

CIPT gives rise to a well-behaved perturbative series:

$$A^{(n)}(a_\tau) \equiv \frac{1}{2\pi i} \oint_{|x|=1} \frac{dx}{x} (1 - 2x + 2x^3 - x^4)\, a(-s)^n = a_\tau^n + \cdots$$

a = 0.11	$A^{(1)}(a)$	$A^{(2)}(a)$	$A^{(3)}(a)$	$A^{(4)}(a)$	δ_P
$\beta_{n>1} = 0$	0.14828	0.01925	0.00225	0.00024	**1.20578**
$\beta_{n>2} = 0$	0.15103	0.01905	0.00209	0.00020	**1.20537**
$\beta_{n>3} = 0$	0.15093	0.01882	0.00202	0.00019	**1.20389**
$\beta_{n>4} = 0$	0.15058	0.01865	0.00198	0.00018	**1.20273**
$O(a^4)$	0.16115	0.02431	0.00290	0.00015	1.22665

Uncertainty only related to the unknown K_n $(n \geq 5)$ coefficients

Modelling a better behaved FOPT (Beneke – Jamin)

- **Large higher-order K_n corrections could cancel the g_n ones**

  Happens in the "large-β_0" approximation (UV renormalon chain)

- D = 4 corrections very suppressed in R_τ

  ➡ **n = 2 IR renormalons can do the job ($K_n \approx - g_n$)**

- No sign of renormalon behaviour in known coefficients

  ➡ **n = -1,2,3 renormalons + linear polynomial**

  5 unknown constants fitted to K_n ($2 \leq n \leq 5$). $K_5 = 283$ assumed

- **Borel summation:** large renormalon contributions. Smaller α_s

  Same result with Modified (conformal mapping) CIPT (Fischer – Caprini)

Nice model of higher orders. But too many different possibilities …

(Descotes-Genon – Malaescu)

Non-perturbative contributions

$$R_\tau = N_C\, S_{EW}\left(1+\delta_P+\delta_{NP}\right)$$

	δ_{NP}	
Davier et al '08	-0.0059 ± 0.0014	ALEPH data
ALEPH '05	-0.0043 ± 0.0019	
OPAL '99	-0.0024 ± 0.0025	
CLEO '95		
Maltman-Yavin '08	0.012 ± 0.018	Phenom. analysis
Braaten et al '92	-0.009 ± 0.005	Theory estimate
Beneke-Jamin '08	-0.007 ± 0.003	Theory estimate

$\delta_P = 0.2066 \pm 0.0070$ (Davier et al '08)

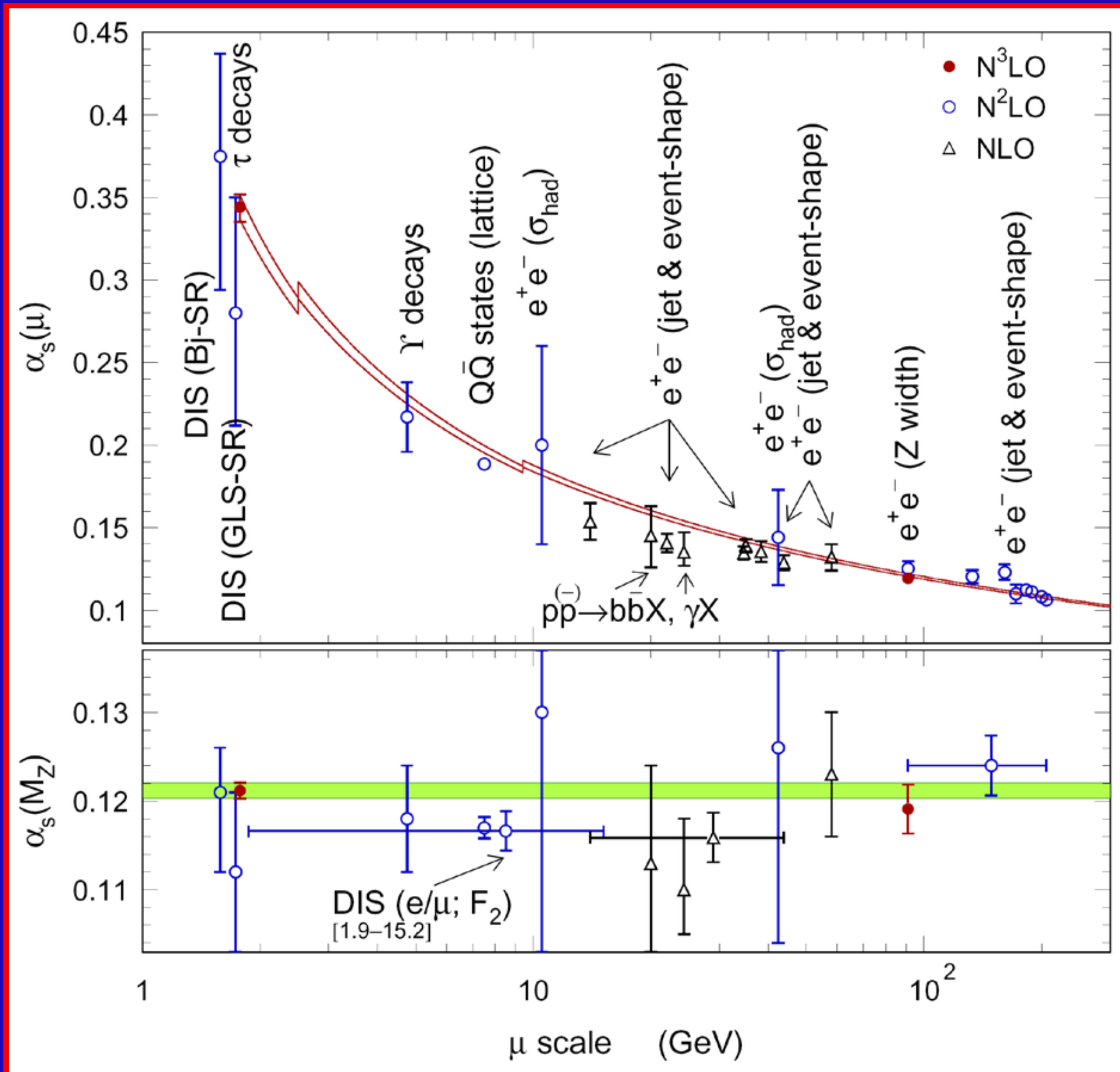
Small "Duality violations" –OPE uncertainties– (Cata – Golterman – Peris '08)

$$\delta_{DV} = 2\pi i \oint_{|x|=1} dx\,(1-x)^2(1+2x)\left[\Pi^{(0+1)}(x m_\tau^2) - \Pi^{(0+1)}_{OPE}(x m_\tau^2)\right]$$

$$R_{\tau,V} = 1.783 \pm 0.011 \quad ; \quad R_{\tau,A} = 1.695 \pm 0.011 \quad ; \quad R_{\tau,V+A} = 3.478 \pm 0.010$$

Davier et al

ALEPH



$$\alpha_s(m_\tau^2) = 0.344 \pm 0.009$$

$$\alpha_s(M_Z^2) = 0.1212 \pm 0.0011$$

$$\alpha_s(M_Z^2)_{Z\,\mathrm{width}} = 0.1193 \pm 0.0028$$

The most precise test of Asymptotic Freedom

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0019 \pm 0.0011_\tau \pm 0.0028_Z$$

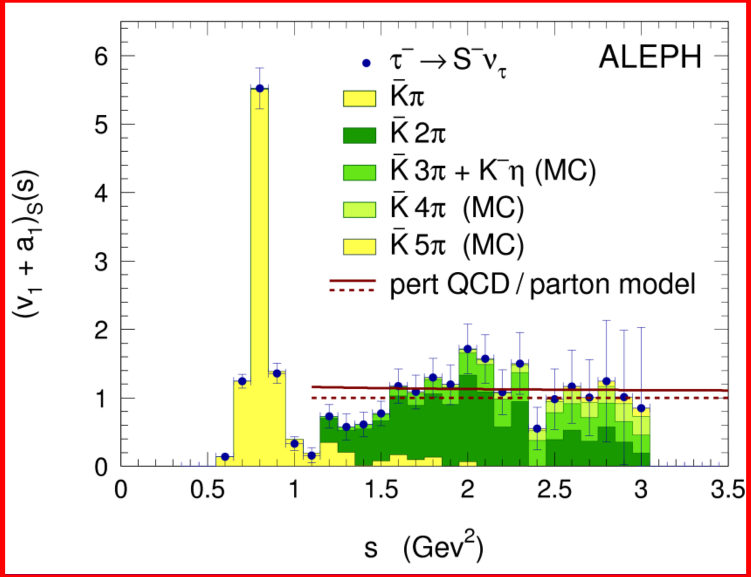
SU(3) Breaking

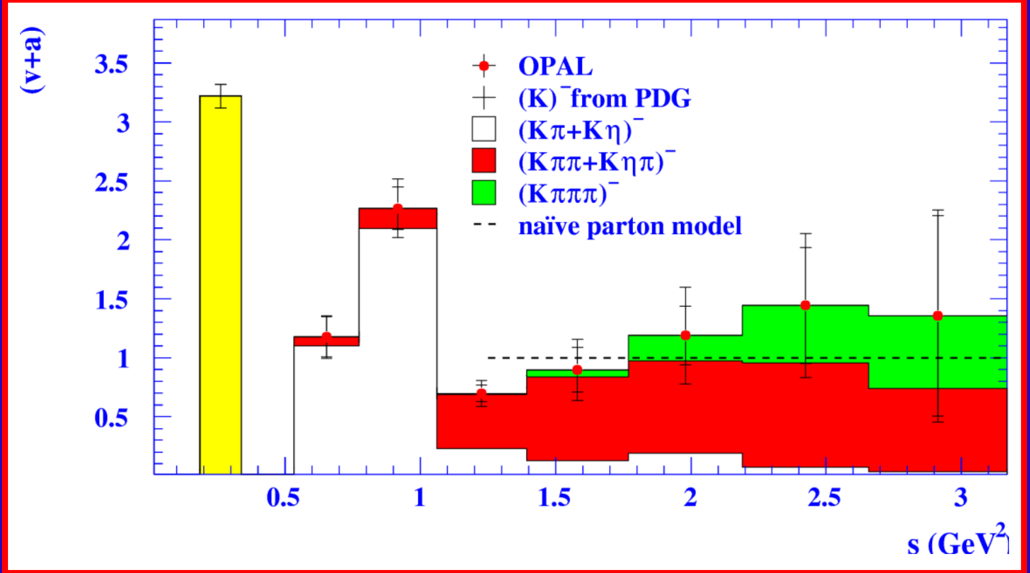
$$R_\tau^{kl} = N_C \; S_{\mathrm{EW}} \left\{ \left(|V_{ud}|^2 + |V_{us}|^2 \right) \left[1 + \delta^{kl(0)} \right] + \sum_{D\geq 2} \left[|V_{ud}|^2 \, \delta_{ud}^{kl(D)} + |V_{us}|^2 \, \delta_{us}^{kl(D)} \right] \right\}$$

⬇

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \approx N_C \; S_{\mathrm{EW}} \sum_{D\geq 2} \left[\delta_{ud}^{kl(D)} - \delta_{us}^{kl(D)} \right]$$

Strange Spectral Function: SU(3) Breaking





(k,l)	ALEPH	OPAL
(0,0)	0.39 ± 0.14	0.26 ± 0.12
(1,0)	0.38 ± 0.08	0.28 ± 0.09
(2,0)	0.37 ± 0.05	0.30 ± 0.07
(3,0)	0.40 ± 0.04	0.33 ± 0.05
(4,0)	0.40 ± 0.04	0.34 ± 0.04

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \approx 24\, \frac{m_s^2(m_\tau^2)}{m_\tau^2}\, \Delta_{kl}(\alpha_s)$$

→ m_s and/or V_{us}

QCD uncertainties

$$\delta R_\tau^{kl} \equiv \frac{R_{\tau,V+A}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau,S}^{kl}}{|V_{us}|^2} \approx 24\, \frac{m_s^2(m_\tau^2)}{m_\tau^2}\, \Delta_{kl}(\alpha_s)$$

Known to $O(\alpha_s^3)$

- **$\Delta_{kl}(\alpha_s)$** gets **longitudinal (J=0)** and **transverse (J=0+1)** contributions
- Divergent QCD series for J=0
- **Longitudinal contribution determined through data:**
  - Kaon pole ($K\to\mu\nu$) (dominant J=0 contribution)
  - Pion pole ($\pi\to\mu\nu$)
  - $(K\pi)_{J=0}$ (S-wave $K\pi$ scattering)
  - …
- Smaller uncertainties

Gámiz et al '03

	$R_{us,A}^{00,L}$	$R_{us,V}^{00,L}$	$R_{ud,A}^{00,L}$
Theory:	-0.144 ± 0.024	-0.028 ± 0.021	$-(7.79 \pm 0.14) \cdot 10^{-3}$
Phenom:	-0.135 ± 0.003	-0.028 ± 0.004	$-(7.77 \pm 0.08) \cdot 10^{-3}$

$$\delta R_{\tau,\mathrm{th}}^{00} \equiv \underbrace{0.1544\,(37)}_{J=0} + \underbrace{0.062\,(15)}_{m_s(m_\tau)\,=\,0.100\,(10)} = 0.216\,(16)$$

$$|V_{us}|^2 = \frac{R^{00}_{\tau,S}}{\frac{R^{00}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{00}_{\tau,\mathrm{th}}}$$

Gámiz-Jamin-Pich-Prades-Schwab

τ data: $R^{00}_{\tau,S} = 0.1615\ (40)$

$R^{00}_{\tau,V+A} = 3.479\ (11)$

PDG 10: $|V_{ud}| = 0.97425\ (22)$

$\delta R^{00}_{\tau,\mathrm{th}} = 0 \quad \Rightarrow \quad |V_{us}| = 0.210\ (3)$

Taking as input (from non τ sources) $m_s(m_\tau) = 100 \pm 10\ \mathrm{MeV}$:

$\delta R^{00}_{\tau,\mathrm{th}} = 0.216\ (16) \quad \Rightarrow \quad |V_{us}| = 0.2164 \pm 0.0027_{\mathrm{exp}} \pm 0.0005_{\mathrm{th}}$

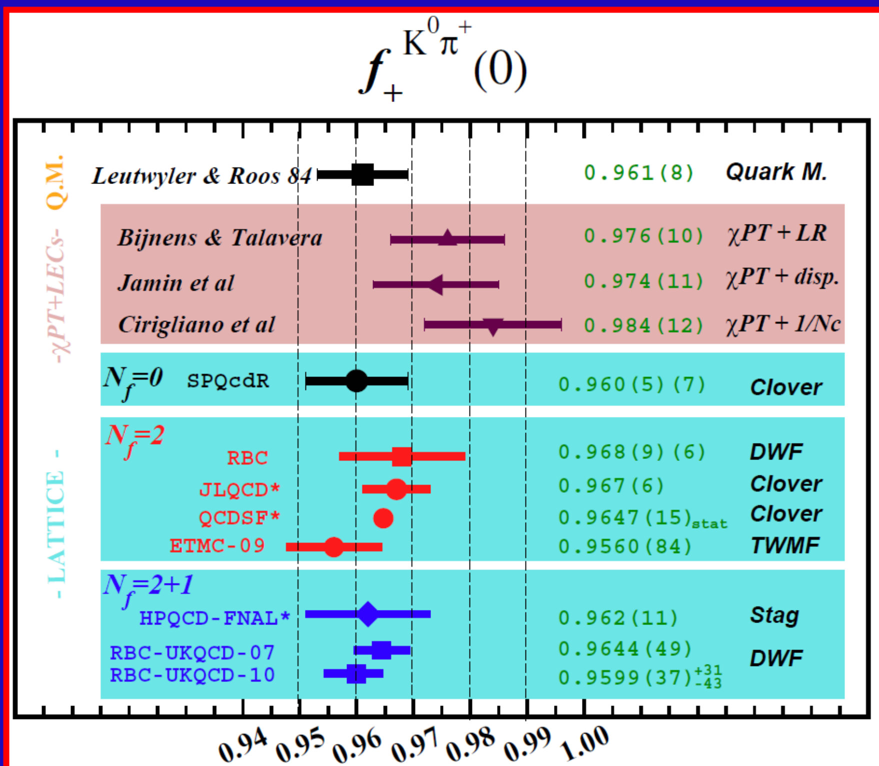
K_{l3}: $|V_{us}| = 0.2241 \pm 0.0024$ $\quad [\, f_+(0) = 0.965 \pm 0.010 \,]$

The τ could give the most precise V_{us} determination

K_{l3} Decays

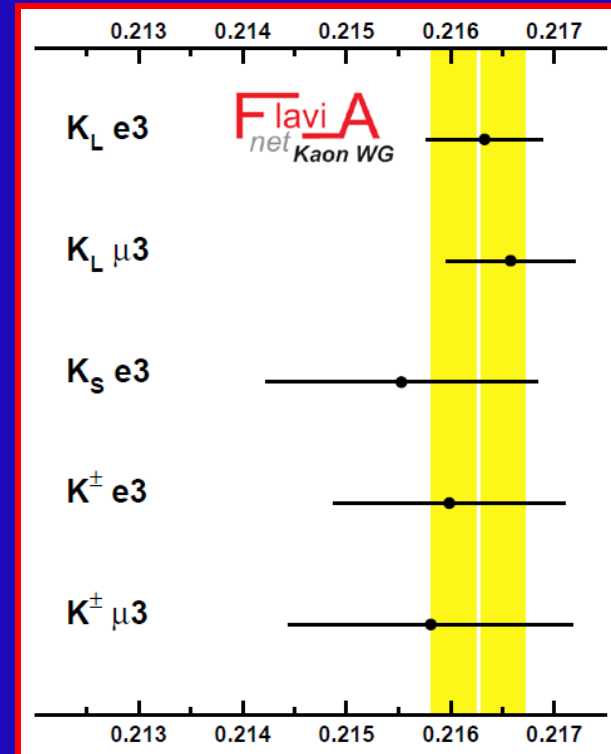
arXiv:1005.2323 [hep-ph]

Large O(p⁶) ChPT correction (Bijnens-Talavera)



O(p⁴)

O(p⁶)



$$|f_+(0)\; V_{us}| = 0.2163 \pm 0.0005$$

$$f_+(0) = 0.965 \pm 0.010 \quad \Longrightarrow \quad |V_{us}| = 0.2241 \pm 0.0024$$

$$f_+(0) = 0.959 \pm 0.005 \quad \Longrightarrow \quad |V_{us}| = 0.2254 \pm 0.0013$$

UKQCD/RBC

Do we have a normalization problem?

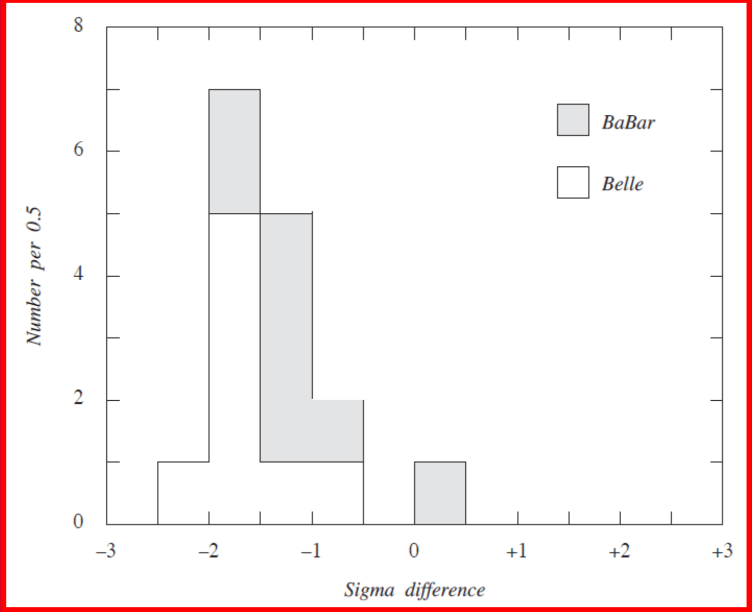
Smaller $\tau \to K$ branching ratios ➡ smaller $R_{\tau,S}$ ➡ smaller V_{us}

$$R^{00}_{\tau,S}\Big|_{\text{OLD}} = 0.1686\ (47) \quad \longrightarrow \quad R^{00}_{\tau,S}\Big|_{\text{NEW}} = 0.1615\ (40)$$

$$|V_{us}|_{\text{OLD}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0005_{\text{th}}$$ ➡ $$|V_{us}|_{\text{NEW}} = 0.2164 \pm 0.0027_{\text{exp}} \pm 0.0005_{\text{th}}$$

PDG 2010:

"Fifteen of the 16 *B-factory* branching fraction measurements are smaller than the non-*B-factory values. The average normalized difference between* the two sets of measurements is -1.36"

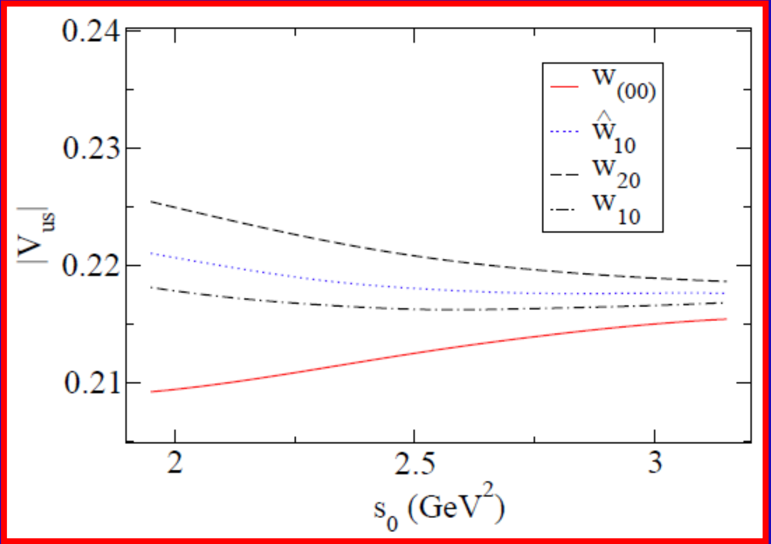


Missing modes ?

More data needed

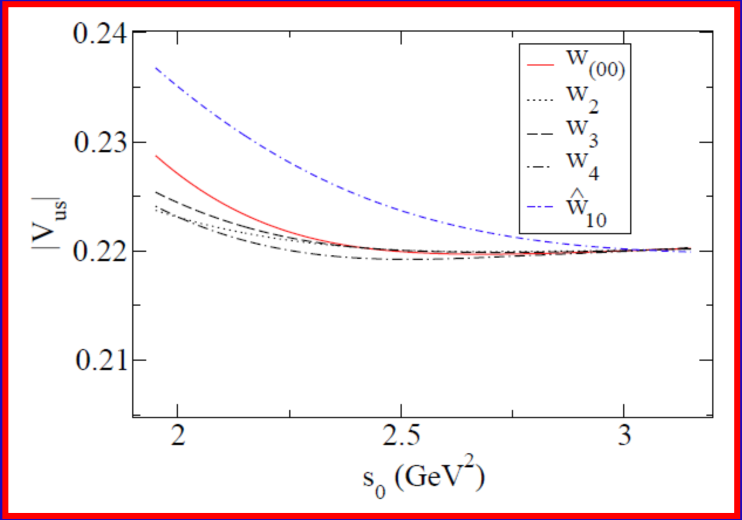
Spectral Moment Analysis

(Maltman et al '09)



$$|V_{us}| = \begin{cases} 0.2180(32)(15) & (\hat{w}_{10}) \\ 0.2188(29)(22) & (w_{20}) \\ 0.2172(34)(11) & (w_{10}) \\ 0.2160(26)(8) & (w_{(00)}) \end{cases}$$

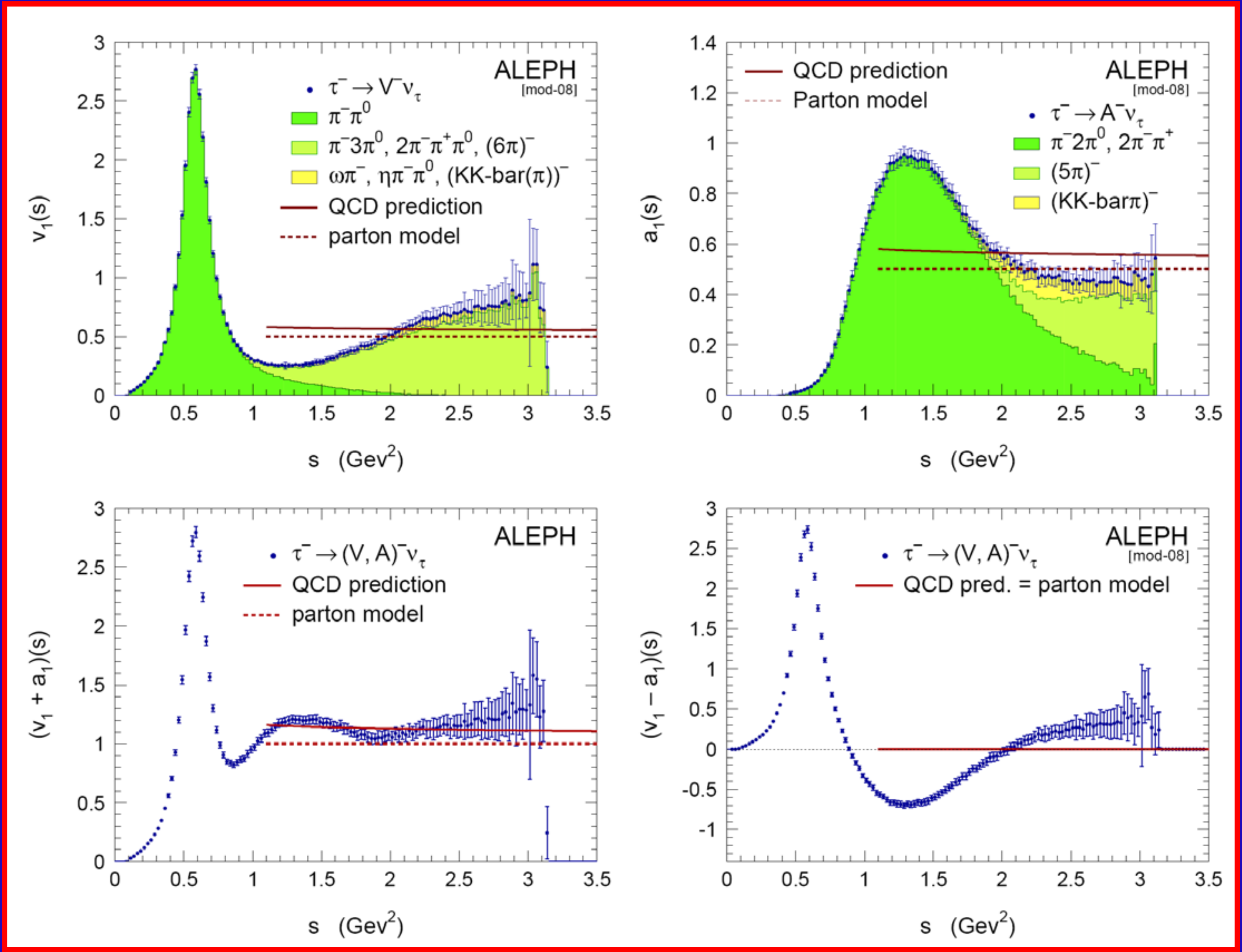
τ + Electroproduction data:



$$|V_{us}| = 0.2208(27)(28)(5)(2)$$

SPECTRAL FUNCTIONS

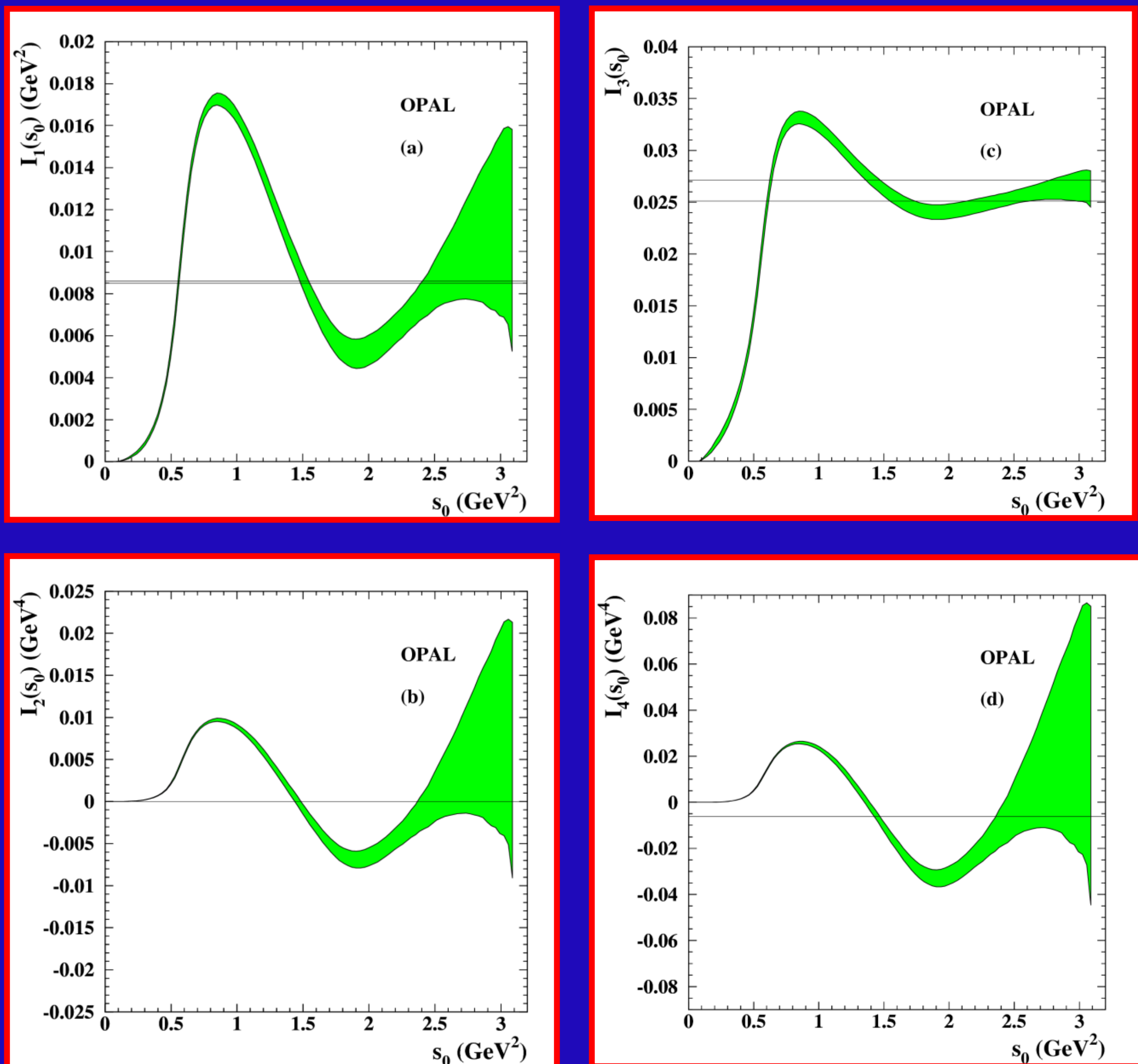
$v_1(s) = 2\pi \, \mathrm{Im}\, \Pi^{(0+1)}_{ud,V}(s)$ $\qquad$ $a_1(s) = 2\pi \, \mathrm{Im}\, \Pi^{(0+1)}_{ud,A}(s)$



Davier et al '08

Chiral Sum Rules

$$\lim_{s\to\infty} s^2 \left[\Pi^{(0+1)}_{ud,V}(s) - \Pi^{(0+1)}_{ud,A}(s) \right] = 0 \qquad ; \qquad \Pi^{(0+1)}_{ud,J}(s) = \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\,\Pi^{(0+1)}_{ud,J}(t)}{t-s-i\varepsilon}$$



When $s_0 \to \infty$

$$I_1 = \int_0^{s_0} ds\; \hat\rho(s) = f_\pi^2$$

$$I_2 = \int_0^{s_0} ds\; s\, \hat\rho(s) = 0$$

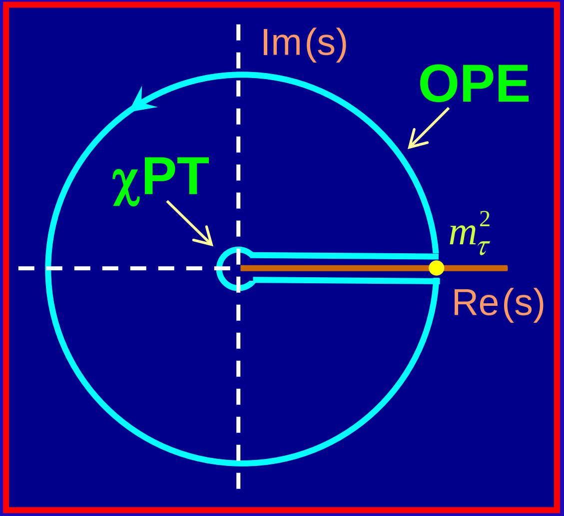
$$I_3 = \int_0^{s_0} \frac{ds}{s}\, \hat\rho(s) = f_\pi^2 \frac{\langle r_\pi^2 \rangle}{3} - F_A$$

$$I_4 = \int_0^{s_0} ds\; s \ln s\; \hat\rho(s)$$

$$= \frac{4\pi f_\pi^2}{3\alpha}\left(m^2_{\pi^0} - m^2_{\pi^\pm} \right)$$

$$\hat\rho \equiv \frac{1}{2}\rho \equiv \frac{1}{2\pi}\,\mathrm{Im}\left[\Pi^{(0+1)}_{ud,V} - \Pi^{(0+1)}_{ud,A} \right]$$

Davier-Girlanda-Höcker-Stern '98



$$\int_{s_{th}}^{s_0} ds\; w(s)\,\rho(s) \;+\; \frac{1}{2\pi i}\oint_{|s|=s_0} ds\; w(s)\,\Pi^{OPE}(s) \;+\; DV[w(s), s_0] \;=\; 2 f_\pi^2\, w(m_\pi^2) \;+\; \mathrm{Res}_{s=0}\,[w(s)\,\Pi(s)]$$

$$DV[w(s), s_0] \;\equiv\; \frac{1}{2\pi i}\oint_{|s|=s_0} ds\; w(s)\,\left(\Pi(s) - \Pi^{OPE}(s)\right) \;=\; \int_{s_0}^{\infty} ds\; w(s)\,\rho(s)$$

González-Prades-Pich '10, Catà-Golterman-Peris '05

$$\Pi(s) \equiv \Pi_{VV}(s) - \Pi_{AA}(s)$$

$$\lim_{s\to\infty} s^2\,\Pi(s) = 0 \quad\rightarrow\quad \Pi^{OPE}(s) = -\frac{O_6}{s^3} + \frac{O_8}{s^4} - \cdots$$

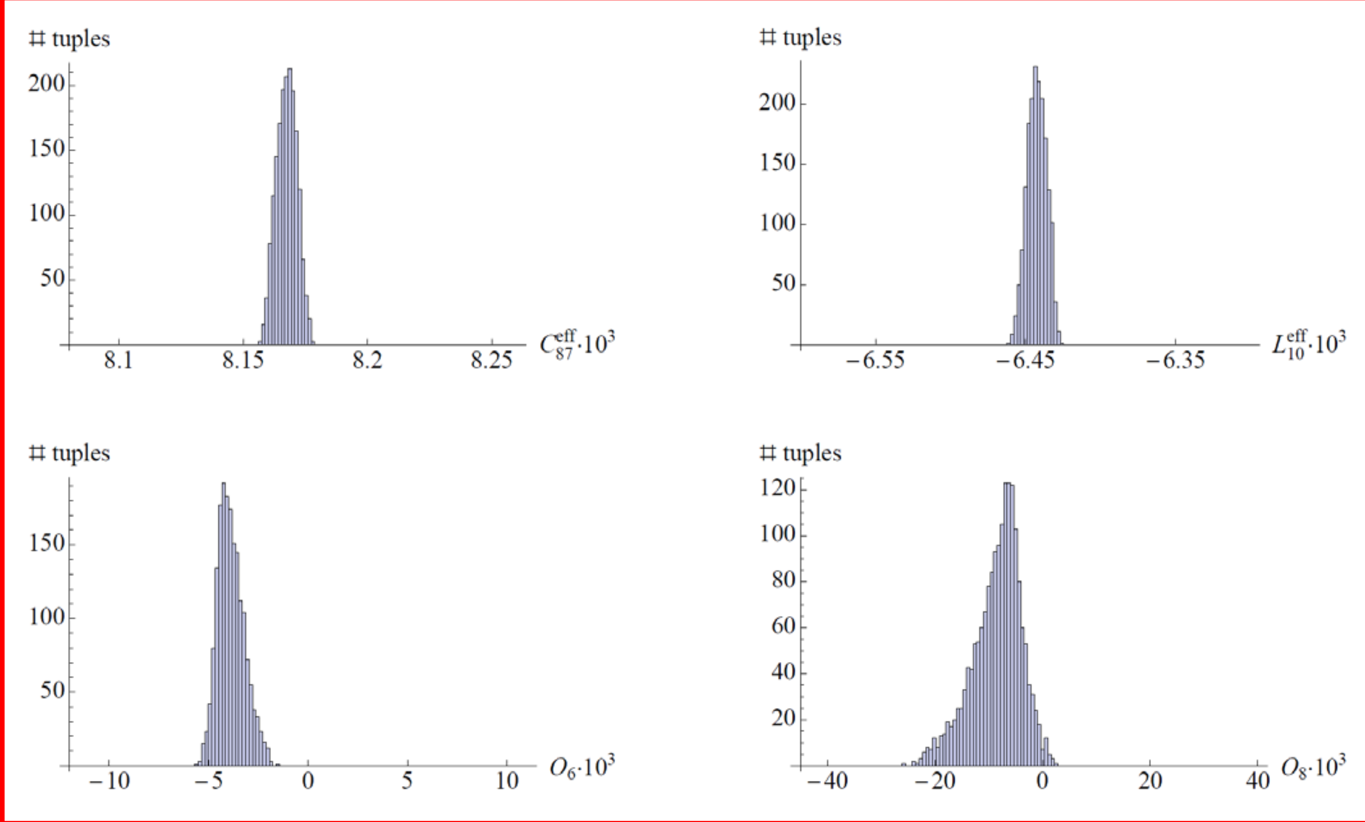
χPT: $$\Pi(t) = \frac{2F^2}{t} - 8L_{10}^r(\mu) - \frac{\Gamma_{10}}{4\pi^2}\left(\frac{5}{3} - \ln\frac{-t}{\mu^2}\right) + \frac{t}{F^2}\,16\,C_{87}^r(\mu) + \cdots$$

Statistical analysis: (González-Prades-Pich '10)

- DV parametrized through $\rho(s \geq s_z) = \kappa\, e^{-\gamma s}\, \sin[\beta\,(s-s_z)]$ (Shifman, Catà-Golterman-Peris)
- Generate 160.000 $(\kappa,\gamma,\beta,s_z)$ tuples
- Data fit (ALEPH) + QCD constraints → 1.789 "acceptable" spectral functions
- Wanted QCD parameters determined for each acceptable spectral function
- $w(s) = s^n\,(s-s_z)^m$ (pinched moments)

Pinched Weights

(González-Prades-Pich ’10)



$$
\begin{aligned}
C_{87}^{\rm eff} &= \left(8.168\,^{+0.003}_{-0.004} \pm 0.12\right)\cdot 10^{-3}\ \mathrm{GeV}^{-2} = \left(8.17 \pm 0.12\right)\cdot 10^{-3}\ \mathrm{GeV}^{-2},\\
L_{10}^{\rm eff} &= \left(-6.444\,^{+0.007}_{-0.004} \pm 0.05\right)\cdot 10^{-3} = \left(-6.44 \pm 0.05\right)\cdot 10^{-3}\ ,\\
\mathcal{O}_6 &= \left(-4.33\,^{+0.68}_{-0.34} \pm 0.65\right)\cdot 10^{-3}\ \mathrm{GeV}^{6} = \left(-4.3\,^{+0.9}_{-0.7}\right)\cdot 10^{-3}\ \mathrm{GeV}^{6}\ ,\\
\mathcal{O}_8 &= \left(-7.2\,^{+3.1}_{-4.4} \pm 2.9\right)\cdot 10^{-3}\ \mathrm{GeV}^{8} = \left(-7.2\,^{+4.2}_{-5.3}\right)\cdot 10^{-3}\ \mathrm{GeV}^{8}\ ,
\end{aligned}
$$

(González-Prades-Pich)

χPT$_2$	χPT$_3$
$\bar{l}_5 = 12.24 \pm 0.21$	$L_{10}^r(M_\rho) = -(4.06 \pm 0.39) \cdot 10^{-3}$
$\bar{l}_6 = 15.22 \pm 0.39$	$L_9^r(M_\rho) = (5.50 \pm 0.40) \cdot 10^{-3}$
$c_{50}^r = (4.95 \pm 0.19) \cdot 10^{-3}\ \mathrm{GeV}^{-2}$	$C_{87}^r(M_\rho) = (4.89 \pm 0.19) \cdot 10^{-3}\ \mathrm{GeV}^{-2}$

Table 1: Results for the χPT LECs obtained at $\mathcal{O}(p^6)$.

χPT$_2$	χPT$_3$
$\bar{l}_5 = 13.30 \pm 0.11$	$L_{10}^r(M_\rho) = -(5.22 \pm 0.06) \cdot 10^{-3}$
$\bar{l}_6 = 15.80 \pm 0.29$	$L_9^r(M_\rho) = (6.54 \pm 0.15) \cdot 10^{-3}$

Table 2: Results for the χPT LECs obtained at $\mathcal{O}(p^4)$.

Lattice , O(p^4)

$$L_{10}^r(M_\rho) = \begin{cases} -(5.2 \pm 0.5) \cdot 10^{-3} & \text{JLQCD} \\ -(5.7 \pm 1.1 \pm 0.7) \cdot 10^{-3} & \text{RBC/UKQCD} \end{cases}$$

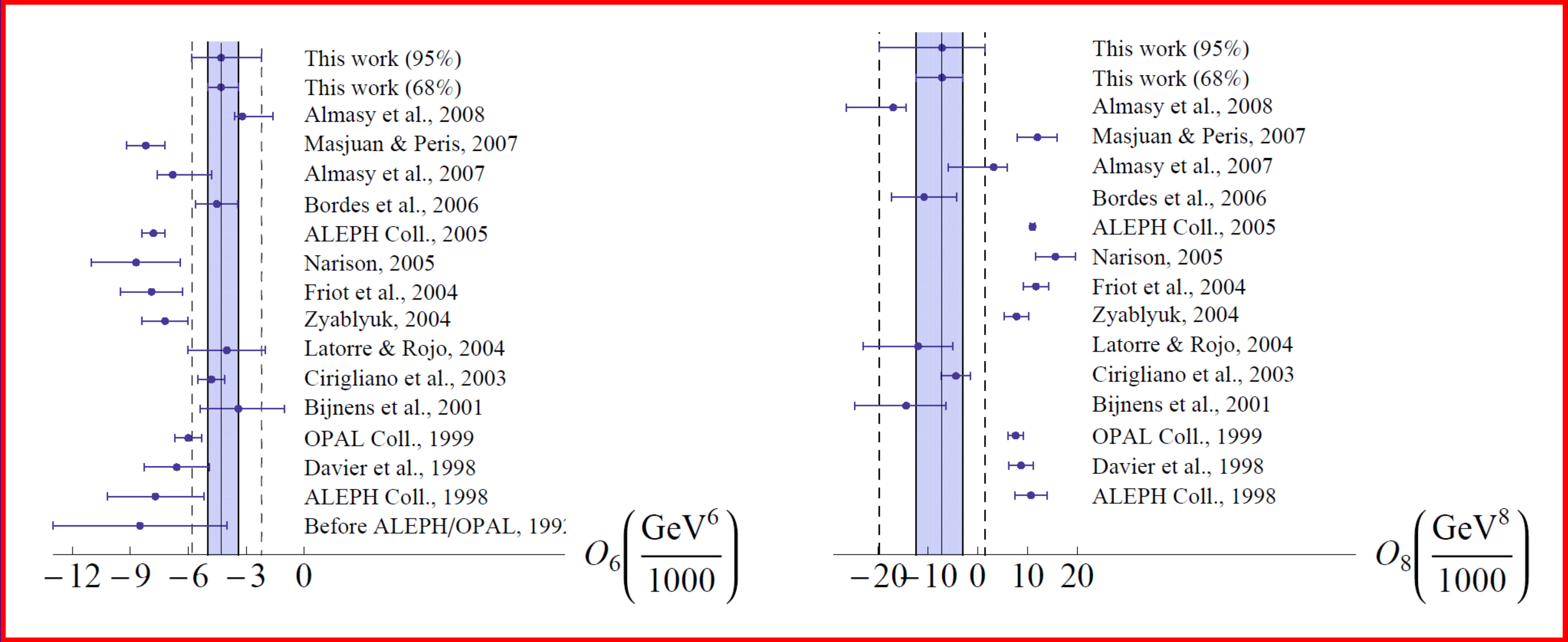
$$\bar{l}_6 = \begin{cases} 14.9 \pm 1.2 \pm 0.7 & \text{ETM} \\ 11.9 \pm 0.7 \pm 1.0 & \text{JLQCD/TWQCD} \end{cases}$$

RχT prediction (NLO):

Pich, Rosell, Sanz-Cillero '08

$$L_{10}^r(M_\rho) = -(4.4 \pm 0.9) \cdot 10^{-3} \quad , \quad C_{87}^r(M_\rho) = (3.6 \pm 1.3) \cdot 10^{-3}\ \mathrm{GeV}^{-2}$$

(González-Prades-Pich '10)



Implications for ε'/ε : Electromagnetic Penguin ($Im A_2$)

SUMMARY

- **Very precise determination of α_s from τ decays**

$$\alpha_s(m_\tau^2) = 0.342 \pm 0.010 \quad \Rightarrow \quad \alpha_s(M_Z^2) = 0.1213 \pm 0.0014$$

Error assessment: Higher perturbative orders, δ_{NP}

- **The τ could give the most precise V_{us} determination**

$$|V_{us}| = 0.2164 \pm 0.0027_{exp} \pm 0.0005_{th}$$

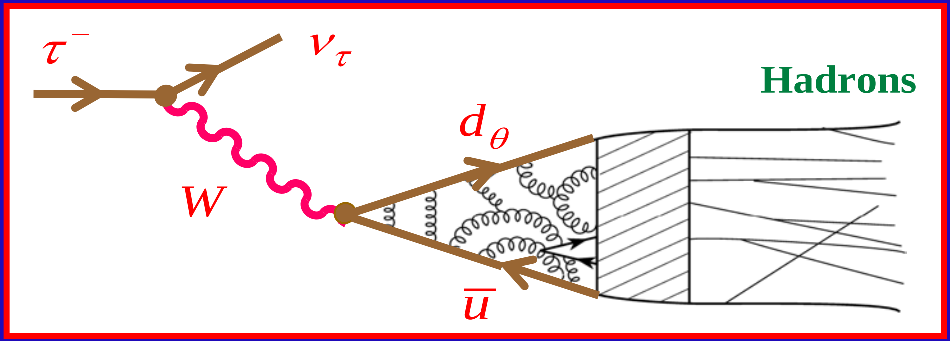
(present τ data)

K_{l3}: $|V_{us}| = 0.2241 \pm 0.0024$ $[\, f_+(0) = 0.965 \pm 0.010 \,]$

Data normalization, unmeasured modes ...

- **Many low-energy QCD tests from decay distributions**

Chiral Dynamics



The 11th International Workshop on Tau Lepton Physics
Manchester, UK, 13-17 September 2010

To the memory
of our friend
Ximo Prades