Charged LFV in a low-energy see-saw mSUGRA model

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- We present new SUSY mechanism for LFV in MSSM $+3N$ (low scale heavy singlet neutrinos), independent of soft SUSY breaking in mSUGRA framework.
- On-mass-shell $\ell \to \ell'\gamma$ amplitude suppressed/forbidden, other amplitudes enhanced.
- Comparison with experiment : $\mu \to e$ conversion, $\mu \to 3e$, $\tau \to 3e/e + 2\mu \ldots$

Standard MSSM+3N LFV

Leptonic part of the superpotential

$$W = Y_e^{ij} E_{iR}^c H_{dL} \cdot L_{jL} + Y_\nu^{ij} N_{iR}^c H_{uL} \cdot L_{jL} + \frac{1}{2} M_M^{ij} N_{iR}^c N_{jR}^c$$

LFV : Borzumati, Masiero PRL (1986) 961;

$$\mathcal{M}_{\tilde{e}}^2 = \begin{pmatrix} M_{\tilde{L}}^2 + (m_e m_e^\dagger) + D_1 \mathbf{1} & m_e (A_e^* - \mu t_\beta \mathbf{1}) \\ (A_e^T - \mu^* t_\beta \mathbf{1}) m_e^\dagger & M_{\tilde{e}}^2 + (m_e^\dagger m_e) + D_2 \mathbf{1} \end{pmatrix}$$

$$(\Delta M_{\tilde{L}}^2)_{ij} \approx -\frac{1}{8\pi^2} (3 m_0^2 + A_0^2) Y_\nu^\dagger Y_\nu \log \frac{M_X}{M_N},$$

$$(A_e)_{ij} \approx -\frac{3}{8\pi^2} A_0 Y_e Y_\nu^\dagger Y_\nu \log \frac{M_X}{M_N},$$

All SUSY LFV studies : LFV induced by soft-SUSY breaking

LFV in low-scale see-saw models

- **New SUSY mechanism: $\mathbf{m_N \gtrsim 1}$ TeV**

- LFV parameters :

$$\Omega_{\ell\ell'} \;=\; \frac{v_u^2}{2m_N^2}(Y_\nu^\dagger Y_\nu)_{\ell\ell'} \;=\; B^*_{\ell N_i} B_{\ell' N_i}$$

- Neutrino mass matrix (m_e diagonal basis)

$$M_\nu \;=\; \begin{pmatrix} 0 & m_D^T \\ m_D & M_M \end{pmatrix}, \qquad \begin{array}{l} M_\nu B^{\nu\dagger} \;=\; 0, \\ m_{n_i} \approx m_{n_j}, \quad i,j > 3 \end{array}$$

· $\nu_\ell^{SM} \;=\; (Bn)_\ell \;=\; (B^\nu \nu)_\ell + (B^N N)_\ell$

· ν masses radiatively induced

- Sneutrino mass matrix

$$\mathcal{M}^2_{\tilde{\nu}} = \begin{pmatrix} H_1 & N & 0 & M \\ N^\dagger & H_2^T & M^T & 0 \\ 0 & M^* & H_1^T & N^* \\ M^\dagger & 0 & N^T & H_2 \end{pmatrix},$$

$$H_1 = m^2_{\tilde{L}} + (\frac{1}{2} M_Z^2 c_{2\beta} \mathbf{1}) + (m_D m_D^\dagger)$$

$$H_2 = m^2_{\tilde{\nu}} + (m_D^\dagger m_D) + (M_M^\dagger M_M)$$

$$M = m_D (A_\nu - \mu \, ct_\beta)$$

$$N = m_D M_M^\dagger, \qquad M_B \equiv \frac{1}{2} B_{IJ} (M_\nu)_{IJ} \rightarrow 0$$

- N-$\tilde{N}$ sector nearly supersymmetric if $m_N \gg m_{SUSY}$ and $Y_\nu \leq 0.2$

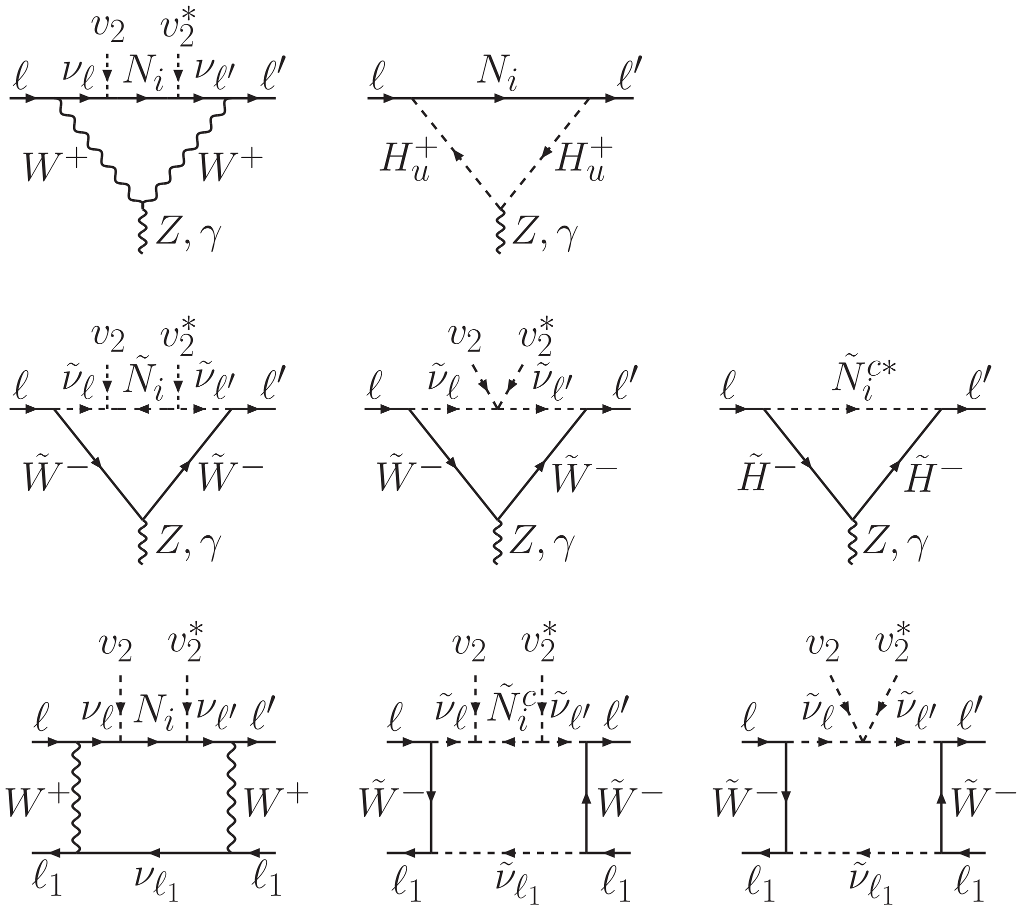
- **Pinpoint the SUSY LFV effects :**

- $\mu \ll M_N$
- $\tilde{M}^2_L$, $\tilde{M}^2_e$, A_e diagonal at M_N scale

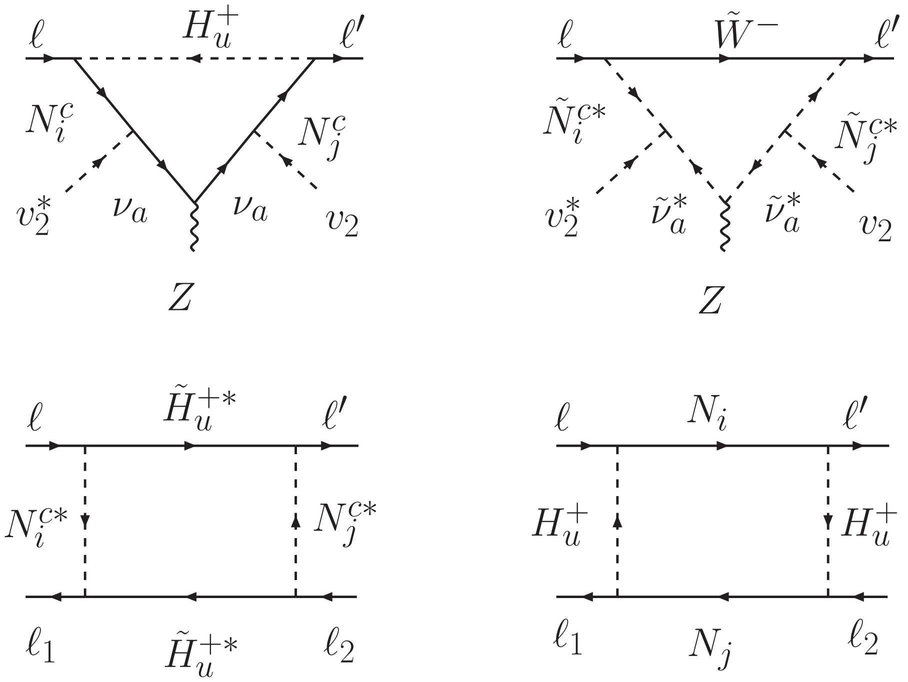
Amplitudes : Dominant contributions

- dominant terms in lowest order in g_W and v_u (Y_ν)

Two Yukawas



Four Yukawas



Amplitudes : structure

$$\mathcal{T}_\mu^{\ell\ell'\gamma} = \frac{e\alpha_W}{8\pi M_W^2}\bar{\ell}'\big(F_\gamma^{\ell\ell'}(q^2\gamma_\mu - \not{q}q_\mu)P_L + G_\gamma^{\ell\ell'} i\sigma_{\mu\nu}q^\nu m_\ell P_R\big)\ell,$$

$$\mathcal{T}_\mu^{\ell\ell' Z} = \frac{g_W\alpha_W}{8\pi c_W}\bar{\ell}'\gamma_\mu P_L \ell F_Z^{\ell'\ell}$$

$$\mathcal{T}_{box}^{\ell\ell'\ell_1\ell_2} = -\frac{\alpha_W^2}{4M_W^2}F_{box}^{\ell\ell'\ell_1\ell_2}\bar{\ell}'\gamma_\mu P_L \ell\bar{\ell}_1\gamma^\mu P_L \ell_2$$

$$\mathcal{T}_{box}^{\ell\ell' qq} = -\frac{\alpha_W^2}{4M_W^2}F_{box}^{\ell\ell' qq}\bar{\ell}'\gamma_\mu P_L \ell\bar{q}\gamma^\mu P_L q, \qquad q = u, d$$

Form factors

$$
\begin{aligned}
(F_\gamma^{\ell\ell'})^N &= \frac{\Omega_{\ell\ell'}}{6s_\beta^2}\ln\underbrace{\frac{m_N^2}{M_W^2}}_{=\lambda_N}, \\
(F_\gamma^{\ell\ell'})^{\tilde{N}} &= \frac{\Omega_{\ell\ell'}}{3s_\beta^2}\sum_{k=1}^{2}\mathcal{V}_{k1}^2 \ln\frac{m_N^2}{\tilde{m}_{\tilde{\chi}_k^2}}, \\
(G_\gamma^{\ell\ell'})^N &= -\Omega_{\ell\ell'}\left(\frac{1}{6s_\beta^2}+\frac{5}{6}\right) \\
(G_\gamma^{\ell\ell'})^{\tilde{N}} &= \Omega_{\ell\ell'}\left(\frac{1}{6s_\beta^2}+g_\gamma\right) \\
g_\gamma &= -\sum_{k=1}^{2}\left[\mathcal{V}_{k1}^2\frac{2M_W^2}{m_{\tilde{\chi}_i}^2}g_{\gamma,1}\left(\frac{m_{\tilde{\nu}}^2}{m_{\tilde{\chi}_i}^2}\right)+\mathcal{V}_{k1}\mathcal{U}_{k1}\frac{\sqrt{2}}{c_\beta}\frac{M_W^2}{m_{\tilde{\chi}_i}^2}g_{\gamma,2}\left(\frac{m_{\tilde{\nu}}^2}{m_{\tilde{\chi}_i}^2}\right)\right]
\end{aligned}
$$

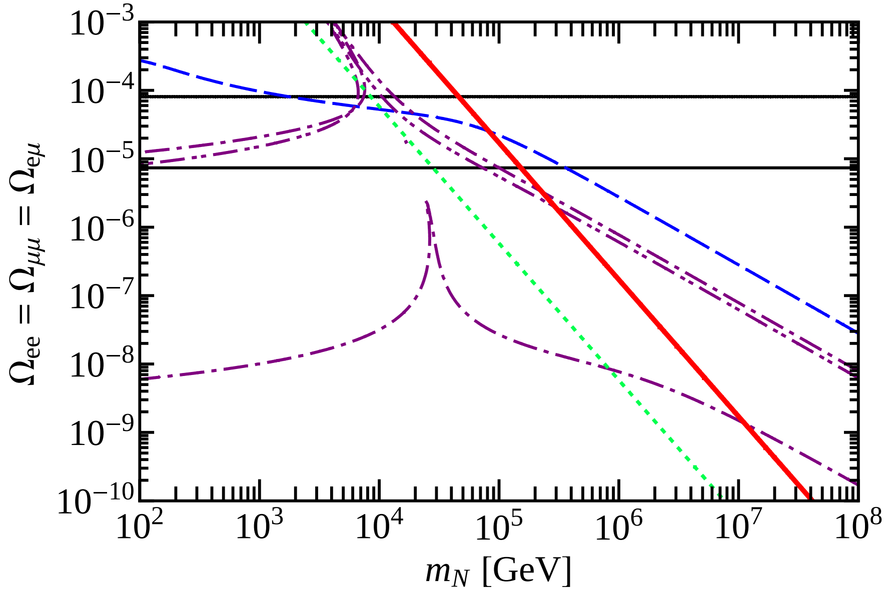
$$
\begin{aligned}
(F_Z^{\ell\ell'})^N &= -\frac{3\Omega_{\ell\ell'}}{2}\ln\frac{m_N^2}{M_W^2} - \frac{\Omega_{\ell\ell'}^2}{2s_\beta^2}\frac{m_N^2}{M_W^2}, \\
(F_Z^{\ell\ell'})^{\tilde{N}} &= \frac{\Omega_{\ell\ell'}}{2}\ln\frac{m_N^2}{\tilde{m}_1^2}\Big(-\frac{1}{2}+2s_W^2+\frac{1}{s_\beta^2}f_Z\Big) \\
f_Z &= \sum_{k,l=1}^{2}\frac{m_{\tilde{\chi}_k}m_{\tilde{\chi}_l}}{M_W^2}(\mathcal{V}_{k2}\mathcal{U}_{k1}\mathcal{U}_{l1}\mathcal{V}_{l2}+\frac{1}{2}\mathcal{V}_{k2}\mathcal{U}_{k2}\mathcal{U}_{l2}\mathcal{V}_{l2}-s_W^2\delta_{kl}\mathcal{V}_{k2}\mathcal{V}_{l2}) \\
(F_{box}^{\ell\ell'\ell_1\ell_2})^N &= -(\Omega_{\ell\ell'}\delta_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\delta_{\ell_2\ell'})+\frac{1}{4s_\beta^4}(\Omega_{\ell\ell'}\Omega_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\Omega_{\ell_2\ell'})\frac{m_N^2}{M_W^2} \\
(F_{box}^{\ell\ell'\ell_1\ell_2})^{\tilde{N}} &= (\Omega_{\ell\ell'}\delta_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\delta_{\ell_2\ell'})\ f_{box}^{\ell}+\frac{1}{4s_\beta^4}(\Omega_{\ell\ell'}\Omega_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\Omega_{\ell_2\ell'})\frac{m_N^2}{M_W^2} \\
f_{box}^{\ell} &= \sum_{k,l=1}^{2}\mathcal{V}_{k1}^2\mathcal{V}_{l1}^2 f_{box,1}^{\ell}(\lambda_{\tilde{\chi}_k},\lambda_{\tilde{\chi}_l},\lambda_{\tilde{\nu}},\lambda_N)+\mathcal{V}_{k2}\mathcal{V}_{k1}\mathcal{V}_{l2}\mathcal{V}_{l1}f_{box,2}^{\ell}()
\end{aligned}
$$

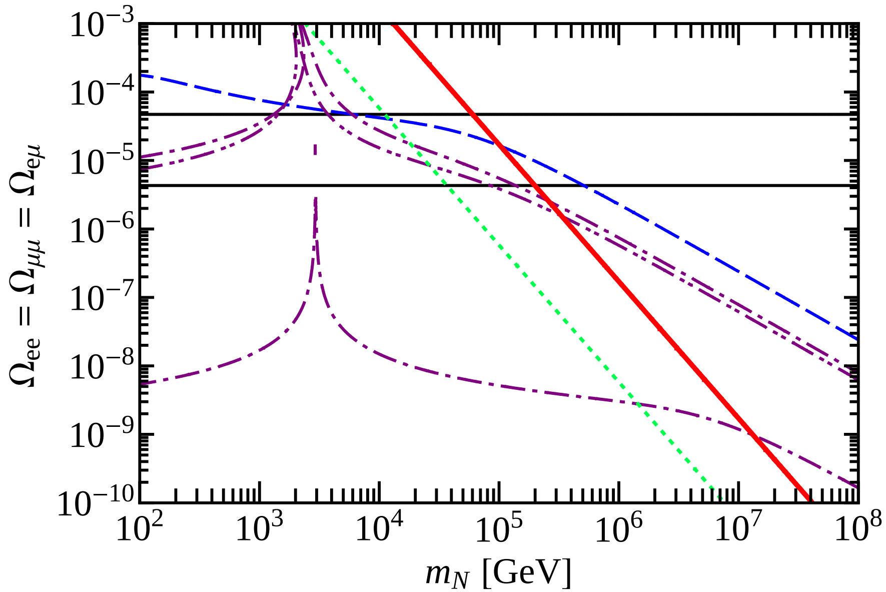
$$
\begin{aligned}
(F_{box}^{\ell\ell' uu})^N &= -4(F_{box}^{\ell\ell' dd})^N = 4\Omega_{e\mu} \\
(F_{box}^{\ell\ell' uu})^{\tilde{N}} &= \sum_{k,l=1}^{2} \mathcal{V}_{k1}^2 \mathcal{V}_{l1}^2 f_{box}^u(\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_l}, \lambda_{\tilde{d}}, \lambda_N) \\
(F_{box}^{\ell\ell' dd})^{\tilde{N}} &= \sum_{k,l=1}^{2} \mathcal{V}_{k1}^2 \mathcal{V}_{l1}^2 f_{box}^d(\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_l}, \lambda_{\tilde{u}}, \lambda_N)
\end{aligned}
$$

SUSY limit; cancelations, enhancements:

- $\tilde{m}^2_{\tilde{\chi}_{1,2}} \stackrel{SL}{\rightarrow} M_W^2, \quad t_\beta \stackrel{SL}{\rightarrow} 1, \quad \mu \stackrel{SL}{\rightarrow} 0$ (Barbieri, Giudice PLB309)
- $(G_\gamma^{\ell\ell'})^N + (G_\gamma^{\ell\ell'})^{\tilde{N}} \stackrel{SL}{=} 0$: Ferrara, Remiddi PLB53 (1974) 347
- box form factors : positive interference
- Y_ν^4 terms : become important when $Y_\nu/g_W \sim 1$ $\left(\Omega_{\ell\ell'}\frac{m_N^2}{M_W^2} = 2(Y^\dagger Y)_{\ell\ell'}/g_W^2\right)$
  (A. Pilaftsis, A.I, NPB437 (1995) 491)

Numerical estimates





$\tan\beta = 3$

$\mu = \tilde{M}_Q = M_{\tilde{\nu}} = 200$ GeV

$M_{\tilde{W}} = 100$ GeV

$\Omega_{\mu e} = \Omega_{ee} = \Omega_{\mu\mu}$, other $\Omega_{\ell\ell'} = 0$

Upper bounds

$B(\mu^- \to e^-\gamma)$	1.2×10^{-11}	[1]
	1×10^{-13}	[2]
$B(\mu^- \to e^-e^-e^+)$	1×10^{-12}	[1]
$R_{\mu e}^{Ti}$	4.3×10^{-12}	[3]
	1×10^{-18}	[4]
$R_{\mu e}^{Au}$	7×10^{-13}	[5]

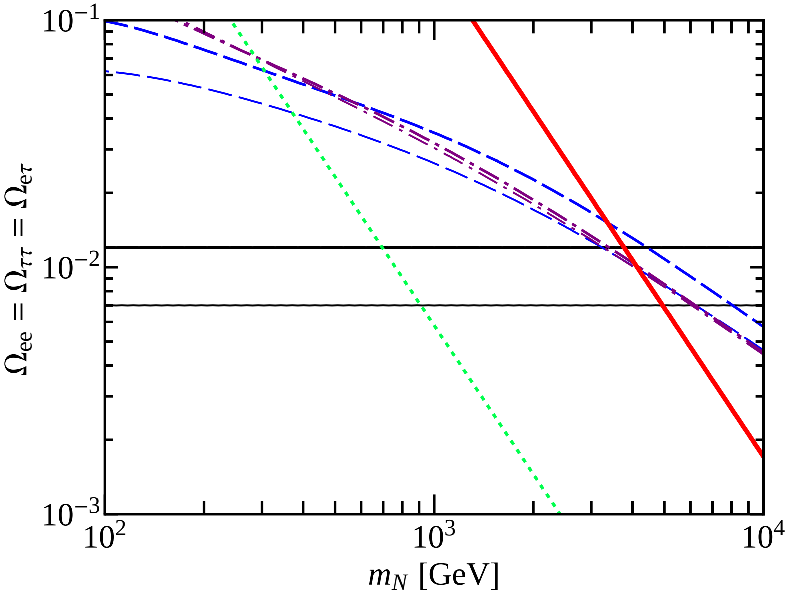
[1] Amsler, PLB 667 (2008) 1

[2] Ritt, NPBPS 162 (2006) 279

[3] Dohmen, PLB 317 (1993) 631

[4] Kuno, NPBPS 149 (2005) 376

[5] Bertl, EPJC 47 (2006) 337



$\Omega_{\tau e} = \Omega_{ee} = \Omega_{\tau\tau}$, other $\Omega_{\ell\ell'} = 0$

Upper bounds

$B(\tau^- \to e^-\gamma)$	3.3×10^{-8}	[1]
$B(\tau^- \to e^-e^-e^+)$	2.7×10^{-8}	[1]
$B(\tau^- \to e^-\mu^-\mu^+)$	2.7×10^{-8}	[1]

[1] Nakamura, JPG 77 (2010) 1

mSUGRA Framework

Boundary conditions and RGEs:

1. SM parameters at M_Z scale (Fusaoka and Koide PRD57 (1998) 3986).
2. Neutrino Yukawa and heavy neutrino masses at heavy neutrino scale m_N, (Pilaftsis PRL95 (081602) 2005, PRD72 (2005) 113001)

$$m_{N_i} = m_N, \qquad Y_n^T = \begin{pmatrix} 0 & a\,e^{-i\pi/4} & a\,e^{-i\pi/4} \\ 0 & b\,e^{-i\pi/4} & b\,e^{-i\pi/4} \\ 0 & c\,e^{-i\pi/4} & c\,e^{-i\pi/4} \end{pmatrix}.$$

3. mSUGRA conditions at gauge unification scale $g_1 = g_2 = g_3$,

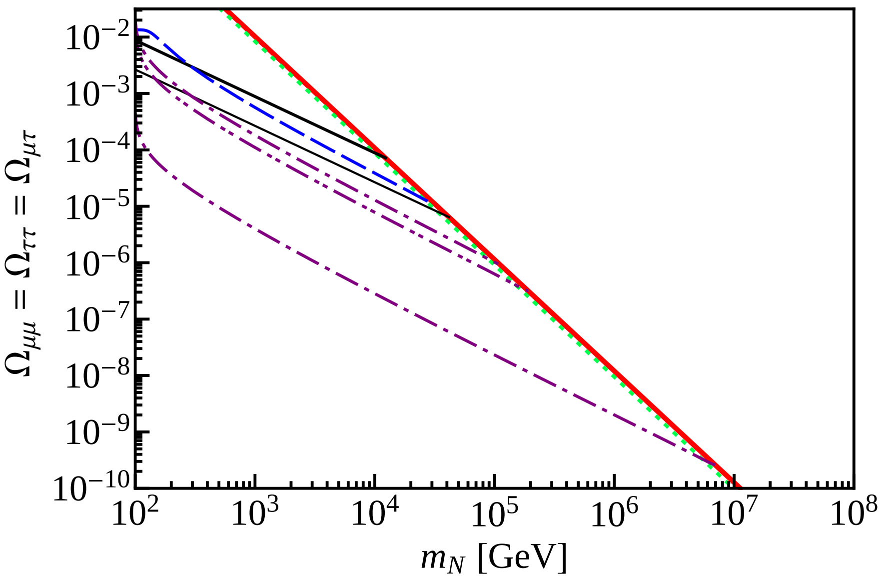
$$\begin{aligned} m^2_{H_1,H_2} &= m_0^2, \qquad m^2_{\tilde{u},\tilde{d},\tilde{e},\tilde{n}} = m_0^2\,\mathbf{1} \\ M_{1,2,3} &= M_0, \qquad A_{u,d,e,n} = A_0\,\mathbf{1}\,. \end{aligned}$$

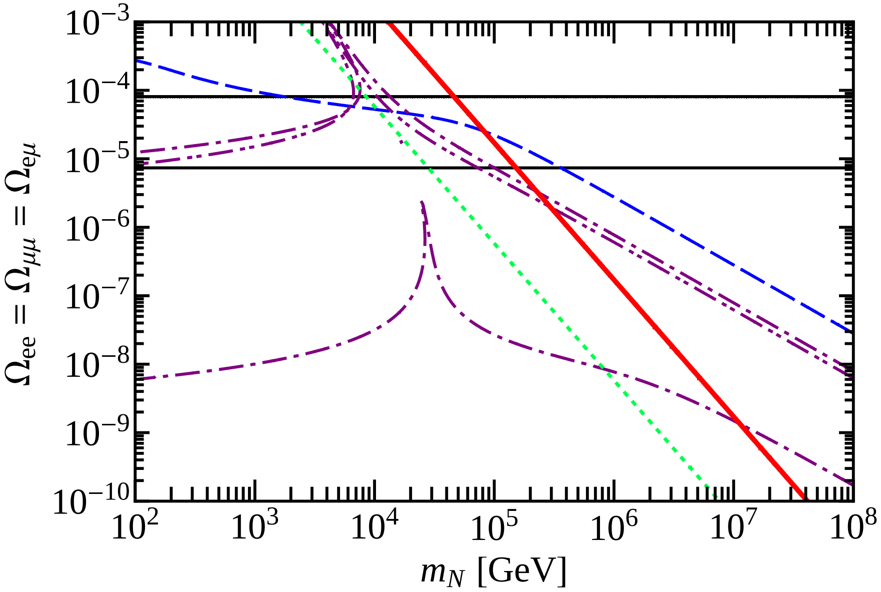
We took

$$m_0 = 100 \text{ GeV}, \qquad M_0 = 250 \text{ GeV}, \qquad A_0 = 100 \text{ GeV}\,.$$

4. MSSM+$3N$ RGE equations (Petcov NPB676 (2004) 453).

Numerical estimates





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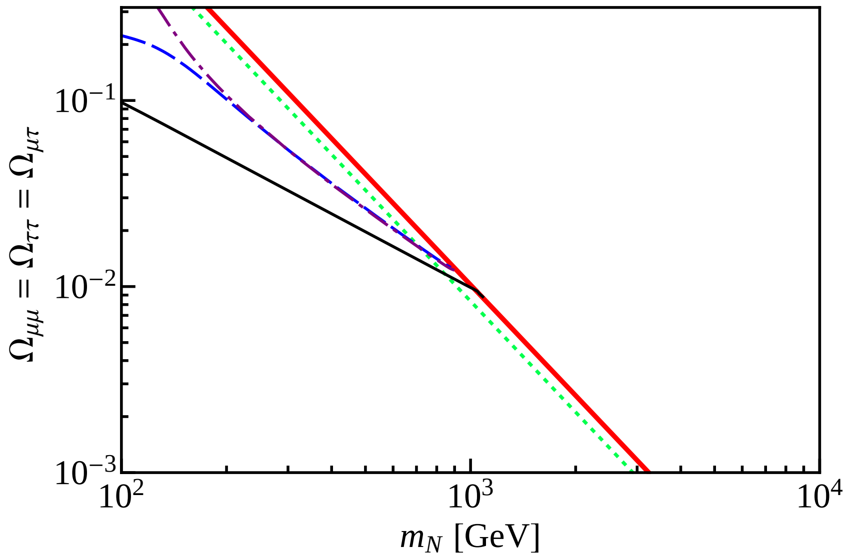

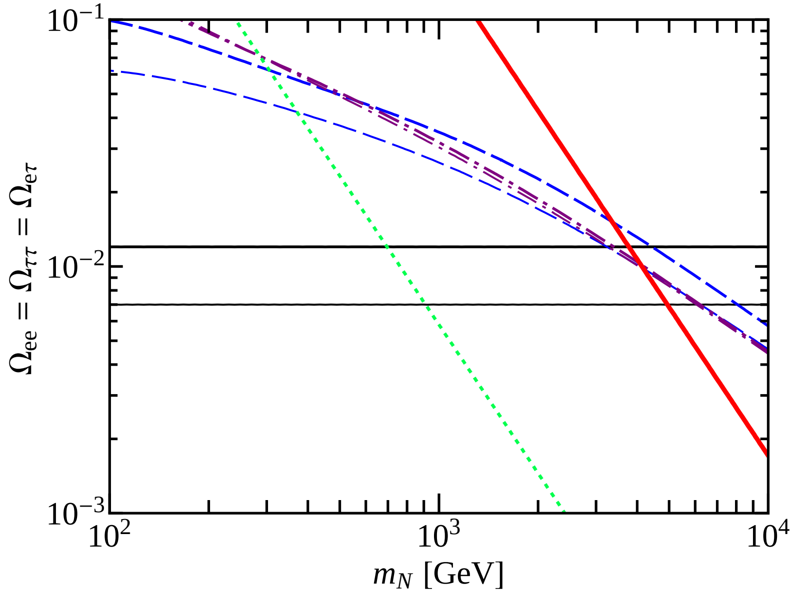

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Summary

- We have shown that in the low-scaled supersymmetric see-saw models sneutrinos might give large effects indenpenent of SUSY breaking mechanism.
- Due to SUSY the $\ell \to \ell' \gamma$ are suppressed.
- That makes $\mu \to e$ conversion especially interesting candidate for finding LFV. $\mu \to 3e$ and $\tau \to 3e$ give complementary information on LFV.
- Inclusion of the mSUGRA boundary conditions strongly influences te final results of the model.