Charged LFV and lepton dipole moments in a low-scale seesaw mSUGRA model

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Based on : A. Pilaftsis, L. Popov, A.I., PRD 87 (2013) 053014, PRD 89 (2014) 015001

- The model: two sources of LFV: soft-SUSY breaking sector; neutrino Yukawa sector: supersymmetric.
- Amplitudes : diagrams, form factor structure
- Numerical results for $\mu \to e$ conversion, $\mu \to 3e$, $\tau \to 3e/e+2\mu \ldots a_\mu$, electron EDM: dominance of Z-boson and box amplitudes, EDM - only from SB phases of A and B.

Motivation

Experiment

Observable	Upper Limit	Future sensitivity
$B(\mu \to e\gamma)$	2.4×10^{-12} [1]	$1-2 \times 10^{-13}$ [6], 10^{-14} [6]
$B(\mu \to eee)$	10^{-12} [2]	10^{-16} [8], 10^{-17} [7]
$R_{\mu e}^{\mathrm{Ti}}$	4.3×10^{-12} [3],	$3-7 \times 10^{-17}$ [10, 9], 10^{-18} [11, 7]
$R_{\mu e}^{\mathrm{Au}}$	7×10^{-13} [4]	$3-7 \times 10^{-17}$ [10, 9], 10^{-18} [11, 7]
$B(\tau \to e\gamma)$	3.3×10^{-8} [5]	$1-2 \times 10^{-9}$ [13, 12]
$B(\tau \to \mu\gamma)$	4.4×10^{-8} [5]	2×10^{-9} [13, 12]
$B(\tau \to eee)$	2.7×10^{-8} [5]	2×10^{-10} [13, 12]
$B(\tau \to e\mu\mu)$	2.7×10^{-8} [5]	10^{-10} [12]
$B(\tau \to \mu\mu\mu)$	2.1×10^{-8} [5]	2×10^{-10} [13, 12]
$B(\tau \to \mu ee)$	1.8×10^{-8} [5]	10^{-10} [12]
d_e	1.05×10^{-27} e cm [14, 15, 16]	$10^{-29} - 10^{-31}$ e cm [16]
a_μ^{exp}	Present sensitivity ($\delta a_\mu / a_\mu$)	Future sensitivity
$(116592089) \times 10^{-11}$	0.54×10^{-6} [14]	0.14×10^{-6} [17, 18]

Table 1: Current upper limits and future sensitivities of CLFV observables, electron EDM and muon MDM.

[1] J. Adam, PRL (MEG) 107 (2011) 171801

[2] U. Bellgart, (SINDRUM) NPB 299 (1988) 1

[3] C. Dohmen, (SINDRUM II) PLB 317 (1993) 631

[4] W. Bertl, EPJ C47 (2006) 337

[5] See A.I., arXiv:1212.5939, Ref. [11]

[6] B.A. Golden (MEG) PhD 2012, J. Adam (MEG) PhD 2012

[7] J.L. Hewett, arXiv:1205.2671

[8] N. Berger, (μ3e) JPCS 408, 122070 (2013)

[9] A. Kurup (COMET) NPPS 218, 38 (2011)

[10] R.J. Abrams (Mu2e) arXiv:1211.7019; E.C. Dukes NPPS 218 (2011) 44

[11] Y. Kuno (PRISM) NPPS 149 (2005) 376; R.J. Barow (PRISM) , NPPS 218 (2011) 44

[12] K. Hayasaka, JPCS 171 (2009) 012079

[13] M. Bona (SuperB), arXiv:0709.0451

[14] J. Beringer (PDG) PRD 86 (2012) 010001

[15] J. Hudson Nature 473 (2011) 493

[16] M. Jung, JHEP 1305 (2013) 168

[17] B.L. Roberts NPPS 218 (2011) 237

[18] G. Venanzoni, arXiv:1203.1501; J.Phys.Conf.Ser. 349 (2012) 012008

Theory

- LFV: found in neutrino oscillations only: sign for a physics BSM: scale not determined;
- CLFV, $d_e > 10^{-33}$ e cm, $(a_\mu^{th} - a_\mu^{exp})/a_\mu^{exp} > (a_\mu$ sensitivity): would be independent sign for a physics BSM
– information on a scale of new physics

Standard MSSM+3N LFV

Leptonic part of the superpotential

$$W = h_e^{ij} E_{iR}^c H_{dL} \cdot L_{jL} + h_\nu^{ij} N_{iR}^c H_{uL} \cdot L_{jL} + \frac{1}{2} M_M^{ij} N_{iR}^c N_{jR}^c$$

LFV : Borzumati, Masiero PRL (1986) 961;

$$\mathcal{M}_{\tilde{e}}^2 = \begin{pmatrix} M_{\tilde{L}}^2 + (m_e m_e^\dagger) + D_1 \mathbf{1} & m_e(A_e^* - \mu t_\beta \mathbf{1}) \\ (A_e^T - \mu^* t_\beta \mathbf{1}) m_e^\dagger & M_{\tilde{e}}^2 + (m_e^\dagger m_e) + D_2 \mathbf{1} \end{pmatrix}$$

$$(\Delta M_{\tilde{L}}^2)_{ij} \approx -\frac{1}{8\pi^2}(3m_0^2 + A_0^2) h_\nu^\dagger h_\nu \log \frac{M_X}{M_N},$$

$$(A_e)_{ij} \approx -\frac{3}{8\pi^2} A_0 h_e h_\nu^\dagger h_\nu \log \frac{M_X}{M_N},$$

Since recently : in SUSY LFV studies LFV induced by soft-SUSY breaking terms only

LFV in low-scale seesaw models (ν_RMSSM)

- **New supersymmetric LFV mechanism: $\mathbf{m_N \gtrsim 1}$ TeV**

- LFV parameters in N sector:

$$\Omega_{\ell\ell'} = \frac{v_u^2}{2m_N^2}(h_\nu^\dagger h_\nu)_{\ell\ell'} = B^*_{\ell N_i} B_{\ell' N_i}$$

- Neutrino mass matrix (m_e diagonal basis; at scale m_N)

$$M_\nu = \begin{pmatrix} 0 & m_D^T \\ m_D & M_M \end{pmatrix}, \qquad \begin{array}{rcl} M_\nu B^{\nu\dagger} & = & 0, \quad m_{n_i} \approx m_{n_j}, \quad i,j > 3, \\ m_D & = & \sqrt{2} M_W s_\beta g^{-1} h_\nu^\dagger \end{array}$$

$$h_\nu = \begin{pmatrix} 0 & 0 & 0 \\ ae^{-\frac{i\pi}{4}} & be^{-\frac{i\pi}{4}} & ce^{-\frac{i\pi}{4}} \\ ae^{\frac{i\pi}{4}} & be^{\frac{i\pi}{4}} & ce^{\frac{i\pi}{4}} \end{pmatrix} \qquad h_\nu = \begin{pmatrix} a^* & b^* & c^* \\ a^* e^{-\frac{2\pi i}{3}} & b^* e^{-\frac{2\pi i}{3}} & c^* e^{-\frac{2\pi i}{3}} \\ a^* e^{\frac{2\pi i}{3}} & b^* e^{\frac{2\pi i}{3}} & c^* e^{\frac{2\pi i}{3}} \end{pmatrix}$$

· $\nu_\ell^{SM} = (Bn)_\ell = (B^\nu \nu)_\ell + (B^N N)_\ell$: B diagonalizes M_ν

· ν masses : sym. breaking; radiatively induced

- Sneutrino mass matrix

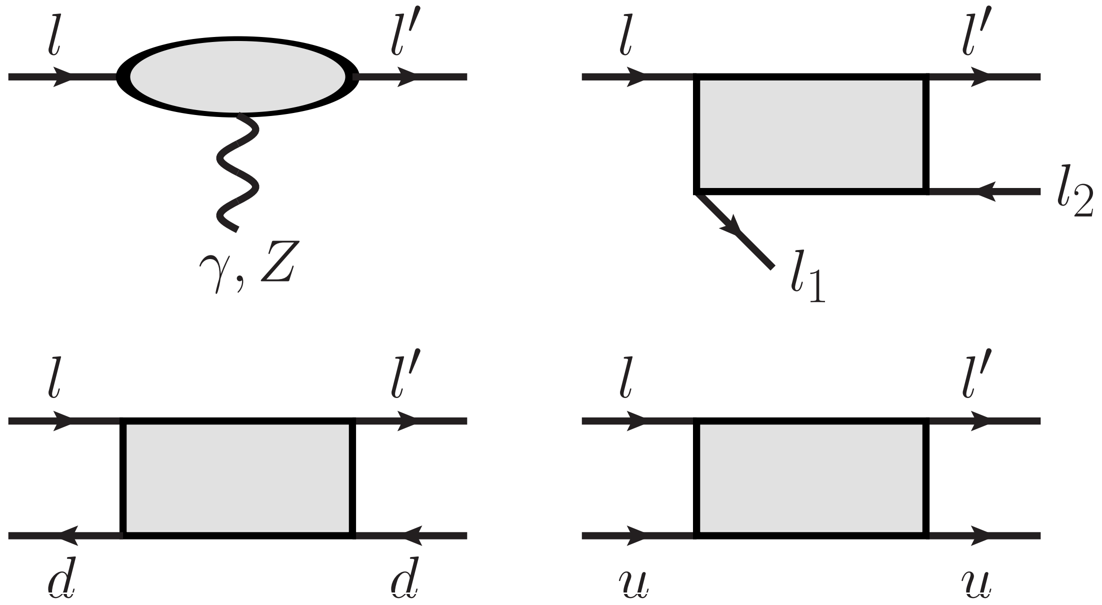
$$M_{\tilde{\nu}}^2 = \begin{pmatrix} H_1 & N & 0 & M \\ N^\dagger & H_2^T & M^T & M_B \\ 0 & M^* & H_1^T & N^* \\ M^\dagger & M_B^\dagger & N^T & H_2 \end{pmatrix}, \qquad M_{\tilde{\nu}}^2 \stackrel{SUSY}{\longrightarrow} \begin{pmatrix} M_\nu M_\nu^\dagger & 0_{6\times 6} \\ 0_{6\times 6} & M_\nu^\dagger M_\nu \end{pmatrix}$$

$$\begin{aligned} H_1 &= m_{\tilde{L}}^2 + (\frac{1}{2}M_Z^2 c_{2\beta}\mathbf{1}) + (m_D m_D^\dagger) \\ H_2 &= m_{\tilde{\nu}}^2 + (m_D^\dagger m_D) + (M_M^\dagger M_M) \\ M &= m_D(A_\nu - \mu\, ct_\beta) \\ N &= m_D M_M^\dagger, \qquad M_B \equiv \frac{1}{2}B_{IJ}(M_\nu)_{IJ} \to 0; b_\nu \end{aligned}$$

- N-$\tilde{N}$ sector nearly supersymmetric if $m_N > m_{SUSY}$ and $h_\nu \leq 0.2$

Amplitudes

Amplitudes : diagrams



We took $\tan\beta < 20$. Neutral Higgs (h, H, A) contr. not taken into account

Amplitudes : structure

$$\mathcal{T}_\mu^{\gamma l' l} = \frac{e\,\alpha_w}{8\pi M_W^2}\,\bar{l}'[(F_\gamma^L)_{l'l}\,(q^2\gamma_\mu - \not{q}q_\mu)P_L + (F_\gamma^R)_{l'l}\,(q^2\gamma_\mu - \not{q}q_\mu)P_R$$

$$+(G_\gamma^L)_{l'l}\,i\sigma_{\mu\nu}q^\nu P_L + (G_\gamma^R)_{l'l}\,i\sigma_{\mu\nu}q^\nu P_R]\,l,$$

$$\mathcal{T}_\mu^{Z l' l} = \frac{g_w\,\alpha_w}{8\pi\cos\theta_w}\,\bar{l}'[(F_Z^L)_{l'l}\,\gamma_\mu P_L + (F_Z^R)_{l'l}\,\gamma_\mu P_R]\,l,$$

$$\mathcal{T}_\gamma^{l l' l_1 l_2} = \frac{\alpha_w^2 s_w^2}{2M_W^2}\{\delta_{l_1 l_2}\,\bar{l}'\,[(F_\gamma^L)_{l'l}\,\gamma_\mu P_L + (F_\gamma^R)_{l'l}\,\gamma_\mu P_R$$

$$+\frac{(\not{p}-\not{p}')}{(p-p')^2}((G_\gamma^L)_{l'l}\,\gamma_\mu P_L + (G_\gamma^R)_{l'l}\,\gamma_\mu P_R)]\,l\;\;\bar{l}_1\gamma^\mu l_2^C\;-\;[l' \leftrightarrow l_1]\}\;,$$

$$\mathcal{T}_Z^{l l' l_1 l_2} = \frac{\alpha_w^2}{2M_W^2}[\delta_{l_1 l_2}\,\bar{l}'((F_Z^L)_{l'l}\,\gamma_\mu P_L + (F_Z^R)_{l'l}\,\gamma_\mu P_R)l$$

$$\times\;\bar{l}_1(g_L^l\,\gamma^\mu P_L + g_R^l\,\gamma^\mu P_R)l_2^C - (l' \leftrightarrow l_1)]\;,$$

$$
\begin{aligned}
\mathcal{T}_{\rm box}^{ll'l_1l_2} &= -\frac{\alpha_w^2}{4M_W^2}\Big(B_{\ell V}^{LL}\,\bar{l}'\gamma_\mu P_L l\;\bar{l}_1\gamma^\mu P_L l_2^C + B_{\ell V}^{RR}\,\bar{l}'\gamma_\mu P_R l\;\bar{l}_1\gamma^\mu P_R l_2^C \\
&\quad + B_{\ell V}^{LR}\,\bar{l}'\gamma_\mu P_L l\;\bar{l}_1\gamma^\mu P_R l_2^C + B_{\ell V}^{RL}\,\bar{l}'\gamma_\mu P_R l\;\bar{l}_1\gamma^\mu P_L l_2^C \\
&\quad + B_{\ell S}^{LL}\,\bar{l}' P_L l\;\bar{l}_1 P_L l_2^C + B_{\ell S}^{RR}\,\bar{l}' P_R l\;\bar{l}_1 P_R l_2^C \\
&\quad + B_{\ell S}^{LR}\,\bar{l}' P_L l\;\bar{l}_1 P_R l_2^C + B_{\ell S}^{RL}\,\bar{l}' P_R l\;\bar{l}_1 P_L l_2^C \\
&\quad + B_{\ell T}^{LL}\,\bar{l}'\sigma_{\mu\nu} P_L l\;\bar{l}_1\sigma^{\mu\nu} P_L l_2^C + B_{\ell T}^{RR}\,\bar{l}'\sigma_{\mu\nu} P_R l\;\bar{l}_1\sigma^{\mu\nu} P_R l_2^C\Big) \\
&\equiv -\frac{\alpha_w^2}{4M_W^2}\sum_{X,Y=L,R}\;\sum_{A=V,S,T} B_{\ell A}^{XY}\,\bar{l}'\Gamma_A^X l\;\bar{l}_1\Gamma_A^Y l_2^C\,,
\end{aligned}
$$

$\mathcal{T}_{\rm box}^{ll'dd}$ and $\mathcal{T}_{\rm box}^{ll'uu}$ have the same structure as $\mathcal{T}_{\rm box}^{ll'l_1l_2}$

- form factors
- new form factors

Form factors

Contributions

1. γ, Z, l-box, sl-box; h, H, A not included
2. Each form factor in principle has heavy neutrino (N), sneutrino ($\tilde{N}$) and soft SUSY breaking SB contributions, for instance

$$(F_\gamma^L)_{l'l} \quad = \quad F^N_{l'l\gamma} + F^{L,\tilde{N}}_{l'l\gamma} + F^{L,\mathrm{SB}}_{l'l\gamma}$$

A.I., A. Pilaftsis, PRD80 (2009) 091902 : N, $\tilde{N}$; γ, Z, l-box, sl-box; $\nu_R MSSM$

M. Hirsch, F. Staub, A. Vicente, Phys.Rev. D85 (2012) 113013, A. Abada, D. Das, A. Vicente, C. Weiland: N, $\tilde{N}$, SB; γ, Z, higgs, l-box, sl-box, but no N-box, MSISM

A.I., A. Pilaftsis, L. Popov, PRD 87 (2013) 5, 053014: N, $\tilde{N}$, SB; γ, Z, l-box, sl-box but no higgs; $\nu_R MSSM$

M. E. Krauss, W. Porod, F. Staub, A. Abada, A. Vicente, C. Weiland, arXiv1312.5318, MSISM: Z not dominant

SUSY limit; cancelations:

- $\tilde{m}^2_{\tilde{\chi}_{1,2}} \stackrel{SL}{\to} M_W^2$, $\quad t_\beta \stackrel{SL}{\to} 1$, $\quad \mu \stackrel{SL}{\to} 0$ (Barbieri, Giudice PLB309)
- $(G_\gamma^{\ell\ell'})^N + (G_\gamma^{\ell\ell'})^{\tilde{N}} \stackrel{SL}{=} 0$: Ferrara, Remiddi PLB53 (1974) 347

Dipole moments

Lagrangian and dipole moments

$$\mathcal{L} = \bar{l}[\gamma(i\partial^\mu + eA^\mu) - m_l - \frac{e}{2m_l}\sigma^{\mu\nu}(F_l + iG_l\gamma_5)\partial_\nu A_\mu]l$$

$$a_l = F_l \qquad d_l = eG_l/m_l$$

Amplitude and dipole moments

$$i\mathcal{T}^{\gamma ll} = \frac{ie\alpha_w}{8\pi M_W^2}[(G_\gamma^L)_{ll} i\sigma_{\mu\nu}q^\nu P_L + G_\gamma^R)_{ll} i\sigma_{\mu\nu}q^\nu P_R]$$

$$a_l = \frac{\alpha_w m_l}{8\pi M_W^2}[(G_\gamma^L)_{ll} + (G_\gamma^R)_{ll}] \qquad d_l = \frac{e\alpha_w}{8\pi M_W^2} i[(G_\gamma^L)_{ll} - (G_\gamma^R)_{ll}]$$

Possible sources of lepton EDM (CPV)

$$A_\nu = h_\nu A_0 e^{i\phi} \qquad -(A_\nu)^{ij}\tilde{\nu}_R^c(h_{uL}^+\tilde{e}_{jL} - h_{uL}^0\tilde{\nu}_{jL})$$

$$b_\nu = B_0 e^{i\theta} m_N \mathbf{1}_3 \qquad (b_\nu)_{ii}\tilde{\nu}_{Ri}\tilde{\nu}_{Ri}$$

$$\Delta_{\rm CP}^{LR} = \tilde{B}_{lkA}^L \tilde{B}_{lkA}^{R*} \qquad \Delta_{\rm CP}^{RL} = \tilde{B}_{lkA}^R B_{lkA}^{L*}$$

Scaling behaviour of MSSM contribution dipole moments

$$a_l^{MSSM} \propto \frac{m_l^2}{M_{SUSY}^2} \tan\beta \,\mathsf{sign}(\mu M_{1,2}) \qquad \text{(checked)}$$

$$d_l \propto \frac{e\, m_l}{M_{SUSY}^2} \tan\beta \sin(\phi_{CP}) \qquad \text{(expected, checked)}$$

$$d_l \propto \frac{e\, m_l\, f(m_0)}{M_N^x} \tan\beta, \quad 2/3 < x < 1 \qquad \text{(found)}$$

mSUGRA Framework

Boundary conditions and RGEs:

1. SM parameters at M_Z scale (Fusaoka and Koide PRD57 (1998) 3986).
2. Neutrino Yukawa and heavy neutrino masses at heavy neutrino scale m_N, (Pilaftsis PRL95 (081602) 2005, PRD72 (2005) 113001, PRD83 (2011) 076007; J. Kersten, A.Y. Smirnov, PRD76 (2007) 073005)

$$
\begin{aligned}
m_{N_i} &= m_N, \\
h_\nu &= \begin{pmatrix} 0 & 0 & 0 \\ ae^{-\frac{i\pi}{4}} & be^{-\frac{i\pi}{4}} & ce^{-\frac{i\pi}{4}} \\ ae^{\frac{i\pi}{4}} & be^{\frac{i\pi}{4}} & ce^{\frac{i\pi}{4}} \end{pmatrix} \qquad h_\nu = \begin{pmatrix} a^* & b^* & c^* \\ a^* e^{-\frac{2\pi i}{3}} & b^* e^{-\frac{2\pi i}{3}} & c^* e^{-\frac{2\pi i}{3}} \\ a^* e^{\frac{2\pi i}{3}} & b^* e^{\frac{2\pi i}{3}} & c^* e^{\frac{2\pi i}{3}} \end{pmatrix}
\end{aligned}
$$

3. mSUGRA conditions at gauge unification scale $g_1 = g_2 = g_3$,

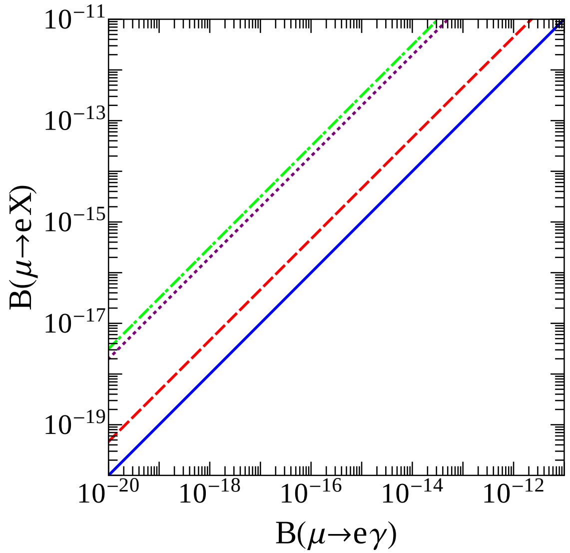
$$
\begin{aligned}
m^2_{H_1,H_2} &= m_0^2, \qquad m^2_{\tilde{u},\tilde{d},\tilde{e},\tilde{n}} = m_0^2\,\mathbf{1} \\
M_{1,2,3} &= M_0, \qquad A_{u,d,e,n} = A_0\, h_{u,d,e,n}\ .
\end{aligned}
$$

4. MSSM$+3N$ RGE equations (P. Chankowski and S. Pokorski, IJMP A17 (2002) 575, S. Petcov et al. NPB676 (2004) 453).

Numerical results for CLFV

Choice of parameters

1. $m_0 = 1000$ GeV, $A_0 = -3000$ GeV, $M_{1/2} = 1000$ GeV
   consistent with $m_h \approx 126$ GeV
   consistent with $m_{\tilde{g}},\ m_{\tilde{q}} > 1$ GeV
   in agreement with lightest neutralino as a dark matter candidate
2. $sign(\mu) > 0$
3. $\tan\beta = 10$ in most of calculations
4. Yukawa parameters:
   model 1: $a = b$, $c = 0$; $a = c$, $b = 0$; $b = c$, $a = 0$
   model 2: $a = b = c$
   Pertubativity condition $Trh_\nu^\dagger h_\nu < 4\pi$:
   model 1: $a < 0.34$
   model 2: $a < 0.23$
5. $m_N < 10$ TeV: consistency with resonant leptogenesis



$B(\mu \to e\gamma)$ $B(\mu \to eee)$ $R^{Ti}_{\mu e}$ $R^{Au}_{\mu e}$

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

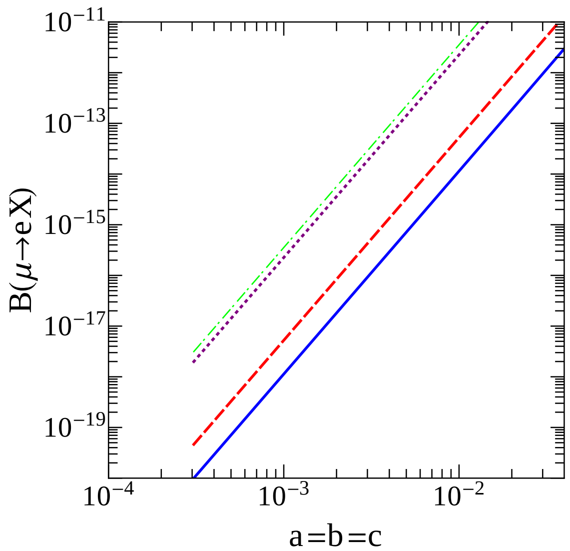
$m_N = 1$ TeV, $\tan\beta = 10$

model 2: $a = b = c$

pertubativity condition $Tr h^\dagger_\nu h_\nu < 4\pi$

quadratic Yukawa dependence

$R^{Au}_{\mu e}$, $R^{Ti}_{\mu e}$, $B(\mu \to eee) > B(\mu \to e\gamma)$



$B(\mu \to e\gamma)$ $B(\mu \to eee)$ $R^{Ti}_{\mu e}$ $R^{Au}_{\mu e}$

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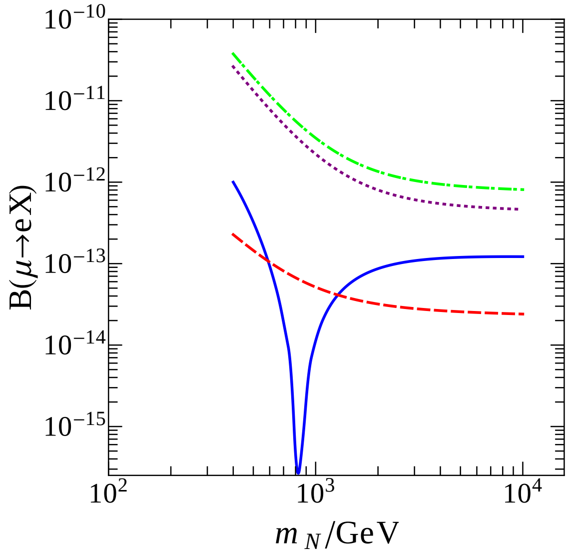


$B(\mu \to e\gamma)$ $B(\mu \to eee)$ $R^{Ti}_{\mu e}$ $R^{Au}_{\mu e}$

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

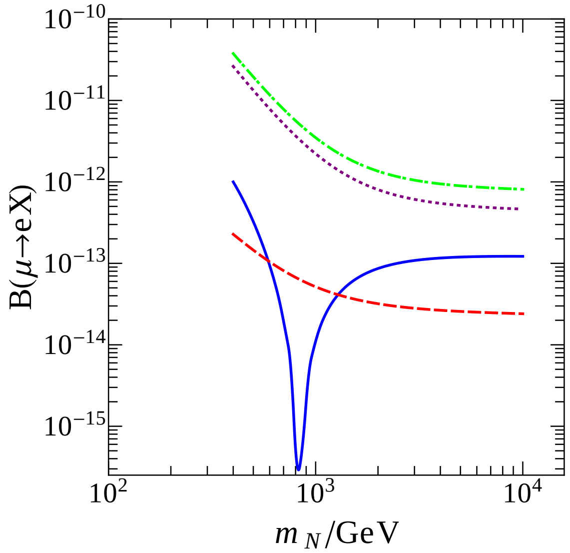
a: $B(\mu \to eee) = 10^{-12}$ for $m_N = 400$ GeV

$\tan\beta = 10$

model 2: $a = b = c$

$B(\mu \to e\gamma)$: cancelation of N, $\tilde{N}$ and SB contributions

$R^{Au}_{\mu e}$, $R^{Ti}_{\mu e} > B(\mu \to e\gamma)$



$B(\mu \to e\gamma)$ $B(\mu \to eee)$ $R^{Ti}_{\mu e}$ $R^{Au}_{\mu e}$

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

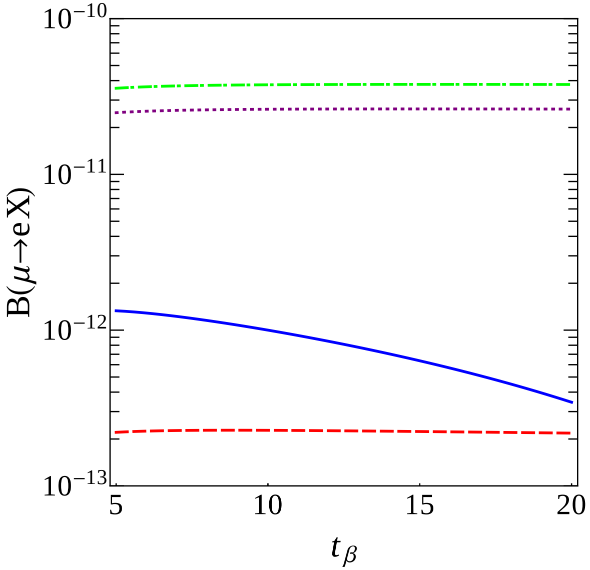
a: $B(\mu \to eee) = 10^{-12}$ for $m_N = 400$ GeV

$\tan\beta = 10$

model 1: $a = b$, $c = 0$

$B(\mu \to e\gamma)$: cancelation of N, $\tilde{N}$ and SB contributions

$R^{Au}_{\mu e}$, $R^{Ti}_{\mu e} > B(\mu \to e\gamma)$



$B(\mu \to e\gamma)$ $B(\mu \to eee)$ $R_{\mu e}^{Ti}$ $R_{\mu e}^{Au}$

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

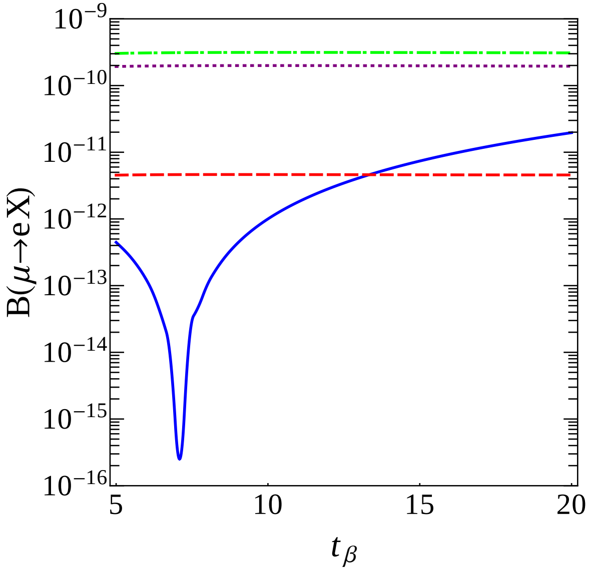
$m_N = 400$ GeV

a: $B(\mu \to eee) = 10^{-12}$ for $\tan\beta = 10$

model 2: $a = b = c$

weak dependence on $\tan\beta$

$R_{\mu e}^{Au},\ R_{\mu e}^{Ti} > B(\mu \to e\gamma)$



$B(\mu \to e\gamma)$ $B(\mu \to eee)$ $R_{\mu e}^{Ti}$ $R_{\mu e}^{Au}$

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

$m_N = 1$ TeV

a: $B(\mu \to eee) = 10^{-12}$ for $m_N = 1$ TeV

model 2: $a = b = c$

$B(\mu \to e\gamma)$: cancelation of N, $\tilde{N}$ and SB contributions

$R_{\mu e}^{Au},\ R_{\mu e}^{Ti} > B(\mu \to e\gamma)$

$$R_1 \equiv \frac{B(l \to l' l_1 l_1^c)}{B(l \to l'\gamma)} \to \frac{\alpha}{3\pi}\left(\ln\frac{m_l^2}{m_{l'}^2} - 3\right)$$

$$R_2 \equiv \frac{B(l \to l' l' l'^c)}{B(l \to l'\gamma)} \to \frac{\alpha}{3\pi}\left(\ln\frac{m_l^2}{m_{l'}^2} - \frac{11}{4}\right)$$

$$R_3 \equiv \frac{R^J_{\mu e}}{B(\mu \to e\gamma)}$$

$$\to 16\alpha^4 \frac{\Gamma_\mu}{\Gamma_{\text{capture}}} Z Z_{eff}^4 |F(-\mu^2)|^2$$

$(G^L_\gamma)_{l'l}$, $(G^R_\gamma)_{l'l}$ only:

$R_1(\tau \to e\mu\mu) = 1/90$,

$R_1(\tau \to e\mu\mu) = 1/419$

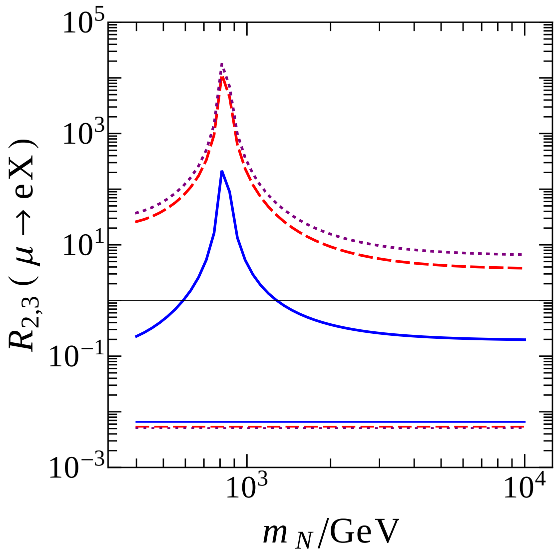
$R_2(\mu \to eee) = 1/159$,

$R_2(\tau \to eee) = 1/91$,

$R_2(\tau \to \mu\mu\mu) = 1/460$

$R_3^{Ti} = 1/198$,

$R_3^{Au} = 1/188$



$R_2(\mu \to eee)$, R_3^{Ti}, R_3^{Au}

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

$m_N = 400$ GeV, $\tan\beta = 10$

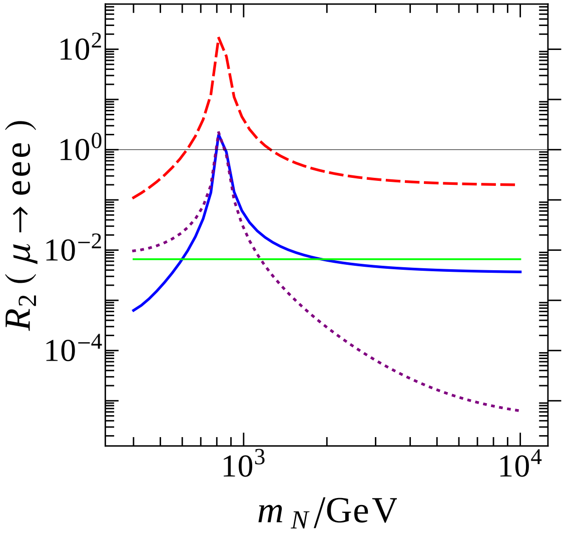
model 1 : $a = b$, $c = 0$

$R_2(\mu \to eee)$: full: $0.2 - 10^2$, $(G^{L,R}_\gamma)_{l'l}$ only: $1/159$

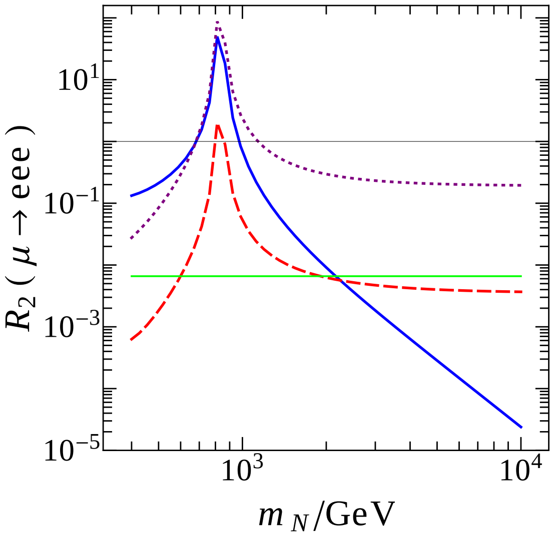
R_3^{Ti}: full: $3 - 10^4$, $(G^{L,R}_\gamma)_{l'l}$ only: $1/198$

R_3^{Au}: full: $6 - 2\times 10^2$, $(G^{L,R}_\gamma)_{l'l}$ only: $1/188$

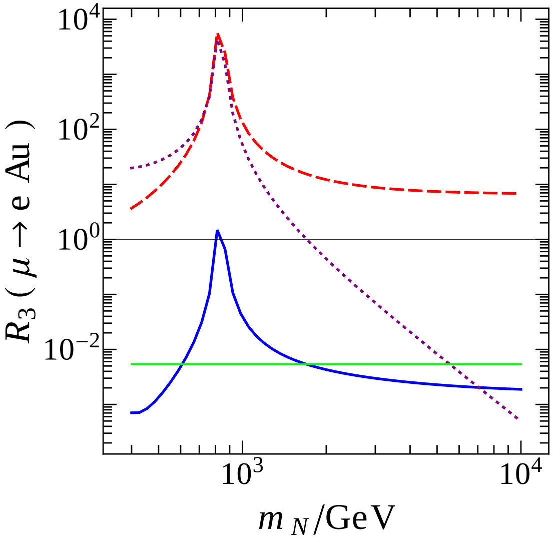
- source of strong enhancement?



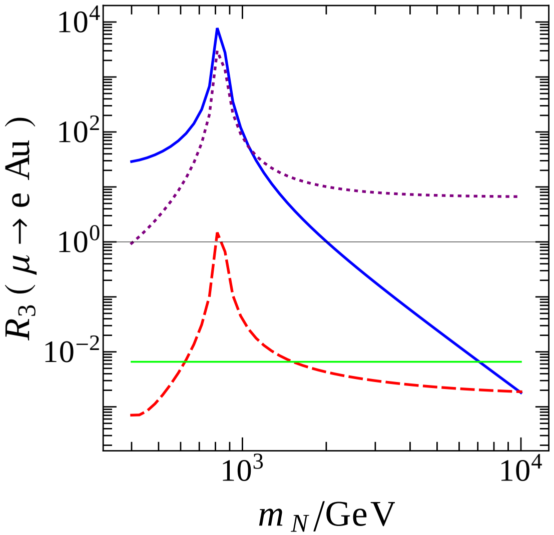
$R_2(\mu \to eee)$: form factor contributions
G_γ and F_γ, F_Z, box, $G_\gamma^{L,R}$ only
$\tan\beta = 10$
a : $B(\mu \to e\gamma) = 10^{12}$ for $m_N = 400$ GeV
- dominance of the F_Z contribution



$R_2(\mu \to eee)$: N, $\tilde{N}$, SB, $G_\gamma^{L,R}$ only
$\tan\beta = 10$
a : $B(\mu \to e\gamma) = 10^{12}$ for $m_N = 400$ GeV
- dominance of N for $m_N < 1$ TeV,
- dominance of SB for $m_N > 1$ TeV



R_3^{Au} : form factor contributions
G_γ and F_γ, F_Z, box, $G_\gamma^{L,R}$ only
$\tan\beta = 10$
a : $B(\mu \to e\gamma) = 10^{12}$ for $m_N = 400$ GeV
- dominance of F_Z cont. for $m_N > 1$ TeV
- dominance of box cont. for $m_N < 1$ TeV

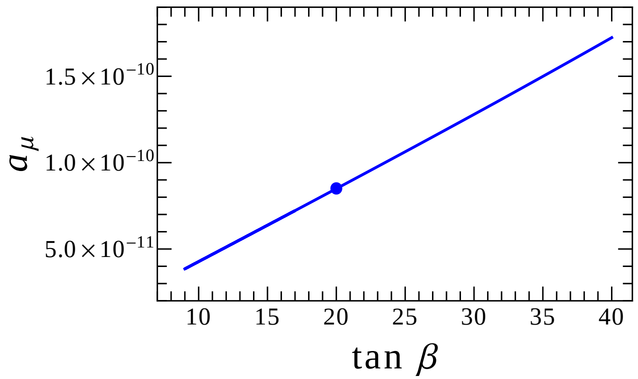


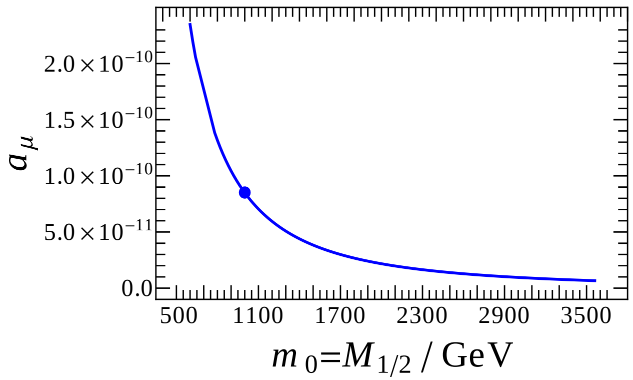
R_3^{Au}: N, $\tilde{N}$, SB, $G_\gamma^{L,R}$ only
$\tan\beta = 10$
a : $B(\mu \to e\gamma) = 10^{12}$ for $m_N = 400$ GeV
- dominance of N cont. for $m_N < 1$ TeV,
- dominance of SB cont. for $m_N > 1$ TeV

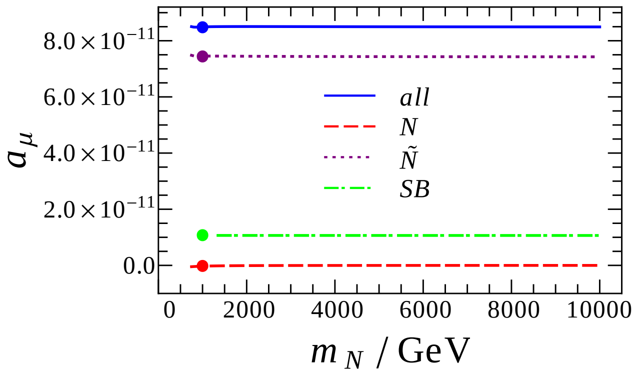
Numerical results for lepton dipole moments

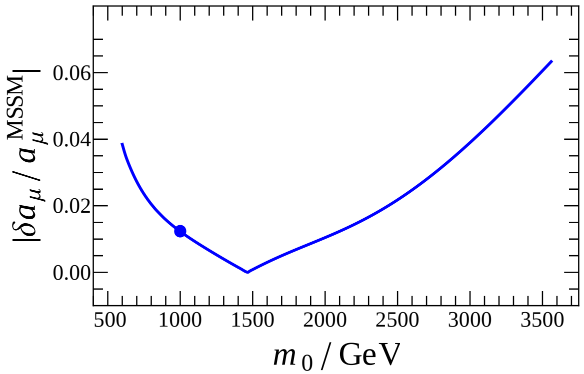
Choice of baseline parameters

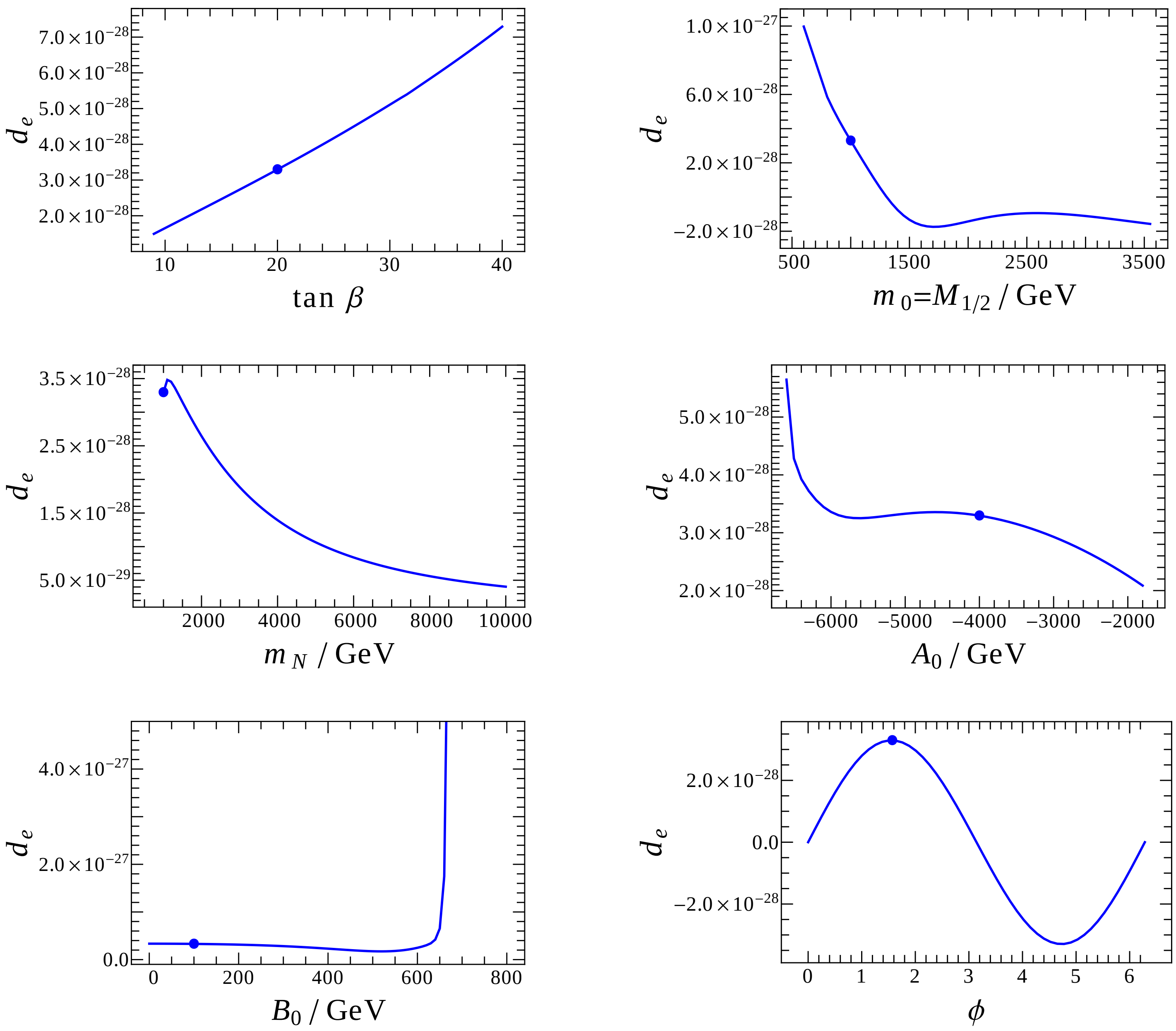
$$m_0 = 1\ \text{TeV}\ , \quad M_{1/2} = 1\ \text{TeV}\ , \quad A_0 = -4\ \text{TeV}\ , \quad \tan\beta = 20\ ,$$
$$m_N = 1\ \text{TeV}\ , \quad B_0 = 0.1\ \text{TeV}\ , \quad a = b = c = 0.05\ ,$$



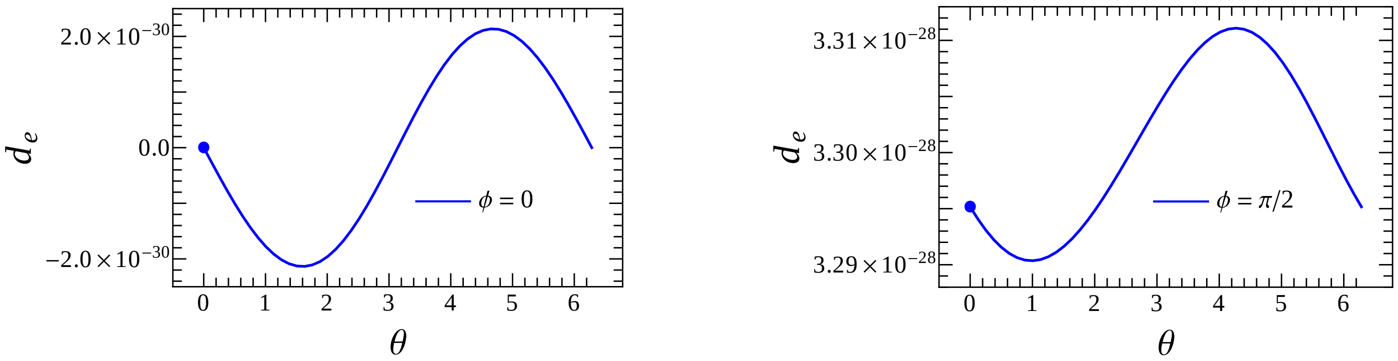








7.0×10^−28
6.0×10^−28
5.0×10^−28
4.0×10^−28
3.0×10^−28
2.0×10^−28
d_e
10
20
30
40
tan β
1.0×10^−27
6.0×10^−28
2.0×10^−28
−2.0×10^−28
d_e
500
1500
2500
3500
m_0=M_1/2 / GeV
3.5×10^−28
2.5×10^−28
1.5×10^−28
5.0×10^−29
d_e
2000
4000
6000
8000
10000
m_N / GeV
5.0×10^−28
4.0×10^−28
3.0×10^−28
2.0×10^−28
d_e
−6000
−5000
−4000
−3000
−2000
A_0 / GeV
4.0×10^−27
2.0×10^−27
0.0
d_e
0
200
400
600
800
B_0 / GeV
2.0×10^−28
0.0
−2.0×10^−28
d_e
0
1
2
3
4
5
6
ϕ



d_e
2.0×10^{-30}
0.0
-2.0×10^{-30}
$\phi = 0$
0
1
2
3
4
5
6
θ
d_e
3.31×10^{-28}
3.30×10^{-28}
3.29×10^{-28}
$\phi = \pi/2$
0
1
2
3
4
5
6
θ

Summary

- We have carefully studied the N, $\tilde{N}$ and soft SB contributions to LFV. For the first time complete set of box diagrams is included. Complete set of chiral amplitudes is included ($B^{LR}_{\ell S}$, $B^{RL}_{\ell S}$) - this decomposition is valid for any model.

- We have shown that in $\mu \to eee$ N Z-boson-mediated graphs dominate for $m_N < 1$ TeV and soft SB Z-boson-mediated graphs dominate for $m_N > 1$ TeV. In $\mu \to e$ conversion in nuclei N box graphs dominate for $m_N < 1$ TeV and soft SB Z-boson-mediated graphs dominate for $m_N > 1$ TeV. It is interesting that the low-scale seesaw model setup strongly influences soft SB part of the amplitude.

- Due to partial cancelation of N and $\tilde{N}$ contributions in magnetic dipole amplitudes the $l \to l'\gamma$ amplitudes are suppressed relative to other CLFV amplitudes.

- Due to pertubativity condition on Yukawa couplings, the CLFV amplitudes are dominated by quadratic Yukawa contributions, while quartic contributions are small.

- The dependence of LFV amplitudes on $\tan\beta$ for $5 \leq \tan\beta \leq 20$ is weak, except for $l \to l'\gamma$ processes. $(B_s \to \mu\mu)$

- Relative to the MSSM with ordinary seesaw mechanism, $l \to l' l_1 l_2$ and $\mu \to e$ conversion branching ratios are enhanced $2-3$ orders of magnitude in the region of parametric space where are no accidental cancelations of amplitudes. Opposed to the high-scale seesaw MSSM models, in the low-scale seesaw MSSM models $l \to l' l_1 l_2$ and $\mu \to e$ may give stronger constraint to the model parameters than $l \to l'\gamma$ processes.

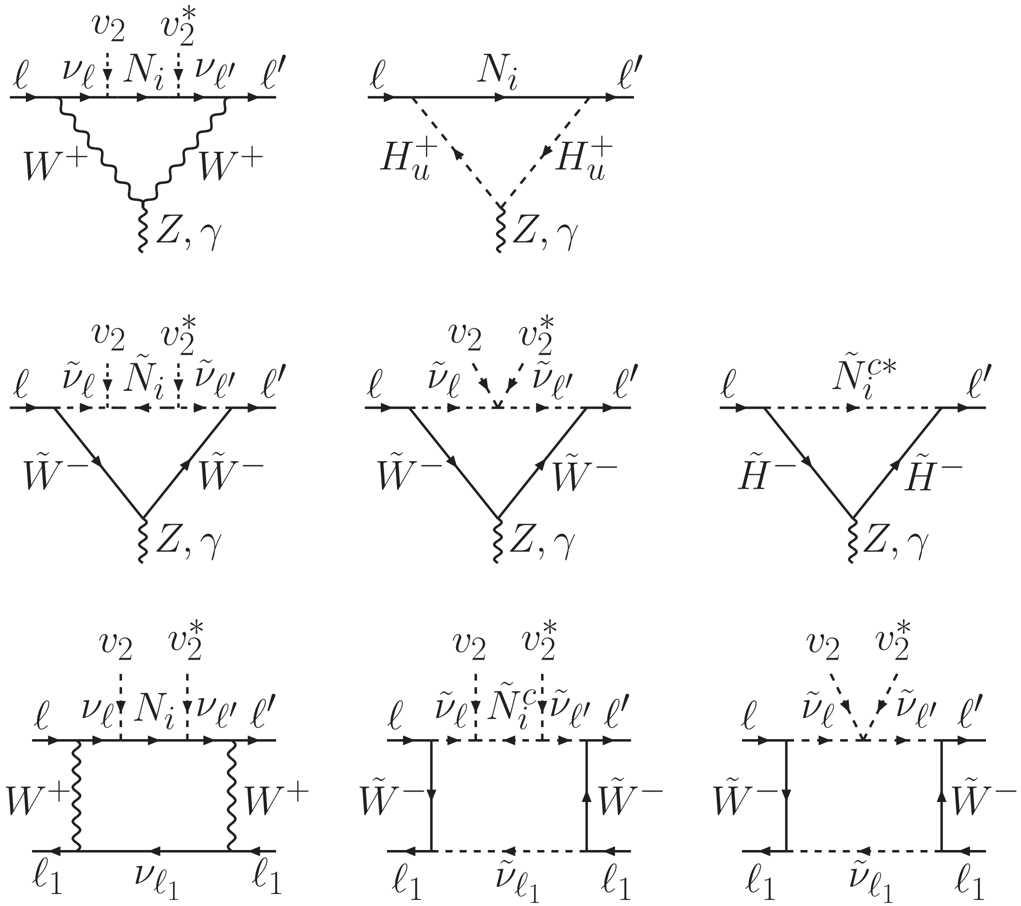
- We made an analysis of the lepton dipole moments, with particular regard to muon magnetic dipole moment a_μ and electron electric dipole moment d_e. Up to our knowledge such analysis has been done for the first time in a model with a low scale seesaw mechanism. We showed that a_μ satisfies scaling behaviour as in MSSM, and the heavy neutrino and sneutrino contributions do not numerically change the MSSM prediction for a_μ. For d_e we found a scaling behaviour which almost agrees with the naive scaling prediction. That is new result. Further, at one loop level, only the additional phases of the soft SUSY breaking bilinear and trilinear couplings induce d_e, while the potential source of CPV from $\nu_R MSSM$ vertices which are not complex conjugate to each other give numerically zero contribution to d_e. That is in accord with the result obtained for the one loop result for d_e in models with high scale seesaw mechanism.

Thank you

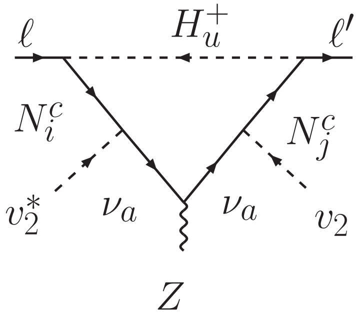
Amplitudes : Dominant contributions

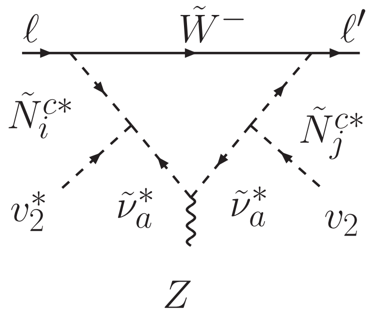
- dominant terms in lowest order in g_W and v_u (Y_ν)

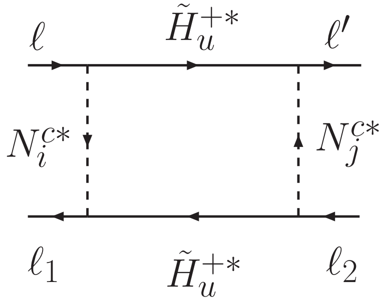
Two Yukawas

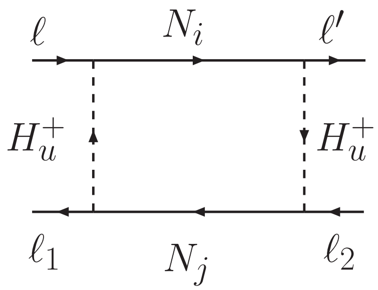


Four Yukawas









Form factors

$$
\begin{aligned}
(F_\gamma^{\ell\ell'})^N &= \frac{\Omega_{\ell\ell'}}{6s_\beta^2}\ln\underbrace{\frac{m_N^2}{M_W^2}}_{=\lambda_N},\\
(F_\gamma^{\ell\ell'})^{\tilde N} &= \frac{\Omega_{\ell\ell'}}{3s_\beta^2}\sum_{k=1}^{2}\mathcal{V}_{k1}^2\ln\frac{m_N^2}{\tilde m_{\tilde\chi_k^2}},\\
(G_\gamma^{\ell\ell'})^N &= -\Omega_{\ell\ell'}(\frac{1}{6s_\beta^2}+\frac{5}{6})\\
(G_\gamma^{\ell\ell'})^{\tilde N} &= \Omega_{\ell\ell'}(\frac{1}{6s_\beta^2}+g_\gamma)\\
g_\gamma &= -\sum_{k=1}^{2}\left[\mathcal{V}_{k1}^2\frac{2M_W^2}{m_{\tilde\chi_i}^2}g_{\gamma,1}(\frac{m_{\tilde\nu}^2}{m_{\tilde\chi_i}^2})+\mathcal{V}_{k1}\mathcal{U}_{k1}\frac{\sqrt{2}}{c_\beta}\frac{M_W^2}{m_{\tilde\chi_i}^2}g_{\gamma,2}(\frac{m_{\tilde\nu}^2}{m_{\tilde\chi_i}^2})\right]
\end{aligned}
$$

$$
\begin{aligned}
(F_Z^{\ell\ell'})^N &= -\frac{3\Omega_{\ell\ell'}}{2}\ln\frac{m_N^2}{M_W^2} - \frac{\Omega_{\ell\ell'}^2}{2s_\beta^2}\frac{m_N^2}{M_W^2},\\
(F_Z^{\ell\ell'})^{\tilde N} &= \frac{\Omega_{\ell\ell'}}{2}\ln\frac{m_N^2}{\tilde m_1^2}\Big(-\frac{1}{2}+2s_W^2+\frac{1}{s_\beta^2}f_Z\Big)\\
f_Z &= \sum_{k,l=1}^{2}\frac{m_{\tilde\chi_k}m_{\tilde\chi_l}}{M_W^2}(\mathcal{V}_{k2}\mathcal{U}_{k1}\mathcal{U}_{l1}\mathcal{V}_{l2}+\frac{1}{2}\mathcal{V}_{k2}\mathcal{U}_{k2}\mathcal{U}_{l2}\mathcal{V}_{l2}-s_W^2\delta_{kl}\mathcal{V}_{k2}\mathcal{V}_{l2})\\
(F_{box}^{\ell\ell'\ell_1\ell_2})^N &= -(\Omega_{\ell\ell'}\delta_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\delta_{\ell_2\ell'})+\frac{1}{4s_\beta^4}(\Omega_{\ell\ell'}\Omega_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\Omega_{\ell_2\ell'})\frac{m_N^2}{M_W^2}\\
(F_{box}^{\ell\ell'\ell_1\ell_2})^{\tilde N} &= (\Omega_{\ell\ell'}\delta_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\delta_{\ell_2\ell'})\ f_{box}^{\ell}+\frac{1}{4s_\beta^4}(\Omega_{\ell\ell'}\Omega_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\Omega_{\ell_2\ell'})\frac{m_N^2}{M_W^2}\\
f_{box}^{\ell} &= \sum_{k,l=1}^{2}\mathcal{V}_{k1}^2\mathcal{V}_{l1}^2 f_{box,1}^{\ell}(\lambda_{\tilde\chi_k},\lambda_{\tilde\chi_l},\lambda_{\tilde\nu},\lambda_N)+\mathcal{V}_{k2}\mathcal{V}_{k1}\mathcal{V}_{l2}\mathcal{V}_{l1}f_{box,2}^{\ell}()
\end{aligned}
$$

$$(F_{box}^{\ell\ell' uu})^N \quad = \quad -4(F_{box}^{\ell\ell' dd})^N \; = \; 4\Omega_{e\mu}$$

$$(F_{box}^{\ell\ell' uu})^{\tilde{N}} \quad = \quad \sum_{k,l=1}^{2} \mathcal{V}_{k1}^2 \mathcal{V}_{l1}^2 f_{box}^u(\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_l}, \lambda_{\tilde{d}}, \lambda_N)$$

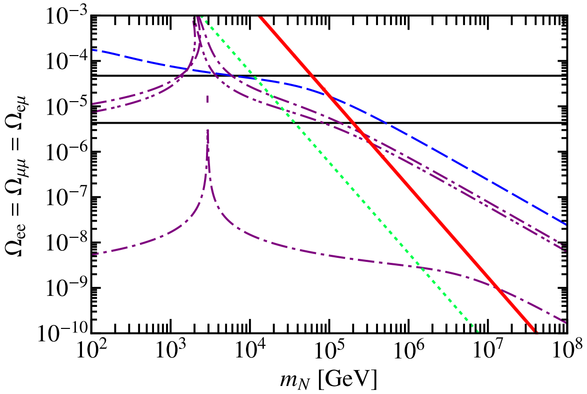
$$(F_{box}^{\ell\ell' dd})^{\tilde{N}} \quad = \quad \sum_{k,l=1}^{2} \mathcal{V}_{k1}^2 \mathcal{V}_{l1}^2 f_{box}^d(\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_l}, \lambda_{\tilde{u}}, \lambda_N)$$

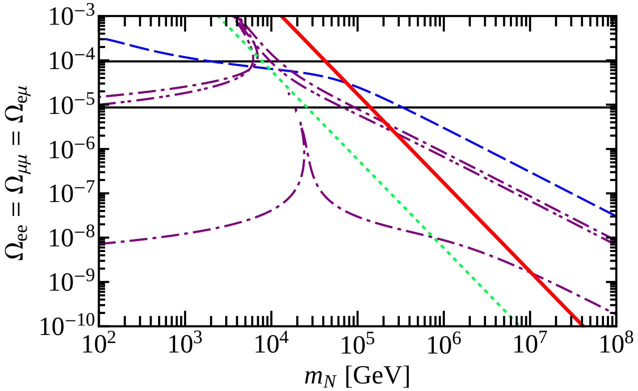
SUSY limit; cancelations, enhancements:

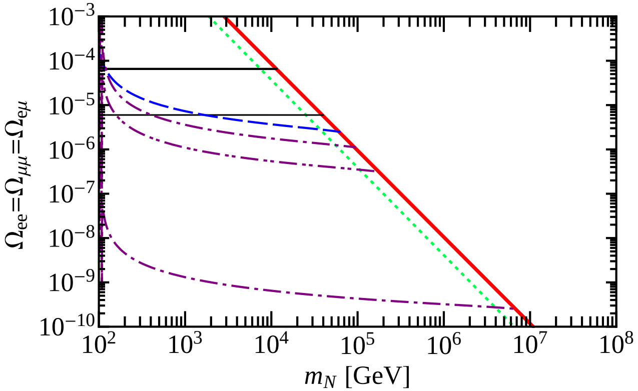
- $\tilde{m}^2_{\tilde{\chi}_{1,2}} \stackrel{SL}{\to} M_W^2, \quad t_\beta \stackrel{SL}{\to} 1, \quad \mu \stackrel{SL}{\to} 0$ (Barbieri, Giudice PLB309)
- $(G_\gamma^{\ell\ell'})^N + (G_\gamma^{\ell\ell'})^{\tilde{N}} \stackrel{SL}{=} 0$: Ferrara, Remiddi PLB53 (1974) 347
- box form factors : positive interference
- Y_ν^4 terms : become important when $Y_\nu / g_W \sim 1$ $\left(\Omega_{\ell\ell'} \frac{m_N^2}{M_W^2} = 2(Y^\dagger Y)_{\ell\ell'}/g_W^2\right)$

  (A. Pilaftsis, A.I, NPB437 (1995) 491)

Numerical estimates







$\tan\beta = 3$

$m_0 = 100$ GeV, $M_0 = 250$ GeV

$A_0 = 100$ GeV

$\Omega_{\mu e} = \Omega_{ee} = \Omega_{\mu\mu}$, other $\Omega_{\ell\ell'} = 0$

Upper bounds

$B(\mu^- \to e^-\gamma)$	1.2×10^{-11}	[1]
	1×10^{-13}	[2]
$B(\mu^- \to e^-e^-e^+)$	1×10^{-12}	[1]
$R^{Ti}_{\mu e}$	4.3×10^{-12}	[3]
	1×10^{-18}	[4]
$R^{Au}_{\mu e}$	7×10^{-13}	[5]

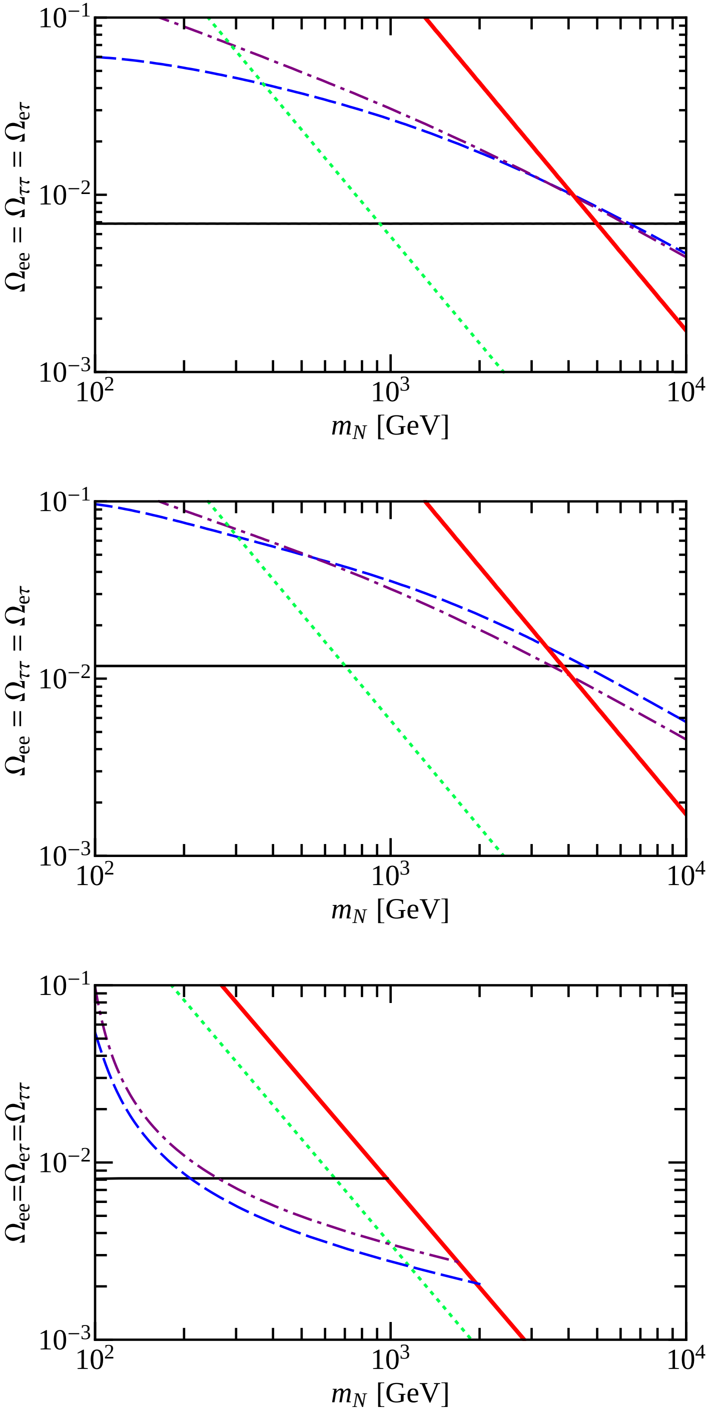
[1] Amsler, PLB 667 (2008) 1

[2] Ritt, NPBPS 162 (2006) 279

[3] Dohmen, PLB 317 (1993) 631

[4] Kuno, NPBPS 149 (2005) 376

[5] Bertl, EPJC 47 (2006) 337



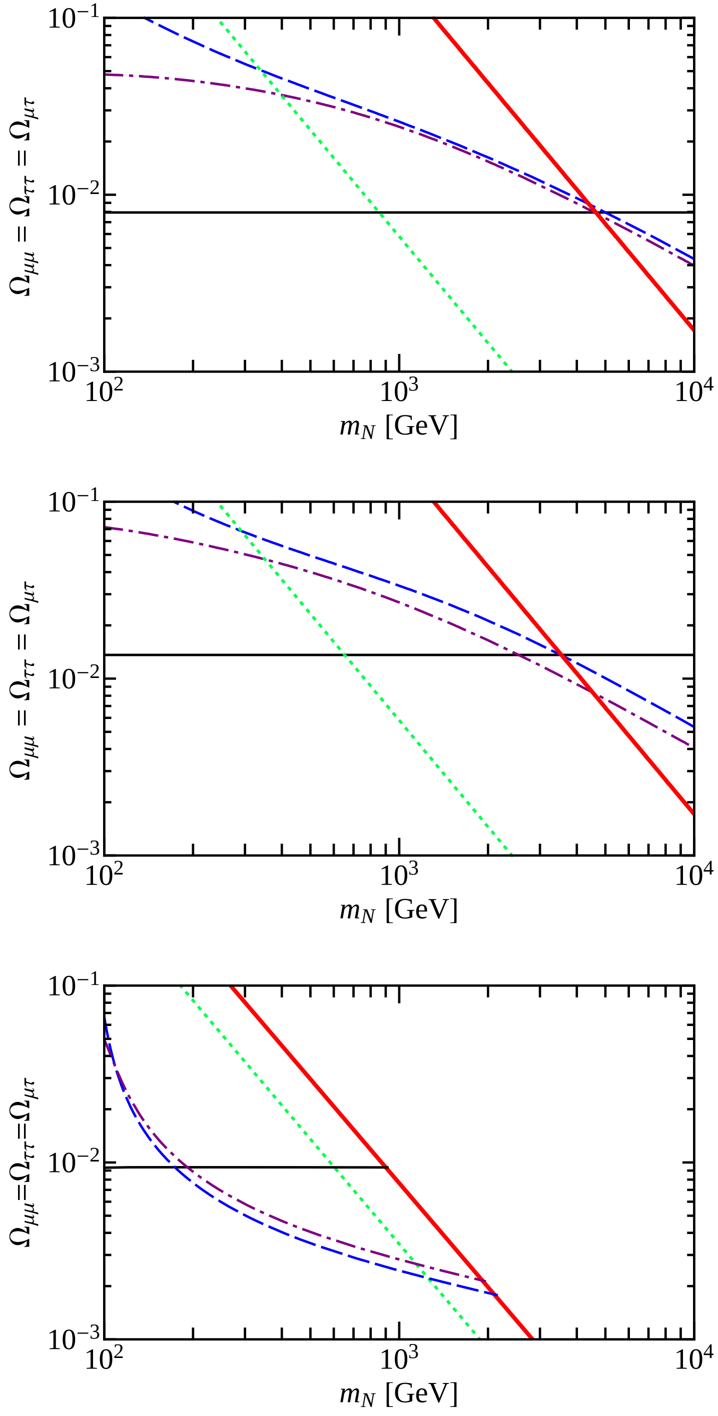
$\Omega_{\tau e} = \Omega_{ee} = \Omega_{\tau\tau}$, other $\Omega_{\ell\ell'} = 0$

Upper bounds

$B(\tau^- \to e^-\gamma)$	3.3×10^{-8}	[1]
$B(\tau^- \to e^-e^-e^+)$	2.7×10^{-8}	[2]
$B(\tau^- \to e^-\mu^-\mu^+)$	2.7×10^{-8}	[2]

[1] Aubert, PRL 104 (2010) 021802

[2] Hayasaka, PRL 687 (2010) 139



$\Omega_{\tau\mu} = \Omega_{\mu\mu} = \Omega_{\tau\tau}$, other $\Omega_{\ell\ell'} = 0$

Upper bounds

$B(\tau^- \to \mu^- \gamma)$	4.4×10^{-8}	[1]
$B(\tau^- \to \mu^- \mu^- \mu^+)$	2.1×10^{-8}	[2]
$B(\tau^- \to \mu^- e^- e^+)$	1.8×10^{-8}	[2]

[1] Aubert, PRL 104 (2010) 021802

[2] Hayasaka, PRL 687 (2010) 139

Summary

- We have shown that in the low-scaled supersymmetric seesaw models sneutrinos might give large effects indenpenent of SUSY breaking mechanism.
- Due to SUSY the $\ell \to \ell'\gamma$ are suppressed.
- That makes $\mu \to e$ conversion especially interesting candidate for finding LFV. $\mu \to 3e$ and $\tau \to 3e$ give complementary information on LFV.
- Inclusion of the mSUGRA boundary conditions strongly influences te final results of the model. Particularly it leads to lo larger theoretical prediction for LFV observables $R_{\mu e}$, $\mu \to 3e$ and $\tau \to 3$ leptons by up to factor of 25. The branching fractions for $\ell \to \ell'\gamma$ variation show smaller variation – they are slightly larger than those obtained in in MSSM+3N without mSUGRA boundary condition, but larger than results obtained in non-supersymmetric version of the model.