FACULTY OF PHYSICS WARSAW UNIVERSITY

INNOVATIVE ECONOMY
NATIONAL COHESION STRATEGY

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Thick–Brane Cosmology

Aqeel Ahmed

University of Warsaw, Warsaw

In collaboration with Bohdan Grzadkowski and Jose Wudka

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Outline

- Introduction and motivation.
- Randall–Sundrum models; a brief review.
- Thick-brane models.
- Cosmology with static thick-branes.
- Cosmology with dynamical thick-branes.
- Summary.

★ AA, B. Grzadkowski and J. Wudka, *"Thick-Brane Cosmology"*, JHEP **04** (2014) 061 [arXiv:1312.3576].

★ AA and B. Grzadkowski, *"Brane modeling in warped extra-dimension"*, JHEP **01** (2013) 177 [arXiv:1210.6708].

★ AA, L. Dulny and B. Grzadkowski, *"Generalized Randall-Sundrum model with a single thick brane"*, EPJC **74** (2014) 2862 [arXiv:1312.3577].

Why extra-dimensions

- (Gauge) Hierarchy problem:
  Why gravity is weaker than other fundamental forces?
- Dark matter/Dark Energy?
- (Fermion) Mass hierarchy problem:
  Why $m_\nu \lesssim 10^{-9}$ GeV $\ll M_t \sim 10^3$ GeV?
- Unification of fundamental forces?
- Matter-anti matter asymmetry
  and so on $\cdots$.

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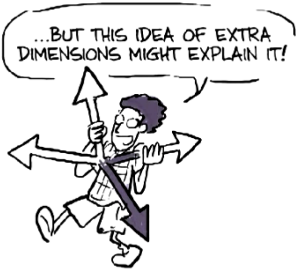
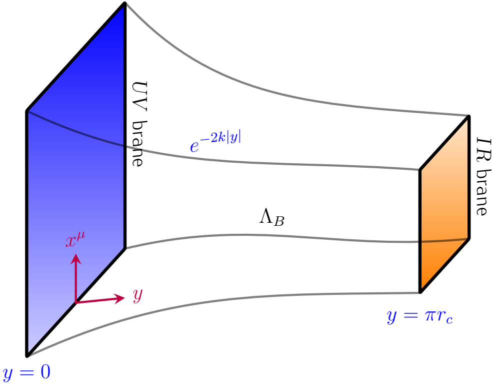


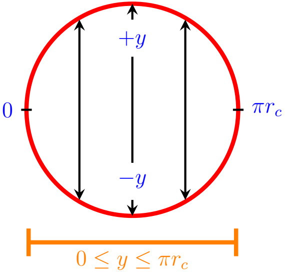
Large extra-dimensions

- Flat extra-dimensions (e.g. ADD, UED).
- Warped extra-dimensions (e.g. RS).

RS model with two branes (RS1)

A 5D model with two D3-branes on S_1/Z_2 orbifold along the extra-dimension. hep-ph/9905221



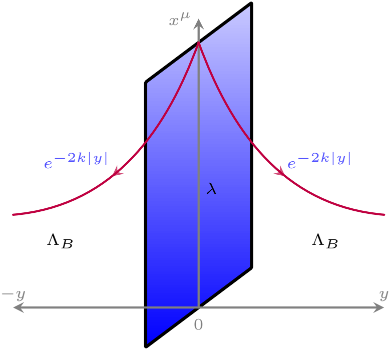


$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

RS2: alternative to compactification

A model with one D3-brane embedded in an infinite extra-dimension. hep-th/9906064

$$S = \int d^4x \int_{-\infty}^{\infty} dy \sqrt{-g} \left\{ 2M_*^3 R - \Lambda_B - \lambda \delta(y) \right\}$$



- Solution respecting 4D Poincaré symmetry:

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$

- Possible if $\lambda = \sqrt{-24 M_*^3 \Lambda_B}$

4D Planck mass: $M_{Pl}^2 \simeq \frac{M_*^3}{k}$

Summarizing RS models

Attractive features

- RS1 offer an elegant and simple solution to the hierarchy problem.
- RS2 provides an alternative to compactification.

Summarizing RS models

Attractive features

- RS1 offer an elegant and simple solution to the hierarchy problem.
- RS2 provides an alternative to compactification.

"Un"-attractive features

- Thin (singular) branes considered in RS-like models are not dynamically generated.
- Presence of a negative tension thin brane in RS1.

Smoothing the RS models

5D scalar-gravity action

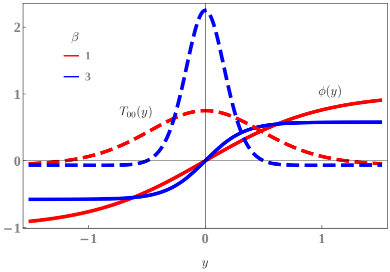
$$S = \int d^5x\sqrt{-g}\left\{2M_*^3 R - \frac{1}{2}g^{MN}\nabla_M\phi\nabla_N\phi - V(\phi)\right\}$$

- Metric ansatz

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

- Scalar field profile

$$\phi(y) = \frac{\kappa}{\sqrt{\beta}}\tanh(\beta y)$$



Background solutions

- Einstein equations

$$24M_*^3\left(\frac{a'}{a}\right)^2=\frac{1}{2}(\phi')^2-V(\phi)$$

$$12M_*^3\left(\frac{a''}{a}\right)+12M_*^3\left(\frac{a'}{a}\right)^2=-\frac{1}{2}(\phi')^2-V(\phi)$$

- Scalar potential ansatz

$$V(\phi)=\frac{1}{2}\left(\frac{\partial W(\phi)}{\partial\phi}\right)^2-\frac{1}{6M_*^3}W(\phi)^2$$

- Superpotential $W(\phi)$ satisfies

$$\phi'=\frac{\partial W(\phi)}{\partial\phi} \qquad \& \qquad \frac{a'}{a}=-\frac{1}{12M_*^3}W(\phi)$$

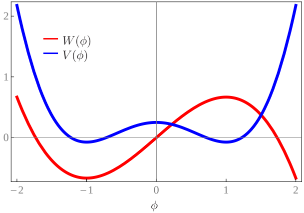
- The scalar field $\phi(y) = \frac{\kappa}{\sqrt{\beta}} \tanh(\beta y)$ for which,

$$W(\phi) = \kappa\sqrt{\beta}\phi\left(1 - \frac{\beta}{3\kappa^2}\phi^2\right)$$

- The warp factor $a(y)$

$$a(y) = \exp\left\{\frac{-\kappa^2}{72M_*^3\beta}\left(\tanh^2(\beta y) + \ln\cosh^4(\beta y)\right)\right\}$$

- Brane limit ($\beta \to \infty$): $a(y) \approx e^{-k|y|}$ where $k = \frac{1}{24M_*^3}\lambda$, with $\lambda \equiv \frac{4}{3}\kappa^2$.



Static thick-brane cosmology

- Metric ansatz for cosmology of thick brane,

$$ds^2 = a^2(\tau, y) g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

$g_{\mu\nu}$ is the 4D conformal metric and $\tau \equiv \int \frac{dt}{a}$ is conformal time.

- Time-independent (static) scalar field and gravity action

$$S = \int d^5x \sqrt{-g} \left\{ 2M_*^3 R - \frac{1}{2} g^{MN} \nabla_M \phi \nabla_N \phi - V(\phi) \right\}$$

- Einstein equations

"overdot" derivative w.r.t. τ and "prime" derivative w.r.t. y and $4M_*^3 = 1$

$$00: \; 3\left[\frac{1}{a^2}\frac{\dot{a}^2}{a^2} - \left(\frac{a''}{a} + \frac{a'^2}{a^2}\right) + \frac{k}{a^2}\right] = \frac{1}{2}\phi'^2 + V(\phi),$$

$$ij: \; \frac{1}{a^2}\left(2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) - 3\left(\frac{a''}{a} + \frac{a'^2}{a^2}\right) + \frac{k}{a^2} = \frac{1}{2}\phi'^2 + V(\phi),$$

$$05: \; \frac{a'}{a}\frac{\dot{a}}{a} - \frac{\dot{a}'}{a} = 0, \qquad 55: \; 3\left[2\frac{a'^2}{a^2} - \frac{1}{a^2}\frac{\ddot{a}}{a} - \frac{k}{a^2}\right] = \frac{1}{2}\phi'^2 - V(\phi).$$

- 05 constraint implies $a(\tau, y)$ has separable form

$$a(\tau, y) = \hat{a}(\tau)\bar{a}(y)$$

- Einstein equations become; $\bar{\Lambda} \equiv$ 4D cosmological constant

$$00: \quad \frac{1}{\hat{a}^2}\frac{\dot{\hat{a}}^2}{\hat{a}^2} + \frac{k}{\hat{a}^2} = \frac{\bar{a}^2}{3}\left[3\left(\frac{\bar{a}''}{\bar{a}} + \frac{\bar{a}'^2}{\bar{a}^2}\right) + \frac{1}{2}\phi'^2 + V(\phi)\right] = \frac{1}{2}\bar{\Lambda},$$

$$ij: \quad \frac{1}{\hat{a}^2}\left(2\frac{\ddot{\hat{a}}}{\hat{a}} - \frac{\dot{\hat{a}}^2}{\hat{a}^2}\right) + \frac{k}{\hat{a}^2} = \bar{a}^2\left[3\left(\frac{\bar{a}''}{\bar{a}} + \frac{\bar{a}'^2}{\bar{a}^2}\right) + \frac{1}{2}\phi'^2 + V(\phi)\right] = \frac{3}{2}\bar{\Lambda},$$

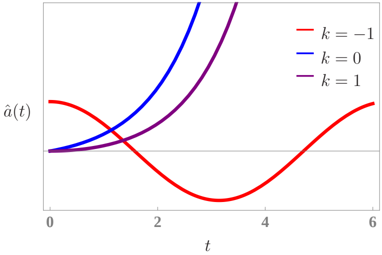
$$55: \quad \frac{1}{\hat{a}^2}\frac{\ddot{\hat{a}}}{\hat{a}} + \frac{k}{\hat{a}^2} = \frac{\bar{a}^2}{3}\left[6\frac{\bar{a}'^2}{\bar{a}^2} - \frac{1}{2}\phi'^2 + V(\phi)\right] = \bar{\Lambda}.$$

- Friedman-like equations follow from 00 and ij components,

$$\frac{\dot{\hat{a}}^2}{\hat{a}^2} - \frac{\bar{\Lambda}}{2}\hat{a}^2 + k = 0 \qquad\qquad \frac{\ddot{\hat{a}}}{\hat{a}} - 2\frac{\dot{\hat{a}}^2}{\hat{a}^2} - k = 0$$

Evolution of the scale factor $\hat{a}$

$$\hat{a}(t) = \sqrt{\frac{2}{|\bar{\Lambda}|}} \begin{cases} \operatorname{sech}\left[\operatorname{arctanh}[\tan(t)]\right] & k=-1 \quad (\bar{\Lambda}<0) \\ e^{t} & k=0 \quad (\bar{\Lambda}>0) \\ \cosh(t) & k=1 \quad (\bar{\Lambda}>0) \end{cases}$$



- For $\bar{\Lambda}=0$, we get the scale factor

$$\hat{a}(t) \propto \begin{cases} t & k=-1 \quad \text{Milne universe} \\ a_0 & k=0 \quad \text{Static universe} \end{cases}$$

Extra-dimensional profiles

- For the warp factor $\bar{a}(y) \equiv e^{A(y)}$, Einstein equations reduced to

$$\begin{aligned} 3A'' + \frac{3}{2}\bar{\Lambda}e^{-2A} &= -\phi'^2 \\ 6A'^2 - 3\bar{\Lambda}e^{-2A} &= \frac{1}{2}\phi'^2 - V(\phi) \end{aligned}$$

  where $\bar{\Lambda}$ is 4D effective cosmological constant.

- For $\bar{\Lambda} = 0$ we recover the static solutions, e.g., AA and Grzadkowski, JHEP **01** (2013) 177.

- For $\bar{\Lambda} \neq 0$ we find de Sitter or anti-de Sitter static solutions.

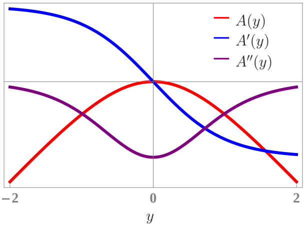
- We consider the warp function $A(y)$ as

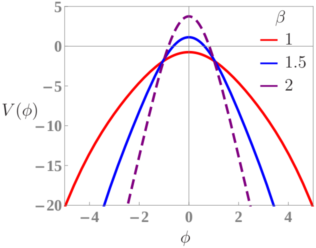
$$A(y) = -\ln\cosh(\beta y)$$

where β is the parameter which controls the thickness of the brane such that

$$\beta \to \infty \quad \Rightarrow \quad A(y) \to -|y| \qquad \text{RS solution}$$

- We found the analytic and numerical exact solutions for the scalar field $\phi(y)$ and $V(\phi)$.





Dynamical thick-brane cosmology

- Metric ansatz for cosmology of thick brane,

$$ds^2 = a^2(\tau, y) g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

- With time–dependent (dynamical) scalar field the Einstein equations are

$$\begin{aligned}
00: &\qquad 3\left[\frac{1}{a^2}\frac{\dot{a}^2}{a^2} - \left(\frac{a''}{a} + \frac{a'^2}{a^2}\right) + \frac{k}{a^2}\right] = \frac{1}{2}\phi'^2 + \frac{1}{2}\frac{1}{a^2}\dot{\phi}^2 + V(\phi), \\
ij: &\qquad \frac{1}{a^2}\left(2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) - 3\left(\frac{a''}{a} + \frac{a'^2}{a^2}\right) + \frac{k}{a^2} = \frac{1}{2}\phi'^2 - \frac{1}{2}\frac{1}{a^2}\dot{\phi}^2 + V(\phi), \\
05: &\qquad \frac{a'}{a}\frac{\dot{a}}{a} - \frac{\dot{a}'}{a} = \frac{1}{3}\phi'\dot{\phi}, \\
55: &\qquad 3\left[2\frac{a'^2}{a^2} - \frac{1}{a^2}\frac{\ddot{a}}{a} - \frac{k}{a^2}\right] = \frac{1}{2}\phi'^2 + \frac{1}{2}\frac{1}{a^2}\dot{\phi}^2 - V(\phi).
\end{aligned}$$

where $k = 0, \ \pm 1$.

Generalized superpotential method

- For more general time-dependent solution, we generalized the superpotential method by defining:

$$\frac{a'}{a} \equiv -\frac{1}{3}W(\phi), \qquad \frac{\dot{a}}{a} \equiv -\frac{1}{3}H(\phi)$$

- From the 05 component of the Einstein equation,

$$\frac{\partial W(\phi)}{\partial \phi} = \phi', \qquad \frac{\partial H(\phi)}{\partial \phi} = \dot{\phi}$$

- The Einstein equations gives the scalar potential $V(\phi)$,

$$V(\phi) = \frac{1}{2}\left(\frac{\partial W(\phi)}{\partial \phi}\right)^2 - \frac{2}{3}W(\phi)^2$$

with the constraint: $\frac{1}{2}\left(\frac{\partial H(\phi)}{\partial \phi}\right)^2 - \frac{1}{3}H(\phi)^2 - 3k = 0$

Generalized superpotential method

- The solution for $H(\phi)$ from the above constraint is (with $k=0$):

$$H(\phi) = H_0 e^{\pm\sqrt{\frac{2}{3}}\phi},$$

where H_0 constant

- Solution for $W(\phi)$ is,

$$W(\phi) = A_0 H(\phi) + W_0,$$

where A_0 & W_0 constants

- The solution for scalar field $\phi(t, y)$ is

$$\phi(t,y) \equiv \phi(\eta) = \mp\sqrt{\frac{3}{2}}\ln\left(-\frac{2}{3}H_0\eta + e^{\mp\sqrt{\frac{2}{3}}\phi_0}\right)$$

where $\eta = ct + dy$ and we choose for simplicity $c = d = 1$.

Generalized superpotential method

- From the superpotential equations, one can find

$$a(t,y) \equiv a(\eta) = a_0\,(1+2b_0\eta)^{1/2}$$

where a_0 & b_0 constants

- The scalar potential $V(\phi)$ is,

$$V(\phi) = -\frac{1}{3}\left(A_0 H_0 e^{\pm\sqrt{\frac{2}{3}}\phi} + 2W_0\right)^2 + \frac{2}{3}W_0^2$$

- The time-independent analogue of these solutions appear in Linear Dilaton warped geometries as discussed independently by Antoniadis et al. PRL 108 (2012) 081602.

Boosted solutions

- Static solutions discussed earlier with $a(y)$ and $\phi(y)$ can be promoted to time-dependent solution through the boost:

$$y \to y' = \gamma(vt + y) \qquad \text{where} \qquad \gamma = 1/\sqrt{1 - v^2}$$

- Let us rewrite the metric as,

$$ds^2 = a^2(z)\left(g_{\mu\nu}dx^\mu dx^\nu + dz^2\right)$$

- One can show that with Lorentz transformations $t' = \gamma(t + vz)$ and $z' = \gamma(vt + z)$;

$$a[\gamma(-vt + z(y))] \qquad \text{and} \qquad \phi[\gamma(-vt + z(y))]$$

is solution!

Summary

- Extra-dimensions could answers some of the most fundamental puzzles of particle physics.
- The smooth generalizations of RS-like models were obtained with a scalar field.
- Static thick-brane cosmology was discussed for different values of spacial curvature k.
- Two classes of dynamical thick-brane cosmology were presented including boosted and twisted solutions.
- A generalized superpotential method was introduced for dynamical thick-brane cosmology.

Thank you for your attention

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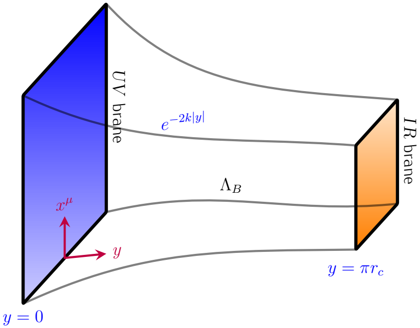
Backup slides

RS1 solutions

$$S_1 = \int d^4x \int_0^{\pi r_c} dy \sqrt{-g} \left\{ 2M_*^3 R - \Lambda_B - \lambda_1 \delta(y) - \lambda_2 \delta(y - \pi r_c) \right\}$$

- Solution preserving 4D Poincaré symmetry:

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$



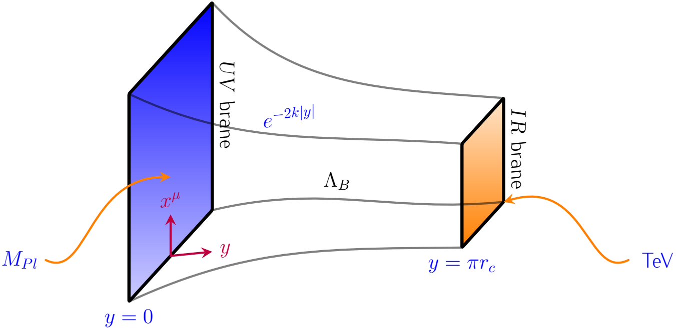
$$k \equiv \sqrt{\frac{-\Lambda_B}{24 M_*^3}}$$

$$\lambda_1 = -\lambda_2 = \sqrt{-24 M_*^3 \Lambda_B}$$

RS solution to hierarchy problem

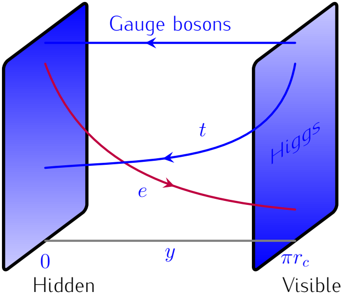
- For $m_0 \sim M_{Pl} \sim 10^{19}$ GeV, the physical mass on IR brane (our brane) is:

$$m_{obs} = e^{-\pi k r_c} m_0 \sim \text{TeV} \qquad \text{with} \qquad k r_c \approx 10$$



RS solution to fermion mass hierarchy

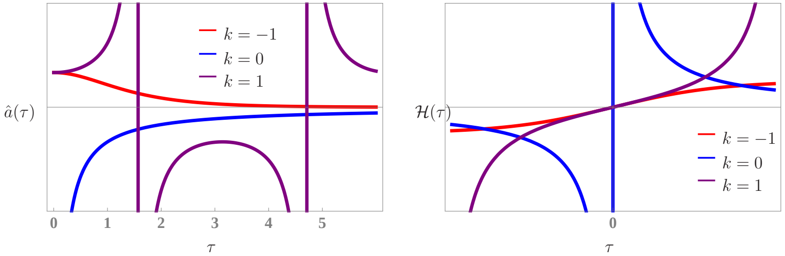
- Why $m_\nu \sim 10^{-9}$ GeV $\ll M_t \sim 10^3$ GeV?
- Higgs field H is localized at visible brane (our brane).
- Geometric localization of fermions.
- Overlap of fermionic fields with Higgs field determines their mass.



- Hubble parameter $\mathcal{H}(\tau) \equiv \dot{\hat{a}}/\hat{a}$

$$\mathcal{H}(\tau) = \begin{cases} \tanh(\tau) & k = -1 \\ \frac{1}{\tau} & k = 0 \\ \tan(\tau) & k = 1 \end{cases}$$

- Graph shows $\hat{a}(\tau)$ and $\mathcal{H}(\tau)$ for $k = 0, \ \pm 1$.



Hierarchy problem

- Why gravity is so weak as compared to the other fundamental forces?

Gravity

$$F = G_N \frac{m_p^2}{r^2} \approx \frac{m_p^2}{M_{Pl}^2}\frac{1}{r^2}$$

$$\alpha_g = \frac{1GeV^2}{M_{Pl}^2} \approx 10^{-38}$$

Electromagnetism

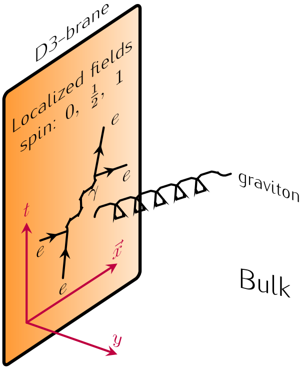
$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} \approx \frac{e^2}{4\pi\epsilon_0}\frac{1}{r^2}$$

$$\alpha_e = \frac{e^2}{4\pi\epsilon_0} \approx \frac{1}{137} \approx 10^{-2}$$

- Why $m_{EW} \sim \alpha_{EW}^{-1/2} \sim 10^3$ GeV $\ll M_{Pl} \sim \alpha_g^{-1/2} \sim 10^{19}$ GeV?

Large extra-dimensions

- Arkani-Hamed, Dimopoulos and Dvali (ADD) proposed that large extra dimensions can solve the hierarchy problem. hep-ph/9803315
- SM fields are localized on the D3-brane while the gravity can propagate to n extra-dimensions.



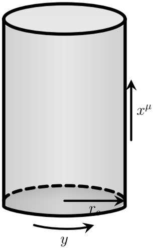
ADD solution to hierarchy problem

- Newton's law in $D = 4 + n$ dimensions:

$$F \approx G^N_{(4+n)} \frac{m_1 m_2}{r^{2+n}} \approx \frac{1}{M_*^{2+n}} \frac{m_1 m_2}{r^{2+n}}.$$

- If the n–dimensions are compactified with size $L = 2\pi r_c$ then the force law will be:

$$F_{4+n} \approx \frac{1}{M_*^{2+n}} \frac{m_1 m_2}{L^n r^2} \qquad r \gg L.$$



- Comparing 4D Newton's law with 4+n-dimensional Newton's law

$$F_4 \approx \frac{1}{M_{Pl}^2} \frac{m_1 m_2}{r^2} \qquad F_{4+n} \approx \frac{1}{M_*^{2+n}} \frac{m_1 m_2}{L^n r^2}$$

$$M_{Pl}^2 \approx M_*^{2+n} L^n$$

- If the fundamental scale $M_* \sim 10^3$ GeV and the 4D Planck scale $M_{Pl} \sim 10^{19}$ GeV, then:

$$L \approx \left(\frac{M_{Pl}^2}{M_*^{2+n}} \right)^{1/n} \approx 10^{32/n}\ \text{TeV}^{-1} \approx 10^{32/n} 10^{-17}\ \text{cm}.$$

$n = 1$	$L \sim 10^{15}$ cm	ruled out,
$n = 2$	$L \sim 10^{-1}$ cm	allowed in 1998,
$n = 3$	$L \sim 10^{-6}$ cm	allowed.

4D effective gravity

- Zero mode solution:

$$ds^2 = e^{-2k|y|}\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + dy^2$$

  where $\hat{g}_{\mu\nu}(x)$ is 4D localized graviton!

- 4D Graviton action

$$S_g = 2M_*^3 \int_{-\infty}^{\infty} dy e^{-2k|y|} \int d^4x \sqrt{-\hat{g}}\hat{R},$$
$$= \frac{2M_*^3}{k} \int d^4x \sqrt{-\hat{g}}\hat{R}.$$

- 4D Plank mass $M_{Pl}^2 = M_*^3/k$ is finite.
- Unlike ADD model $M_{Pl}^2 = M_*^3 y \to \infty$ for $y \to \infty$.
- **Conclusion**: 4D effective gravity is recovered.

Generalized RS2

We consider a single D3-brane embedded in an infinite extra-dimension with different cosmological constants on each side.

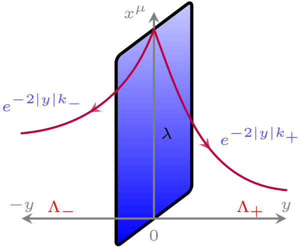
- Action for generalized RS2

$$S = \int d^5x\sqrt{-g}\left\{2M_*^3 R - \Lambda_+\theta(y) - \Lambda_-\theta(-y) - \lambda\delta(y)\right\}$$

- Solution preserving 4D Poincaré symmetry

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$A(y) = \begin{cases} -|y|k_+ & y > 0 \\ -|y|k_- & y < 0 \end{cases}$$



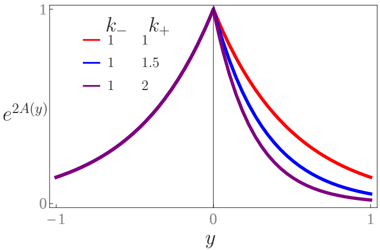
$$k_\pm \equiv \sqrt{\frac{-\Lambda_\pm}{24M_*^3}} \qquad \Lambda_+ \neq \Lambda_-$$

- The above solution is possible if

$$\lambda = \sqrt{-12M_*^3(\Lambda_+ + \Lambda_-)}$$

- The warp function $A(y)$ is

$$A(y) = \begin{cases} -|y|k_+ & y > 0 \\ -|y|k_- & y < 0 \end{cases}$$



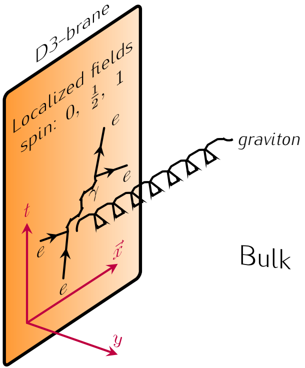
4D Effective gravity

$$M_{Pl}^2 = \frac{M_*^3}{2k_-} + \frac{M_*^3}{2k_+}$$

- 4D gravity can be reproduced.

Branes

- Branes are hypersurfaces with localized energy/fields. (in higher dimensional manifold)
- D-branes play crucial role in string theory. (Localize open strings)
- QFT branes are domain walls (localized energy/fields) and cosmic strings.



- Branes localize spin: $0,\ \frac{1}{2},\ 1.$
- Can spin 2 be localized?
  Only weakly localized!
- Gravity can leak to Bulk.