# 3D $\mathcal{N} = 2$ dualities

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Amariti Antonio

LPT ENS Paris

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## Motivations

| As. free QFTs ↓ IR Strongly Coupled | → | Non pert. techniques, e.g. duality ↓ SUSY useful laboratory |
|---|---|---|

**Example**

**Seiberg** duality in 4D $\mathcal{N} = 1$

↙ ↘

| Electric | Magnetic |
|---|---|
| $G = SU(N_c)$ | $\widetilde{G} = SU(N_f - N_c)$ |
| $N_f\ Q \oplus \widetilde{Q}$ | $N_f\ q \oplus \widetilde{q}$ <br> $M = Q\widetilde{Q}$ |
| $W = 0$ | $W = Mq\widetilde{q}$ |

**3D Case**

'97 **Aharony**
'08 **Giveon**, **Kutasov**
} 10 yrs!!

Why such a long time?

Lackness of techniques

Recently:

↗ Localization — **Kapustin, Willett, Yaakov '09**, **Jafferis '10**, **Hama, Hosomichi, Lee '10**

↘ **ABJM '08** (AdS/CFT)

# Outline

- Useful aspects of $\mathcal{N} = 2$ 3D theories
- **Aharony** and **Giveon-Kutasov** duality
- RG flows between dual pairs
- Localization on the three sphere and RG flows
- Contact terms as a new check of duality
- Generalizations: chiral theories, tensor matter, real groups

# Useful aspects of $\mathcal{N}=2$ (2+1)-dim. theories

Algebra: $\{Q_\alpha, \tilde{Q}_\beta\} = \sigma^\mu_{\alpha,\beta} P_\mu + 2i\epsilon_{\alpha,\beta} Z$ ($Z$ central charge)

Multiplets:

| | | |
|---|---|---|
| Vector | $V = (A_\mu, \lambda_\alpha, \tilde{\lambda}_\alpha, \sigma, D)$ | ($\sigma$ from dim red of $A_3$) |
| Chiral (charged) | $\Phi_R = (\phi_R, \psi_R, F_R)$ | (rep. $R$ of $G$) |

Coulomb Branch(CB)($\langle\sigma\rangle$)

↓

Dual photon $F_{\mu\nu,i} = \epsilon_{\mu\nu\lambda}\partial^\lambda \varphi_i$
Chiral $a_i = \sigma_i + i\varphi_i$
$e^{a_i}$ coordinate on CB (UV mon.)

Global Symmetries (Abelian)

↓

Axial $U(1)_A$
$U(1)_R$ rotates $Q_\alpha$
Top. $U(1)_J$ (shifts $\varphi$)

CS action: $S_{CS} = \frac{k}{4\pi}\int Tr(A\wedge dA - \frac{2}{3}A^3 - \lambda\tilde{\lambda} + 2\sigma D)$ w/ $k \in Z$

Real masses
Coupling $|\sigma^i T^i_R \phi_R|^2$
$\langle\sigma\rangle$ real mass for $\phi$
Real masses also from $V_{bckg.}$

→

By integrating out fermions w/ real mass
$$k^{eff}_{ij} = k_{ij} + \frac{1}{2}\sum_I c_i(\psi_I)c_j(\psi_I)sgn(m_I)$$

## Aharony duality

$G = U(N_c)_0$
$N_f(\geq N_c)\ Q \oplus \tilde{Q}$
$W = 0$

$\leftrightarrow$

$\tilde{G} = U(\tilde{N}_c = N_f - N_c)_0$
$N_f\ \ q \oplus \tilde{q}$
Singlets: $M = Q\tilde{Q},\ T,\ \tilde{T}$
$W = Mq\tilde{q} + T\tilde{t} + T\tilde{t}$

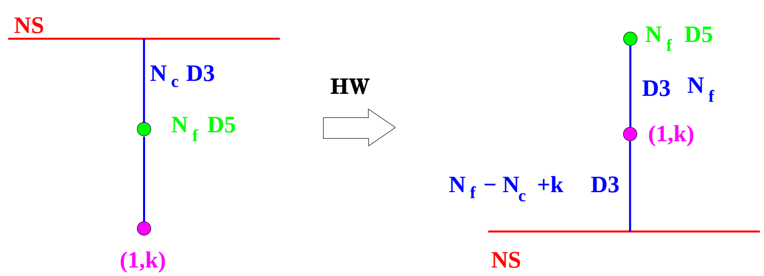
Where $T, \tilde{T}$ ($t, \tilde{t}$) are monopoles w/ magn. flux $(\pm 1, 0 \ldots, 0)$ in the Cartan of $G$ ($\tilde{G}$)

| Electric | $U(N_c)$ | $U(\tilde{N}_c)$ | $SU(N_f)_L$ | $SU(N_f)_R$ | $U(1)_A$ | $U(1)_R$ | $U(1)_J$ |
|---|---|---|---|---|---|---|---|
| $Q \oplus \tilde{Q}$ | $N_c \oplus \bar{N}_c$ | $-$ | $\bar{N}_f \oplus 1$ | $1 \oplus N_f$ | 1 | $\Delta$ | 0 |
| Magnetic | | | | | | | |
| $q \oplus \tilde{q}$ | $-$ | $\tilde{N}_c \oplus \bar{\tilde{N}}_c$ | $1 \oplus N_f$ | $\bar{N}_f \oplus 1$ | $-1$ | $1 - \Delta$ | 0 |
| $M (= Q\tilde{Q})$ | 1 | 1 | $\bar{N}_f$ | $N_f$ | 2 | $2\Delta$ | 0 |
| $T \oplus \tilde{T}$ | 1 | 1 | 1 | 1 | $2N_f$ | $\Delta_T$ | $1 \oplus -1$ |

with $\tilde{N}_c = N_f - N_c$ and $\Delta_T = N_f(1 - \Delta) - N_c + 1$.

## Giveon Kutasov duality

$$U(N_c)_k\ w/\ N_f\, Q \oplus \tilde{Q}\,,\ W = 0 \leftrightarrow U(N_f - N_c + |k|)_{-k}\ w/\ N_f\, q \oplus \tilde{q}\,,\ M\,,\ W = Mq\tilde{q}$$




NS (0123455)
$N_c$ D3 (01260)
$N_f$ D5 (012789)
(1,k)$(012(37)_\theta 89)$
$g_s k = \tan\theta$

## From Aharony to Giveon Kutasov **Willett, Yaakov '11**

Note: the masses in the following are real (weak gauging of the global sym.)

Electric theory: $G = U(N_c)_0$
$N_f$ light and $k$ heavy $(m > 0)$ $Q$ and $\tilde{Q}$
$W = 0$

$G = U(N_c)_k$
$N_f$ $Q$ and $\tilde{Q}$
$W = 0$

$m \to \infty$

Magnetic theory: $\tilde{G} = U(N_f + k - N_c)_0$
$N_f$ light and $k$ heavy $(m < 0)$ $q$ and $\tilde{q}$
$k^2 + 2N_f k$ heavy mesons $(m > 0)$
$N_f^2$ light mesons, $T, \widetilde{T}$ heavy $(m < 0)$
$W = Mq\tilde{q} + tT + \tilde{t}\tilde{T}$

$\tilde{G} = U(N_f - N_c + k)_{-k}$
$N_f$ $q$ and $\tilde{q}$
$M = Q\tilde{Q}$ $(N_f \times N_f)$
$W = Mq\tilde{q}$

## From Giveon Kutasov to Aharony

**Intriligator, Seiberg '13**
**Khan, Tatar '13, A. '13**

Electric flow

| |
|---|
| $G = U(N_c)_{-k}$ w/ $N_f$ light and<br>$k$ heavy ($m > 0$) $Q$ and $\tilde{Q}$, $W = 0$ |

$m \to \infty$

| |
|---|
| $G = U(N_c)_0$ w/ $N_f$ $Q$ and $\tilde{Q}$, $W = 0$ |

Magnetic flow

| |
|---|
| $G = U(N_f - N_c + 2k)_k$<br>$N_f$ light and $k$ heavy ($m < 0$) $q$ and $\tilde{q}$<br>$k^2 + 2N_f k$ heavy mesons ($m > 0$)<br>$N_f^2$ light mesons<br>**Dual vacuum**:<br>$\sigma_i = \begin{cases} 0 & i = 1, \ldots, N_f \\ m & i = N_f + 1, \ldots, N_f + k \\ -m & i = N_f + k + 1, \ldots N_f + 2k \end{cases}$<br>$W = Mq\tilde{q}$ |

$m \to \infty$

| |
|---|
| Three sectors:<br><br>$G_1 = U(N_c)_0$ w/ $N_f$ $q$ and $\tilde{q}$<br>$G_2 = U(k)_k$ w/ k $q$<br>$G_3 = U(k)_{-k}$ w/ k $\tilde{q}$<br><br>$U(k)$ **sectors dual to singlets**<br>w/ quantum number of $T, \tilde{T}$<br>$W = Mq\tilde{q} + tT + \tilde{t}\tilde{T}$ |

## Localization and duality

3D $\mathcal{N}=2$ QFT on curved background (here $S^3$) preserving some SUSY.
Compute $Z = e^{-S}$ (w/ $Q-$**exact** term), matrix integral (1-loop exact):

$$Z_{S^3} = Z_{1-loop}^{singlets}(\mu) \int [dG(\sigma)] e^{i\pi(kTr\sigma^2 + \lambda Tr\sigma)} \prod_{r_i} Z_{1-loop}^{r_i}(\mu, \rho_i(\sigma)) Z_{1-loop}^{V}(\sigma)$$

$Z_{1-loop}$: ratios of eigenvalues of $\Delta_\Phi$ and $\Delta_\psi$ (in math. Hyperbolic $\Gamma$ functions)
$\mu$: complex combinations of real masses and $R$-charges ($\mu = \sum m_i q_i + i\Delta$).

Example: **Aharony** duality (**Van der Bult '06**)

$$Z_{U(N_c)_0}(\vec{\mu}, \vec{\nu}; \lambda) = \int \frac{\prod_{i=1}^{N_c} d\sigma_i \; e^{i\pi\lambda\sigma_i} \prod_{\alpha=1}^{N_f} Z(\sigma_i + \mu_\alpha) Z(-\sigma_i + \nu_\alpha)}{\prod_{i \neq j} Z(\sigma_i - \sigma_j)}$$

$$= \prod_{\alpha,\beta=1}^{N_f} Z^M(\mu_\alpha + \nu_\beta) Z^T\left(\frac{\vec{\mu} \oplus \vec{\nu}}{2} + \frac{\lambda}{2}\right) Z^{\tilde{T}}\left(\frac{\vec{\mu} \oplus \vec{\nu}}{2} - \frac{\lambda}{2}\right) Z_{U(\tilde{N}_c)_0}(i - \vec{\nu}, i - \vec{\mu}; -\lambda)$$

w/ $\vec{\mu} \oplus \vec{\nu} \equiv \sum_{\alpha,\beta=1}^{N_f} \mu_\alpha + \nu_\beta$

## From **Aharony** to **Giveon Kutasov** on $Z_{S^3}$ **Willet, Yaakov '11**

Large positive axial masses to $k$ $Q$ and $\tilde{Q}$

Large negative axial masses to $k$ $q$ and $\tilde{q}$

Large positive masses to $k(k+2N_f)$ $M_{\alpha,\beta}$, $T$ and $\tilde{T}$

| |
|---|
| $SU(N_f)^2$ unbroken |
| mass $\to \infty$ |

Large mass limit on both sides of the partition function through the formula

$$\lim_{m\to\infty} Z_{1-loop}(m,x) = exp^{-\frac{i\pi}{2}\text{sign}(m)(m+x-i)^2}$$

Final expression (in agreement w/**Van der Bult**)

$$Z_{U(N_c)_k}(\vec{\mu},\vec{\nu},\lambda) = e^{\phi(\mu,\nu,k,\lambda)} Z_{U(N_f-N_c+|k|)_{-k}}(i-\vec{\nu}, i-\vec{\mu}, -\lambda) \prod_{\alpha,\beta=1}^{N_f} Z^M(\mu_\alpha+\nu_\beta)$$

Correct relation between the dual **Giveon-Kutasov** phases.

Extra phase $e^{\phi(\mu,\nu,k,\lambda)}$ related to the global CS contact terms (discussed later).

## From **Giveon Kutasov** to **Aharony** on $Z_{S^3}$ **A. '13**

Electric Theory:

$$\mu_\alpha = \begin{cases} m_\alpha + m_A + i\Delta & \alpha = 1, \dots, N_f \\ m \quad + m_A + i\Delta & \alpha = N_f + 1, \dots, N_f + k \end{cases}$$
$$\nu_\beta = \begin{cases} \tilde{m}_\beta + m_A + i\Delta & \beta = 1, \dots, N_f \\ m \quad + m_A + i\Delta & \beta = N_f + 1, \dots, N_f + k \end{cases}$$

Magnetic dual:

-**Masses** to $q$, $\tilde{q}$ and $M$

-**Shifts** on $\sigma_i$

- Dual gauge symmetry broken in three sectors.
  - One vector-like sector:

$$\prod_{\alpha,\beta=1}^{N_f} Z^M(\mu_\alpha + \nu_\beta) Z_{U(N_f-N_c)_0}(i - \vec{\nu}, i - \vec{\mu}, -\lambda)$$

  - Two chiral-like $U(k)_{\pm k}$ sectors (w/ new massles d.o.f.).
    Re-absorb $m$ by shifting the effective FI.
    Integrals computed analytically (**Van der Bult**) $\to Z^T$ and $Z^{\tilde{T}}$.
- Expected expression for the **Aharony** duality, w/o extra phases.

## Contact terms as a new check of duality

**Closset, Dumitrescu, Festuccia, Komargodski, Seiberg '12**

Phase $\phi$ in $Z_{S^3}$. $\phi = 0$ in **Aharony**, $\phi \neq 0$ in **Giveon Kutasov**. Why?

$\phi$ related to the contact terms of 2 pt. functions. Consider $J^a_\mu$ and $J^b_\nu$:

$$\langle J^a_\mu(p) J^b_\nu(-p)\rangle = \tau^{ab}\left(\frac{p^2}{\mu^2}\right)\frac{\delta_{\mu\nu}p^2 - p_\mu p_\nu}{16|p|} - i\kappa^{ab}\left(\frac{p^2}{\mu^2}\right)\frac{\epsilon_{\mu\nu\lambda}p^\lambda}{2\pi}$$

$\kappa^{ab}$ is a contact term: $\delta$-function in $x$ space induced by a CS $A^a \wedge dA^b$.
During an **RG flow**

$$\kappa^{ab}_{UV} = \lim_{p\to\infty}\left(\frac{p^2}{\mu^2}\right) \qquad \kappa^{ab}_{IR} = \lim_{p\to 0}\left(\frac{p^2}{\mu^2}\right)$$

one can always add a counterterm $-i\delta k^{ab}\frac{\epsilon_{\mu\nu\lambda}p^\lambda}{2\pi}$ keeping $k^{ab} \equiv \kappa^{ab}_{UV} - \kappa^{ab}_{IR}$ fixed.

EXAMPLE: phase in the **Giveon Kutasov** duality

Integrate out matter, obtain a pure $U(N_c)_k$ CS theory w/ contact terms.
Integrate out $\lambda$, left w/ $\mathcal{L}_{IR} = A_\mu \wedge dA_\nu + A^J_\mu \wedge dA_\nu + A^J_\mu \wedge dA^J_\nu$.
Integrate out $A_\mu \to$ and obtain the final expression.
Matching w/ $\phi$ from the partition function.

## Generalizations

Dualities w/ chiral matter and $k \in \frac{\mathbf{Z}}{2}$ (**Benini, Cremonesi, Closset '11**)

Flow from Aharony duality w/ different masses to $Q$ and $\tilde{Q}$

- Opposite flow as from **Giveon Kutasov** to **Aharony** (**A., Klare '14**);
- Check the contact terms in these cases (**A., Klare** ).

Dualities with tensor matter

**Niarchos '09**
**Kapustin, Kim, Park '11**
**Kim, Park '13**

Analogous of **Kutasov, Schwimmer, Seiberg** in 4d.

Limiting case: free duals w/ accidental symmetries (**Agarwal, A., Siani '12**).
Exact relations (**Van der Bult**) between the partition functions.
Matching of the contact terms (**A., Klare**).

New dualities between $U(N_c)_k$ and $SP(2N_c)_k$ gauge theories (**A., Klare**).

## More on Aharony duality:

Classical CB lifted by instantons

Only monopoles w/ fluxes $(\pm 1, 0 \ldots, 0)$ unlifted

At quantum level effective $W$:

$$W_{eff} = (N_f - N_c + 1)(T\tilde{T} detM)^{\frac{1}{N_f - N_c + 1}}$$

The origin $T = \tilde{T} = M = 0$ is singular if $N_f > N_c \rightarrow$ dual description necessary